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PREVENTIVE MAINTENANCE POLICIES WITH RELIABILITY THRESHOLDS FOR TABLE SAW MACHINE

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Abstract

Preventive maintenance policies are essential practical guide for effective maintenance of industrial machines. In this study, system reliability is estimated and used as the condition variable on reliability-based preventive maintenance models to formulate preventive maintenance policies for Table Saw machine which has an increasing hazard rate. Inventory holding cost is introduced as part of the repair cost to complement the actual cost of maintenance. The inter-failure times of the machine was modeled as Weibull distribution and the shape parameters estimate were obtained. Three preventive maintenance policies were obtained for the machine from respective preventive maintenance models with predetermined fixed level of reliability, variable reliability and a combination of both. Result from the third policy with critical reliability level which combines both fixed and unfixed reliability levels is noted as the optimal preventive maintenance policy for the machine in terms of extended lifespan and minimum maintenance cost.

Keywords: Reliability threshold, Preventive Maintenance, Weibull distribution, Table Saw machine, failure distribution.

I. Introduction

Maintenance encompasses a range of activities carried out on facilities or equipment to either restore them to good working condition or to ensure they remain in an acceptable working state. It includes technical procedures aimed at achieving/maintaining satisfactory operation of machines or parts. It serves the primary purpose of ensuring that equipment and facilities can perform their designated tasks as scheduled and under specified conditions. It also helps in preventing unexpected failures and disruptions in operations. Maintenance is vital for the operational efficiency and reliability of facilities and equipment. Neglecting maintenance can lead to increased downtime, decreased performance, and higher repair costs, among others. Therefore, it is essential to prioritize maintenance in the overall strategy of any facility or system, noting that industrial facilities and equipment could be preventively or correctively maintained. *Preventive Maintenance* is proactive and aims to prevent breakdowns and failures. It involves scheduled inspections, repairs, and replacements to keep equipment in good condition while *Corrective Maintenance (Repair)* is reactive and focuses on fixing equipment after a failure or breakdown has occurred. Its goal is to restore equipment to working condition as quickly as possible. Maintenance can be classified as *Perfect Maintenance (As-Good-as-New)* which suggests that after maintenance, equipment is restored to a state identical to when it was new. In reality, achieving this level of restoration is either impracticable or costly. *Imperfect Maintenance* which includes *Better-than-Old or Worse-than-New* acknowledges that maintenance efforts generally result in equipment being in a better condition than it was before the maintenance but not as good as when it was brand new, *same-as-old* ensures that equipment are restored to performance level before maintenance. While *Worse-than-Old* implies that maintenance does not improve the equipment's condition, but left it in a rather worse state than it was before maintenance. Hence, maintenance falls between perfect maintenance and Worse-than-Old, [1]. This is because the failure nature of repairable systems depends on the repair history of the system, [2]. In summary, maintenance is a critical aspect of facility and equipment management that can significantly impact the overall performance and longevity of assets. The choice between preventive and corrective maintenance strategies depends on the specific needs and goals of a facility. Accordingly, [3] proposed a reliability assessment approach based on multi-deterioration measurement and failure analysis for effective maintenance while [4] proposed a reliability centered predictive maintenance policy using reliability threshold.

PM models are utilized to obtain two kinds of maintenance policies according to their maintenance criteria; the time-dependent PM policy, which determines a PM schedule based on the system age and as concepts of minimal repair and imperfect maintenance; [5] [6] and [7]. For instance, [8] constructed a PM model and applied to selective PM manufacturing system to ensure reliability and minimize the total cost of maintenance and failure losses. While the applicability of the competitive failure model with multiple shock types in the degradation process based on threshold variation was undertaken by [9]. In [10], the lifetime-reward-maximizing maintenance policies under perfect and imperfect maintenance conditions was analyzed and the tradeoff between the system's virtual age and the decision maker's reward rate was investigated. The concept of age reduction factor to formulate imperfect PM policies has been widely used; see, [8], [12]and [13]. Accordingly, [14] investigated the failure data of a marine diesel engine to estimate the reliability of the cylinder liner which was then utilized to develop a reliability-based PM strategy for the diesel engine. The use of estimated reliability as the condition variable to develop three reliability-based PM models with consideration of different scenarios which can assist in evaluating the maintenance cost for each scenario was undertaken by [13]. The proposed approach provides optimal reliability thresholds and PM schedules in advance by which the system availability and quality can be ensured and the organizational resources can be well prepared and managed. In [15], a sequential preventive maintenance was obtained for 8hp-pml gold engine cassava grinding machine with a two parameter Weibull failure distribution. The resulting PM and replacement plan provide effective maintenance schedule that guarantees optimum performance of the machine at specified cost levels., Furthermore, [16] formulated a geometric imperfect preventive maintenance and replacement (GIPMAR) model for aging repairable systems due to age and prolong usage that would meet users need in three phases: within average life span, beyond average life span and beyond initial replacement age of system. The work extended the PM model of [17] to provide PM/replacement schedules for aging repairable systems which was not provided for in earlier models. In another development, [18] proposed a knowledge-based framework that exploits fuzzy logic to generate precise cost implication decisions from an optimal maintenance and replacement schedule using data from a locally fabricated 8HP-PML Gold engine cassava grinding machine whose failure distribution followed the Weibull distribution function, while in the same year, [19] relaxed the assumption of an information-symmetric system, where both the manufacturer's expected profit and the system's expected profit are maximize. Also, [20] studied the stress–strength reliability of a failure profile in which the components of the system are affected by the internal environmental factors and their effect under various scenarios.

Following from [9] and [13], this work seeks to formulate PM and replacement policies for mechanically repairable systems with increasing hazard rate considering three cases with a view to determining an optimal policy that minimizes total cost and extended life cycle of the machine.

II. Methods

2.1 Notations and meaning

θ - shape parameter of Hazard intensity function, $\theta > 0$ of the Weibull distribution
λ - Deteriorating parameter of hazard intensity function, where $\lambda > 0$ of the Weibull distribution
α_j - Age reduction factor, where $\alpha \in (0,1)$
$R(t)$- Reliability function without PM at time, t
y_j - PM interval between successive PM actions, where $j = 1, 2, \ldots, N-1$
Z_j - Effective age of the system after jth PM action, $j = 1, 2, \ldots, N$
T_w - System lifetime until replacement, where $T_w = \sum_{j=1}^{N} y_j$
R_T^* - The optimal reliability threshold for performing the jth PM action in policy 2
N - The number of scheduled PM actions for models 1, 2 and 3
N_r - Number of minimal repair actions until system replacement
C_r - Cost of minimal repair action
C_h - Cost of holding spare parts (inventory cost)
C_{r^*} - Joint cost for minimal repair action and holding cost of spare part
C_m - Cost of preventive maintenance action
C_i - Cost of system replacement

2.2 Assumptions of the model

- The machine fails randomly
- Failure occurs at the end of time, t given the lifetime distribution of the machine, f(t).
- Spare parts of the machine are readily available in the warehouse with holding cost, C_h
- Failure process is an increasing failure rate
- The required time for PM activities and minimal repairs is negligible
- The cost parameters C_i, C_r, C_m are constants
- PM activities restore the system to "better-than-old" state.

2.3 Choice of failure distribution function

There are several probability distributions for modeling the failure rate of repairable systems depending on the failure mechanism. Theoretical consideration is given to both the probabilistic arguments of the failure mode and failure mechanism and practically, the success of modeling empirical failure data through Goodness-of -fit test.

2.3.1 Goodness- of- fit test

There are various tests for assessing the goodness- of- fit of a probability distribution to sample data. These include Chi-square test, kolmogorov Smirnov test, Anderson-Darling test, Cramer-Von Mises test and Mann-Scheuer-Fertig test, etc. This study uses Easyfit software to perform the Chi-squared Goodness-of-fit test because of its simplicity and adaptability. The Easyfit software result in Table 2.1 shows that the 2-parameter Weibull distribution with rank 1 is the best-fit model for the data set.

Table 2.1: Extract of Goodness-of-fit test of Probability Distributions of Interest

Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-squared	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Weibull	0.10264	2	0.87318	1	0.62162	1
Gen. Pareto	0.11014	1	0.87596	2	0.7244	2
Uniform	0.11057	3	0.90513	3	N/A	
Levy (2P)	0.11093	4	0.9133	4	0.9851	3

2.4 The two-parameter Weibull distribution

2.4.1 The density function

The probability distribution function of a two-parameter Weibull distribution is given by;

$$f(z;\theta,\lambda)=\frac{\lambda}{\theta}\left(\frac{z}{\theta}\right)^{\lambda-1}e^{-\left(\frac{z}{\theta}\right)^{\lambda}},\ \lambda>0,\ \theta>0,\ Z>0 \quad (1)$$

Whereθ = scale parameter, λ =shape parameter

2.4.2 Reliability function, $R(t)$

The reliability function of the two-parameter Weibull distribution is given by;

$$R(t)=-P(T\le t)=P(T>t)=1-\int_0^t f(t)dt \quad (2)$$

Substituting for $f(t)$ in Eq (2), we have;

$$R(t)=1-\int_0^t\left(\frac{\lambda}{\theta}\right)\left(\frac{t}{\theta}\right)^{\lambda-1}e^{-\left(\frac{t}{\theta}\right)^{\lambda}}dt=1-\left(\frac{\lambda}{\theta}\right)\int_0^t\left(\frac{t}{\theta}\right)^{\lambda-1}e^{-\left(\frac{t}{\theta}\right)^{\lambda}}dt$$

Since $$F(t)=1-e^{-\left(\frac{t}{\theta}\right)^{\lambda}}$$

$$R(t)=1-F(t)=1-\left(1-e^{-\left(\frac{t}{\theta}\right)^{\lambda}}\right)$$

$$\therefore R(t)=e^{-\left(\frac{t}{\theta}\right)^{\lambda}} \quad (3)$$

2.4.3 Failure rate, *h(t)* of two-parameter Weibull distribution

The failure rate, *h(t)* during a given interval of time $t=[t_1,t_2]$ shows the probability that a failure per unit time occurs in the interval (t_1,t_2), conditioned on the event that no failure has occurred at or before time, t_1. This means that $T>t_{1.}$. The failure rate can be defined as follows:

$$h(t)=\frac{R(t_1)-R(t_2)}{(t_2-t_1)R(t_1)}=\frac{F(t_2)-F(t_1)}{(t_2-t_1)R(t_1)} \quad (4)$$

Taking the limit of the failure rate at the interval, $(t_{1,}\Delta t+1)$ as Δt approaches zero, where $t=t_1$and $(t+\Delta t)=t_2$ gives the hazard function, $h(t)$, as follows;

$$h(t)=\lim_{\Delta t\to 0}\frac{R(t)-R(t+\Delta t)}{(\Delta t)\times R(t)} \quad (5)$$

$$=\lim_{\Delta t\to 0}\frac{F(t+\Delta t)-F(t)}{(\Delta t)}\times\frac{1}{R(t)} \quad (6)$$

But $$h(t)=\lim_{\Delta t\to 0}\frac{F(t+\Delta t)-F(t)}{(\Delta t)}=f(t)$$

$$\therefore h(t)=\frac{f(t)}{R(t)}$$

Where $$f(t)=\left(\frac{\lambda}{\theta}\right)\left(\frac{t}{\theta}\right)^{\lambda-1}e^{-\left(\frac{t}{\theta}\right)^{\lambda}}$$

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\lambda}}$$

$$h(t) = \frac{\left(\frac{\lambda}{\theta}\right)\left(\frac{t}{\theta}\right)^{\lambda-1} e^{-\left(\frac{t}{\theta}\right)^{\lambda}}}{e^{-\left(\frac{t}{\theta}\right)^{\lambda}}}$$

$$\therefore h(t) = \lambda\theta^{-\lambda}t^{\lambda-1} \quad (7)$$

And the cumulative hazard rate is given as;

$$H(t) = \int_0^t h(t)dt = \int_0^t \lambda\theta^{-\lambda}t^{\lambda-1}dt = \theta^{-\lambda}t^{\lambda} \quad (8)$$

2.4.4 Estimating the parameters of Weibull distribution

Easyfit software will be used to determine the appropriate failure function from the observed data and also obtain the parameters estimates of the fit distribution.

2.4.5 Evaluation of the Weibull parameters

As part of the preliminary analysis, the scale and shape parameters estimate of the Weibull distribution were also obtained for the failure data of Table Saw machine with the aid of the Easyfit software as; $\lambda = 3.8548$ and $\theta = 202.92$.

2.5 Preventive maintenance policies

We seek to modify and adapt the models in [13] to obtain suitable maintenance policies for mechanically repairable systems with increasing hazard rate using Table Saw machine as case study.

2.5.1 Policy 1: System undergoes PM activity whenever the reliability reaches the predetermined threshold

Let the effective age of the system just before the jth PM activity be given as;

$$Z_j = y_j + \alpha_{j-1}Z_{j-1};\ j = 1, 2, \dots, N \quad (9)$$

Where,

Z_j is the effective age of the system

y_j is the interval between successive PM activities

α_{j-1} is the age reduction factor; $0 < \alpha_0 < \alpha_1 < \cdots < \alpha_N$

$\alpha_j Z_j$ denotes the effective age of the system immediately after the jth PM activity.

Let the associated expected cost rate per unit time for performing PM activities according to [13] be given as;

$$C(N) = \frac{1}{T_w}[C_i + C_r N_r + (N-1)C_m] \quad (10)$$

$$T_w = \sum_{j=1}^{N} y_j = \sum_{j=1}^{N} Z_j - \sum_{j=1}^{N} \alpha_{j-1}Z_{j-1} = \left(Z_N + \sum_{j=1}^{N-1} Z_j\right) - \sum_{j=1}^{N-1} \alpha_j Z_j = Z_N + \sum_{j=1}^{N-1}(1-\alpha_j)Z_j$$

and $N_r = \sum_{j=1}^{N}\int_{\alpha_{j-1}Z_{j-1}}^{Z} h(t) = \sum_{j=1}^{N}[H(t)]_{\alpha_{j-1}Z_{j-1}}^{Z_j}$

Therefore,

$$C(N) = \frac{C_i + (N-1)C_m + C_r\sum_{j=1}^{N}\int_{\alpha_{j-1}Z_{j-1}}^{Z_j} h(t)}{Z_N + \sum_{j=1}^{N-1}(1-\alpha_j)Z_j} \quad (11)$$

Let the reliability threshold at the end of each PM cycle be:

$$R_T = R(\alpha_{j-1}Z_{j-1})R_m\left(Z_j \middle| Z_{j-1}\right)$$

This is on the assumption that the system age can be proportionally reduced by imperfect PM actions to $\alpha_{j-1}Z_{j-1}$immediately after the *(j-1)*th PM activity, [21]. $R_m(Z_j|Z_{j-1})$ is the reliability of *jth* PM cycle at age, t given that the system was maintained at Z_{j-1}. This is equivalent to the product of probability of survival until $\alpha_{j-1}Z_{j-1}$.
The optimization problem is to minimize:

$$C(N) = \frac{C_i+(N-1)C_m+C_r\sum_{j=1}^{N}\int_{\alpha_{j-1}Z_{j-1}}^{Z_j} h(t)}{Z_N+\sum_{j=1}^{N-1}(1-\alpha_j)Z_j} \tag{12}$$

Subject to: $R(\alpha_{j-1}Z_{j-1})R_m\,(Z_j|Z_{j-1})$.

According to [22], the reliability function of an initial system is given as; $R(t) = e^{-\int_0^t h(t)dt}$.
The failure density, $f(t)$ and the reliability function, $R(t)$ can be derived from the knowledge of the hazard function, $h(t)$. Then, $R(t) = exp\left[-\int_0^t h(t)dt\right]$ and $f(t) = h(t) \times R(t)$. Since $R(t) = 1 - F(t)$, $R'(t) = -F'(t)$ (by differentiating both sides). Therefore,

$$h(t) = \frac{f(t)}{R(t)} = \frac{F'(t)}{R(t)} = \frac{-R'(t)}{R(t)}$$

Integrating both sides, we have;
$\int_0^t h(t)dt = -\int_0^t \frac{R'(t)}{R(t)}dt = -[ln\,R\,(t) - ln\,R\,(0)]$, under the boundary condition, $R(0) = 1$, since the component will not fail before time $t = 0$, once it is put into operation.
Since $\ln R(0) = \ln 1 = 0$, we see that $-\int_0^t h(t)dt = ln\,R\,(t) \Rightarrow \int_0^t h(t)dt = ln\,R\,(t)$. Taking exponent of both sides,

$$e^{-\int_0^t h(t)dt} = e^{ln\,R(t)} \Rightarrow e^{-\int_0^t h(t)dt} = R(t)$$

Since the cumulative hazard intensity at time, t is given as $H(t) = \int_0^t h(t)dt$, we can say that

$$R(t) = R_T = e^{-H(Z_j)} \tag{13}$$

Substitute Eq (8) in (13), we have; $R_T = e^{-\theta^{-\lambda}Z^{\lambda}}$
Taking the ln of both sides, we have,

$$ln(R_T) = -\theta^{-\lambda}Z^{-\lambda} \implies \left[\frac{-ln(R_T)}{\theta^{-\lambda}}\right]^{1/\lambda} = \theta[ln(R_T)]^{1/\lambda} \tag{14}$$

Considering the jth PM action,

$$Z_j = y_j + \alpha_j Z_j \Rightarrow y_j = Z_j - \alpha_j Z_j \Rightarrow y_j = Z_j(1-\alpha_j) \tag{15}$$

2.5.2 Policy 2: System with non-fixed Reliability Threshold considered as a decision variable
This policy states that a reliability threshold is not predetermined but considered as a decision variable with additional inventory holding cost
Let R be the reliability threshold which is a decision variable, then the cost function is;

$$C(N,R) = \frac{C_i+(N-1)C_m+C_r\sum_{j=1}^{N}\int_{\alpha_{j-1}Z_{j-1}}^{Z_j} h(t)}{\sum_{j=1}^{N} y_j} \tag{16}$$

We introduce inventory cost, C_h, which is the cost of holding spare parts of the machine readily available in the warehouse. This will reduce the time of ordering spare part which increases downtime and reduces the operational time of the machine, hence, productivity. Therefore, our new cost of minimal repair is $C_{r^*} = C_r + C_h$. Eq (16) becomes;

$$C(N,R) = \frac{C_i + (N-1)C_m + C_{r^*}\sum_{j=1}^{N}\int_{\alpha_{j-1}Z_{j-1}}^{Z_j} h(t)}{\sum_{j=1}^{N} y_j}$$
$$= \frac{C_i + (N-1)C_m + C_{r^*}\sum_{j=1}^{N}[H(Z_j) - H(\alpha_{j-1}Z_{j-1})]}{Z_N + \sum_{j=1}^{N-1}(1-\alpha_j)Z_j} \quad (17)$$

The term under summation sign in the numerator of (Eq 17) can be written as follows;

$$= \theta^{-\lambda}\sum_{j=1}^{N}\left[Z_j^{\lambda} - \left(\alpha_{j-1}^{\lambda}Z_{j-1}^{\lambda}\right)\right] = \theta^{-\lambda}\left[\sum_{j=1}^{N}Z_j^{\lambda} - \sum_{j=1}^{N}\left(\alpha_{j-1}^{\lambda}Z_{j-1}^{\lambda}\right)\right] = \theta^{-\lambda}\left[\left(Z_N^{\lambda} + \sum_{j=1}^{N-1}Z_j^{\lambda}\right) - \sum_{j=1}^{N-1}\left(\alpha_j^{\lambda}Z_j^{\lambda}\right)\right]$$
$$= \theta^{-\lambda}\left[\left(Z_N^{\lambda} + \sum_{j=1}^{N-1}Z_j^{\lambda}\right) - \sum_{j=1}^{N-1}\left(\alpha_j^{\lambda}Z_j^{\lambda}\right)\right] = \theta^{-\lambda}Z_N^{\lambda} + \sum_{j=1}^{N-1}\theta^{-\lambda}Z_j^{\lambda} - \theta^{-\lambda}\sum_{j=1}^{N-1}\left(\alpha_j^{\lambda}Z_{lj}^{\lambda}\right)$$
$$= \theta^{-\lambda}Z_N^{\lambda} + \sum_{j=1}^{N-1}H(Z_j) - \theta^{-\lambda}\sum_{j=1}^{N-1}\left(\alpha_j^{\lambda}Z_j^{\lambda}\right)$$

Since the effective age, Z_j at the replacement point, $Z_N = 0$, then, Z_N vanishes in the numerator and the denominator. Hence, Eq (17), becomes;

$$C(N,R) = \frac{C_i + (N-1)C_m + C_{r^*}\left\{\sum_{j=1}^{N-1}\left[-\ln(R_T) + \theta^{-\lambda}\alpha_j^{\lambda}Z_j^{\lambda}\right]\right\}}{\sum_{j=1}^{N-1}\left(1-\alpha_j\right)Z_j}$$
$$= \frac{C_i + (N-1)C_m + C_{r^*}\left\{\sum_{j=1}^{N-1}\left[-\ln(R_T) + \alpha_j^{\lambda}H(Z_j)\right]\right\}}{\sum_{j=1}^{N-1}\left(1-\alpha_j\right)Z_j}$$
$$= \frac{C_i + (N-1)C_m + C_{r^*}\left\{\sum_{j=1}^{N-1}\left[-\ln(R_T) + \alpha_j^{\lambda}\ln(R_T)\right]\right\}}{\sum_{j=1}^{N-1}\left(1-\alpha_j\right)Z_j}$$
$$= \frac{C_i + (N-1)C_m - C_{r^*}\ln(R_T)\left[\sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right)\right]}{\sum_{j=1}^{N-1}\left(1-\alpha_j\right)Z_j}$$
$$C(N,R) = \frac{C_i + (N-1)C_m - C_{r^*}\ln(R_T)\left[\sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right)\right]}{\left[\frac{-\ln(R_T)}{\theta^{-\lambda}}\right]^{1/\lambda}\sum_{j=1}^{N-1}(1-\alpha_j)} \quad (18)$$

Differentiating Eq (18) w.r.t. R_T, equating it to zero and writing R_T with respect to other terms, we have;

$$R_T^{\bullet} = \exp\left\{\frac{C_i + (N-1)C_m}{(1-\lambda)C_{r^*}\sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right)}\right\} \quad (19)$$

2.5.3 Policy 3: System with optimal combination of reliability threshold in policies 1 and 2

The aim of this policy is to combine the PM policy with fixed reliability threshold, R_N and unfixed reliability thresholds, R_j with a view to obtaining the total minimum cost per unit time compared to the previous two models

By substituting Eq (14) in Eq (16), we have;

$$\left(C, R_N, R_j\right) = \frac{C_i + (N-1)C_m + C_{r^*}\left\{\sum_{j=1}^{N}\left[-\ln(R_T) - \alpha_j^{\lambda}H(Z_j)\right]\right\}}{Z_N + \sum_{j=1}^{N-1}\left(1-\alpha_j\right)Z_j}$$
$$= \frac{C_i + (N-1)C_m + C_{r^*}\left\{\sum_{j=1}^{N}\left[-\ln(R_T) - \alpha_j^{\lambda}\ln(R_T)\right]\right\}}{Z_N + \sum_{j=1}^{N-1}\left(1-\alpha_j\right)Z_j}$$

$$= \frac{C_i + (N-1)C_m - C_{r^*}\sum_{j=1}^{N}\left(1-\alpha_j^{\lambda}\right) ln(R_T)}{Z_N + \sum_{j=1}^{N-1}(1-\alpha_j)Z_j} \tag{20}$$

Since $\sum_{j=1}^{N}\left(1-\alpha_j^{\lambda}\right) ln(R_T) = \left[ln(R_N) + \sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right) ln(R_T)\right]$

and $Z_j = \left[\frac{-ln(R_T)}{\theta^{-\lambda}}\right]^{1/\lambda} = \theta[-ln(R_T)]^{1/\lambda}$

Eq (20) will now become;

$$C(N, R_N, R_j) = \frac{C_i + (N-1)C_m - C_{r^*}\left[ln(R_N) + \sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right) ln(R_T)\right]}{Z_N + \sum_{j=1}^{N-1}(1-\alpha_j)\left[\frac{-ln(R_j)}{\theta^{-\lambda}}\right]^{1/\lambda}}$$

$$= \frac{C_i + (N-1)C_m - C_{r^*}\left[ln(R_N) + \sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right) ln(R_j)\right]}{\left[\frac{-ln(R_N)}{\theta^{-\lambda}}\right]^{1/\lambda} + \sum_{j=1}^{N-1}(1-\alpha_j)\left[\frac{-ln(R_j)}{\theta^{-\lambda}}\right]^{1/\lambda}} \tag{21}$$

Differentiating Eq (21) w.r.t. R_j, equating it to zero and making C the subject, we have;

$$C = \frac{\lambda C_{r^*}\sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right)}{\sum_{j=1}^{N-1}(1-\alpha_j)\left[\frac{-ln(R_j)}{\theta^{-\lambda}}\right]^{1/\lambda - 1}} = \frac{\lambda C_{r^*}\left(1-\alpha_j^{\lambda}\right)}{\theta(1-\alpha_j)[-ln(R_j)]^{1/\lambda - 1}} \tag{22}$$

Similarly, differentiating Eq (21) w.r.t. R_N and making C the subject, we have;

$$C = \frac{\lambda C_{r^*}}{\theta[-ln(R_N)]^{1/\lambda - 1}} \tag{23}$$

Equating Eqs (22) and (23), we have;

$$\frac{\lambda C_{r^*}\left(1-\alpha_j^{\lambda}\right)}{\theta(1-\alpha_j)[-ln(R_j)]^{1/\lambda - 1}} = \frac{\lambda C_{r^*}}{\theta[-ln(R_N)]^{1/\lambda - 1}}$$

$$[-ln(R_N)]^{1/\lambda - 1}\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right] = [-ln(R_j)]^{1/\lambda - 1}$$

$$-ln(R_N)\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{\lambda}{1-\lambda}} = -ln(R_j)$$

$$ln(R_j) = ln(R_N)\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{\lambda}{1-\lambda}} \tag{24}$$

$$\therefore R_j^* = R_N \, exp\left\{\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{\lambda}{1-\lambda}}\right\} \tag{25}$$

From Eq (21), we have;

$$C\left\{\left[\frac{-ln(R_N)}{\theta^{-\lambda}}\right]^{1/\lambda} + \sum_{j=1}^{N-1}(1-\alpha_j)\left[\frac{-ln(R_j)}{\theta^{-\lambda}}\right]^{1/\lambda}\right\} = C_i + (N-1)C_m - C_{r^*}\left[ln(R_N) + \sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right) ln(R_j)\right]$$

Note that: $[-ln(R_j)]^{1/\lambda - 1} = [-ln(R_N)]^{1/\lambda - 1}\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]$

$$[-ln(R_j)]^{1/\lambda} = [-ln(R_N)]^{1/\lambda - 1}\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right] \times [-ln(R_j)]$$

Substituting for $ln\, R_j$, we have;

$$\left[-ln\left(R_j\right)\right]^{1/\lambda} = \left[-ln(R_N)\right]^{1/\lambda-1}\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]\times\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{\lambda}{1-\lambda}}\left[-ln(R_N)\right]$$

$$\left[-ln\left(R_j\right)\right]^{1/\lambda} = \left[-ln(R_N)\right]^{1/\lambda}\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{1}{1-\lambda}} \quad (26)$$

Substituting Eqs (22), (23) and (24) in (26) where necessary, we have;

$$\frac{\lambda C_{r^*}}{\theta\left[-ln(R_N)\right]^{1/\lambda-1}}\left\{\left[\frac{-ln(R_N)}{\theta^{-\lambda}}\right]^{1/\lambda}+\sum_{j=1}^{N-1}\left(1-\alpha_j\right)\left[\frac{-ln(R_N)}{\theta^{-\lambda}}\right]^{1/\lambda}\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{1}{1-\lambda}}\right\}=$$

$$C_i+(N-1)C_m-C_{r^*}\left[ln(R_N)+ln(R_N)\sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right)\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{\lambda}{1-\lambda}}\right]$$

$$\Rightarrow C_i+(N-1)C_m-C_{r^*}\,ln(R_N)\left[1+\sum_{j=1}^{N-1}\left(1-\alpha_j^{\lambda}\right)\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{\lambda}{1-\lambda}}\right]$$

$$+\lambda C_{r^*}\left[1+\sum_{j=1}^{N-1}\left(1-\alpha_j\right)\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{1}{1-\lambda}}\right]\left[-ln(R_N)\right]=0$$

$$C_i+(N-1)C_m-C_{r^*}\,ln(R_N^*)\left\{1+\sum_{j=0}^{N-1}\left[\frac{\left(1-\alpha_j\right)^{\lambda}}{1-\alpha_j^{\lambda}}\right]^{\frac{1}{\lambda-1}}-\lambda\left[1+\sum_{j=0}^{N-1}\left[\frac{1-\alpha_j^{\lambda}}{\left(1-\alpha_j\right)^{\lambda}}\right]^{\frac{1}{1-\lambda}}\right]\right\}=0$$

$$C_i+(N-1)C_m-C_{r^*}\,ln(R_N^*)\left\{(1-\lambda)\sum_{j=0}^{N-1}\left[\frac{\left(1-\alpha_j\right)^{\lambda}}{1-\alpha_j^{\lambda}}\right]^{\frac{1}{\lambda-1}}\right\}=0$$

$$ln(R_N^*)=\frac{C_i+(N-1)C_m}{C_{r^*}(1-\lambda)\sum_{j=0}^{N-1}\left[\frac{\left(1-\alpha_j\right)^{\lambda}}{1-\alpha_j^{\lambda}}\right]^{\frac{1}{\lambda-1}}}$$

$$\therefore R_N^*=exp\left\{\frac{C_i+(N-1)C_m}{C_{r^*}(1-\lambda)\sum_{j=0}^{N-1}\left[\frac{\left(1-\alpha_j\right)^{\lambda}}{1-\alpha_j^{\lambda}}\right]^{\frac{1}{\lambda-1}}}\right\} \quad (27)$$

Substituting Eq (27) in Eq (25), we have;

$$R_j^*=exp\left\{\frac{C_i+(N-1)C_m}{C_{r^*}(1-\lambda)\sum_{j=0}^{N-1}\left[\frac{\left(1-\alpha_j\right)^{\lambda}}{1-\alpha_j^{\lambda}}\right]^{\frac{1}{\lambda-1}}}\right\}\times exp\left\{\left[\frac{1-\alpha_j^{\lambda}}{1-\alpha_j}\right]^{\frac{\lambda}{1-\lambda}}\right\} \quad (28)$$

III. Results

Table 3.1: Values of y_j with fixed reliability threshold

N	α_j	y_j	R_t
1	0.3333	0.8281	0.9000
2	0.4000	0.4047	0.9000
3	0.4286	0.3101	0.9000
4	0.4444	0.2852	0.9000
5	0.4545	0.2583	0.9000
6	0.4615	0.2189	0.9000
7	0.4667	0.1943	0.9000
8	0.4706	0.1744	0.9000
9	0.4737	0.1626	0.9000
10	0.4762	0.1610	0.9000
11	0.4783	0.1587	0.9000
12	0.4800	0.1516	0.9000
13	0.4815	0.1511	0.9000
14	0.4828	0.1508	0.9000
15	0.4839	0.1491	0.9000
16	0.4848	0.1489	0.9000
17	0.4857	0.1484	0.9000
18	0.4865	0.1475	0.9000
		4.2035	

Table 3.2: Values of y_j with variable reliability threshold

N	$1-\alpha_j^{\lambda}$	y_j	R_t
1	0.5943	0.5340	0.8586
2	0.5287	0.4897	0.8979
3	0.5013	0.4824	0.8970
4	0.4862	0.4048	0.8985
5	0.4766	0.3426	0.8947
6	0.4700	0.3199	0.8783
7	0.4652	0.3044	0.8662
8	0.4615	0.2902	0.8569
9	0.4586	0.2689	0.8496
10	0.4562	0.2376	0.8437
11	0.4543	0.2220	0.8389
12	0.4526	0.2131	0.8368
13	0.4513	0.1923	0.8320
14	0.4501	0.1901	0.8312
15	0.4490	0.1827	***0.8307***
16	0.4481	0.1792	0.8992
17	0.4473	0.1749	0.8989
18	0.4466	0.1721	0.8987
		5.1920	

Table 3.3: Values of y_j under the combined policies

S/N	$\frac{1-\alpha)^{\lambda}}{1-\alpha^{\lambda}}^{\frac{1}{\lambda-1}}$	R_n	$\frac{1-\lambda}{1-\alpha^{\lambda}}^{\frac{\lambda}{\lambda-1}}$	R_j	y_j
1	0.3507	0.1650	5.4455	0.8987	0.5340
2	0.2960	0.1505	5.9674	0.8981	0.4816
3	0.2748	0.1448	6.1969	0.8975	0.4833
4	0.2636	0.1418	6.3260	0.8970	0.4057
5	0.2566	0.1399	6.4088	0.8963	0.3436
6	0.2518	0.1385	6.4664	0.8957	0.3208
7	0.2483	0.1381	6.5088	0.8988	0.3053
8	0.2457	0.1364	6.5413	0.8923	0.2912
9	0.2437	0.1348	6.5671	0.8850	0.2698
10	0.2420	0.1334	6.5879	0.8791	0.2385
11	0.2406	0.1324	6.6052	0.8743	0.2229
12	0.2395	0.1320	6.6197	0.8735	0.2140
13	0.2385	0.1311	6.6321	***0.8695***	0.1932
14	0.2377	0.1354	6.6428	0.8997	0.1910
15	0.2370	0.1351	6.6521	0.8985	0.1836
16	0.2363	0.1348	6.6603	0.8980	0.1801
17	0.2358	0.1346	6.6675	0.8976	0.1758
18	0.2353	0.1344	6.6740	0.8972	0.1730
					5.2077

Table 3.4: Optimal solution of system performance under policies 1, 2 and 3

System performance	Policy 1	Policy 2	Policy 3
System Lifespan	4.2035	5.192	5.2077
Minimum reliability	0.9000	0.8310	0.8700
Maintenance cost	6.3248	6.1146	5.5316
Number of PM actions	18	18	18

3.1 Application of PM policies with reliability threshold to the maintenance of Table Saw machine
The distribution of the inter failure times of the Table Saw machine was modeled as a two parameter Weibull distribution as given in Eq (1), the shape and scale parameters of the distribution were obtained in section 2.4.5 as $\theta = 202.92$; and $\lambda = 3.8548$, respectively. The results of policies 1, 2 and 3 are contained in Tables 3.1, 3.2 and 3.3. The y_j values are the length of operating time of the machine before PM or replacement maintenance (as the case may be) at respective policy.

3.2 Expected maintenance cost per unit time for policies 1, 2 and 3

The different maintenance costs ratios were obtained as; $\frac{C_{r^*}}{C_m} = 8.0$ and $\frac{C_i}{C_m} = 2.8$. Eqs (18) was used to obtain the expected maintenance cost for each policy as; Policy 1: $C(N) = 6.3248$, Policy 2**:** $(N) = 6.1146$, Policy 3 **:** $C(N) = 5.5316$

IV. Discussion

I. Policy 1: When the Table Saw machine has a fixed reliability threshold

The fixed reliability threshold for proper functioning of the machine was set at 0.9000. Eqs (9) and (14) were used to obtain the operating time (life span) of the machine, y_j before next PM activity. The age improvement factor, $\alpha_j = \frac{j}{(2j+1)}$, ([23] and [8]), increases marginally with the frequency of PM. The values of α_j, y_j, and R_t are shown in Table 3.1. The bold last value of y_j in column 3 is the total lifespan of the machine over 18 PM cycles equal 4.2035 years. The column of R_t is the fixed reliability threshold for all maintenance actions. It took 0.8281unit of the operating time before the first PM activity, 0.4047 unit of working time before the next PM, and so on. Generally, it is observed that the lifespan of the machine keeps decreasing in spite of the fact that the fixed reliability limit is not violated. Hence, maintenance engineers need not rely on this policy.

II. Policy 2: When the Table Saw machine has an unfixed reliability threshold

Eqs (14), (15) and (19) were used to obtained the lifespan of the machine, y_j after each PM and the corresponding reliability index, R(t) which serves as a check for replacement. The total lifespan of the machine over 18 PM cycles equal 5.1920 years. The minimum reliability threshold set at $R_T = 0.8310$ is always set a value below the fixed reliability (0.9000) used in policy 1, see [13]. The y_j column which is the operational time before PM, shows a gradual decrease of PM intervals from 0.5340 unit time in the first cycle of operation before next PM to 0.4897 unit of operational time before the 2nd PM to 0.4824 units of operational time before the third PM activity and so on. The 15th PM calls for replacement maintenance of the failed component of the machine because its reliability value is below the threshold (0.8310) of the machine. After the replacement maintenance, it is observed that the values of the machine's reliability increase to within the tolerance level again illustrating the import of PM actions.

III. Policy 3: A Combination of policies 1 and 2 for the Table Saw machine

Eqs (9) and (28) were used to obtain the lifespan of the machine, y_j. The minimum reliability threshold, $R_T = 0.8700$ while the lifespan equals 5.2077 years. Eq (27) gives columns 2 and 3, Eq (28) gives columns 4 and 5, while Equations (14) and (15) give the last column, all in Table 3.3. The y_j column has 0.5340 unit of operating time before first PM and 0.4816 unit of operating time before the 2nd PM, and 0.4833 unit of operating time before the 3rd PM activity, and so on. Replacement maintenance is required at the 13th PM because its reliability value is below the threshold (0.8700). It is observed that after the appropriate replacement maintenance action, the critical reliability level rises to 0.8997, even above the initial level at the beginning of operation. This underscores the need for PM and a justification to recommend policy 3 for effective maintenance management.

The lifespan of the machine with regard to Policies 1, 2 and 3 in Table 3.4 are 4.2035, 5.1920, and 5.2077, respectively. The minimum reliability values of these three policies are 0.900, 0.8310 and 0.8700, respectively, and the associated respective cost of maintaining this machine in a lifecycle of 18 PM's are 6.3248, 6.1146 and 5.5316, respectively. These results agree with [22] who showed that PM model based on the unfixed reliability threshold led to a lower expected maintenance cost and longer system lifespan.

Therefore, policy 3 which combines fixed predetermined reliability threshold and the variable threshold values as decision variable is the recommended optimum maintenance policy for the Table Saw machine. It yielded the minimum cost of maintenance as well as the longest lifespan of the machine over the PM cycles under consideration.

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