PROFIT ANALYSIS OF REPAIRABLE WARM STANDBY SYSTEM UNDER IMPERFECT SWITCH

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Abstract

In this paper, the performance of two non identical units repairable system are analyzed by using regenerative point graphical technique. Generally, the system has one operative unit and one warm standby unit. Fuzzy concept is used to find the reliability measures under imperfect switch. Regenerative point graphical technique and semi markov process are used to evaluate the reliability measures. Primary, secondary and tertiary circuits are used to describe the base state. The system is repaired by the available technician when any unit is failed or switch is under imperfect mode. The priority in repair is given to switch before working units. In this paper, the failure time and repair time follow general distributions. The tables are used to explore the reliability measures such that mean time to system failure, availability and profit values.

Keywords: Base state, warm standby, imperfect switch, availability and profit values.

I. Introduction

To meet the increasing demand for their products, manufacturers need to constantly improve their products which can be achieved by improving their production procedures. The MTSF, availability and profitability of two non identical units system with priority in switch repair are discussed by using the regenerating point graphical technique under specific circumstances. In the real time system, it is impossible to ignore the concept of spare unit switching failure when it is utilized to replace a failed operative unit.

Balagurusamy [1] described the terms related to the system meantime, failure, repair, redundancy, maintainability and availability. Gopalan and Bhanu [3] examined the behaviour of two unit repairable system under online preventive maintenance using regenerative point technique. Gupta and Singh [4] threw light on the profit and availability values of redundant system with imperfect switch. Lim [7]. Kumar and Sirohi [6] examined the reliability, availability and profit values of two unit system under cold standby approach with delays in repair of partially failed unit. Kumar and Goel [5] evaluated the behaviour of two unit cold standby system under general distribution. Taneja et al. [10] discussed on the comparative study of profit values two reliability models under varying demands. Bhardwaj et al. [2] analyzed the reliability measures of two unit cold standby system under standby failure and arbitrary distribution for repair and replacement times. Sadeghi and Roghanian [8] discussed on the availability and profit values of the two dissimilar unit repairable system under two cases of imperfect switch. Singh et al. [9] evaluated the behaviour of the complex repairable system with two subsystems connected in series with switch facility.

II. System Assumptions

To describe the system, there are following assumptions

- The system has two distinct units where unit *A* is in operative mode and unit *B* is in warm standby mode.
- There is an imperfect switching.
- The sequence of repair is switch, operative main unit and warm standby unit.
- A technician is available to repair the failed unit.
- Failure time and repair time follow the general distribution.
- The repaired unit functions just like a brand-new one.

III. System Notations

To explain the juice plant, there are following notations

$i \xrightarrow{Sr} j$	r^{th} directed simple path from state '*i*' to state '*j*' where '*r*' takes the positive integral values for different directions from state '*i*' to state '*j*'.
$\xi \xrightarrow{sff} i$	A directed simple failure free path from state ξ to state '*i*'.
$m - cycle$	A circuit (may be formed through regenerative or non regenerative / failed state) whose terminals are at the regenerative state '*m*'.
$m - \overline{cycle}$	A circuit (may be formed through the unfailed regenerative or non regenerative state) whose terminals are at the regenerative '*m*' state.
$U_{k,k}$	Probability factor of the state '*k*' reachable from the terminal state '*k*' of '*k*' cycle.
$U_{\overline{k,k}}$	The probability factor of state '*k*' reachable from the terminal state '*k*' of $k\ \overline{cycle}$.
μ_i	Mean sojourn time spent in the state '*i*' before visiting any other states.
μ_i'	Total unconditional time spent before transiting to any other regenerative state while the system entered regenerative state '*i*' at t=0.
η_i	Expected waiting time spent while doing a job given that the system entered to the regenerative state '*i*' at t=0.
A / a	The first unit is in the operative state/failed state.
$B / \overline{B} / b$	The second unit is in the operative state/ standby state/ failed state.
	The third unit is in the operative state/reduced state/failed state.
λ_1, λ_2	Fixed failure rate of the unit A/B respectively.
p / q	Probability of switch properly working/not working.
S/s	The switch is in perfect mode/Imperfect mode.
θ_1, θ_2	Fixed repair rate of the unit A/B after respectively.
γ	Fixed repair rate of the switch.
	Upstate/ reduced state/ failed state.

IV. Circuits Descriptions

Primary, secondary and tertiary circuits are used to find the base state such that

Table 1: *Circuit Descriptions*

i	(C1)	(C2)	(C3)
0	(0, 2, 0) (0, 1, 2, 0) (0, 2, 4, 3, 0)	Nil Nil (4, 3, 4) (3, 4, 3)	Nil Nil Nil Nil
1	1, 2, 0, 1	(0,2,0) (0,2,4,3,0) (2,0,2) (2,4,3,0,2)	Nil (4,3,4) (3,4,3) Nil (4,3,4) (3,4,3)
2	(2,0,2) (2,0,1,2) (2,4,3,0,2)	Nil Nil (3,4,3) (4,3,4)	Nil Nil Nil Nil
3	(3,0,2,4,3)	(0,2,0) (0,1,2,0) (2,0,2)	Nil Nil Nil
4	(4, 3, 0, 2, 4)	(0,2,0) (2,0,2)	Nil Nil

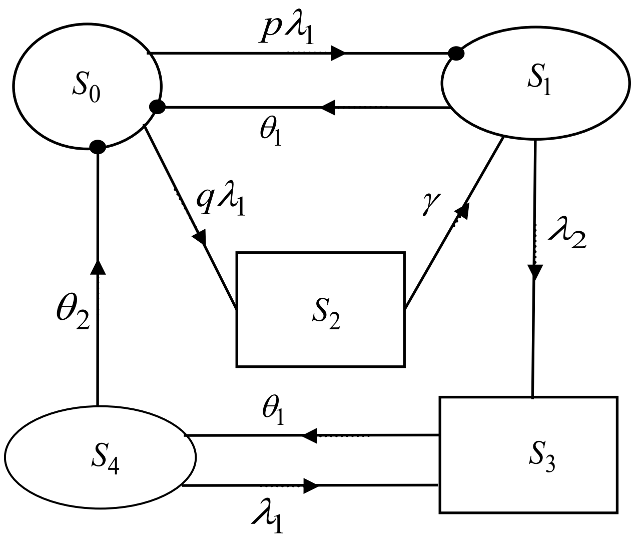

Figure 1 *State Transition Diagram*

where, $S_0 = A\bar{B}S$, $S_1 = aBS$, $S_2 = a\bar{B}s$, $S_3 = abS$, $S_4 = AbS$

V. Transition Probabilities

There are following transition probabilities

$$p_{0,1} = p,\ p_{0,2} = q,\ p_{1,0} = \theta_1 /(\theta_1 + \lambda_2),\ p_{1,3} = \lambda_2 /(\theta_1 + \lambda_2)$$

$$p_{2,1} = p_{3,4} = 1,\ p_{4,0} = \theta_2 /(\theta_2 + \lambda_1),\ p_{4,3} = \lambda_1 /(\theta_2 + \lambda_1) \tag{1}$$

It has been conclusively established that

$$p_{0,1} + p_{0,2} = 1,\ p_{1,0} + p_{1,3} = 1,\ p_{2,1} = 1,\ p_{3,4} = 1,\ p_{4,0} + p_{4,3} = 1$$

VI. Mean Sojourn Time

For the particular state, it becomes

$$\mu_0 = 1/\lambda_1,\ \mu_1 = 1/(\theta_1 + \lambda_2),\ \mu_2 = 1/\gamma,\ \mu_3 = 1/\theta_1$$

$$\mu_4(t) = 1/(\theta_2 + \lambda_1) \tag{2}$$

VII. Evaluation of Parameters

Using the circuit table, '0' is used as the base state to calculate the reliability using the regenerative point graphical technique. The probability factors of all the reachable states from the base state '0' are given below

$$U_{0,0} = (0,1,0) = 1,\ U_{0,1} = p_{0,1},\ U_{0,2} = p_{0,2}$$

$$U_{0,3} = p_{0,1}p_{1,3} + p_{0,2}p_{2,1}p_{1,3},\ U_{0,4} = p_{0,1}p_{1,3}p_{3,4} + p_{0,2}p_{2,1}p_{1,3}p_{3,4}$$

I. Mean Time to System Failure

The regenerative un-failed states (i=0, 1) to which the system can transit (with initial state 0) before entering to any failed state (using base state ξ=0) then MTSF becomes

$$T_0 = \left[\sum_{i=0}^{1} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr(sff)} i\right) \right\} \cdot \mu_i}{\Pi_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[1 - \Sigma Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr(sff)} 0\right) \right\}}{\Pi_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$T_0 = \frac{U_{0,0}\mu_0 + U_{0,1}\mu_1}{[1 - (1,0,1)]} \tag{3}$$

II. Availability of the system

The system is available for use at regenerative states j=0, 1, 4 with ξ=0 then the availability of system is defined as

$$A_0 = \left[\sum_{j} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} j\right) \right\} \cdot f_j \cdot \mu_j}{\Pi_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{4} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} i\right) \right\} \cdot \mu_i'}{\Pi_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$A_0 = \frac{U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,4}\mu_4}{U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4} \tag{4}$$

III. Busy Period of the Technician

The Technician is busy due to repair of the failed unit at regenerative states *j*= 1, 2, 3, 4 with ξ = 0 then the fraction of time for which the server remains busy is defined as

$$B_0 = \left[\sum_j Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} j\right) \right\} . \eta_j}{\Pi_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{4} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} i\right) \right\} . \mu_i'}{\Pi_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$B_0 = \frac{U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4}{U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4} \tag{5}$$

IV. Estimated number of visits made by the Technician

The technician visits at regenerative states *j*= 1, 2 with ξ=0 then the number of visits by the repairman is defined as

$$V_0 = \left[\sum_j Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} j\right) \right\}}{\Pi_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{4} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} i\right) \right\} . \mu_i'}{\Pi_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$V_0 = \frac{U_{0,1}\mu_1 + U_{0,2}\mu_2}{U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4} \tag{6}$$

V. Profit Analysis

The profit of the system is analyzed by

$$P = E_0 A_0 - E_1 B_0 - E_2 V_0 \tag{7}$$

where, $E_0 = 15000$ (Revenue per unit uptime of the system)

$E_1 = 500$ (Cost per unit time for which technician is busy due to repair)

$E_2 = 100$ (Cost per visit of the technician)

VIII. Discussion

Regenerative point graphical technique plays an important role to determine the reliability metrics of the repairable system. Here, λ_i are the failure rate and θ_j are the repair rate. Let all λ_i=λ (*i*=1, 2) and θ_j=θ (*j*=1, 2) then tables 2, 3 and 4 describe the nature of mean time to system failure, availability and profit values of the two unit system under imperfect switch having an increasing trend corresponding to increment in repair rate (θ).

In these tables, the values of parameters change such that λ=0.3, 0.4, 0.5 and *θ*=0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95 respectively. When λ=0.3 changes into 0.4, 0.5 then MTSF, availability and profit values have decreasing trends.

Table 2: *MTSF vs. Repair Rate* (θ)

θ ↓	λ=0.3	λ=0.4	λ=0.5
0.5	3.628692	3.333333	3.090278
0.55	3.675035	3.368794	3.132184
0.6	3.720609	3.403509	3.173516
0.65	3.765432	3.4375	3.214286
0.7	3.809524	3.47079	3.254505
0.75	3.852901	3.503401	3.294183
0.8	3.895582	3.535354	3.333333
0.85	3.937583	3.566667	3.371965
0.9	3.97892	3.59736	3.410088
0.95	4.019608	3.627451	3.447712

Table 3: *Availability vs. Repair Rate* (θ)

θ ↓	λ=0.3	λ=0.4	λ=0.5
0.5	0.623324	0.604782	0.542904
0.55	0.628307	0.609813	0.547959
0.6	0.633159	0.614717	0.552904
0.65	0.637887	0.619499	0.557741
0.7	0.642494	0.624164	0.562476
0.75	0.646985	0.628716	0.56711
0.8	0.651365	0.633159	0.571646
0.85	0.655637	0.637497	0.576089
0.9	0.659806	0.641734	0.58044
0.95	0.663876	0.645873	0.584703

Table 4: *Profit vs. Repair Rate* (θ)

θ ↓	λ=0.3	λ=0.4	λ=0.5
0.5	2438.338	2386.076	2019.52
0.55	2467.262	2415.876	2049.467
0.6	2495.431	2444.927	2078.759
0.65	2522.874	2473.257	2107.417
0.7	2549.618	2500.892	2135.461
0.75	2575.691	2527.857	2162.912
0.8	2601.117	2554.178	2189.787
0.85	2625.919	2579.875	2216.104
0.9	2650.121	2604.972	2241.881
0.95	2673.744	2629.49	2267.135

IX. Conclusion

The performance of the two non identical unit system is discussed using the regenerative point graphical technique. The above tables explore that when the repair rate increases then the MTSF, availability and profit values also increase but when the failure rate increases then the MTSF, availability and profit values decrease.

It is clear that RPGT is helpful for industries to analyze the behaviour of the products and components of a system.

X. Future Scope

It is observed that the role of the regenerative point graphical technique for the two non identical unit system will be beneficial and also used by the management, manufacturers and the persons engaged in reliability engineering and working on analyzing the nature and performance analysis of the system like soft drink, paper industry.

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