ACCEPTANCE SAMPLING PLAN BASED ON TRUNCATED LIFE TESTS FOR RAYLEIGH DISTRIBUTION

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Abstract

This paper addresses the problem of designing an acceptance sampling plan for a truncated life test where the lifetime of the product follows a generalized Rayleigh distribution. The study identifies the minimum sample sizes needed to ensure the specified mean life for various acceptance numbers, confidence levels, and ratios of the fixed experiment time to the specified mean life. The operating characteristic values of the sampling plans, along with the producer's risk, are discussed. Additionally, tables are provided to facilitate the application of these sampling plans, and a numerical example is included to illustrate the use of these tables.

Keywords: Consumer's risk; normal distribution, Rayleigh distribution, operating characteristic curve.

I. Introduction

Lifetime is an important quality variable of a product. Sampling plans used to determine the acceptability of a product, with respect to its lifetime, are known as reliability sampling plans. In most life test sampling plans, a common constraint is the duration of the total time spent on the test. When the lifetime of a product is expected to be high, it might be exceedingly time consuming to wait until all items fail. Therefore, it is usual to terminate a life test by a pre-assigned time t and note the number of failures. One purpose of these tests is to set a confidence limit on the mean life. If a confidence limit on the mean life is set, it is then desired to establish a specified mean life, μ_0, with a given probability of at least P^*. The decision to accept the specified mean life occurs if and only if the observed number of failures at the end of the pre-determined time t does not exceed a given acceptance number c. That is, if the number of failures exceeds c, one can terminate the test before the time t and reject the lot. Such a test is called the truncated life test. The problem considered is that of finding the smallest sample size necessary to assure a certain mean life based on the truncated life test. The sampling plan consists of the number of items n on test, the acceptance number c, and the ratio t/μ_0. For a fixed P^*, such a sampling plan is characterized by the triplet $(n, c, t/\mu_0)$.

Acceptance sampling plans based on truncated life tests were developed by [1] for exponential distribution; by [2] for Weibull distribution; by [3] for gamma distribution; by [4] for half logistic distribution, and by [5] for log-logistic distribution. The present paper extends these to the generalized Rayleigh distribution.

The rest of this paper is organized as follows. In the next section we introduce the generalized Rayleigh distribution briefly. In the third section, an acceptance sampling plan for the truncated life test based on the generalized Rayleigh distribution is developed, and some tables are then established. In the fourth section, a numerical example is provided to illustrate the use of the sampling plan. Some conclusions are made in the final section.

2. Generalized Rayleigh Distribution

The Rayleigh distribution was originally derived by Rayleigh (1880) in connection with a problem in the field of acoustics, and was used to be a lifetime distribution in reliability for recent years. The Rayleigh distribution is a special case of the Weibull distribution and has wide applications, such as in communication engineering [6] in life testing of electro-vacuum devices [7]. The probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of the Rayleigh distribution, respectively, are given by

$$f(t;b) = \frac{t}{b^2} e^{-\frac{t^2}{2b^2}} \tag{1}$$

and

$$F(t;b) = 1 - e^{-\frac{t^2}{2b^2}},\ t > 0 \tag{2}$$

where b>0 is the scale parameter. An important characteristic of the Rayleigh distribution is that its failure rate is an increasing linear function of time. This property makes it a suitable model for components that possibly have no manufacturing defects but age rapidly [4] with time. [8] considered a generalized version of the Rayleigh distribution. The generalized Rayleigh distribution has p.d.f.

$$f_k(t;\beta) = \frac{2}{\Gamma(k+1)\beta^{k+1}} t^{2k+1} e^{\frac{t^2}{\beta}},\ t > 0 \tag{3}$$

where $k \geq 0$ is the shape parameter and $\beta > 0$ is the scale parameter. When $k = 0$ and set $\beta = 2b^2$, the generalized Rayleigh distribution reduces to the Rayleigh distribution in equation (1). The c.d.f. of T is given by

$$F_k(t;\beta) = \Gamma_{I,k+1}\left(\frac{t^2}{\beta}\right) \tag{4}$$

where $\Gamma_{I,\kappa}(\nu) = 1/\Gamma(\kappa)\int_0^{\nu} x^{\kappa-1}e^{-x}dx$ is known as the incomplete gamma function. If k is an integer, the c.d.f. in equation (3) reduces to

$$F_k(t;\beta) = 1 - \sum_{j=0}^{k} \frac{(t^2/\beta)^j e^{(-t^2/\beta)}}{j!} \tag{5}$$

The rth moment of the generalized Rayleigh distribution is

$$E(T^r) = \frac{\Gamma(k+r/2+1)}{\Gamma(k+1)} \beta^{r/2},\ r = 1,2,\ldots \tag{6}$$

Then, the mean and variance of the generalized Rayleigh distribution are $\mu = m\sqrt{\beta}$ and $\sigma^2 = (k + 1 - m^2)\beta$, respectively, where $m = \Gamma(k + 3/2)/\Gamma(k + 1)$.

3. Design of Acceptance Sampling Plan

In the postulated sampling plan, a lot is sentenced to be bad if the true mean life of items, μ, is below the specified value μ_0. In other words, a lot is sentenced to be good if the true mean life of items is at least μ_0. Therefore, the consumer's risk is the probability of accepting a bad lot, and the producer's risk is the probability of rejecting a good lot.

In this study, we fixed the consumer's risk not to exceed $1 - P^*$. It should be noted here that we consider a lot of infinitely large size so that binomial distribution theory can be applied and the acceptance or rejection of the lot are equivalent to the acceptance or rejection of the hypothesis $\mu \geq \mu_0$. We assume that the lifetime follows a generalized Rayleigh distribution. The hypothesis $\mu \geq \mu_0$ is then equivalent to $\beta \geq \beta_0$. Given a value of $P^*(0 < P^* < 1)$, a value of t/μ_0, and a value of the acceptance number c, then the required n is the smallest positive integer satisfying the inequality.

$$\sum_{i=0}^{c} \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq 1 - P^* \tag{7}$$

where $p_0 = F_k(t;\beta_0)$ is given by equation (3) and it is the probability that the lifetime does not exceed t if the true mean life is $\mu_0 = m\sqrt{\beta_0}$. Since the c.d.f. $F_k(t,\beta)$ depends only on the ratio $t/\sqrt{\beta}$, the experimenter needs to specify only this ratio. If the number of observed failures is less than or equal to c, then from equation (4) we can make the confidence statement that $F_k(t;\beta) \leq F_k(t;\beta_0)$ with probability P^*. Since the first derivative of $F_k(t;\beta)$ with respect to $t/\sqrt{\beta}$ is positive, the c.d.f $F_k(t;\beta)$ is a monotonically increasing function of $t/\sqrt{\beta}$. It follows that

$$F_k(t;\beta) \leq F_k(t;\beta_0) \iff \beta \geq \beta_0 \text{ (or } \mu \geq \mu_0) \tag{8}$$

The minimum values of n satisfying equation (4) can be obtained and are given in Table 1 for $k = 0, P^* = 0.75, 0.90, 0.95$, and $t/\mu_0 = 0.4, 0.6, 0.8, 1.0, 1.5$, and 3.0. This choice of t/μ_0 is consistent with the corresponding tables of [3] for a gamma distribution and [9] for a half logistic distribution. For the cases of small values of p_0 and large values of n, the binomial distribution is approximated by Poisson distribution with parameter $\lambda_0 = np_0$. Thus, equation (4) can be rewritten as

$$\sum_{i=0}^{c} \frac{e^{-\lambda_0}\lambda_0^i}{i!} \leq 1 - P^* \tag{9}$$

The operating characteristic function of the sampling plan $(n, c, t/\sqrt{\beta_0})$ is the probability of accepting a lot and is given by

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{10}$$

Table 1: *Minimum sample size n to be tested for a time t in order to assert with probability P^* and acceptance number c that μ≥μ_0 (k=0)*

P^*	c	t/μ_0					
		0.4	0.6	0.8	1.0	1.5	2.0
0.75	0	12	5	3	2	1	1
	1	22	11	6	4	3	2
	2	33	15	9	7	4	3
	3	43	20	12	9	5	4
	4	52	25	15	11	7	5
	5	62	29	18	13	8	6
	6	72	34	21	15	9	8
0.90	0	19	9	5	3	2	1
	1	32	15	9	6	3	2
	2	44	20	12	8	5	4
	3	55	26	15	11	6	5
	4	66	31	19	13	8	6
	5	77	36	22	15	9	7
	6	87	41	25	17	10	8
0.95	0	24	11	6	4	2	1

1	39	18	10	7	4	3
2	52	24	14	10	5	4
3	64	29	18	12	7	5
4	75	35	21	14	8	6
5	87	40	24	17	10	7
6	98	46	27	19	11	8

where $p = F_k(t;\beta) = \Gamma_{I,k+1}\big((t^2/\beta_0)\beta_0/\beta\big)$. If the producer's risk α is given and a sampling plan ($n, c, t/\mu_0$) is adopted, one interesting question is what value of $\mu/\mu_0(> 1)$ will insure the producer's risk equal to or less than α. For a given producer's risk, say $\alpha = 0.05$, the smallest values of μ/μ_0 satisfying the inequality $L(p) \geq 0.95$ were found and are given in Table 2 for $k = 0, P^* = 0.75,0.90,0.95,0.99$ and the selected values of t/μ_0. Tables 1 and 2 can be generated for any other value of k. A computer program is available with the authors. A numerical example is provided in the next section to illustrate the use of these tables.

4. An Example

Assume that the lifetime distribution of the test items is a generalized Rayleigh distribution with shape parameter $k = 0$ and we wish to establish that the mean life μ_0 is at least 1000 hours with probability $P^* = 0.90$. Suppose the experimenter desires to terminate the life test at $t = 600$ hours and set an acceptance number of $c = 3$. The required n is the entry in Table 1 corresponding to the values of $P^* = 0.90, c = 3$ and $t/\mu_0 = 600/1000 = 0.6$. This number is $n = 26$. That is, 26 items have to be put on test. If no more than three failures are observed during 600 hours, then the experimenter can assert that the mean life of the items is at least 1000 hours with a confidence level of 0.90.

Table 2: *Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of* $0.05(k = 0)$

P^*	c	t/μ_0					
		0.4	0.6	0.8	1.0	1.5	2.0
0.75	0	5.43	5.25	5.43	5.54	5.87	7.83
	1	2.76	2.89	2.79	2.77	3.49	3.53
	2	2.22	2.20	2.22	2.39	2.49	2.62
	3	1.96	1.96	1.96	2.07	2.06	2.22
	4	1.79	1.82	1.82	1.88	2.06	1.99
	5	1.70	1.70	1.72	1.77	1.86	1.84
	6	1.63	1.64	1.66	1.68	1.72	2.05
0.90	0	6.83	7.05	7.00	6.78	8.31	7.83
	1	3.34	3.40	3.47	3.48	3.49	3.53
	2	2.58	2.57	2.60	2.59	2.91	3.32
	3	2.22	2.26	2.23	2.33	2.37	2.74
	4	2.03	2.04	2.08	2.09	2.28	2.41
	5	1.90	1.91	1.94	1.93	2.05	2.20
	6	1.73	1.74	1.76	1.74	1.89	1.93
0.95	0	7.67	7.79	7.67	7.83	8.31	7.83
	1	3.69	3.74	3.67	3.79	4.15	4.65
	2	2.80	2.82	2.83	2.94	2.91	3.32
	3	2.40	2.39	2.47	2.45	2.64	2.74
	4	2.16	2.18	2.20	2.18	2.28	2.41
	5	2.02	2.02	2.03	2.08	2.22	2.20
	6	1.91	1.93	1.92	1.95	2.03	2.05

In general, almost all the values of the required number n tabulated by us are found to be less than those tabulated by [4] for a gamma distribution and for a half logistic distribution. The table 2 gives the values of μ/μ_0 in order that the producer's risk may not exceed $\alpha = 0.05$. Thus, for the above example, the values of μ/μ_0 for $c = 0,1,2,3,4,5$ are 7.05,3.40,2.57,2.26,2.04 and 1.91, respectively. The consideration of the actual mean life necessary in order to ship 95% of the lots will play the key role in deciding which c to be selected. For example, if the actual mean life μ is about 2000 hours (that is, the value of μ/μ_0 is about 2.0) based on the production conditions, and the experimenter desires to terminate the test at $t = 400$ hours under the producer's risk 0.05 and the consumer's risk 0.10 (or $P^* = 0.90$), it follows that $c = 4$ from Table 2. The required n in Table 1 corresponding to the values of $P^* = 0.90, c = 4$ and $t/\mu_0 = 0.4$ is $n = 66$. Hence, a sampling plan $(n, c, t/\mu_0) = (66,4,0.4)$ is taken.

5. Conclusion

In this paper develops an acceptance sampling plan based on a truncated life test, assuming the life distribution of test items follows a generalized Rayleigh distribution. The provided tables facilitate the practical application of the suggested plans, enabling practitioners to implement them conveniently. The real time applications of the proposed sampling plan are given using the industrial data. The proposed sampling plan can also be used in testing of software.

Discloser statement

The authors declare no potential conflict of interest.

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