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RECTIFYING INSPECTION FOR DOUBLE SAMPLING PLANS WITH FUZZY LOGIC UNDER ZERO-INFLATED POISSON DISTRIBUTION USING IN PYTHON

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Abstract

Acceptance sampling is a statistical quality control technique used in manufacturing to determine whether to accept or reject a batch of products based on the number of defects obtain in a sample. Among the various sampling plans, the double sampling plan more effective because it often delivers more reliable results in selecting quality lots than other plans. In most of the real life situation, it is not easy found the product as strictly defective or non-defective. In some situation, quality of the product can be classified several types which are expressed as good, almost good, bad, not so bad and so on. This is causes fuzzy logic comes into play. Fuzzy set theory is most powerful mathematical tool, it can deal incomplete and imprecise information. In this paper Double Sampling Plans (DSPs) are derived when non conformities are say imprecise and these imprecisions are model through ZIP distribution. It analyzes, the effectiveness of these sampling plans by comparing vital metrics such as Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) using both fuzzy and crisp environments. These findings are appraised as both numerically and graphically, showing that whether the process quality is either extremely good or very bad, the AOQ curve will be lower, the plan's able to effectively control product quality.

Keywords: Acceptance Double Sampling Plan, FAOQ Curve, FATI curve, ZIP Distribution, Fuzzy Parameter.

I. Introduction

Quality control in industries often meets troubles when handling with data that such as high number of zero values, which usual sampling plans may not manage properly. This paper presents a new approach that combines fuzzy logic and double sampling plan constructed on the basis of Zero-Inflated Poisson (ZIP) distribution. Fuzzy logic enhances the handling of uncertainty and incorrect data, making it easy to deal with the ambiguous limit with zero and non-zero values in ZIP distributions. The method, which was developed by Zadeh and Kosko's (1965) original work, improves quality control by providing a versatile and adaptable method. This technique tries to enhancing decision-making in quality control processes, specifically in circumstances where technological innovation might lead to rare zero defects. Utilizing fuzzy logic, the proposed sampling plan provides an accurate strategy to identify defects in industrial processes, even when handling complex information. The Zero-Inflated Poisson (ZIP) distribution is used in agriculture, epidemiology, econometrics, public health, process control, medicine, and manufacturing. Especially helpful for studying data containing extra zeros. Bohning, et.al., [1] analyzed dental epidemiology intervention effects using the ZIP model. Buckley [2] developed flexible probability theory under uncertainty. Duncan AJ [3] Quality Control and Industrial Statistics is a most impact guide that combines statistical methods with practical approaches to quality control in industrial circumstances.

Chakraborty [4] gives a fuzzy optimization method for single sample plans that minimizes inspection and manages customer risk using Poisson distribution. Ezzatallah and Bahram [5] suggested fuzzy Poisson-based acceptance double sampling for not clear faulty rates. Kavithanjali, et.al., [6] reviewing a SQC methods in single and double-sampling plans, pointing at possible effects on quality. Kavithanjali and Sheik Abdullah [7] gives an innovative technique by combination of fuzzy logic with Zero-Inflated Poisson distribution for single sampling plans, improving quality control and risk management in ambiguity distribution plans. The Python implementation gives practical value, making it useful for real-world circumstances. Kaviyarasu and Asif T Thottathil [8] deals the application of Zero-Inflated Poisson distribution in designing optimal acceptance sampling plans for quality control in manufacturing with a focus on special type double sampling plans. Lotfi A. Zadeh [9] provide fuzzy sets and the degree of membership, which is basis for the employment of conventional theory of sets in fuzzy controller. Lambert [10] illustrated how ZIP regression might enhance data analysis in manufacturing by removing extra zeros from count data. Malathi and Muthulakshmi [11] studied fuzzy logic in double-sampling plans to overcome uncertainty in quality evaluations.

McLachlan and Peel [12] provided insights into finite mixture models, which are critical for analysing complicated data with diverse populations. Naya and et.al., [13] employed ZIP models to demonstrate that age has a substantial effect on the incidence of black patches. Ridout, Demetrio, and Hinde [14] assess count data model excess zeros by using horticultural examples. Schilling and Neubauer [15] provide a comprehensive and authoritative guide on acceptance sampling plan, offering useful insights for quality control in numerous industries. Tamaki F, Kanagawa and Ohta H [16] deals a unique method applying fuzzy logic to attribute-based sampling inspection plans, improving decision-making under ambiguity. For over-dispersed data, Xie, He, and Goh [17] show that the ZIP distribution is better than the Poisson distribution for statistical process control.

The techniques used will be discussed in the sections that follow, along with the results of our study and an explanation of their importance for quality control practitioners. We did our analysis using Python and powerful libraries like NumPy, Pandas, SciPy, and Matplotlib to help with statistics and data visualization. We hope that our work will make a major contribution to the evolving scene of statistical methods meant to solve the issues raised by demanding distributions in industrial environments.

II. Methodology

2.1 Basic Definitions

2.1.1 Fuzzy Number: A fuzzy number $(\widetilde{N})$ is a fuzzy set on the real line R, characterized by a membership function $\mu_N: R \to [0,1]$, that satisfies the following conditions:

- $(\widetilde{N})$ is normal, meaning there exists some x such that $\mu_N(x) = 1$.
- $(\widetilde{N})$ is convex, meaning for any $x_1, x_2 \in R$ and $\lambda \in [0,1], \mu_N(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_N(x_1), \mu_N(x_2))$.
- The membership function μ_N is upper semi-continuous, meaning the set $\{x \in R \mid \mu_N(x) \geq \alpha\}$ is closed for every $\alpha \in (0,1]$.

- The support of $(\widetilde{N})$, defined as $\mathrm{Sup}(\widetilde{N}) = \{x \in R \mid \mu_N(x) > 0\}$, is bounded.

2.1.2 Triangular Fuzzy Number: A triangular fuzzy number $(\widetilde{N})$ is defined by a triplet (a, b, c), where $a < b < c$. The membership function $\mu_{(\widetilde{N})}(x)$ is given by:

$$\mu_{(\widetilde{N})}(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \le x \le b \\ \frac{c-x}{c-b} & \text{if } b < x \le c \\ 0 & \text{otherwise} \end{cases}$$

This function forms a triangular shape with $[a, c]$ as the base and the peak at $x = b$.

2.1.3 α-Cut of Fuzzy : The α-cut of a fuzzy set $\widetilde{N}$ is a crisp set of values where the membership function is at least α. It is defined as:

$$N[\alpha] = \{x \in R \mid \mu_N(x) \ge \alpha\}$$

The fuzzy number $\widetilde{N}[\alpha]$ can be represented by its lower and upper bounds as $N^L[\alpha]$ and $N^U[\alpha]$, where:

$$N^L[\alpha] = \inf\{x \in R \mid \mu_N(x) \ge \alpha\}$$
$$N^U[\alpha] = \sup\{x \in R \mid \mu_N(x) \ge \alpha\}$$

2.1.4 ZIP Distribution: The Zero-Inflated Poisson (ZIP) distribution, define as ZIP (φ, λ), is used when there is an more number of zero counts. The probability mass function (p.m.f.) is found in Lambert [10] and Mclachlan [12]:

$$P(D = d \mid \varphi, \lambda) = \begin{cases} \varphi + (1-\varphi)e^{-\lambda} & \text{if } d = 0 \\ (1-\varphi)\frac{e^{-\lambda}\lambda^d}{d!} & \text{if } d = 1,2,\ldots \end{cases}$$

In this distribution:

- φ represents the probability of extra zeros.
- λ is the mean of the underlying Poisson distribution.

The ZIP distribution mean is $(1-\varphi)\lambda$, and the variance is $\lambda(1-\varphi)(1+\varphi\lambda)$.
To extend the ZIP distribution to a fuzzy setting, we replace λ with a fuzzy number $\tilde{\lambda} > 0$. The fuzzy p.m.f. can be represented as:

$$\tilde{P}(d \mid \alpha) = \begin{cases} \varphi + (1-\varphi)e^{-\lambda} & \text{if } d = 0 \\ (1-\varphi)\frac{e^{-\lambda}\lambda^d}{d!} & \text{if } d = 1,2,\ldots \end{cases}$$

Where λ belongs to the $\alpha-$ cut of $\tilde{\lambda}$.

2.2 Python Programming

Python was an indispensable part of this study as it helped creating of statistical quality control plans. was specifically used to calculate the upper and lower limits for the Fuzzy AOQ Band and the Fuzzy ATI tables. Moreover, Python's extensive plotting capabilities were utilized to show important metrics, which including the Fuzzy Average Outgoing Quality (AOQ), the fuzzy probability of acceptance, and the Average Total Inspection (ATI) curve. Python was chosen for its strong numerical computation capabilities and a massive library that could otherwise allow for the incorporation of complex statistical strategies into the research environment.

III. Operating procedure for RDSPs

Let us consider a circumstance where we analyse the N- lot size for defects with Zero-Inflated Poisson (ZIP) distribution. These are general steps of the typical double sampling plan.

Step 1:

- Take a random sample of size n_1 and count the number of defective items (D_1).
- c_1 is the acceptance number for the first sample.
- c_2 is the acceptance number for both combined samples.

Step 2:

- Accept the lot if $D_1 \le c_1$.
- Reject the lot if $D_1 > c_2$.
- If $c_1 < D_1 \le c_2$, proceed to Step 3 .

Step 3:

- Take a random sample from second sample n_2 and count the number of defective items (D_2).
- Add D_1 and D_2 together.
- Accept the lot if $D_1 + D_2 \le c_2$, otherwise reject it.

Step 4:

- The random variables D_1 and D_2 follows the ZIP distribution with parameter $\lambda_1 = n_1 p$ and $\lambda_2 = n_2 p$, given a large sample size and a small probability p.
- Let P_a stand for the acceptance probability of the lot based onto the combined samples.
- $\tilde{P}_a^I$is for the acceptance probability after the first sample and $\tilde{P}_a^{II}$ for the second sample.

Thus, the overall probability of acceptance is:

$$\tilde{P}_a = \tilde{P}_a^I . \tilde{P}_a^{II}$$

Using the ZIP distribution p.m.f, the number of nonconforming items in the lot is given by

$$\mathrm{P(D = d} \mid \varphi, \lambda) = \tilde{P}(\mathrm{\ d}) = \begin{cases} \varphi + (1-\varphi)e^{-\lambda}, & \text{Whend } = 0 \\ (1-\varphi)\dfrac{e^{-\lambda}\lambda^d}{d!}, & \text{Whend } = 1,2,\dots,0 < \varphi < 1, \lambda > 0 \end{cases}$$

Given a sample size of n_1, the probability of finding no deficiencies will be

$$\tilde{P}(D = 0) = \tilde{P}_a^I(\alpha) = \varphi + (1-\varphi)e^{-n_1 p} \quad (1)$$

Given a sample size of n_2, the probability of finding one deficiencies will be

$$\tilde{P}(D = 1, D_1 + D_2 \le 1) = \tilde{P}_a^{II}(\alpha) = (1-\varphi)e^{-\mathrm{n}p} n_2 p \quad (2)$$

A DSP only accepts a lot if a sample of size n_1 has no faults and a sample of size n_2 has one defect or less. Thus, DSP's $\hat{P}_a(\alpha)$ will be provided by

$$\tilde{P}_a(\alpha) = \tilde{P}_a^I(\alpha) + \tilde{P}_a^{II}(\alpha)$$

IV. Fuzzy Average Outgoing Quality (FAOQ) under ZIP distribution

In acceptance sampling programs, rectification inspection is used to improve the quality of the lot. When the lot is approved, any faulty items in the sample are replaced with non-defective ones. If the batch is rejected, the whole batch is thoroughly inspected, and any faulty goods are replaced with new ones. Then by the following steps Involved operating procedure for AOQ of double sampling plan the acceptable quality level can be obtained from the double sampling plan as given below.

4.1 Operating Methodology for Fuzzy Average Outgoing Quality(FAOQ)

Step 1: Initial Assumptions

- The lot size is N, and the probability of a faulty item is $\tilde{p}$.
- A sample of size n_1 is taken from the batch.

Step 2: First-Stage Sampling

- The probability of the lot being approved after the first step is $\tilde{P}_a^I$.
- If rejected with probability $1 - \tilde{P}_a^I$, a second stage of sampling is conducted.

Step 3: Results of First Stage Acceptance

- After checking n1 items, the remaining N-n₁ items have an average defect rate of $\tilde{p}(N - n_1)$.

Step 4: Second-stage Sampling

- A second sample of size n_2 is chosen.
- If a lot is rejected beyond this step, all faulty components are replaced to ensure zero defects.
- If passed, the remaining N-n_1-n_2 items have an average of $\tilde{p}(N - n_1 - n_2)$ faulty items.

Step 5: Probabilities of Outcomes

The probabilities of every possible result are:

$\tilde{P}_a^{I}$: Acceptance lot after first stage with probability $\tilde{p}(N - n_1)$ of having defective products.
$\tilde{P}_a^{II}$: Lot allowed after the second stage, $\tilde{p}(N - n_1 - n_2)$ defective pieces.
$\tilde{P}_a^{III}$: The lots are 100% checked and this means that no defective items get to make their way through the production line. The probability of satisfying: $\tilde{P}_a^{I} + \tilde{P}_a^{II} + \tilde{P}_a^{III} = 1$.

From using the fuzzy mean definition, the α-cut of FAOQ is as follows.

$$FAOQ(\alpha) = \left\{ \frac{[P_a^{I}(N - n_1) + P_a^{II}(N - n_1 - n_2)].p}{N} \middle| \, p \in \tilde{p}(\alpha) \right\}$$

$$= [FAOQ^{L}(\alpha), FAOQ^{U}(\alpha)]$$

Where

$$FAOQ^{L}(\alpha) = \min \left\{ \frac{[P_a^{I}(N - n_1) + P_a^{II}(N - n_1 - n_2)] \cdot p}{N} \middle| \, p \in \tilde{p}(\alpha) \right\} \text{ and}$$

$$FAOQ^{U}(\alpha) = \max \left\{ \frac{[P_a^{I}(N - n_1) + P_a^{II}(N - n_1 - n_2)].p}{N} \middle| \, p \in \tilde{p}(\alpha) \right\} \text{ for } 0 \le \alpha \le 1$$

Illustration 1

Take into consideration that $\tilde{P} = (0.01, 0.02, 0.03), \mathrm{N} = 200, \mathrm{n}_1 = 10, \mathrm{n}_2 = 10, \mathrm{c}_1 = 0, \mathrm{c}_2 = 1, \tilde{\lambda} = n\tilde{p}$, $\mathrm{n} = \mathrm{n}_1 + \mathrm{n}_2$, $\varphi = 0.0001$, $\tilde{P}[\alpha] = [0.01 + 0.01\alpha, 0.03 - 0.01\alpha]$.

A sample of size n_1, the probability of obtained no defectives will be the equation (1) from

$$\tilde{P}_a^{I}(\alpha) = \varphi + (1 - \varphi)\mathrm{e}^{-10p}$$

A sample size of n_2, the probability of obtained one defectives will be the equation (2) from

$$\tilde{P}_a^{II}(\alpha) = (1 - \varphi)\mathrm{e}^{-20p} 10p$$

$$FAOQ(\alpha) = \left\{ \frac{[\varphi + (1 - \varphi)\mathrm{e}^{-10p}(200 - 10) + (1 - \varphi)\mathrm{e}^{-20p}10p(200 - 10 - 10)] \cdot p}{200} \right\}$$

$$FAOQ(\alpha) = \varphi + (1 - \varphi)\mathrm{e}^{-10p}(0.95)p + (1 - \varphi)\mathrm{e}^{-20p}(9)p^2$$

$$= [FAOQ^{L}(\alpha), FAOQ^{U}(\alpha)]$$

Afterward, using examining function

$$f(p) = \varphi + (1 - \varphi)\mathrm{e}^{-10p}(0.95)p + (1 - \varphi)\mathrm{e}^{-20p}(9)p^2$$

We will have the α cut in the following functions

$$FAOQ^{L}(\alpha) = \varphi + (1-\varphi)e^{-10(0.01+0.01\alpha)}(0.95)(0.01+0.01\alpha) + (1-\varphi)e^{-20(0.01+0.01\alpha)}(9)(0.01+0.01\alpha)^{2}$$

and

$$FAOQ^{U}(\alpha) = \varphi + (1-\varphi)e^{-10(0.03-0.01\alpha)}(0.95)(0.03-0.01\alpha) + (1-\varphi)e^{-20(0.03-0.01\alpha)}(9)(0.03-0.01\alpha)^{2}$$

From $\alpha = 0$, find that FAOQ(0) varies between 0.0094 and 0.0256, showing that 94 to 256 products are predicted to be faulty each lot in this process. When $\alpha = 1$, we get FAOQ(1) = 0.0180. Figure 1 represents the FAOQ, which displays increases in the input quality process.

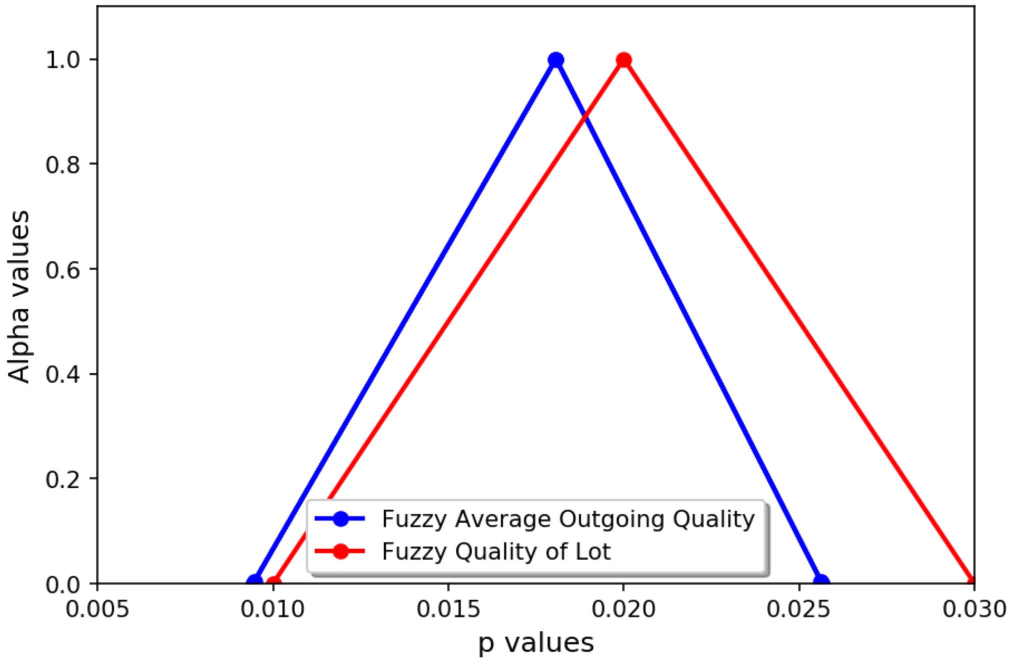

Figure 1: *Comparison of $\tilde{p}$ and fuzzy AOQ with parameters $c_1 = 0$, $c_2 = 1$, and sample sizes $n_1 = n_2 = 10$,* $\tilde{P} = (0.01, 0.02, 0.03)$

FAOQ is a function of lot quality, and when the quality changes, consequently changes the FAOQ. When the FAOQ is plotted against the proportion of bad items in the input lot, the result appears as a diagram with upper and lower boundaries known as the FAOQ band. According upon the established structure for $\tilde{P}$, discussed in the next section.

$$\tilde{P}^{I}_{a,m} = \tilde{P}_{m}(D_1 \le c_1)(\alpha)$$

$$\tilde{P}^{II}_{a,m} = \tilde{P}_{m}(c_1 < D_1 \le c_2 \,.\, D_1 + D_2 \le c_2)(0) \text{ and}$$

$$FAOQ(\alpha) = [FAOQ^{L}_{m}(\alpha), FAOQ^{L}_{m}(\alpha)]$$

$$FAOQ(\alpha) = \left\{ \frac{[P^{I}_{a,m}(N-n_1) + P^{II}_{a,m}(N-n_1-n_2)]p}{N} \mid p \in \tilde{p}(\alpha) \right\}$$

Where

$$\mathrm{FAOQ}^{L}_{m}(\alpha) = \min\left\{ \frac{[P^{I}_{a,m}(N-n_1) + P^{II}_{a,m}(N-n_1-n_2)]p}{N} \middle| \; p \in \tilde{p}(\alpha) \right\} \text{and}$$

$$\mathrm{FAOQ}_m^U(\alpha) = \max\left\{\frac{\left[P_{\alpha,m}^{I}(N-n_1) + P_{\alpha,m}^{II}(N-n_1-n_2)\right]p}{N}\middle|\ p \in \tilde{p}(\alpha)\right\}$$

Illustration 2

Considering the following case scenario: $\mathrm{N} = 200, \mathrm{n}_1 = 20, \mathrm{n}_2 = 20, \mathrm{c}_1 = 0, \mathrm{c}_2 = 1, \tilde{\lambda} = n\tilde{p},\ \mathrm{n} = \mathrm{n}_1 + \mathrm{n}_2,\ \mathrm{b}_2 = 0.01,\ \ \mathrm{b}_3 = 0.02, \varphi = 0.0001$. From these parameters, the next findings have been generated.

$$\tilde{p}(\alpha) = [m + 0.01\alpha, m + 0.02 - 0.01\alpha]$$

$$FAOQ(\alpha) = \varphi + (1-\varphi)\mathrm{e}^{-20p}(0.9)p + (1-\varphi)\mathrm{e}^{-40p}(16)p^2$$

The α-cut of FAOQ at α=0 will be determined as follows

$$\mathrm{FASN}(0) = \begin{cases} \mathrm{FAOQ}^{*},\ \mathrm{FAOQ}^{**} & ,0 \le k < 0.03 \\ \mathrm{FAOQ}^{*}, 0.021970 & ,0.03 \le k < 0.0406 \\ \mathrm{FAOQ}^{**}, 0.021970 & ,0.0406 \le k < 0.05 \\ \mathrm{FAOQ}^{**},\ \mathrm{FAOQ}^{*} & ,0.05 \le k < 0.98 \end{cases}$$

$$FAOQ^{*} = \varphi + (1-\varphi)\mathrm{e}^{-20m}(0.9)m + (1-\varphi)\mathrm{e}^{-40m}(16)m^2$$

$$FAOQ^{**} = \varphi + (1-\varphi)\mathrm{e}^{-20(m+0.02)}(0.9)(m+0.02) + (1-\varphi)\mathrm{e}^{-40(m+0.02)}(16)(m+0.02)^2$$

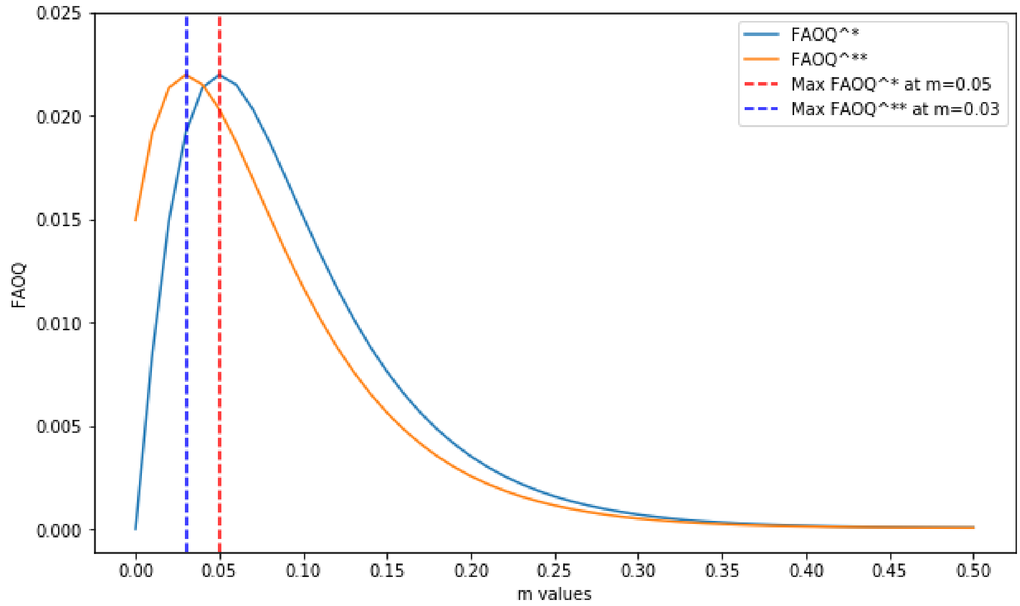

Figure 2: *FAOQ band with the following parameters: N=200, n_1=20, n_2=20, c_1= 0, c_2=1, b_2=0.01, b_3=0.02*

Figure 2 and Table 1 illustrate the FAOQ band for a sampling plan that includes a fuzzy parameter. The outcome show that the FAOQ operates effectively when the proportion of defective items in the incoming lot is either good or bad. The average outgoing quality limit (AOQL) is an important measure of sampling plan performance since it provides the highest expected to percentage of faulty items in inspected lots. The FAOQL, or maximum FAOQ value, is the worst- case scenario for the FAOQ, occurring at a defect level represented as $\tilde{p}^{*}$. In Instance 2, the results obtained are $\tilde{p}^{*} = 0.03, 0.05$ and FAOQL = 0.02197, 0.02197.

Table 1: *FAOQ with N=200, n_1=20, n_2=20, c_1= 0, c_2=1, b_2=0.01, b_3=0.02*

m	FAOQ*	FAOQ**	m	FAOQ*	FAOQ**
0.00	0.0000	0.01494	0.25	0.00158	0.001146
0.01	0.00844	0.01916	0.26	0.00135	0.000974
0.02	0.01494	0.02135	0.27	0.00115	0.000829
0.03	0.01916	0.02197	0.28	0.00097	0.000705
0.04	0.02135	0.02149	0.29	0.00083	0.0006
0.05	0.02197	0.02031	0.3	0.00071	0.000512
0.06	0.02149	0.01872	0.31	0.0006	0.000437
0.07	0.02031	0.01694	0.32	0.00051	0.000374
0.08	0.01872	0.01512	0.33	0.00044	0.00032
0.09	0.01694	0.01336	0.34	0.00037	0.000275
0.10	0.01512	0.0117	0.35	0.00032	0.000238
0.11	0.01336	0.01019	0.36	0.00028	0.000206
0.12	0.0117	0.00883	0.37	0.00024	0.000179
0.13	0.01019	0.00763	0.38	0.00021	0.000157
0.14	0.00883	0.00656	0.39	0.00018	0.000138
0.15	0.00763	0.00564	0.4	0.00016	0.000123
0.16	0.00656	0.00483	0.41	0.00014	0.00011
0.17	0.00564	0.00413	0.42	0.00012	0.000099
0.18	0.00483	0.00353	0.43	0.00011	0.000091
0.19	0.00413	0.00301	0.44	0.000099	0.000083
0.2	0.00353	0.00257	0.45	0.000091	0.000077
0.21	0.00301	0.00219	0.46	0.000083	0.000072
0.22	0.00257	0.00186	0.47	0.000077	0.000069
0.23	0.00219	0.00158	0.48	0.000072	0.000065
0.24	0.00186	0.00135	0.49	0.000069	0.000063

V. Fuzzy Average Total Inspection (FATI) for DSPs under ZIP distribution

The Fuzzy Average Total Inspection (FATI) is an important technique for rectifying inspection for sampling plans with a fuzzy parameter. The following steps are used to compute the FATI:

- If the lot is accepted in the first inspection stage, the number of inspected items is n_1. The probability of this happening (fuzzy probability) is $\tilde{P}_a^I(\alpha)$.
- If the lot is accepted in the second inspection stage, the total number of inspected items is n_1+n_2 .The probability of this happening is $\tilde{P}_a^{II}(\alpha)$.
- If the lot is not accepted at all, all items (N) are inspected. The probability of this happening is $1 - \tilde{P}_a^I(\alpha) - \tilde{P}_a^{II}(\alpha)$.

Finally, the random variable of inspected items has a fuzzy probability function as Table 2 consequently, the FATI according to the definition of a fuzzy mean is as follows:

Table 2: *Fuzzy probability function for the number of inspected items*

Total items under inspection	n_1	n_2	N
Probability with fuzzy	$\tilde{P}_a^I(\alpha)$	$\tilde{P}_a^{II}(\alpha)$	$1 - \tilde{P}_a^I(\alpha) - \tilde{P}_a^{II}(\alpha)$

$$\mathrm{FATI}(\alpha) = \{n_1 p_a^I + (n_1 + n_2) p_a^{II} + N p_a^{III}\}$$

$$= \{N - (N - n_1) p_a^I - (N - n_1 - n_2) p_a^{II} \mid p \in \tilde{p}(\alpha)\}$$

$$= [FATI^L(\alpha), FATI^U(\alpha)]$$

In which $FATI^L(\alpha) = \min\{N - (N - n_1) p_a^I - (N - n_1 - n_2) p_a^{II}\}$

$$F_{ATI}{}^L(\alpha) = \max\{N - (N - n_1) p_a^I - (N - n_1 - n_2) p_a^{II}\} \text{ For } 0 \le \alpha \le 1$$

To calculate FATI in illustration 1 using N=200, as follows:

$$FATI = \{200 - \varphi + (1 - \varphi)\mathrm{e}^{-10\mathrm{p}}(190) - (1 - \varphi)\mathrm{e}^{-20\mathrm{p}}(1800p)\}$$

The lower and higher limits of the α cut are shown below

$$FATI^L(\alpha) = \{200 - \varphi + (1 - \varphi)\mathrm{e}^{-(0.1+0.1\alpha)}(190) - (1 - \varphi)\mathrm{e}^{-(0.2+0.2\alpha)}1800(0.01 + 0.01\alpha)\}$$

$$FATI^U(\alpha) = \{200 - \varphi + (1 - \varphi)\mathrm{e}^{-(0.3-0.1\alpha)}(190) - (1 - \varphi)\mathrm{e}^{-(0.6-0.2\alpha)}1800(0.03 - 0.01\alpha)\}$$

For $\alpha = 0$, FATI[0] = [13.36230, 29.6256], and when $\alpha = 1$, FATI[1] = [20.3275, 20.3275]. This implies that we estimate monitoring 20 items from each accepted batch. The figure 3 displays the fuzzy average total inspection using Example 1.

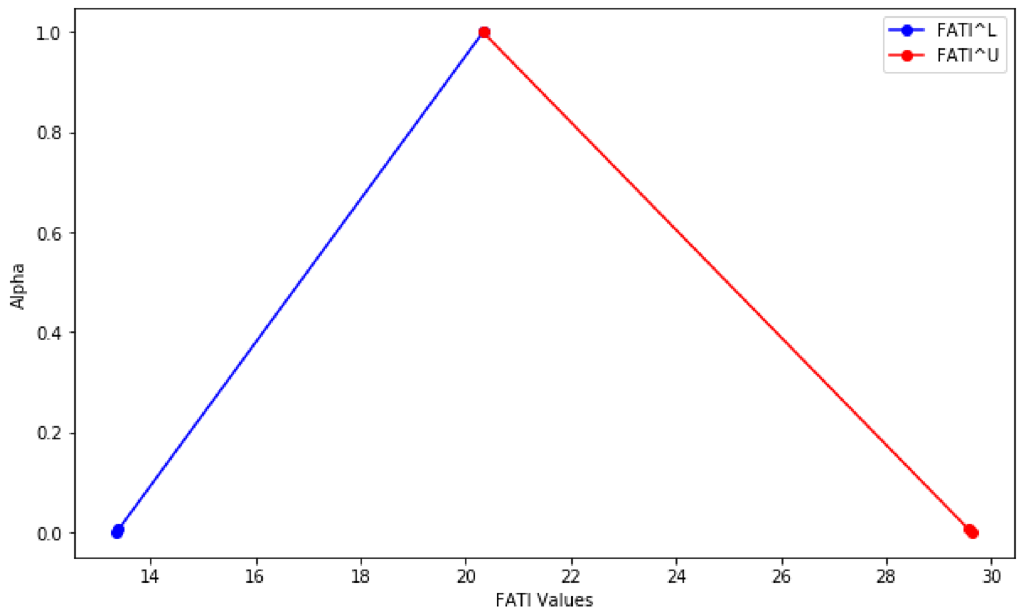

Figure 3: *FATI for DSP parameters N=200, c_1=0, c_2=1, n_1=n_2=10*

As formulated based on the structural definition given for $\tilde{p}$ in this section, the FATI band can be plotted on the basis of $\tilde{p}$ with the range upper and lower boundaries. The width of this band depends on the amount of variation in the proportion parameter as a result of this. A lesser level of uncertainty produces a smaller band, and when the proportion parameter is highly accurate, the upper and lower limits are similar, suggesting that the AOQ and ATI curves return to their classic shape. The FATI band increases in proportion to the number of defective items in the input batch.

Illustration 3

Considering the following case scenario: $N = 200, n_1 = 10, n_2 = 10, c_1 = 0, c_2 = 1, \tilde{\lambda} = n\tilde{p}$, $n = n_1 + n_2$, $b_2 = 0.01$, $b_3 = 0.02, \varphi = 0.0001$. From these parameters, the next findings have been calculating the FATI band.

$$\tilde{p}(\alpha) = [m + 0.01\alpha, m + 0.02 - 0.01\alpha]$$

$$FATI = \{200 - \varphi + (1-\varphi)e^{-10p}(190) - (1-\varphi)e^{-20p}(1800p)\}$$

For $\alpha = 0$ we get

$$FATI^L(0) = \{200 - \varphi + (1-\varphi)e^{-10m}(190) - (1-\varphi)e^{-20m}(1800m)\}$$

$$FATI^U(0) = \{200 - \varphi + (1-\varphi)e^{-(10\,m+0.2)}(190) - (1-\varphi)e^{-(20\,m+0.4)}1800(m+0.02)\}$$

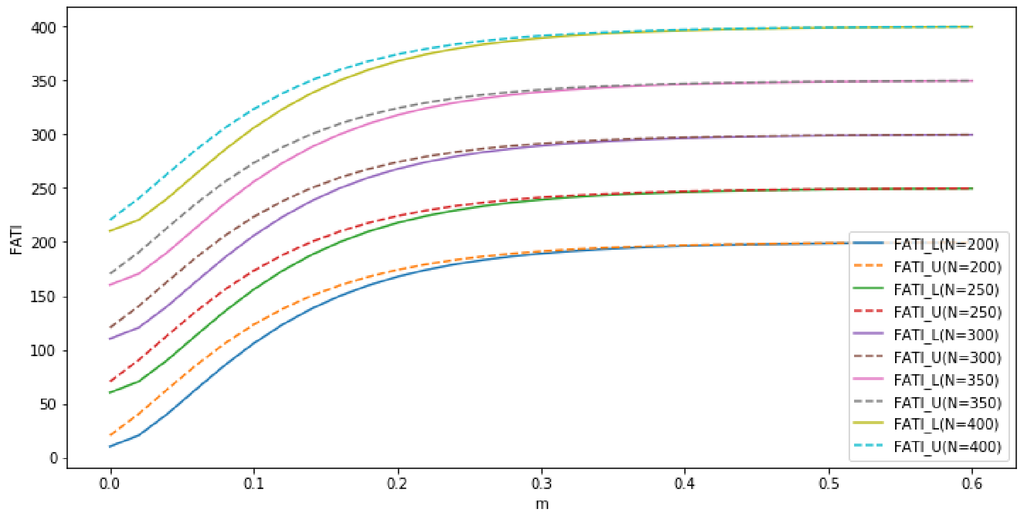

Figure 4: *FATI bands for DSP with fuzzy parameter c_1=0, c_2=1, n_1=n_2=10*

Figure 4 and Table 3 show five FATI bands for N = 200, 250, 300, 350, and 400. These bands illustrate how FATI grows with the amount of defective products. The data demonstrate that when process quality reduces, the FATI band decreases. It has also been found that when process quality is good, FATI approaches the sample size, however when process quality is very low and most lots are rejected, leading FATI to get near the size of the entire lot.

Table 3: *FATI for DSPs with fuzzy parameter c_1=0, c_2=1, n_1=n_2=10*

M	FATI for N = 200	FATI for N = 250	FATI for N = 300	FATI for N = 350	FATI for N = 400
0	10.1890, 20.4883	60.1890, 70.4883	110.1890, 120.4883	160.1890, 170.4883	210.1890, 220.4883
0.02	20.4883, 40.4462	70.4883, 90.4462	120.4883, 140.4462	170.4883, 190.4462	220.4883, 240.4462
0.04	40.4462, 63.3326	90.4462, 113.3326	140.4462, 163.3326	190.4462, 213.3326	240.4462, 263.3326
0.06	63.3326, 85.6678	113.3326, 135.6678	163.3326, 185.6678	213.3326, 235.6678	263.3326, 285.6678

0.08	85.6678, 105.8358	135.6678, 155.8358	185.6678, 205.8358	235.6678, 255.8358	285.6678, 305.8358
0.1	105.8358, 123.2538	155.8358, 173.2538	205.8358, 223.2538	255.8358, 273.2538	305.8358, 323.2538
0.12	123.2538, 137.8836	173.2538, 187.8836	223.2538, 237.8836	273.2538, 287.8836	323.2538, 337.8836
0.14	137.8836, 149.9492	187.8836, 199.9492	237.8836, 249.9492	287.8836, 299.9492	337.8836, 349.9492
0.16	149.9492, 159.7796	199.9492, 209.7796	249.9492, 259.7796	299.9492, 309.7796	349.9492, 359.7796
0.18	159.7796, 167.7240	209.7796, 217.7240	259.7796, 267.7240	309.7796, 317.7240	359.7796, 367.7240
0.2	167.7240, 174.1105	217.7240, 224.1105	267.7240, 274.1105	317.7240, 324.1105	367.7240, 374.1105
0.22	174.1105, 179.2281	224.1105, 229.2281	274.1105, 279.2281	324.1105, 329.2281	374.1105, 379.2281
0.24	179.2281, 183.3220	229.2281, 233.3220	279.2281, 283.3220	329.2281, 333.3220	379.2281, 383.3220
0.26	183.3220, 186.5948	233.3220, 236.5948	283.3220, 286.5948	333.3220, 336.5948	383.3220, 386.5948
0.28	186.5948, 189.2117	236.5948, 239.2117	286.5948, 289.2117	336.5948, 339.2117	386.5948, 389.2117
0.3	189.2117, 191.3058	239.2117, 241.3058	289.2117, 291.3058	339.2117, 341.3058	389.2117, 391.3058
0.32	191.3058, 192.9835	241.3058, 242.9835	291.3058, 292.9835	341.3058, 342.9835	391.3058, 392.9835
0.34	192.9835, 194.3294	242.9835, 244.3294	292.9835, 294.3294	342.9835, 344.3294	392.9835, 394.3294
0.36	194.3294, 195.4108	244.3294, 245.4108	294.3294, 295.4108	344.3294, 345.4108	394.3294, 395.4108
0.38	195.4108, 196.2812	245.4108, 246.2812	295.4108, 296.2812	345.4108, 346.2812	395.4108, 396.2812
0.4	196.2812, 196.9829	246.2812, 246.9829	296.2812, 296.9829	346.2812, 346.9829	396.2812, 396.9829
0.42	196.9829, 197.5494	246.9829, 247.5494	296.9829, 297.5494	346.9829, 347.5494	396.9829, 397.5494
0.44	197.5494, 198.0075	247.5494, 248.0075	297.5494, 298.0075	347.5494, 348.0075	397.5494, 398.0075
0.46	198.0075, 198.3785	248.0075, 248.3785	298.0075, 298.3785	348.0075, 348.3785	398.0075, 398.3785
0.48	198.3785, 198.6793	248.3785, 248.6793	298.3785, 298.6793	348.3785, 348.6793	398.3785, 398.6793
0.50	198.6793, 198.9234	248.6793, 248.9234	298.6793, 298.9234	348.6793, 348.9234	398.6793, 398.9234
0.52	198.9234, 199.1219	248.9234, 249.1219	298.9234, 299.1219	348.9234, 349.1219	398.9234, 399.1219

Conclusion

In conclusion, the integration of acceptance double sampling plans with integrates of fuzzy logic and ZIP distribution greatly improves the majority of methods in quality control. The use of the FAOQ band which has an upper and lower limits captures the fluctuation in defect proportions. The scenario of fuzzy logic resembles that ambiguity in the probability of defects is allowed and that results in most effective and accurate controlling. The ZIP distribution, is suited for higher number of defect free items, very similar to the stated FAOQ and FATI behavior. Acceptance double sampling plans enhance reliability by manage defect rejection and acceptance risks. This combination method gives an adequate basis for monitoring and calculating defect percentage, resulting in enhanced quality control and approaches to decision-making.

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