OPTIMIZATION OF A TWO-WAREHOUSE INVENTORY MANAGEMENT FOR DETERIORATING ITEMS WITH TIME AND RELIABILITY-DEPENDENT DEMAND UNDER CARBON EMISSION CONSTRAINTS

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Abstract

The main objective of this study is to demonstrate how a company's inventory management can be significantly impacted by its ability to provide reliable, high-quality products and to balance stock availability in order to maintain customer satisfaction. Such measures can ultimately lead to an increase in a company's market share, efficiency, and profitability. In order to analyze the impact of reliability and time-based demand rate on inventory management system, an economic order quantity (EOQ) model with two-warehouse is established. Complete backlog allows for the consequences of constant degradation and shortages. The holding and degradation costs are considered while analyzing the effect of carbon emissions. This study's primary goal is to optimize overall cost while maintaining item reliability and total cycle time. Analytical optimization is used to yield an algorithm for the inventory model that determines the optimal output. A numerical example-based sensitivity analysis using MATLAB Software version R2021b is also presented to illustrate the effect of carbon emission and validation of the model.

Keywords: Two-Warehouse, Inventory Management, Deteriorating Items, Time and Reliability-Dependent Demand, Carbon Emission Constraints, Optimization.

1. Introduction and literature survey

Inventory management is a crucial part of supply chain operations because it ensures that the right amount of items are available when needed to meet demand. However, managing inventories of perishables, chemicals, or other items that deteriorate over time presents unique challenges. Traditional inventory models typically prioritize ordering, holding, and shortfall charges in an effort to keep costs as low as possible. Unfortunately, critical components such as the environmental impact of carbon emissions and the reliability of the supply chain are often overlooked by these models. The need to shift inventory management practices toward more environmentally friendly ones stems from the growing emphasis on sustainability. The carbon emissions resulting from transportation, storage, and other logistical activities significantly augment a business's environmental footprint. Thus, in order to achieve both environmental and economic objectives, carbon emission constraints must now be incorporated into inventory models. Furthermore, the demand for degrading commodities is sometimes influenced by time and the reliability of the inventory system. While time-dependent demand reflects the seasonal or cyclical character of particular commodities, reliability-dependent demand considers how supply chain performance influences demand and consumer satisfaction. When these components are integrated, inventory models become more realistic and useful.

Two-warehouse inventory models have been extensively studied in the literature, with a particular emphasis on cost optimization and operational performance. The purpose behind this concept is to manage inventory between a primary warehouse (central storage) and a secondary warehouse (regional or local storage). Several significant studies in this sector include: In order to lower holding and transportation costs, Hartley [7] addressed the distribution of stock between two warehouses and developed one of the first two-warehouse inventory models. Demand-dependent costs were added to the two-warehouse model by Banerjee and Gupta [1], providing a foundation for more adaptable inventory control. Jha and Shanker [9] looked at partial backordering in a two-warehouse system and emphasized the trade-offs between holding costs and stockout concerns. These studies laid the groundwork for two-warehouse inventory management, but they neglected to account for item deterioration and environmental influences. For the review, we can assess the research conducted by Yadav, K.K., et al. [26] and Debnath, B.K., et al. [30].

Degrading inventory models address the issues caused by items that eventually become worthless or useless. The primary goal of this research has been to devise strategies to mitigate the impact of degradation on inventory costs. Among the significant inputs are: Ghare and Schrader [5] initially introduced an exponential decay model for inventory systems, providing a theoretical framework for handling deteriorating items. Wee [13] proposed an inventory model with lost sales and partial backordering for degrading items with time-dependent demand. Bakker et al. [2] stressed the need of including degradation rates in procedures for making decisions regarding inventory after analyzing a number of models for degrading inventory. Yadav et al.'s [27] study provides significant insights into the use of optimization algorithms for managing supply chains during the COVID-19 pandemic, proposing a strategic approach to tackling logistical issues. Baloni et al.'s [31] study advances mathematical modeling in oncology by integrating time delays into homogeneous tumor models, offering essential insights to improve predictions of tumor growth and treatment methods. Despite the extensive research on the topic, few studies have combined degraded inventory models with two-warehouse systems and environmental constraints.

Two significant components of demand variability"time- and reliability-dependent demand"are an essential part of inventory management. Time-dependent demand models provide for cyclical, trend-based, and seasonal variations in demand: Silver et al. [12] proposed many inventory control models that account for time-varying demand patterns and provide practical guidance on how to manage these variations. Hill and Omar [8] developed an economic order quantity (EOQ) model for time-dependent demand that highlights the importance of adaptive inventory techniques. When evaluating client demand, demand models that depend on supply chain reliability take this into consideration: Kumar and Uthayakumar [10] investigated inventory systems with reliability-dependent demand, demonstrating how supply chain performance impacts customer ordering decisions. Roy et al. [11] integrated reliability into an inventory model for degraded items to gain insight into the relationship between demand and dependability in supply chain management. However, there is a lack of models in the academic literature that address both time- and reliability-dependent demand simultaneously in a two-warehouse scenario. They previously developed a mathematical inventory model [14-23] that incorporates a demand rate influenced by reliability.

Carbon emissions have only lately been taken into account in inventory management due to growing environmental concerns and legal pressure. Notable contributions include: A framework emphasizing the significance of sustainable practices was developed by Chaabane et al. [4] in order to integrate carbon footprint assessment into supply chain planning. Benjaafar et al. [3] used inventory models that take carbon emissions into account to draw attention to the trade-offs between environmental and economic objectives. Reducing carbon emissions is an essential component of sustainable supply chain strategies, according to Govindan et al. [6], who studied green supply chain management methodologies. While these studies provide valuable insights into the environmental aspects of supply chain management, they don't specifically address the challenges posed by two-warehouse systems or deteriorating commodities. Several significant and previously published studies [24-25] and [32] address the impact of carbon emissions.

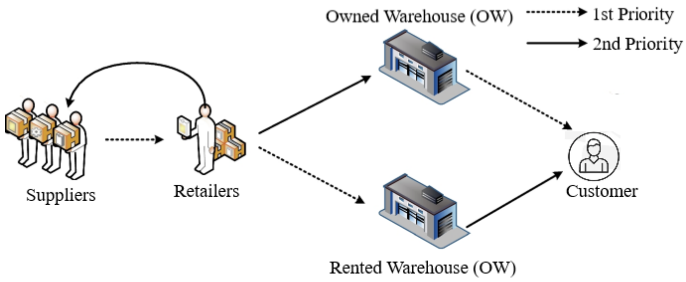

Figure 1: *Sample system framework for managing inventory across two warehouses*

2. Research gap and our contributions

According to the currently published literature, many research studies have been conducted using analytical inventory optimization. They first created a mathematical inventory model [7]. The created problem was then solved in the solution phase using the analytical optimization approach. However, in this study, the inventory model with two-warehouses in a different way. The proposed problem was solved using analytical optimization approach. There are not many studies that use analytical optimization approach for inventory costs. Consequently, Fig. 1 represents the sample system model of the two-warehouse inventory model similar to [33]. The following lists our research's principal contributions:

1. The proposed model is developed considering the impact of carbon emission.
2. When creating the proposed model, inventory criteria such as total cost, and reliability and time-based demand are considered.
3. To manage the inventory in effective manner, then considering the two-warehouses facilities.

The proposed model aims to minimize the whole cost, which includes holding, ordering, degradation, and carbon emission charges. By integrating these elements, the model seeks to provide a comprehensive and durable approach to inventory management for deteriorating items.

Table 1: *The comparison of our current work with previously published work*

Source	Demand	Deterioration	Two-Warehouse	Shortage	Carbon Emission	Optimization Techniques
Kumar, S., and Uthayakumar, R. [10]	Reliability dependent	Yes	No	Yes	No	Analytical
Adak, S., and Mahapatra, G. S. [15]	Time, reliability and advertisement dependent	Yes	No	No	Yes	Analytical
Mahapatra, G. S., et al. [16]	Price, stock, reliability and advertisement dependent	Yes	No	Yes	No	Analytical
Adak, S., and Mahapatra, G. S. [18]	Time and reliability dependent	Yes	No	Yes	No	Analytical
Rajput, N., et al. [21]	Reliability dependent	Yes	No	Yes	No	Analytical
Mahapatra, G. S., et al. [23]	Time and reliability dependent	Yes	No	Yes	No	Analytical
Kumar, K. [24]	Stock dependent	Yes	Yes	Yes	Yes	Analytical
Sharma, A., et al. [32]	Time dependent	Yes	Yes	Yes	Yes	Analytical
Present paper	Time and reliability dependent	Yes	Yes	Yes	Yes	Analytical

3. Presumptions and notations

3.1. Presumptions

The following presumptions were used in the formulation of the mathematical model.

1. The function of demand rate is both time and product reliability dependent, which is $f(t,r) = atr^b$, where $a, b > 0$ are constants.

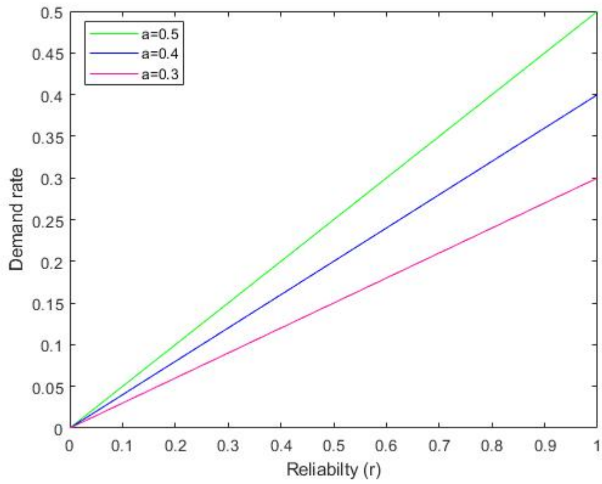

Figure 2: *Correlation between demand rate and product reliability*

2. Shortages is fully backlogged.
3. The filling rate is infinite, and the lead time is zero.
4. The time horizon of the inventory system is infinite.
5. While the rental warehouse (RW) has an infinite capacity, the closely-held warehouse (OW) has a mounted capability of W units.
6. The products of OW are consumed solely once intense, the products unbroken in RW.
7. he rate of deterioration is constant, which is k; $k > 0$.

3.2. Notations

Table 2 is provided a description of the notations utilised for the constructed mathematical model.

Table 2: *Notations*

Notation	Units	Description
a	Constant	Coefficient of demand function
b	Constant	Coefficient of demand function
k	Constant	Coefficient of deterioration rate.
C_s	\$/unit	Shortage cost.
U	capability constraint	The owned warehouse capacity
A	\$/unit	The ordering cost
C_e	\$/unit	Carbon emissions cost for holding items
$I_r(t)$	Units	Rented warehouse Inventory (Stock) level at a time t.
$I_o(t)$	Units	Owned warehouse Inventory (Stock) level at a time t.
$I_s(t)$	Units	Stock out Inventory (Stock) level at a time t.
C_d	\$/Units	Deterioration cost.
h	\$/Units	Holding cost.
Q	Units	The number of orders placed in each cycle.
$TC(r,T)$	\$/Units	The function for total inventory cost.

Table 3: *Decision-makingparameters*

Notation	Units	Description
r	–	Reliability of item of the inventory system.
T	Years	Length of the cycle.

4. Mathematical Model Formulation

In the starting, Q units of deteriorated goods were ordered. Thus, Q it represents the inventory quantity at time zero. At the time interval $t \in [0, t_1]$, the joint effect of deterioration and demand decreases the inventory level in RW to drop until it reaches zero. Also, at the same time interval $t \in [0, t_1]$, only the deterioration effect decreases OW's inventory level. Again, at the time interval $t \in [t_1, t_2]$, the joint impact of deterioration and demand reduces OW's inventory (stock) level to drop until it reaches zero. The period of shortage is $t \in [t_2, T]$ (See Figure 3).

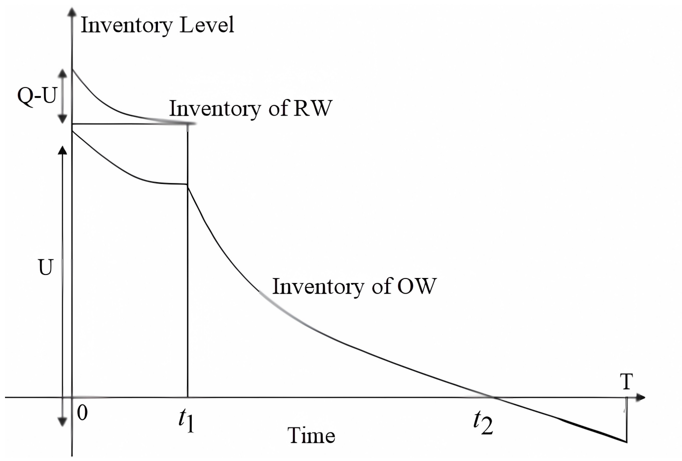

Figure 3: *A graphical depiction of the inventory management system involving two warehouses with complete backlogging.*

The stock level at $t = 0$ to $t = T$ is characterised in the differential equations as follows:

$$\frac{dI_r(t)}{dt} + kI_r(t) = -f(t,r); \quad t \in [0, t_1] \tag{1}$$

with the boundary conditions (B.C.) $I_r(t_1) = 0$ and $I_r(0) = Q - U$.

$$\frac{dI_o(t)}{dt} + kI_o(t) = 0; \quad t \in [0, t_1] \tag{2}$$

with the boundary conditions (B.C.) $I_o(0) = U$.

$$\frac{dI_o(t)}{dt} + kI_o(t) = -f(t,r); \quad t \in [t_1, t_2] \tag{3}$$

with the boundary conditions (B.C.) $I_o(t_2) = 0$.

$$\frac{dI_s(t)}{dt} = -f(t,r); \quad t \in [t_2, T] \tag{4}$$

with the boundary conditions (B.C.) $I_s(t_2) = 0$.

The equations (5), (6), (7) and (8) are the solutions of equations (1), (2), (3) and (4), respectively.

Note that here only taking two terms of exponential functions and remaining are negligible to solve the above four equations (5)-(8).

$$I_r(t) = -\left[\frac{ar^b - ar^b(1-(kt^2))}{2k} + \frac{k^2t^4}{2k}\left(1 + \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right)\right] \tag{5}$$

$$I_o(t) = U\left[1 - \frac{kt^2}{2} + \frac{k^2t^4}{4}\right] \tag{6}$$

$$I_o(t) = -\frac{1}{k}\left[ar^b - ar^b\left(1 - \frac{kt^2}{2} + \frac{k^2t^4}{4}\right)\left(1 + \frac{kt_2^2}{2} + \frac{k^2t_2^4}{4}\right)\right] \tag{7}$$

$$I_s(t) = \frac{ar^bt_2^2}{2} - \frac{ar^bt^2}{2} \tag{8}$$

Using the continuity at $t = t_1$ in equations (6) and (7), we get (9)

$$U = -\frac{1}{k}\left[1 + \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right]\left[ar^b - ar^b\left(1 - \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right)\left(1 + \frac{kt_2^2}{2} + \frac{k^2t_2^4}{4}\right)\right] \tag{9}$$

Using the boundary condition $I_r(0) = Q - U$ in (5), we get (10)

$$Q = U - \frac{1}{k}\left[ar^b - ar^b\left(1 + \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right)\right] \tag{10}$$

From equations (9) and (10), we get (11)

$$Q = -\frac{1}{k}\left[1 + \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right]\left[ar^b - ar^b\left(1 - \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right)\left(1 + \frac{kt_2^2}{2} + \frac{k^2t_2^4}{4}\right)\right] - \frac{1}{k}\left[ar^b - ar^b\left(1 + \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right)\right] \tag{11}$$

The overall cost per cycle comprises the following components:

1. **Ordering cost per cycle :**

$$OC = A \tag{12}$$

2. **Deterioration cost per cycle:**

$$DC = C_d \cdot k\left[\int_0^{t_1} I_r(t)dt + \int_0^{t_1} I_o(t)dt + \int_{t_1}^{t_2} I_o(t)dt\right]$$

$$DC = -C_d \cdot k\Bigg\{\frac{1}{240}\Big\{ar^b(t_1-t_2)\big(3k^3t_1^4t_2^4 + 3k^3t_1^3t_2^5 + 3k^3t_1^2t_2^6 + 3k^3t_1t_2^7 + 3k^3t_2^8 + 6k^2t_1^4t_2^2 + 6k^2t_1^3t_2^3 - 4k^2t_1^2t_2^4 - 4k^2t_1t_2^5 - 4k^2t_2^6 + 12kt_1^4 + 12kt_1^3t_2 - 8kt_1^2t_2^2 - 8kt_1t_2^3 + 52kt_2^4 - 40t_1^2 - 40t_1t_2 + 80t_2^2\big) + ar^bt^3(3k^3t_1^6 - 4k^2t_1^4 + 52kt_1^2 + 80)\Big\} - \frac{t_1}{60k}\left[ar^b - ar^b\left(\frac{k^2t_1^4}{4} - \frac{kt_1^2}{2} + 1\right)\left(1 + \frac{kt_1^2}{2} + \frac{k^2t_2^4}{4}\right)\left(1 + \frac{kt_1^2}{2} + \frac{k^2t_1^4}{4}\right)(3k^2t_1^4 - 10kt_1^2 + 60)\right]\Bigg\} \tag{13}$$

3. **Holding cost per cycle:**

$$HC = h\left[\int_0^{t_1} I_r(t)dt + \int_0^{t_1} I_o(t)dt + \int_{t_1}^{t_2} I_o(t)dt\right]$$

$$HC = -h\Bigg\{\frac{1}{240}\Big\{ar^b(t_1-t_2)\big(3k^3t_1^4t_2^4+3k^3t_1^3t_2^5+3k^3t_1^2t_2^6+3k^3t_1t_2^7+3k^3t_2^8+6k^2t_1^4t_2^2+6k^2t_1^3t_2^3$$
$$-4k^2t_1^2t_2^4-4k^2t_1t_2^5-4k^2t_2^6+12kt_1^4+12kt_1^3t_2-8kt_1^2t_2^2-8kt_1t_2^3+52kt_2^4-40t_1^2-40t_1t_2+80t_2^2)$$
$$+ar^bt^3(3k^3t_1^6-4k^2t_1^4+52kt_1^2+80)\Big\}-\frac{t_1}{60k}\left[ar^b-ar^b\left(\frac{k^2t_1^4}{4}-\frac{kt_1^2}{2}+1\right)\left(1+\frac{kt_1^2}{2}+\frac{k^2t_2^4}{4}\right)\right.$$
$$\left.(3k^2t_1^4-10kt_1^2+60)\left(1+\frac{kt_1^2}{2}+\frac{k^2t_1^4}{4}\right)\right]\Bigg\} \quad (14)$$

4. **Carbon Emission cost per cycle:**

$$CEC = C_e\left[\int_0^{t_1} I_r(t)dt+\int_0^{t_1} I_o(t)dt+\int_{t_1}^{t_2} I_o(t)dt\right]$$

$$CEC = C_e\Bigg\{\frac{1}{240}\Big\{ar^b(t_1-t_2)\big(3k^3t_1^4t_2^4+3k^3t_1^3t_2^5+3k^3t_1^2t_2^6+3k^3t_1t_2^7+3k^3t_2^8+6k^2t_1^4t_2^2+6k^2t_1^3t_2^3$$
$$-4k^2t_1^2t_2^4-4k^2t_1t_2^5-4k^2t_2^6+12kt_1^4+12kt_1^3t_2-8kt_1^2t_2^2-8kt_1t_2^3+52kt_2^4-40t_1^2-40t_1t_2+80t_2^2)$$
$$+ar^bt^3(3k^3t_1^6-4k^2t_1^4+52kt_1^2+80)\Big\}-\frac{t_1}{60k}\left[ar^b-ar^b\left(\frac{k^2t_1^4}{4}-\frac{kt_1^2}{2}+1\right)\left(1+\frac{kt_1^2}{2}+\frac{k^2t_2^4}{4}\right)\right.$$
$$\left.\left(1+\frac{kt_1^2}{2}+\frac{k^2t_1^4}{4}\right)(3k^2t_1^4-10kt_1^2+60)\right]\Bigg\} \quad (15)$$

5. **Shortage cost:**

$$SC = C_s\int_{t_2}^{T} I_s(t)dt = -C_sar^b\left[\frac{(T-t_2)^2(T+2t_2)}{6}\right] \quad (16)$$

Consequently, the retailer's total relevant inventory costs can be stated as follows:

$$TC(r,T) = \frac{1}{T}[OC+DC+HC+CEC+SC]$$

$$TC(r,T) = A+(-C_d\cdot k-h-C_e)\Bigg\{\frac{1}{240}\Big\{ar^b(t_1-t_2)\big(3k^3t_1^4t_2^4+3k^3t_1^3t_2^5+3k^3t_1^2t_2^6+3k^3t_1t_2^7+3k^3t_2^8$$
$$+6k^2t_1^4t_2^2+6k^2t_1^3t_2^3-4k^2t_1^2t_2^4-4k^2t_1t_2^5-4k^2t_2^6+12kt_1^4+12kt_1^3t_2-8kt_1^2t_2^2$$
$$-8kt_1t_2^3+52kt_2^4-40t_1^2-40t_1t_2+80t_2^2)+ar^bt^3(3k^3t_1^6-4k^2t_1^4+52kt_1^2+80)\Big\}-\frac{t_1}{60k}\left[ar^b-ar^b\left(\frac{k^2t_1^4}{4}-\frac{kt_1^2}{2}+1\right)\right.$$
$$\left.\left(1+\frac{kt_1^2}{2}+\frac{k^2t_2^4}{4}\right)\left(1+\frac{kt_1^2}{2}+\frac{k^2t_1^4}{4}\right)(3k^2t_1^4-10kt_1^2+60)\right]\Bigg\}-C_sar^b\left[\frac{(T-t_2)^2(T+2t_2)}{6}\right] \quad (17)$$

Let $t_1 = mt_2,\ 0 < m < 1$, then we get equation (18) from equation (17).

$$TC(r,T) = A + (-C_d \cdot k - h - C_e)\Bigg\{\frac{1}{240}\Bigg\{ar^b(mt_2 - t_2)\big(3k^3(mt_2)^4t_2^4 + 3k^3(mt_2)^3t_2^5 + 3k^3(mt_2)^2t_2^6 + 3k^3mt_2t_2^7 + 3k^3t_2^8 + 6k^2(mt_2)^4t_2^2 + 6k^2(mt_2)^3t_2^3 - 4k^2(mt_2)^2t_2^4 - 4k^2mt_2t_2^5 - 4k^2t_2^6 + 12k(mt_2)^4 + 12k(mt_2)^3t_2 - 8k(mt_2)^2t_2^2 - 8kmt_2t_2^3 + 52kt_2^4 - 40(mt_2)^2 - 40mt_2^2 + 80t_2^2\big) + ar^bt^3(3k^3(mt_2)^6 - 4k^2(mt_2)^4 + 52k(mt_2)^2 + 80)\Bigg\} - \frac{t_1}{60k}\left[ar^b - ar^b\left(\frac{k^2(mt_2)^4}{4} - \frac{k(mt_2)^2}{2} + 1\right)\left(1 + \frac{k(mt_2)^2}{2} + \frac{k^2t_2^4}{4}\right)\left(1 + \frac{k(mt_2)^2}{2} + \frac{k^2(mt_2)^4}{4}\right)(3k^2(mt_2)^4 - 10k(mt_2)^2 + 60)\right]\Bigg\} - C_sar^b\left[\frac{(T-t_2)^2(T+2t_2)}{6}\right] \quad (18)$$

Let $t_2 = nT,\ 0 < n < 1$, then we get equation (19) from equation (18).

$$TC(r,T) = A + (-C_d \cdot k - h - C_e)\Bigg\{\frac{1}{240}\Bigg\{ar^b(mnT - nT)\big(3k^3(mnT)^4nT^4 + 3k^3(mnT)^3(nT)^5 + 3k^3(mnT)^2(nT)^6 + 3k^3m(nT)(nT)^7 + 3k^3(nT)^8 + 6k^2(mnT)^4(nT)^2 + 6k^2(mnT)^3(nT)^3 - 4k^2(mnT)^2(nT)^4 - 4k^2mnT(nT)^5 - 4k^2(nT)^6 + 12k(mnT)^4 + 12k(mnT)^3nT - 8k(mnT)^2(nT)^2 - 8kmnT(nT)^3 + 52k(nT)^4 - 40(mnT)^2 - 40(mnT)^2 + 80(nT)^2\big) + ar^bt^3(3k^3(mnT)^6 - 4k^2(mnT)^4 + 52k(mnT)^2 + 80)\Bigg\} - \frac{t_1}{60k}\left[ar^b - ar^b\left(\frac{k^2(mnT)^4}{4} - \frac{k(mnT)^2}{2} + 1\right)\left(1 + \frac{k(mnT)^2}{2} + \frac{k^2nT^4}{4}\right)\left(1 + \frac{k(mnT)^2}{2} + \frac{k^2(mnT)^4}{4}\right)(3k^2(mnT)^4 - 10k(mnT)^2 + 60)\right]\Bigg\} - C_sar^b\left[\frac{(T-nT)^2(T+2nT)}{6}\right] \quad (19)$$

5. Analytical Optimization Methodology

The optimal values of r and T for minimizing the total inventory costs per unit time is any solution which satisfies simultaneously the equations $\frac{\partial TC(r,T)}{\partial r} = 0$ and $\frac{\partial TC(r,T)}{\partial T} = 0$ for which also satisfies the conditions $\frac{\partial^2 TC(r,T)}{\partial r^2} > 0$, $\frac{\partial^2 TC(r,T)}{\partial T^2} > 0$ and $\left[\frac{\partial^2 TC(r,T)}{\partial r^2} \cdot \frac{\partial^2 TC(r,T)}{\partial T^2} - \frac{\partial^2 TC(r,T)}{\partial r \partial T}\right] > 0$. Using these optimal values of and , the optimal values of can be obtained from above equations.

The aforementioned problems are extremely nonlinear. We have solved the aforementioned problems using an *algorithm* (mentioned below) with the support of MATLAB software (version R2021b) in order to get a solution to non linearity.

Algorithm:

Step 1: Input the values of all the system parameters.
Step 2: Build the functions $TC(r,T)$ given by the equations (10).
Step 3: Find out the optimal values of the decision variables r^* and T^* using the above optimal conditions.
Step 4: Minimize the objective function $TC^*(r,T)$ using r^* and T^*.
Step 5: Calculate the optimal values of $TC^*(r,T)$.
Step 6: Stop.

6. Numerical Illustration

In this part, a numerical example with graphical representations is provided to demonstrate the suggested model. To gauge his best ordering choices, take a look at a retailer of decaying commodities. In order to illustrate the above solution procedure, we consider the following examples:

6.1. Example

A numerical example is given in this section to show how the model works. The following parameter values are used as input:

$$a = 0.005,\ b = 0.02,\ m = 0.5,\ n = 0.5,\ k = 0.2\ Q = 446\ Units,\ A = 2000,$$

$$C_d = 4.0\ \$/unit,\ h = 6.0\ \$/unit,\ C_e = 0.02\ \$/unit,\ C_s = 20.0\ \$/unit.$$

The optimal solution obtained is as below:

$$r^* = 7\%,\ T^* = 14.0835\ weeks,\ TC^* = \$157.5451/cycle.$$

7. Sensitivity analysis

Sensitivity analysis is used in this part to investigate how changes in the parameter's values affect the optimum values. In order to do these studies, one parameter was changed by ±5% and ±10% at a time while remaining at its original value for the other parameters. The following table 3 is demonstrate the outcomes of the sensitivity analysis.

Table 4: *Effects of changing parameters of various kinds on the total cost and optimal solution.*

Parameters	% Change	% change in optimal value r^*	T^*	TC^*
a	-10%	7.9379	14.2271	155.9346
	-5%	7.9689	14.1524	156.7686
	+5%	7.9785	14.0167	158.3036
	+10%	7.9887	13.9533	159.0294
b	-10%	7.7247	14.1074	157.2758
	-5%	7.5300	14.0957	157.4071
	+5%	7.5144	14.0712	157.6851
	+10%	7.4951	14.0589	157.8240
m	-10%	8.5317	14.4785	153.9983
	-5%	8.0970	14.3002	155.5692
	+5%	7.4332	13.8314	159.9597
	+10%	7.7621	13.5525	162.7882
n	-10%	8.8974	15.6681	140.7720
	-5%	8.0178	14.8333	149.1925
	+5%	7.4691	13.4056	165.8533
	+10%	6.9579	12.7923	174.0883
k	-10%	8.8778	14.6177	151.8048
	-5%	8.1907	14.3412	154.7248
	+5%	7.6089	13.8398	160.3066
	+10%	8.1921	13.6093	163.0060

Table 5: *Effects of changing parameters of various kinds on the total cost and optimal solution (Continue).*

Parameters	% Change	% change in optimal value r^*	T^*	TC^*
h	-10%	7.6383	14.2213	155.8464
	-5%	7.6519	14.1504	156.7162
	+5%	7.7463	14.0185	158.3574
	+10%	7.5684	13.9579	159.1237
C_e	-10%	7.2917	14.0848	157.5303
	-5%	7.3169	14.0845	157.5341
	+5%	7.9907	14.0816	157.5669
	+10%	7.9835	14.0814	157.5694
C_d	-10%	7.5185	14.0928	157.4356
	-5%	7.9942	14.0865	157.5096
	+5%	7.7212	14.0782	157.6081
	+10%	7.5155	14.0743	157.6541
C_s	-10%	8.0570	14.0778	157.7645
	-5%	8.0218	14.0798	157.6643
	+5%	7.4901	14.0855	157.4453
	+10%	7.4331	14.0877	157.3441
A	-10%	7.7104	13.9385	143.2794
	-5%	7.7151	14.0125	150.4348
	+5%	7.9615	14.1490	164.6474
	+10%	7.7081	14.2140	171.6880

Following are a few insights drawn from the sensitivity analysis's observations.

1. If the parameters a and b are increased, the total average inventory cost (TC) rises due to the higher demand rate. At the same time, the cycle length decreases, while the product's reliability increases in relation to a but decreases in relation to b.

2. If the parameters m and n are increased, the total average inventory cost (TC) rises quickly due to the extended running time of the warehouses. Simultaneously, both the cycle length and the product reliability decrease.

3. If the deterioration rate (k) increases, the total average inventory cost (TC) rises rapidly due to the higher amount of product waste. At the same time, the cycle length and the product reliability decrease.

4. If the holding cost (h), deterioration cost (k), and carbon emission cost (C_e) increase, the total average inventory cost (TC) rises due to the higher associated expenses. At the same time, the cycle length decreases, but the product reliability improves gradually.

5. If the ordering cost (A) increases, the total average inventory cost (TC) rises rapidly due to the higher cost per order. At the same time, both the cycle length and product reliability increase.

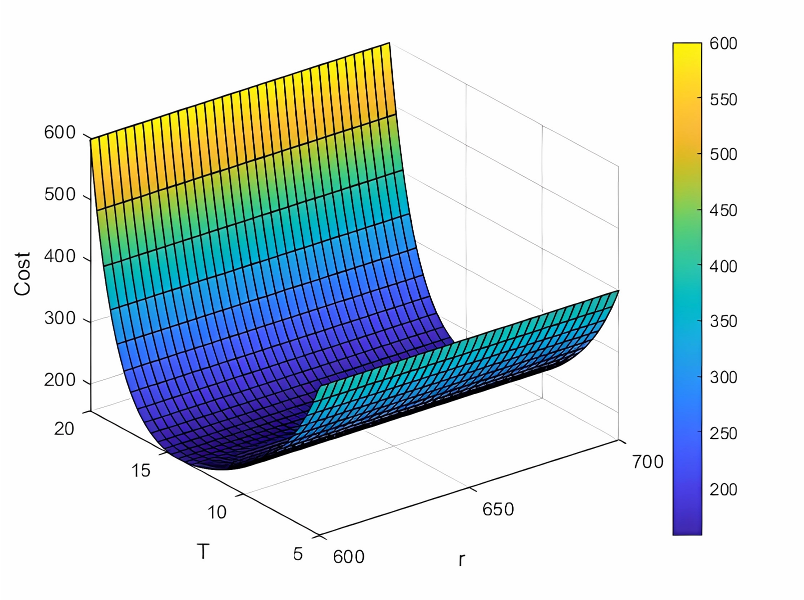

Figure 4: *Convexity of $TC(r, T)$ with respect to r and T*

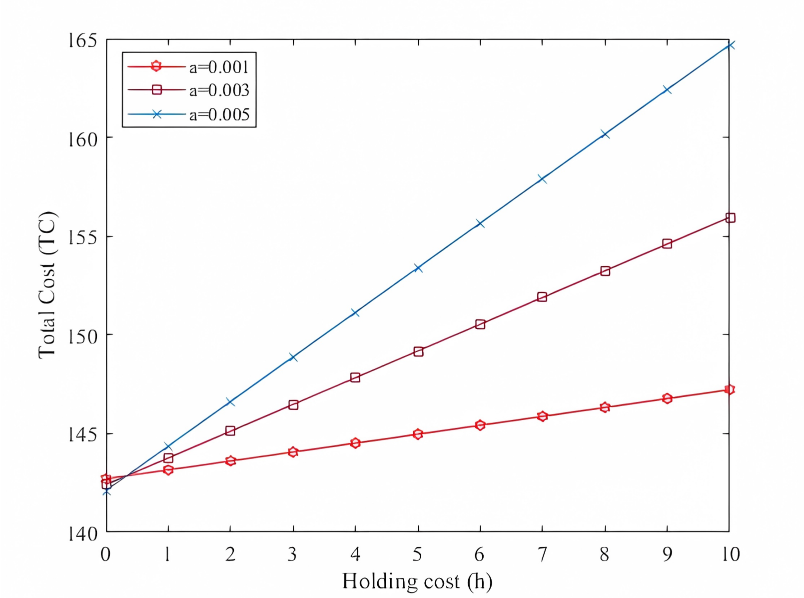

Figure 5: *Relation between Total Cost and Holding Cost using different values of a.*

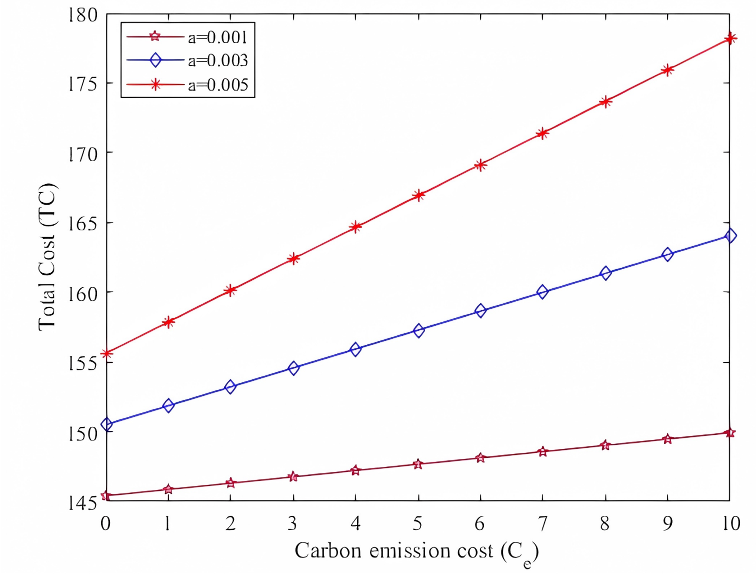

Figure 6: *Relation between Total Cost and Carbon Emission Cost using different values of a.*

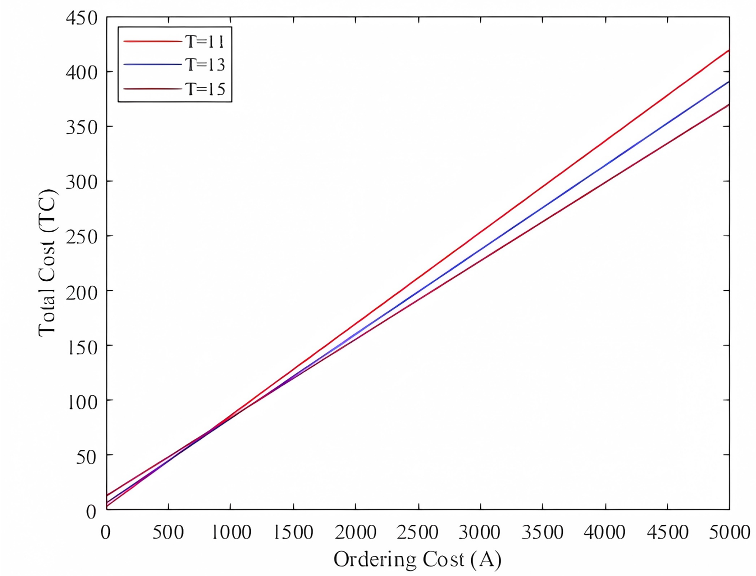

Figure 7: *Relation between Total Cost and Ordering Cost using different values of T.*

8. Conclusion and Future Directions

This research has designed and optimized a two-warehouse inventory management system for deteriorating items, taking into account time and reliability-dependent demand alongside carbon emission constraints. The findings indicate that managing deterioration rates, demand variability, and environmental regulations effectively can notably enhance cost efficiency and inventory reliability. By refining inventory allocation between warehouses and applying effective replenishment strategies, the system can lower total average inventory costs while complying with carbon emission regulations.

The findings emphasize the crucial role of incorporating sustainability into inventory management practices. The model offers important insights into more effective management of deteriorating items, showing that a thorough evaluation of deterioration rates and carbon constraints can significantly enhance overall performance. Additionally, a sensitivity analysis based on numerical examples using MATLAB Software version R2021b is provided to demonstrate the impact of carbon emissions and validate the model.

Future research could investigate different aspects to enhance the model's relevance and accuracy:

1. Expanded Constraints: Explore additional variables like changing transportation costs, supply chain disruptions, or shifts in regulations that could affect the optimization process.

2. Analytical Optimization Technique: Explore analytical optimization methods to more effectively manage trade-offs between cost, reliability, and environmental impact.

3. Industry-Specific Applications: Implement the developed model in specific industries or sectors to assess its robustness and adaptability across different types of deteriorating items and operational settings.

By exploring these future directions, researchers and practitioners can expand on the groundwork established by this study to create more thorough and flexible inventory management strategies.

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