STOCHASTIC BEHAVIOUR OF AN ELECTRONIC SYSTEM SUBJECT TO MACHINE AND OPERATOR FAILURE

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Abstract

A stochastic model is developed by assuming the human (operator) redundancy in cold standby. For constructing this model, one unit is taken as electronic system which consists of hardware and software components and another unit is operator (human being). The system can be failed due to hardware failure, software failure and human failure. The failed hardware component goes under repair immediately and software goes for upgradation. The operator is subjected to failure during the manual operation. There are two separate service facilities in which one repairs/upgrades the hardware/software component of the electronic system and other gives the treatment to operator. The failure rates of components and operator are considered as constant. The repair rates of hardware/software components and human treatment rate follow arbitrary distributions with different pdfs. The state transition diagram and transition probabilities of the model are constructed by using the concepts of semi-Markov process (SMP) and regenerative point technique (RPT). These same concepts have been used for deriving the expressions (in steady state) for reliability measures or indices. The behavior of some important measures has been shown graphically by taking the particular values of the parameters.

Keywords: Electronic System, Operator Failure, Reliability Measures, Stochastic Analysis, semi-Markov Process, Regenerative Point Technique

I. Introduction

The electronic systems containing h/w and s/w components are playing an impactful role in transforming modern society into a digitalization world. All types of modern industries have become dependent on these systems to furnish various jobs with higher accuracy timely. With electronic systems, the digital world can't ignore the impactful role of manpower in doing work. As these systems require hardware and software, the failure of these components during operation is undeniable. Due to these failures, the job cannot be completed in time or the losses can occur in terms of finances, development lives etc. Therefore, the reliability of these systems becomes very important for the completion of jobs.

Many techniques related to reliability improvement have been described by many researchers. The cold standby redundancy techniques have also been included by the engineers. In the modern society, many research works on reliability modelling of redundant systems have been done to do job accurately. In [14], Sridharan and Mohanavadivu checked the stochastic behavior of two-unit standby system having regular and expert repairmen. The aging properties of the residual life length of k-out-of-n system with independent non-identical components has been discussed in [5]. The idea of switch failure and equipment maintenance has been used in reliability analysis of a two-unit cold standby system [12]. The same type of system has been described in [6] with consideration of human error failure. In [13], Salah and Sherbeny assumed different types of failures in analyzing two non-identical unit system stochastically. The concept of Markov process has been used in reliability analysis of system [11].

Many reliability models for non-identical units have been discussed by engineers and researchers. In [7], Malik and Upma described the non-identical units under preventive maintenance. The reliability measures of two dissimilar units using Gumbel-Hougaard Family Copula have been determined in [1]. The provision of rest and switching device for two non-identical unit standby system have been elaborated [3]. In [2], Kadyan et al. studied non-identical repairable system with working of simultaneous cod standby units. The failure of repairman for system of two-non identical units has been considered by Kumar and Nandal [4] in their research work. During h/w repair, assumption of server failure has been considered in stochastic analysis of computer system by Malik and Yadav [8]. In [9], [10] and [15], the authors described the stochastic analysis of a computer system subject to failure of service facility. However, the human being (operator) has not been considered in redundancy in the above-mentioned research by discussing the reliability modelling of electronic system.

Thus, in this paper, the authors tried the use of manpower (operator) in reliability modelling of electronic system. Here, one unit is taken as electronic system which consists of hardware and software components and another unit is operator (human being). The system can be failed due to hardware failure, software failure and human failure. The failed hardware component goes under repair immediately and software goes for upgradation. The operator is subjected to failure during the manual operation. There are two separate service facilities in which one repairs/upgrades the hardware/software component of the electronic system and other gives the treatment to operator. The failure rates of components and operator are considered as constant. The repair rates of hardware/software components and human treatment rate follow arbitrary distributions with different pdfs. The state transition diagram and transition probabilities of the model are constructed by using the concepts of semi-Markov process (SMP) and regenerative point technique (RPT). These same concepts have been used for deriving the expressions (in steady state) for reliability measures or indices. The behavior of some important measures has been shown graphically by taking the particular values of the parameters.

II. Abbreviations and Notations

MTSF	Mean Time to System Failure
SMP	Semi-Markov Process
RPT	Regenerative Point Technique
MST	Mean Sojourn Time
a/b	Probability of hardware/software failure
$x_1/x_2/\ \mu$	Hardware/software/ operator failure rates
$\alpha/\beta/\gamma$	Hardware repair/software up-gradation/operator treatment rates
f(t)/F(t)	pdf/cdf of hardware repair time

g(t)/G(t)	pdf/cdf of software repair time
s(t)/S(t)	pdf/cdf of human treatment time
pdf/cdf	Probability density function/Cumulative density function
$q_{ij}(t)$/ $Q_{ij}(t)$	pdf/cdf of passage time from regenerative state S_i to a regenerative state S_j or to a failed state S_j without visiting any other regenerative state in $(0, t]$
$q_{ij.kr}(t)$/ $Q_{ij.kr}(t)$	pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting states S_k and S_r once in $(0, t]$
$p_{ij}/p_{ij.kr}$	Steady state probability of transition from state S_i to state S_j directly/via states S_k and S_r once
μ_i	MST in state S_i which is given by $\mu_i = E(T_i) = \int_0^\infty \mathrm{P}(T_i > t)dt$ where T_i denotes the sojourn time in state S_i.
m_{ij}	Contribution to MST(μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$
$\emptyset_i(t)$	cdf of first passage time from regenerative state S_i to a failed state
$A_i(t)$	Probability that the system is in up-state at instant 't' given that the system entered in regenerative state S_i at $t = 0$
$M_i(t)$	Probability that the system up initially in regenerative state S_i is up at time t without visiting any other state
$R_i(t)$	Expected number of hardware repairs in the interval $(0, t]$ given that the system entered in regenerative state S_i at $t = 0$.
$U_i(t)$	Expected number of software up-gradations in the interval $(0, t]$ given that the system entered in regenerative state S_i at $t = 0$.
$T_i(t)$	Expected number of treatments given to the human in the interval $(0, t]$ given that the system entered in regenerative state S_i at $t = 0$.
Ⓢ/©	Standard notation for Laplace-Stieltjes convolution/Laplace convolution
*/**	Symbol for Laplace Transform (LT)/Laplace Stieltjes Transform (LST)
P	Profit function of system
Z_0	System revenue per unit up-time
Z_1/Z_2	Repair/up-gradation cost per unit time due to hardware failure/software failure
Z_3	Operator Treatment cost of per unit time

III. Assumptions and State Descriptions

To describe the system the following assumptions are made:

a) There are two units in which one is an electronic system made up of hardware and software and other unit is a human being (operator).
b) The components of electronic system and operator are repaired by separate servers.
c) The h/w repairs, s/w up-gradation and treatments are perfect.
d) The failure rates of components and operator are constant.
e) The arbitrary distributions are taken for h/w repair, s/w up-gradation and operator treatment rates.

The description of all states is given in the Table 1

Table 1: *Description of all states in the system model*

States	Description
S_0	Electronic system is in working and operator is in spare
S_1	h/w component is under repair and operator is working manually
S_2	s/w is under up-gradation and operator is working manually
S_3	h/w component is under repair continuously and operator is under treatment
S_4	s/w is under up-gradation continuously and operator is under treatment
S_5	Electronic system is in working and operator is under treatment continuously
S_6	h/w component is under repair continuously and operator is working manually
S_7	s/w is under up-gradation continuously and operator is working manually
S_8	h/w component is under repair and operator is under treatment continuously
S_9	s/w is under up-gradation continuously and operator is under treatment continuously

All the states are presented in the state transition diagram as shown in Figure 1.

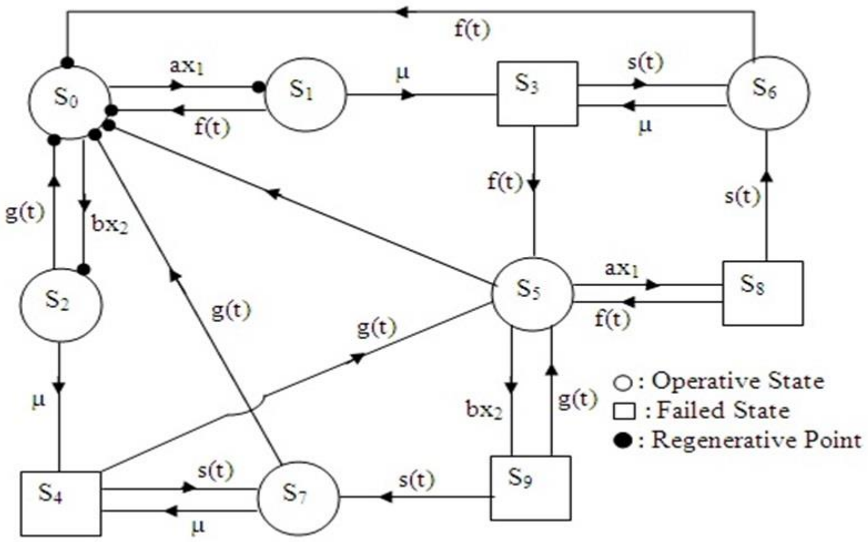

Figure 1: *State transition Diagram of system model*

IV. Reliability Measures

I. Transition Probabilities

The arbitrary distributions are considered as: $f(t) = \alpha e^{-\alpha t}$, $g(t) = \beta e^{-\beta t}$ and $s(t) = \gamma e^{-\gamma t}$.
Using probabilistic arguments, the differential transition probabilities for state S_0 are given by
$dQ_{01}(t) = ax_1 e^{-(ax_1+bx_2)t}dt$, $dQ_{02}(t) = bx_2 e^{-(ax_1+bx_2)t}dt$
The following results: $p_{ij} = \lim_{s\to 0} \emptyset_{ij}^{**}(s) = \emptyset_{ij}^{**}(0) = \int_0^\infty dQ_{ij}(t) = \int_0^\infty q_{ij}(t)dt$, have been used to determine the transition probabilities. These are obtained as
$p_{01} = \frac{ax_1}{ax_1+bx_2}$, $p_{02} = \frac{bx_2}{ax_1+bx_2}$, $p_{10} = p_{60} = \frac{\alpha}{\alpha+\mu}$, $p_{13} = p_{63} = \frac{\mu}{\alpha+\mu}$, $p_{20} = p_{70} = \frac{\beta}{\beta+\mu}$,
$p_{24} = p_{74} = \frac{\mu}{\beta+\mu}$, $p_{35} = p_{85} = \frac{\alpha}{\alpha+\gamma}$, $p_{36} = p_{86} = \frac{\gamma}{\alpha+\gamma}$, $p_{45} = p_{95} = \frac{\beta}{\beta+\gamma}$, $p_{47} = p_{97} = \frac{\gamma}{\beta+\gamma}$,
$p_{50} = \frac{\gamma}{ax_1+bx_2+\gamma}$, $p_{58} = \frac{ax_1}{ax_1+bx_2+\gamma}$, $p_{59} = \frac{bx_2}{ax_1+bx_2+\gamma}$, $p_{15.3} = p_{65.3} = p_{13}p_{35}$, $p_{16.3} = p_{66.3} = p_{13}p_{36}$
$p_{25.4} = p_{75.4} = p_{24}p_{45}$, $p_{27.4} = p_{77.4} = p_{24}p_{47}$, $p_{55.8} = p_{58}p_{85}$, $p_{56.8} = p_{58}p_{86}$, $p_{55.9} = p_{59}p_{95}$,
$p_{57.9} = p_{59}p_{97}$

From these, we have
$p_{01} + p_{02} = p_{10} + p_{13} = p_{20} + p_{24} = p_{35} + p_{36} = p_{45} + p_{47} = p_{50} + p_{58} + p_{59} = 1$

$p_{60} + p_{63} = p_{70} + p_{74} = p_{85} + p_{86} = p_{95} + p_{97} = p_{10} + p_{15.3} + p_{16.3} = 1$
$p_{50} + p_{55.8} + p_{56.8} + p_{55.9} + p_{57.9} = p_{60} + p_{65.3} + p_{66.3} = p_{70} + p_{75.4} + p_{77.4} = 1$

II. MST

The MSTs for all states are determined as follows:
$\mu_0 = \frac{1}{ax_1+bx_2},\ \mu_1 = \frac{1}{\alpha+\mu} = \mu_6,\ \mu_2 = \frac{1}{\beta+\mu} = \mu_7,\ \mu_3 = \frac{1}{\alpha+\gamma} = \mu_8,\ \mu_4 = \frac{1}{\beta+\gamma} = \mu_9,\ \mu_5 = \frac{1}{ax_1+bx_2+\gamma},$
$\mu_1' = \frac{\alpha+\mu+\gamma}{(\alpha+\mu)(\alpha+\gamma)} = \mu_6',\ \mu_2' = \frac{\beta+\mu+\gamma}{(\beta+\mu)(\beta+\gamma)} = \mu_7',\ \mu_5' = \frac{(\beta+\gamma)(\alpha+\gamma+ax_1)+bx_2(\alpha+\gamma)}{(\alpha+\gamma)(ax_1+bx_2+\gamma)(\beta+\gamma)}$

III. Reliability and MTSF

Let $\emptyset_i(t)$ be the c.d.f. of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have following recursive relations for $\emptyset_i(t)$:
$\emptyset_i(t) = \sum_j Q_{ij}(t)Ⓢ\emptyset_j(t) + \sum_k Q_{ik}(t)$
where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is a failed state to which the state S_i can transit directly. Thus, the following equations are obtained as:
$\emptyset_0(t) = Q_{01}(t)Ⓢ\emptyset_1(t) + Q_{02}(t)Ⓢ\emptyset_2(t)$
$\emptyset_1(t) = Q_{10}(t)Ⓢ\emptyset_0(t) + Q_{13}(t)$
$\emptyset_2(t) = Q_{20}(t)Ⓢ\emptyset_0(t) + Q_{24}(t)$
Taking Laplace Stieltjes Transform of above equations, we get
$\emptyset_0^{**}(s) = Q_{01}^{**}(s)\emptyset_1^{**}(s) + Q_{02}^{**}(s)\emptyset_2^{**}(s)$
$\emptyset_1^{**}(s) = Q_{10}^{**}(s)\emptyset_0^{**}(s) + Q_{13}^{**}(s)$
$\emptyset_2^{**}(s) = Q_{20}^{**}(s)\emptyset_0^{**}(s) + Q_{24}^{**}(s)$
Solving for $\emptyset_0^{**}(s)$ by Cramer Rule, we have
$\emptyset_0^{**}(s) = \frac{\Delta_1}{\Delta}$
Where $\Delta = \begin{vmatrix} 1 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) \\ -Q_{10}^{**}(s) & 1 & 0 \\ -Q_{20}^{**}(s) & 0 & 1 \end{vmatrix}$ and
$\Delta_1 = \begin{vmatrix} 0 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) \\ Q_{13}^{**}(s) & 1 & 0 \\ Q_{24}^{**}(s) & 0 & 1 \end{vmatrix}$
Now, we have $R^*(s) = \frac{1-\emptyset_0^{**}(s)}{s}$
The reliability of the system model can be obtained by

$$R(t) = L^{-1}[R^*(s)]$$

The MTSF is given by
$MTSF = \lim_{s\to 0} R^*(s) = R^*(0) = \frac{N_1}{D_1}$, where $N_1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2$ and
$D_1 = p_{01}p_{13} + p_{02}p_{24}$

IV. Availability

Let $A_i(t)$ be the probability that the system is in up-state at epoch 't' given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as
$A_i(t) = M_i(t) + \sum_j q_{ij}^{(n)}(t)©A_j(t)$
where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. Thus, the following equations are obtained as:
$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$
$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{15.3}(t)©A_5(t) + q_{16.3}(t)©A_6(t)$
$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + q_{25.4}(t)©A_5(t) + q_{27.4}(t)©A_7(t)$
$A_5(t) = M_5(t) + q_{50}(t)©A_0(t) + [q_{55.8}(t) + q_{55.9}(t)]©A_5(t) + q_{56.8}(t)©A_6(t) + q_{57.9}(t)©A_7(t)$

$A_6(t) = M_6(t) + q_{60}(t)©A_0(t) + q_{65.3}(t)©A_5(t) + q_{66.3}(t)©A_6(t)$
$A_7(t) = M_7(t) + q_{70}(t)©A_0(t) + q_{75.4}(t)©A_5(t) + q_{77.4}(t)©A_7(t)$
where $M_0(t) = e^{-(ax_1+bx_2)t}$, $M_1(t) = M_6(t) = e^{-\mu t}\bar{F}(t)$,$M_2(t) = M_7(t) = e^{-\mu t}\bar{G}(t)$, and
$M_5(t) = e^{-(ax_1+bx_2)t}\bar{S}(t)$
Taking LT of above equations and solving for $A_0^*(s)$, the steady state availability is calculated by
$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}$
where, $N_2 = p_{01}[p_{57.9}(\mu_1 p_{20} + \mu_2 p_{15.3}) + (1 - p_{27.4})\{\mu_1(p_{50} + p_{56.8}) + \mu_5 p_{15.3}\}] + p_{02}[p_{56.8}(\mu_2 p_{10} + \mu_1 p_{25.4}) + (1 - p_{16.3})\{\mu_2(p_{50} + p_{57.9}) + \mu_5 p_{25.4}\}] + \mu_0[p_{57.9}p_{20}(1 - p_{16.3}) + (1 - p_{27.4})\{p_{50}(1 - p_{16.3}) + p_{56.8}p_{10}\}]$,

$D_2 = (p_{01}\mu_1' + \mu_0' + p_{02}\mu_2')[p_{57.9}p_{20}(1 - p_{16.3}) + (1 - p_{27.4})\{p_{50}(1 - p_{16.3}) + p_{56.8}p_{10}\}] + \mu_5'\{p_{15.3}p_{01}(1 - p_{27.4}) - p_{25.4}p_{02}(1 - p_{16.3})\} + \mu_1'[p_{16.3}p_{01}\{p_{57.9}p_{20} + (1 - p_{27.4})(p_{50} + p_{58})\} + p_{25.4}p_{02}p_{56.8}] + \mu_2'[p_{27.4}p_{02}\{p_{56.8}p_{10} + (1 - p_{16.3})(p_{50} + p_{59})\} + p_{15.3}p_{01}p_{57.9}]$
and $\mu_i = M_i^*(0), i = 0,1,2,5$

V. Expected Number of Hardware Repairs

Let $R_i(t)$ be the expected number of the hardware repairs by the server in the interval $(0, t]$given that the system entered regenerative state S_i at $t = 0$. The expected number of the hardware repairs is given by $R_0(\infty) = \lim_{s\to 0} sR_0^{**}(s)$
The recursive relations for $R_i(t)$ are given as:
$R_i(t) = \sum_j Q_{ij}^{(n)}(t)Ⓢ[\delta_j + R_j(t)]$
Where $S_j$is any regenerative state to which the regenerative state S_i can transit through n transitions and $\delta_j = 1$, if S_j is the regenerative state where server does job afresh, otherwise $\delta_j = 0$. Thus, the following equations are obtained as:
$R_0(t) = Q_{01}(t)ⓈR_1(t) + Q_{02}(t)ⓈR_2(t)$
$R_1(t) = Q_{10}(t)Ⓢ[1 + R_0(t)] + Q_{15.3}(t)Ⓢ[1 + R_5(t)] + Q_{16.3}(t)ⓈR_6(t)$
$R_2(t) = Q_{20}(t)ⓈR_0(t) + Q_{25.4}(t)ⓈR_5(t) + Q_{27.4}(t)ⓈR_7(t)$
$R_5(t) = Q_{50}(t)ⓈR_0(t) + Q_{55.8}(t)Ⓢ[1 + R_5(t)] + Q_{55.9}(t)ⓈR_5(t) + Q_{56.8}(t)ⓈR_6(t) + Q_{57.9}(t)ⓈR_7(t)$
$R_6(t) = Q_{60}(t)Ⓢ[1 + R_0(t)] + Q_{65.3}(t)Ⓢ[1 + R_5(t)] + Q_{66.3}(t)ⓈR_6(t)$
$R_7(t) = Q_{70}(t)ⓈR_0(t) + Q_{75.4}(t)ⓈR_5(t) + Q_{77.4}(t)ⓈR_7(t)$
Taking Laplace Stieltjes Transform of above relation and solving for $R_0^{**}(s)$. The expected number of the hardware repairs is given by
$R_0(\infty) = \lim_{s\to 0} sR_0^{**}(s) = \frac{N_3}{D_2}$
where $N_3 = p_{01}[p_{57.9}p_{20}(1 - p_{16.3}) + (1 - p_{27.4})\{p_{50}(1 - p_{16.3}) + p_{56.8}p_{10}\}] + p_{58}\{p_{15.3}p_{01}(1 - p_{27.4}) + p_{25.4}p_{02}(1 - p_{16.3})\}$ and D_2 is same as calculated in availability.

VI. Expected Number of Software Up-gradations

Let $U_i(t)$ be the expected number of the software up-gradations by the server in the interval $(0, t]$ given that the system entered regenerative state S_i at $t = 0$. The expected number of the software up-gradations is given by $U_0(\infty) = \lim_{s\to 0} sU_0^{**}(s)$
The recursive relations for $U_i(t)$ are given as:
$U_i(t) = \sum_j Q_{ij}^{(n)}(t)Ⓢ[\delta_j + U_j(t)]$
Where $S_j$is any regenerative state to which the regenerative state S_i can transit through n transitions and $\delta_j = 1$, if S_j is the regenerative state where server does job afresh, otherwise $\delta_j = 0$. Thus, the following equations are obtained as:
$U_0(t) = Q_{01}(t)ⓈU_1(t) + Q_{02}(t)ⓈU_2(t)$
$U_1(t) = Q_{10}(t)ⓈU_0(t) + Q_{15.3}(t)ⓈU_5(t) + Q_{16.3}(t)ⓈU_6(t)$
$U_2(t) = Q_{20}(t)Ⓢ[1 + U_0(t)] + Q_{25.4}(t)Ⓢ[1 + U_5(t)] + Q_{27.4}(t)ⓈU_7(t)$
$U_5(t) = Q_{50}(t)ⓈU_0(t) + Q_{55.8}(t)ⓈU_5(t) + Q_{55.9}(t)Ⓢ[1 + U_5(t)] + Q_{56.8}(t)ⓈU_6(t) + Q_{57.9}(t)ⓈU_7(t)$
$U_6(t) = Q_{60}(t)ⓈT_0(t) + Q_{65.3}(t)ⓈU_5(t) + Q_{66.3}(t)ⓈU_6(t)$

$U_7(t) = Q_{70}(t)\circledS[1 + U_0(t)] + Q_{75.4}(t)\circledS[1 + U_5(t)] + Q_{77.4}(t)\circledS U_7(t)$

Taking Laplace Stieltjes Transform of above relation and solving for $R_0^{**}(s)$. The expected number of the software up-gradationsis given by

$U_0(\infty) = \lim_{s\to 0} sU_0^{**}(s) = \frac{N_4}{D_2}$

where $N_4 = p_{02}[p_{57.9}p_{20}(1 - p_{16.3}) + (1 - p_{27.4})\{p_{50}(1 - p_{16.3}) + p_{56.8}p_{10}\}] + p_{59}\{p_{15.3}p_{01}(1 - p_{27.4}) + p_{25.4}p_{02}(1 - p_{16.3})\}$ and D_2 is same as calculated in availability.

VII. Expected Number of Treatments given to Operator

Let $T_i(t)$ be the expected number of the treatments given to human by the server in the interval $(0, t]$ given that the system entered regenerative state S_i at $t = 0$. The expected number of the treatments given to operator is given by $T_0(\infty) = \lim_{s\to 0} sT_0^{**}(s)$

The recursive relations for $U_i(t)$ are given as:

$T_i(t) = \sum_j Q_{ij}^{(n)}(t)\circledS[\delta_j + T_j(t)]$

Where S_j is any regenerative state to which the regenerative state S_i can transit through n transitions and $\delta_j = 1$, if S_j is the regenerative state where server does job afresh, otherwise $\delta_j = 0$". Thus, the following equations are obtained as:

$T_0(t) = Q_{01}(t)\circledS T_1(t) + Q_{02}(t)\circledS T_2(t)$

$T_1(t) = Q_{10}(t)\circledS T_0(t) + Q_{15.3}(t)\circledS T_5(t) + Q_{16.3}(t)\circledS[1 + T_6(t)]$

$T_2(t) = Q_{20}(t)\circledS T_0(t) + Q_{25.4}(t)\circledS T_5(t) + Q_{27.4}(t)\circledS[1 + T_7(t)]$

$T_5(t) = Q_{50}(t)\circledS[1 + T_0(t)] + [Q_{55.8}(t) + Q_{55.9}(t)]\circledS T_5(t) + Q_{56.8}(t)\circledS[1 + T_6(t)] + Q_{57.9}(t)\circledS[1 + T_7(t)]$

$T_6(t) = Q_{60}(t)\circledS T_0(t) + Q_{65.3}(t)\circledS T_5(t) + Q_{66.3}(t)\circledS[1 + T_6(t)]$

$T_7(t) = Q_{70}(t)\circledS T_0(t) + Q_{75.4}(t)\circledS T_5(t) + Q_{77.4}(t)\circledS[1 + T_7(t)]$

Taking Laplace Stieltjes Transform of above relation and solving for $R_0^{**}(s)$. The expected number of the treatments given to human is given by

$T_0(\infty) = \lim_{s\to 0} sT_0^{**}(s) = \frac{N_5}{D_2}$

where $N_5 = p_{01}\{p_{57.9}(p_{15.3} + p_{16.3}p_{20}) + p_{16.3}(1 - p_{27.4})(p_{50} + p_{58})\} + p_{02}\{p_{56.8}(p_{24} - p_{27.4}p_{13}) + p_{27.4}(1 - p_{16.3})(p_{50} + p_{59})\}$ and D_2 is same as calculated in availability.

V. Profit Analysis

The profit function in the time t is given by

P(t) = Expected revenue in (0, t] – expected total cost in (0, t]

In steady state, the profit of the system model can be obtained by the following formula:

$P = Z_0A_0(\infty) - Z_1R_0(\infty) - Z_2U_0(\infty) - Z_3T_0(\infty)$

where $Z_0 =$ ₹ 5000, $Z_1 =$ ₹ 2000, $Z_2 =$ ₹ 1500, $Z_3 =$ ₹ 1000

VI. Application

The application of the present research work is described in the car washing machines. In modern era, most of the people have at least one vehicle in their home for easy service. Therefore, the Vehicle Washing Shops/Vehicle Service Shops are opened within 2KM circle in most of cities. The vehicle is washed with the help of automatic washing machine as shown in the Figure 2, just standing the vehicle under that machine. Due to short circuit, hardware failure or sudden error in software, the machine can be stopped. A server is facilitated for hardware repair and/or software up-gradation of automatic washing machine. After the failure of the machine, the vehicle is washed by a human. During the washing of vehicle, there is possibility that human can be hurt by any part of vehicle; therefore, another service facility has been given for treatment of human being.

Figure 2: *Automatic Car Washing Machine*

VII. Numerical Illustration

Suppose that in a vehicle washing shop there is an automatic car washing machine and a labor. It is obvious that the vehicle washing machine can fail due to h/w component or s/w component with probabilities 'a' or 'b' respectively. The respective failure rates of h/w, s/w and human being are taken as x_1, x_2 and μ. The repair rates of h/w and s/w are assumed as α and β respectively. The labor undergoes treatment with rate γ.

The reliability measures are determined for arbitrary values of the following parameters:

$x_1 = 0.15, x_2 = 0.003, \mu = 0.002, \alpha = 2, \beta = 3, \gamma = 6, a = 0.6$ and $b = 0.4$

$Z_0 =$ ₹ $5000, Z_1 =$ ₹ $2000, Z_2 =$ ₹ $1500, Z_3 =$ ₹ 1000.

The particular values of the transition probabilities are as follows:

$p_{01} = 0.98 = p_{50}, p_{02} = 0.02, p_{10} = 0.99 = p_{20}, p_{13} = 0.01 = p_{24} = p_{58} = p_{59},\ p_{35} = 0.25$

$p_{36} = 0.75, p_{45} = 0.33, p_{47} = 0.67$

The particular values of the MST's are as follows:

$\mu_0 = 10.96, \mu_1 = 0.5 = \mu_1', \mu_2 = 0.33 = \mu_2', \mu_3 = 0.12, \mu_4 = 0.11, \mu_5 = 0.16, \mu_5' = 0.17$

Thus, MTSF $= 11456.66$, Availability $= 0.999848$ and Profit $=$ ₹ 4886.86

VIII. Graphical Study of Reliability Measures

Some important reliability measures such as MTSF, availability and profit have been studied w.r.t h/w failure rate. The graph of MTSF vs h/w failure rate has been shown in the Figure 3. The behavior of availability vs h/w failure rate has been presented in Figure 4. In the similar way, profit analysis has been shown in Figure 5.

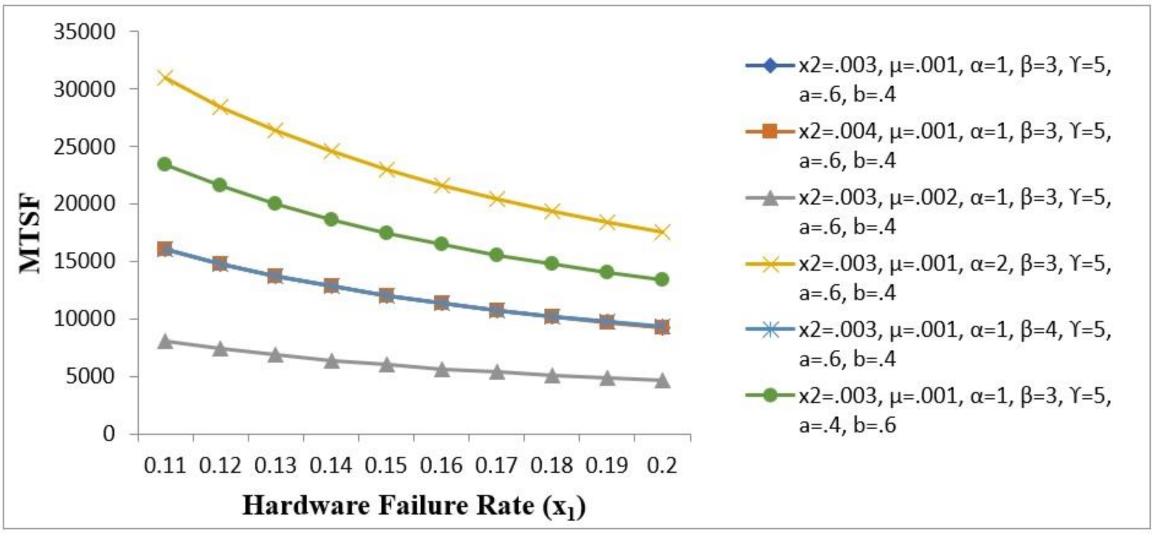

Figure 3: *MTSF Vs Hardware Failure Rate* (x_1)

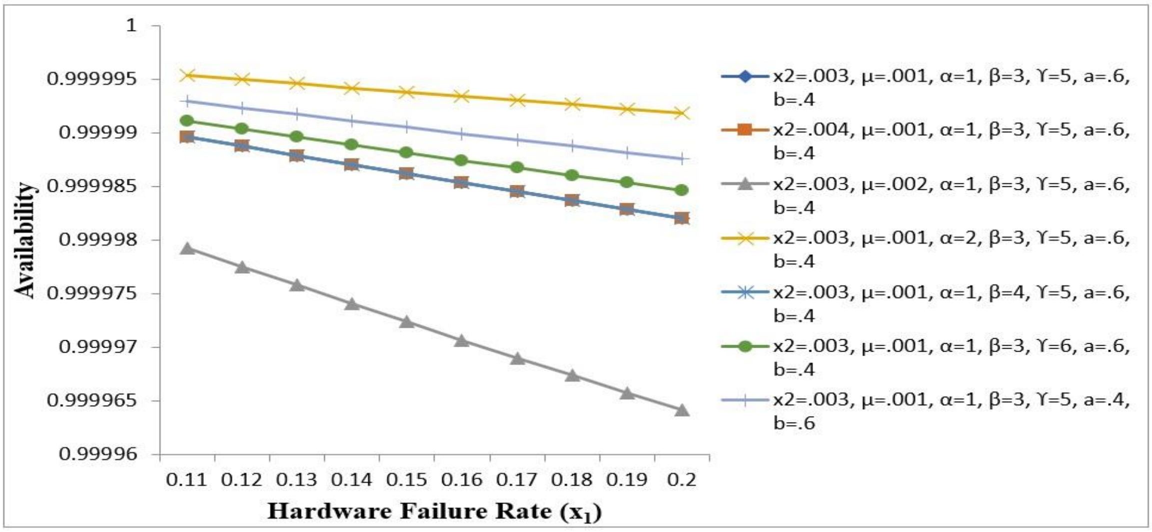

Figure 4: *Availability Vs Hardware Failure Rate* (x_1)

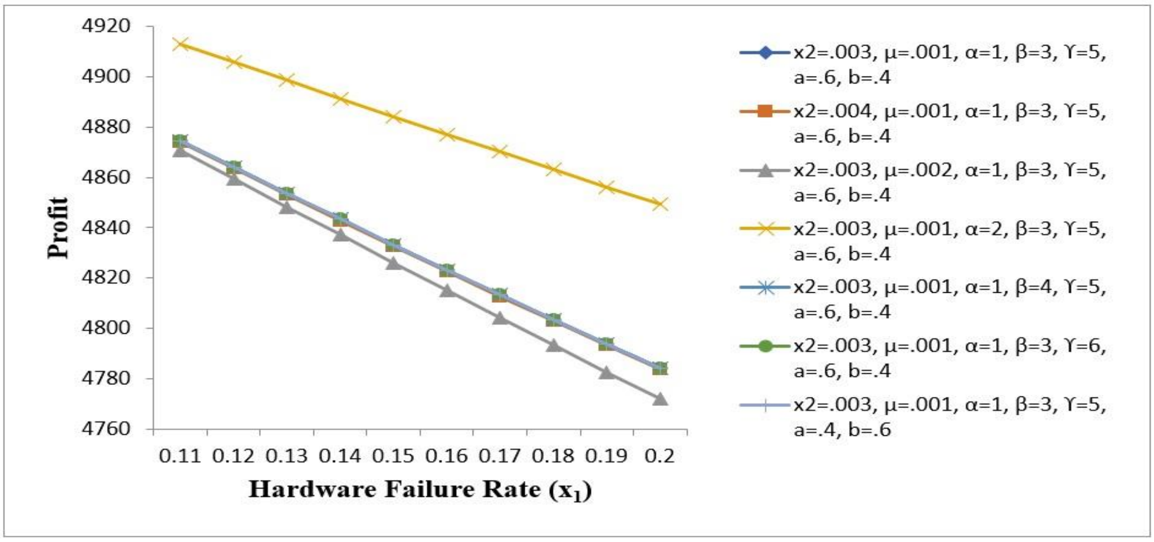

Figure 5: *Profit Vs Hardware Failure Rate* (x_1)

IX. Conclusion

A reliability model of an electronic system with operator failure has been analyzed in present study. The use of SMP and RPT has been incorporated in determining the transition probabilities and various reliability measures. These reliability measures have been explored graphically for the different values of parameters. Figure 3 concludes that MTSF decline according to inclined behavior of components failures and operator failure and MTSF increases with increments in component repair rates and treatment rate. Also, MTSF is very high when hardware repair rate increases from μ=1 to μ=2. Availability and Profit function shows the approximate same behavior as MTSF shows. This nature can be seen if the Figure 4 and Figure 5. There are various future scopes of the present study so that we can make model impactful. The inspection policy for machine can be considered before going under hardware repair as well as software upgradation. The replacement of hardware/software can be done if hardware is not repairable and software is not working. The concept of power failure and timing of power restoration can be considered during the stochastic model. The idea of arbitrary distributions of failure rates of components can be considered.

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