ANALYSIS OF TWO VACATION POLICIES UNDER RETRIAL ATTEMPTS, MARKOVIAN ENCOURAGED ARRIVAL QUEUING MODEL

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Abstract

In this study,Markovian queuing models, which follow encouraged arrival rates and exponential service rates, are used in a variety of systems, including manufacturing, production, telecommunications, computers, and transportation. Every one has a hectic schedule and little free time in the modern world. Because the customer's arrival is unpredictable, they cannot complete their task in the allotted time because they cannot predict it. The encouraged arrival, idle server state, busy server state, vacation state, and breakdown and repair state conditions for a single-server Markovian queuing system were all taken into consideration. Vacation time grows acceleratory, and vacation policies abound. This Markovian-encouraged arrival queuing model takes into account customer impatience and retrial efforts to ensure service completion. We calculate the combined probability of these states and compare first-come, first-served with bulk service. The different performance measures have also been explained.

Keywords: Two vacation, encouraged arrival, reneging, retrial queue, Markov model

1. Introduction

We consider the single server to be encouraged for working vacations, breakdowns, and repairs where the server may be down at any time. A Poisson distribution governs customer arrival rates, whereas an exponential distribution governs customer service rates. Arriving customers will join the orbit group if they discover that the server is too busy serving another customer. Kalyanaraman and Sundaramoorthy [1] demonstrated that a Markovian has dependent arrival and breakdown. A single vacation on an unstable bulk server is detailed in Haridass and Arumuganathan [2]. Batch arrivals on an infinite server are investigated in Daw and Pender [3]. Examples of batch queues with inadequate identification are explored in Bar-Lev et al[4].
A generalized bulk queue with the Poisson model is addressed in Neuts [5]. Parveen and Begum [6] Investigated a general bulk queuing approach with a dual working vacation. In Li and Zhao [7], a retrial model with a consistent retrial rate, breakdowns, and dissatisfied customers were studied. A dual server queuing model for bulk arrival and service was examined in Kumar and Shinde [8]. Working vacations on bulk arrival queues combined with reneging and interruptions were studied in Vijaya Laxmi and Rajesh [9]. Dual service and two vacations are investigated in the bulk arrival Markovian system by Srivastava et al [10]. They investigated batch arrival and retrial queues using a dual vacation policy and the Markovian queuing model Singh and

Srivastava [11]. Som and Seth [12] Explored an $M/M/1/N$ system along with encouraged arrivals. Reduced wait times in an M/M/1/N encouraged customer arrivals, as seen in Khan and Paramasivam [13]. Reneging customers were observed in Som [14] when the M/M/c/N model was used in conjunction with encouraged arrivals.

2. Model elaboration

The following presumptions have been taken into account:

- The customers follow the First-Come-First-Serve rule
- Customers arrive according to an encouraged arrival with mean $\frac{1}{\lambda*(1+\omega)}$. ω representing the offered value and accelerated distribution with mean $\frac{1}{\mu}$.
- Customers will enter the retrial queue with probability 's'
- After a few tries, the customer notices the accelerated distribution with an average of 1- φ and attempts the request from the retrial space.
- When a customer attempts to receive service after a certain amount of time has passed, with probability δ_0
- The server will fail time following the accelerated distribution under the fail rate ν_1 and from the service to the customer under the repair rate ν_2.

We have given that the random variable $J(t)$ describes the absolute customers of the system at period 't', and $R(t) = 0, 1, 2, 3 \ldots$ consider the states, a server is free, a server is busy, the server is on vacation and the server is in breakdown & repair state at period 't'.

3. Analysis of system-size distribution

From first state $0, j \geq 0$ when the server is idle state:

$$\lambda * (1+\omega)\, \acute{P}_{0,0} = \mu \acute{P}_{1,0} \tag{1}$$

$$(\lambda * (1+\omega) + l\psi\delta_0)\, \acute{P}_{0,\,l} = l\mu \acute{P}_{1,\,l} \tag{2}$$

$$(\lambda * (1+\omega) + j\varphi\delta_0)\, \acute{P}_{0,j} = j\mu \acute{P}_{1,j} \tag{3}$$

From second-state $1, j \geq 0$ when the -.server is busy state:

$$(\nu_1 + \Delta_0 + s\lambda * (1+\omega) + \mu)\, \acute{P}_{1,0} = (1-s)\, \varphi \acute{P}_{1,1} + \nu_2 \acute{P}_{3,0} + \lambda * (1+\omega) \acute{P}_{2,0} + \varphi\delta_0 \acute{P}_{0,1}$$

$$+\lambda * (1+\omega)\acute{P}_{0,0}\, (1 - \Delta_0 + s\lambda * (1+\omega) + l\mu + \nu_1)\, P_{1,\,l} \tag{4}$$

$$= \nu_2 \acute{P}_{3,\,l} + \mu \acute{P}_{2,\,l} + \lambda * (1+\omega)\, \acute{P}_{0,l} + s\lambda * (1+\omega)\, \acute{P}_{1,\,l-1}$$

$$+(1-s)(l+1)\varphi P_{1,\,l+1} + (l+1)\varphi\delta_0 \acute{P}_{0,\,l+1}\, (1 - \Delta_0 + \nu_1 + j\mu + (1-s)j\varphi + s\lambda * (1+\omega))\, \acute{P}_{1,j} \tag{5}$$

$$= \mu \acute{P}_{2,j} + \nu_2 \acute{P}_{3,j} + \lambda * (1+\omega)\, \acute{P}_{0,j} + s\lambda * (1+\omega)\, \acute{P}_{1,-1}$$

$$+s\lambda * (1+\omega)\, (j+1)\, \varphi \acute{P}_{1,j+1} + (j+1)\, \varphi\delta_0 \acute{P}_{0,j+1} \tag{6}$$

From third state 2, when the server is on vacation:

$$2\lambda * (1+\omega)\,\acute{P}_{2,0} = \Delta_0 \acute{P}_{0,1} \tag{7}$$

$$(\mu + \lambda * (1+\omega))\,\acute{P}_{2,\,l} = (1-\Delta_0)\,P_{1,\,l} + \lambda * (1+\omega)\,\acute{P}_{1,\,l-1} \tag{8}$$

$$\mu \acute{P}_{2,j} = (1-\Delta_0)\,\acute{P}_{1,j} + \lambda * (1+\omega)\,\acute{P}_{1,j-1} \tag{9}$$

For fourth state 3, when the- server breakdown and is repaired state:

$$\nu_2 \acute{P}_{3,0} = \nu_1 \acute{P}_{1,0} \tag{10}$$

$$\nu_2 \acute{P}_{3,\,l} = \nu_1 \acute{P}_{1,\,l} \tag{11}$$

$$\nu_2 \acute{P}_{3,j} = \nu_1 \acute{P}_{1,j} \tag{12}$$

The following conclusions may be drawn from equations (1), (2), and (3).

$$\left.\begin{array}{c} \acute{P}_{1,0} = \left(\frac{\lambda*(1+\omega)}{\mu}\right)\acute{P}_{0,0} \\ \acute{P}_{1,m} = \left(\frac{\lambda*(1+\omega)+l\varphi\delta_0}{l\mu}\right)\acute{P}_{0,\,l} \\ \acute{P}_{1,j} = \left(\frac{\lambda*(1+\omega)+j\varphi\delta_0}{j\mu}\right)\acute{P}_{0,j} \end{array}\right\} \tag{13}$$

Using (4), now with j = 0 and (10), we obtain

$$(\nu_1 + \Delta_0 + \breve{s} * (1+\omega) + \mu)\,\acute{P}_{1,0} = (1-s)\,\varphi\acute{P}_{1,1} + \nu_1\acute{P}_{1,0} + \lambda * (1+\omega)\acute{P}_{2,0} + \varphi\delta_0\acute{P}_{0,1}$$

$$+\lambda * (1+\omega)\acute{P}_{0,0}\,(\Delta_0 + \breve{s} * (1+\omega) + \mu)\,\acute{P}_{1,0} = (1-s)\,\varphi\acute{P}_{1,1}$$

$$+\lambda * (1+\omega)\acute{P}_{2,0} + \varphi\delta_0\acute{P}_{0,1} + \lambda * (1+\omega)\acute{P}_{0,0}$$

If we use (1), we obtain

$$\left(\frac{\Delta_0\lambda * (1+\omega) + s(\lambda * (1+\omega))^2}{\mu}\right)\acute{P}_{0,0} = (1-s)\varphi\acute{P}_{1,1} + \lambda * (1+\omega)\acute{P}_{2,0} + \varphi\delta_0\acute{P}_{0,1}$$

When k=1, use (13)

$$\begin{array}{c} \acute{P}_{1,1} = \left(\frac{\lambda*(1+\omega)+\varphi\delta_0}{\mu}\right)\acute{P}_{0,1} \\ \acute{P}_{1,1} = a_1\acute{P}_{0,1} \\ \left(\frac{\Delta_0\lambda*(1+\omega)+s(\lambda*(1+\omega))^2}{\mu}\right)\acute{P}_{0,0} = \end{array} \tag{14}$$

$$(1-s)\varphi\, a_1\acute{P}_{0,1} + \lambda * (1+\omega)\acute{P}_{2,0} + \varphi\delta_0\acute{P}_{0,1}$$

If we use (7)

$$\left(\frac{\Delta_0\lambda * (1+\omega) + s(\lambda * (1+\omega))^2}{\mu}\right)\acute{P}_{0,0} = \left[(1-s+a_1+\delta_0)\,\varphi + \frac{\Delta_0}{2}\right]\acute{P}_{0,1}$$

$$\acute{P}_{0,1} = \frac{2\left(\Delta_0\lambda * (1+\omega) + s(\lambda * (1+\omega)\Delta)^2\right)}{\mu\left[(1-s+a_1+\delta_0)\,\varphi + \frac{\Delta_0}{2}\right]}\acute{P}_{0,0}\acute{P}_{0,1} = \frac{b_2}{c_2}\acute{P}_{0,0} \tag{15}$$

Applying j=1 in (6)

$$(1-\Delta_0+\nu_1+\mu+(1-s)\varphi+s\check{} * (1+\omega))\,\acute{P}_{1,1} = \mu\acute{P}_{2,1}+\nu_2\acute{P}_{3,1}+\lambda * (1+\omega)\acute{P}_{0,1}$$

$$+s\check{} * (1+\omega)\acute{P}_{1,0}+2s\check{} * (1+\omega)\varphi\acute{P}_{1,2}+2\varphi\delta_0\acute{P}_{0,2}$$

To do this, use (12) and assign j=1.

$$(1-\Delta_0+\nu_1+(1-s)\,\varphi+s\lambda\,(1+\omega)+\mu)\,\acute{P}_{1,1} = \mu\acute{P}_{2,1}+v_1\acute{P}_{1,1}+\lambda\,(1+\omega)\,\acute{P}_{0,1}+s\lambda\,(1+\omega)\,\acute{P}_{1,0}$$

$$+2s_1\lambda\,(1+\omega)\,\acute{P}_{1,2}+2\varphi\delta_0\acute{P}_{0,2}\,[1-\Delta_0+(1-s)\,\varphi+\mu+s\lambda\,(1+\omega)]\,\acute{P}_{1,1}$$

$$=\mu\acute{P}_{2,1}+\lambda\,(1+\omega)\,\acute{P}_{0,1}+s_{1,0}\acute{P}_{1,0}+2s_1\lambda\,(1+\omega)\,\acute{P}_{1,2}+2\varphi\delta_0\acute{P}_{0,2}$$

If we use (9) for $j=1$ and (14), we get

$$\begin{aligned}
\mu\acute{P}_{2,1} &= (1-\Delta_0)\,\acute{P}_{1,1}+\lambda\,(1+\omega)\,\acute{P}_{1,0}\\
&((1-s)\,\varphi+\mu+s\check{}\,(1+\omega))\,a_1\acute{P}_{0,1}\\
&=\lambda\,(1+\omega)\,\acute{P}_{1,0}+\lambda\,(1+\omega)\,\acute{P}_{0,1}+s\check{}\,(1+\omega)\,\acute{P}_{1,0}\\
&+2s\check{}\,(1+\omega)\,\delta_0\acute{P}_{1,2}+2\varphi\delta_0\acute{P}_{0,2}\\
&((1-s)\varphi\,a_1+\mu a_1+sa_1-\lambda\,(1+\omega))\,\acute{P}_{0,1}\\
&=\lambda\,(1+\omega)\,\acute{P}_{1,0}+s\check{}\,(1+\omega)\,\acute{P}_{1,0}\\
&+2s\check{}\delta_0\acute{P}_{1,2}+2\varphi\delta_0\acute{P}_{0,2}
\end{aligned} \tag{16}$$

Now, we use (13) put $j=2$, and we obtain

$$\acute{P}_{1,2} = \left(\frac{\lambda\,(1+\omega)+2\varphi\delta_0}{2\mu}\right)\acute{P}_{0,2} = \frac{b_1}{c_1}\acute{P}_{0,2}$$

Additionally, using (1) and (15), we can solve (16) to obtain

$$\acute{P}_{0,2} = \frac{b_3}{c_3}\acute{P}_{0,0} \tag{17}$$

$$b_3 = \frac{\begin{array}{c} s\check{}\,(1+\omega)\,a_1\;b_2+\varphi\,(1-s)\,a_1\;b_2\\ +\mu a_1\;b_2-\lambda\,(1+\omega)\,b_2\end{array}}{c_2}$$

$$\frac{-(\lambda\,(1+\omega))^2(1+s)}{\mu}$$

$$c_3 = 2s\check{}\,(1+\omega)\,\delta_0\;b_1/c_1+2\delta_0\varphi$$

When you enter j=2 in equations 6, 9, and 12, we get

$$\begin{aligned}
&(1-\Delta_0+\nu_1+2\mu+2\,(1-s)\,\varphi+s\check{}\,(1+\omega))\,\acute{P}_{1,2}\\
&=\mu\acute{P}_{2,2}+\nu_2\acute{P}_{3,2}+\lambda\,(1+\omega)\,\acute{P}_{0,2}\\
&+s\lambda\,(1+\omega)\,\acute{P}_{1,1}+3s\check{}\,(1+\omega)\,\varphi\acute{P}_{1,3}+3\varphi\delta_0\acute{P}_{0,3}\ldots
\end{aligned} \tag{18}$$

$$\mu\acute{P}_{2,2} = (1-\Delta_0)\,\acute{P}_{1,2}+\lambda\,(1+\omega)\,\acute{P}_{1,1} \tag{19}$$

$$\nu_2\acute{P}_{3,2} = \nu_1\acute{P}_{1,2} \tag{20}$$

If we use (19) & (20) in (18), we have

$$\begin{gathered}(2*\mu+2\,(1-s)\,\varphi+\breve{s}\,(1+\omega))\,\acute{P}_{1,2}\\ =\lambda\,(1+\omega)\,(1+s)\,\acute{P}_{1,1}+\lambda\,(1+\omega)\,\acute{P}_{0,2}\\ +3\breve{s}\,(1+\omega)\,\varphi\acute{P}_{1,3}+3\varphi\delta_0\acute{P}_{0,3}\end{gathered} \tag{21}$$

$$\begin{gathered}(2*\mu+2\,(1-s)\,\varphi+\breve{s}\,(1+\omega))\,\frac{b_1}{c_1}\acute{P}_{0,2}\\ =\lambda\,(1+\omega)\,(1+s)\,\acute{P}_{1,1}+\lambda\,(1+\omega)\,\acute{P}_{0,2}\\ +3\breve{s}\,(1+\omega)\,\varphi\acute{P}_{1,3}+3\varphi\delta_0\acute{P}_{0,3}\end{gathered}$$

$$\begin{gathered}\left((2*\mu+2(1-s)\varphi+\breve{s}\,(1+\omega))\frac{b_1}{c_1}-\lambda\,(1+\omega)\right)\acute{P}_{0,2}\\ =\lambda\,(1+\omega)\,(1+s)\acute{P}_{1,1}+3\breve{s}\,(1+\omega)\,\varphi\acute{P}_{1,3}+3\varphi\delta_0\acute{P}_{0,3}\end{gathered}$$

$$\begin{gathered}\left((2*\mu+2(1-s)\varphi+\breve{s}\,(1+\omega))\frac{b_1}{c_1}-\lambda\,(1+\omega)\right)\frac{b_3}{c_3}\acute{P}_{0,0}\\ =\lambda\,(1+\omega)\,(1+s)a_1\acute{P}_{0,1}+3\breve{s}\,(1+\omega)\,\varphi\acute{P}_{1,3}+3\varphi\delta_0\acute{P}_{0,3}\end{gathered}$$

$$\begin{gathered}\left((2*\mu+2(1-s)\varphi+\breve{s}\,(1+\omega))\frac{b_1}{c_1}-\lambda\,(1+\omega)\right)\frac{b_3}{c_3}\acute{P}_{0,0}\\ =\lambda\,(1+\omega)\,(1+s)a_1\frac{b_2}{c_2}\acute{P}_{0,0}+3\breve{s}\,(1+\omega)\,\varphi\acute{P}_{1,3}+3\varphi\delta_0\acute{P}_{0,3}\end{gathered}$$

$$\begin{gathered}\begin{pmatrix}\left\{(2*\mu+2\,(1-s)\,\varphi+\breve{s}\,(1+\omega))\,\frac{b_1}{c_1}-\lambda\,(1+\omega)\right\}\frac{b_3}{c_3}\\ -\lambda\,(1+\omega)\,(1+s)a_1\frac{b_2}{c_2}\end{pmatrix}\acute{P}_{0,0}\\ =3\breve{s}\,(1+\omega)\,\varphi\acute{P}_{1,3}+3\varphi\delta_0\acute{P}_{0,3}\end{gathered}$$

If, we Put $j=3$ in (13), and we have

$$\acute{P}_{1,3}=\left(\frac{\lambda(1+\omega)+j'\delta_0}{j^{-}}\right)\acute{P}_{0,3}=\frac{b_4}{c_4}\acute{P}_{0,3}$$

$$\begin{gathered}(\left\{(2*\mu+2(1-s)\varphi+\breve{s}\,(1+\omega))\frac{b_1}{c_1}-\lambda\,(1+\omega)\right\}\frac{b_3}{c_3}\\ -\lambda\,(1+\omega)\,(1+s)a_1\frac{b_2}{c_2}\\ =\left(3s'\frac{b_4}{c_4}+3\varphi\delta_0\right)\acute{P}_{0,3}\end{gathered}$$

$$\acute{P}_{0,3}=\frac{\left(\left\{(2*\mu+2(1-s)\varphi+\breve{s}\,(1+\omega))\frac{b_1}{c_1}-\lambda\,(1+\omega)\right\}\frac{b_3}{c_3}-\lambda\,(1+\omega)\,(1+s)a_1\frac{b_2}{c_2}\right)}{\left(\frac{3\lambda(1+\omega)\varphi\,b_4}{c_4}+3\varphi\delta_0\right)}\acute{P}_{0,0}$$

$$\acute{P}_{0,3}=\frac{b_5}{c_5}\acute{P}_{0,0}$$

In general, we get

$$\acute{P}_{0,n}=\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots \tag{22}$$

If we use (13), we get

$$\acute{P}_{1,j}=\left(\frac{\lambda\,(1+\omega)+j\psi\delta_0}{j\mu}\right)(\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots) \tag{23}$$

If, we use (9)

$$\acute{P}_{2,j} = (1-\Delta_0)\left(\frac{\lambda\,(1+\omega)+j\Sigma\delta_0}{j*\mu^2}\right)$$
$$\left(\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots\right)+\frac{\lambda\,(1+\omega)\,\acute{P}_{1,j-1}}{\mu} \tag{24}$$

Similarly, if, we use (12), we get

$$\acute{P}_{3,j} = \left(\frac{\nu_1}{\nu_2}\right)\left(\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots\right) \tag{25}$$

Now, if we use equations (22), (23), (24), and (25), we obtain

$$\acute{P}_{r,n} = \acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots \text{for} s=0\left(\frac{\lambda\,(1+\omega)+j'\delta_0}{j}\right)\left(\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots\right),$$
$$\text{for} s=1\,(1-\Delta_0)\left(\frac{\lambda\,(1+\omega)+j'\delta_0}{j*\mu^2}\right)\left(\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots\right)+\frac{\lambda\,(1+\omega)\,\acute{P}_{1,j-1}}{\mu},$$
$$\text{for} s=2,\left(\frac{\nu_1}{\nu_2}\right)\left(\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots\right),\text{for} s=3.$$

To calculate the value of $\acute{P}_{0,0}$, the normalization function, we have

$$\sum_{s=0}^{3}\sum_{j=0}^{\infty}P_{i,s}=1$$
$$\left(\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}\right)\left(1+\frac{\lambda\,(1+\omega)+j\varphi\delta_0}{j*\mu^2}+\frac{\nu_1}{\nu_2}\right)$$
$$\frac{+\lambda\,(1+\omega)}{\mu}\acute{P}_{1,j-1}=1$$
$$\acute{P}_{0,0}\left(\frac{b_2}{c_2}+\frac{b_3}{c_3}+\frac{b_5}{c_5}\right)\left(1+\frac{\lambda\,(1+\omega)+j\varphi\delta_0}{j*\mu^2}+\frac{\nu_1}{\nu_2}\right)$$
$$\frac{+\lambda\,(1+\omega)}{\mu}P_{1,j-1}=1$$
$$\acute{P}_{0,0}=\frac{1-\frac{\lambda(1+\omega)}{\mu}\acute{P}_{1,j-1}}{\left(\frac{b_2}{c_2}+\frac{b_3}{c_3}+\frac{b_5}{c_5}\right)\left(1+\frac{\lambda(1+\omega)+j'\delta_0}{j*\mu^2}+\frac{\nu_1}{\nu_2}\right)}$$

4. Validation of the model

(i) When the server is free:

$$\acute{P}_0 \quad = \textstyle\sum_{j=0}^{\infty}\acute{P}_{0,j}$$
$$=\frac{1-\frac{\lambda(1+\omega)}{\mu}\acute{P}_{1,j-1}}{\left(1+\frac{\lambda(1+\omega)+j'\delta_0}{j*\mu^2}+\frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2}+\frac{b_3}{c_3}+\frac{b_5}{c_5}\right)}$$
$$\left[\acute{P}_{0,1}+\acute{P}_{0,2}+\acute{P}_{0,3}+\cdots\right]$$

(ii) When the server is busy:

$$\acute{P}_1 \quad = \textstyle\sum_{j=0}^{\infty}\acute{P}_{1,j}$$

$$= \frac{1 - \frac{\lambda(1+\omega)}{\mu} \acute{P}_{1,j-1}}{\left(1 + \frac{\lambda(1+\omega)+j'\delta_0}{j*\mu^2} + \frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2} + \frac{b_3}{c_3} + \frac{b_5}{c_5}\right)}$$

$$\left(\frac{\lambda\,(1+\omega)+j'\delta_0}{j*\mu}\right) \times \left[\acute{P}_{0,1} + \acute{P}_{0,2} + \acute{P}_{0,3} + \cdots\right]$$

(iii) When the server is on vacation:

$$\acute{P}_2 \quad = \textstyle\sum_{j=0}^{\infty} P_{2,j}$$

$$= \frac{1 - \frac{\lambda(1+\omega)}{\mu} \acute{P}_{1,j-1}}{\left(1 + \frac{\lambda(1+\omega)+j'\delta_0}{j*\mu^2} + \frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2} + \frac{b_3}{c_3} + \frac{b_5}{c_5}\right)}$$

$$\times \left[(1-\Delta_0)\left(\frac{\lambda\,(1+\omega)+j'\delta_0}{j*\mu^2}\right)\right]$$

$$\left(\acute{P}_{0,1} + \acute{P}_{0,2} + \acute{P}_{0,3} + \cdots\right) + \frac{\lambda\,(1+\omega)\,\acute{P}_{1,j-1}}{\mu}$$

(iv) When the server is in a breakdown and repaired state:

$$\acute{P}_3 = \textstyle\sum_{j=0}^{\infty} P_{3,j}$$

$$= \frac{1 - \frac{\lambda(1+\omega)}{\mu} \acute{P}_{1,j-1}}{\left(1 + \frac{\lambda(1+\omega)+j'\delta_0}{j*\mu^2} + \frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2} + \frac{b_3}{c_3} + \frac{b_5}{c_5}\right)}\left(\frac{v_1}{v_2}\right)$$

$$\times \left(\acute{P}_{0,1} + \acute{P}_{0,2} + \acute{P}_{0,3} + \cdots\right)$$

5. Conclusion

The Markovian Encouraged Arrival Queuing Model has been developed with the inclusion of customer retry efforts, balking, and reneging behavior. The four system states idle state, busy state, vacation state, breakdown state, and repair state have all been taken into consideration utilizing the concept of encouraged arrival. We have examined and verified the possibilities of the various conditions. Neural networks, communication systems, post offices, and supermarkets can all benefit from using this model to reduce the reneging and balking behavior of their customers.

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