ANALYSIS OF A THREE-NODE SERIES QUEUE WITH ENCOURAGED ARRIVAL

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Abstract

This article deals with the three node series queues with encouraged arrival. We increase the expected number of subscribers by using encouraged arrival in this study. Performance metrics is developed by analytic method. After developing the governing-equations and utilizing the Burke's theorem, we resolve the steady-state probabilities and performance metrics of the three-node series queuing system. The study of learning series queues has received substantial interest in a variety of sectors, including manufacturing lines, computer systems, tollgates, telecommunications, and others. Researchers are becoming interested in the series queuing model because of its real-world application. A series queue is a line that runs through a chain of service stations, with subscribers always going along a single track from station to station studied a finite series queue and the view of approximate decomposition.

Keywords: Series queues, Burke's theorem, Encouraged arrival, Governing equations.

I. INTRODUCTION

This article deals with the three node series queues with encouraged arrival. A series queue is a line that runs through a chain of service stations, with subscribers always going along a single track from station to station studied a finite series queue and the view of approximate decomposition. A queue with the Poisson input and exponential service, and disclosed that the exit also Poisson with similar constraint as in input considered [4]. A double-node series queue with general arrivals and analyzed the performance metrics of a limited series queueing system analyzed [14]. An approximation method have predicted [3]. An admission control policy by learning a two-phase with no-space series queueing system analyzed in [5]. A limited series queue by utilizing the idea of critical paths discussed in [11]. Markovian modeling, they have attained the expected, variance of subscribers queue length and expected subscribers delay in the series queue analysed in [13]. They also governed the generating function for attaining the subscriber's queue-size distribution. A double-stage series retrial queueing model with a batch arrival process and attained results studied [10]. Using a fuzzy simulation results with the optimization of a series queue is analyzed in [1]. The server assigning problem without buffers series queueing system considered in [12]. In view of input flow Markovian and service is generalized distribution with multiple stages with heterogeneous subscribers in [9]. They derived performance metrics and also solved the condition of ergodicity. Utilizing matrix-analytic technique has analyzed a series queueing system with heterogeneous servers in [7]. They estimated obvious outcomes for performance indices has taken into attention a series queue with double heterogeneous service providers and also obtained a closed form solution in [6].The performance metrics of multi-service provider series queues in which input is Markovian and service is phase-type discussed in [2]. An M/M/1/N encouraged arrival quality control queueing system discussed in [15]. The research is systematized as in the following sections: Introduction in section 1.

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Model description is in section 2. Governing equations are in section 4. Burke's theorem in section 5. Section 6 contains Performance measures. Numerical examples are solved in section 7. Results are in section 8. Conclusions are concluded in section 9.

II. MODEL DESCRIPTION

A series queueing model with 3 service nodes is considered with a Poisson input λ the subscribers from external arrive to the node T1. After concluding the service at the nodeT1, the subscribers will go to the node T2 from the node T2 for getting service, in front of the node T3 they join queue. The subscribers leave the system only after the achievement of service at node T3. The queue assumed to be unlimited in capacity and at nodes T1, T2 and T3 service-time follows exponential distributions with parameters μa, μb and μc. It is distinguished that one subscriber can access service from each node at once.

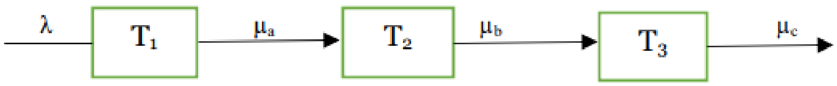

Figure 1: *Three - Phase Series Queue*

To estimate the steady-state probability P, K, L, M nodes of K subscribers at T1 and M subscribers respectively at nodes T2 and T3. Where K, L, M ≥ 0. Utilizing the state-transition figures the governing equations can be written (fig.2 and fig.3).

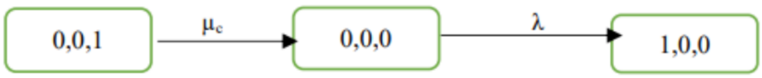

Figure 2.1: *represents that from state(0,0,1) only the (0,0,0)th state can be attained.*

Figure 2.2: *represent that only the (K,0,0)th state can be reached only from the states (K-1,0,0)th or (K,0,1)th and arrival to the (K,0,0)th state becomes the (K+1,0,0)th state and the departure (K,0,0)th state becomes (K-1,1,0)th state*

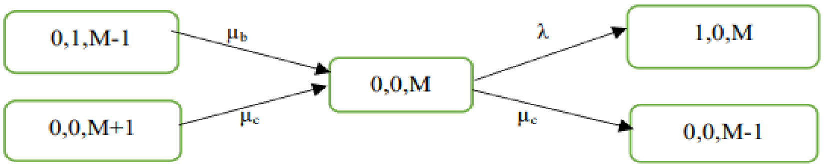

Figure 2.3: *Presents the (0,0,M)th state can be reached only from the states (0,1,M-1)th or (0,0,M+1)th state and arrival to the (0,0,M)th state becomes the (1,0,M)th state and the departure of (0,0,M)th state becomes (0,0,M-1)th state*

Figure 2.4: *Presents the $(0,L,0)^{th}$ state can be reached only from the states $(1,L-1,0)^{th}$ or $(0,L,1)^{th}$ and arrival to the $(0,L,0)^{th}$ state becomes the $(1,L,0)^{th}$ state and the departure of $(0,L,0)^{th}$ state becomes $(0,L-1,1)^{th}$ state*

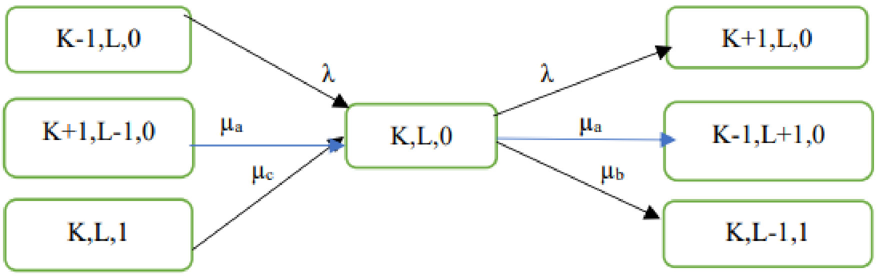

Figure 2.5: *Presents the $(K,L,0)^{th}$ state can be reached only from the states $(K-1,L,0)^{th}$ or $(K+1,L-1,0)^{th}$ or (K,L,1) and arrival to the $(K,L,0)^{th}$ state becomes the $(K+1,L,0)^{th}$ state and the departure of $(K,L,0)^{th}$ state becomes $(K-1,L+1,0)^{th}$ state or $(K,L-1,1)^{th}$ state*

Figure 2: *State Transition Diagram*

Figure 2 represents that from state (0,0,1) only the (0,0,0)th state can be attained.

The only state (1,0,0) into which the middle state (0,0,0) can traverse and the mid state cannot traverse to any other state expect (1,0,0)th state .

Similarly from (M-1,0,0) and (M,0,1) states, only the (M,0,0)th state can be reached.

The subscribers can move either to state (M+1,0,0)th or to (M-1,1,0) state from the (M,0,0)th state.

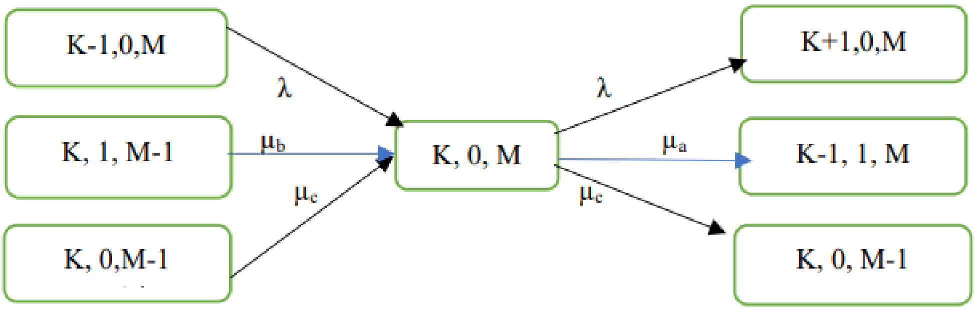

Figure 3.1: *Presents (K,0,M)th state can be reached only from the states (K-1,0,M)th or (K,1,M-1)th or (K,0,M-1) and arrival to the (K,0,M)th state becomes the (K+1,0,M)th state and the departure of (K,0,M)th state becomes (K-1,1,M)th state or (K,0,M-1)th state*

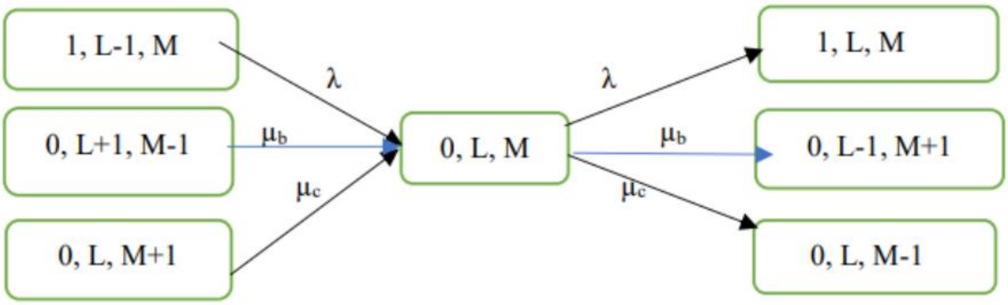

Figure 3.2:*Presents the (0,L,M)th state can be reached only from the states (1,L-1,M)th or (0,L+1,M-1)th or (0,L,M+1) and arrival to the (0,L,M)th state becomes the (1,L,M)th state and the departure of (0,L,M)th state becomes (0,L-1,M+1)th state or (0,L,M-1)th state .*

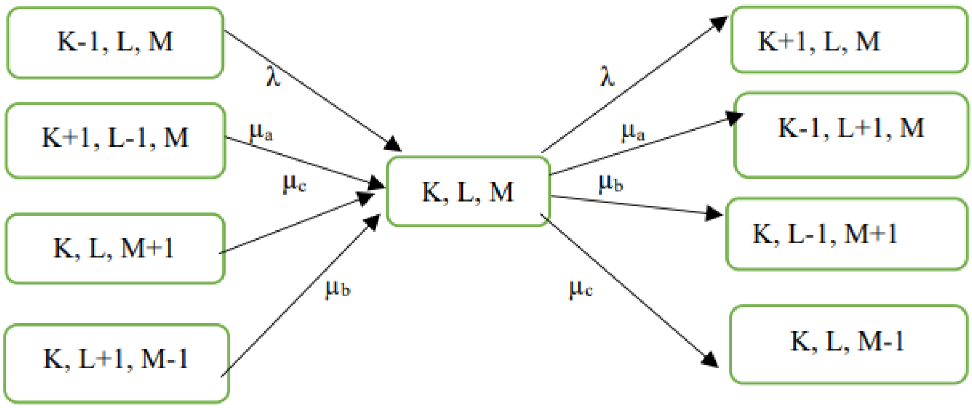

Figure 3.3: *Presents the (K,L,M)th state can be reached only from the states (K-1,L,M)th or (K+1,L-1,M)th or (K,L,M+1)th or (K,L+1,M-1)th and arrival to the (K,L,M)th state becomes the (K+1,L,M)th state and the departure of (K,L,M)th state becomes (K-1,L+1,M)th state or (K,L-1,M+1)th state or (K,L,M-1)th state.*

III. NOTATIONS

The notations are used to formulate the series queue with 3-nodes T1, T2, T3. Where $P_{K,L,M}$ denotes the steady-state probabilities that there are K number of subscribers at node T1, L-number of subscribers in T2 and M-subscribers in T3. States of the model are given by (K, L, M).

P_K − Probability that there are K subscribers at the node T_1

P_L − Probability that there are L subscribers at the node T_2

P_M − Probability that M subscribers at the node T_3

$P_{0,0,0}$ − Probability that no subscribers at any node T_1, T_2 and T_3.

$P_{K,0,0}$ − The probability that K subscribers at node T_1 and no subscribers at nodes T_2 and T_3.

$P_{0,L,0}$ − Probability that no subscribers at node T_1 and T_3, but L subscribers are at node T_2.

$P_{0,0,M}$ − Probability that no subscribers at node T_1 and T_2, but M subscribers are at node T_3.

$P_{K,L,0}$ − Probability that K subscribers at node T_1 and L subscribers at node T_2 no subscribers at node T_3.

$P_{K,0,M}$ - Probability K subscribers are at node T_1 and no subscribers at node T_2 and also M subscribers at node T_3.

$P_{0,L,M}$ − Probability that no subscribers at node T_1, but L subscribers at node T_2 and M subscribers at node T_3.

$P_{K,L,M}$ − Probability that k subscribers at node T_1, L subscribers at node T_2 and M subscribers at T_3.

IV. GOVERNING EQUATIONS

By utilizing the transition figure in figure 2.1, 2.2, 2.3, 2.4, 2.5 and figure 3.1, 3.2, 3.3 we developed the steady state equations as below;

$$\lambda(1+\gamma)P_{0,0,0} = \mu_c P_{0,0,1} \quad (1)$$

$$(\lambda(1+\gamma)+\mu_c)P_{0,0,M} = \lambda(1+\gamma)P_{0,1,M-1} + \mu_a P_{0,0,L+1} \quad (2)$$

$$(\lambda(1+\gamma)+\mu_a)P_{K,0,0} = \lambda(1+\gamma)P_{K-1,0,0} + \mu_c P_{K,0,1} \quad (3)$$

$$(\lambda(1+\gamma)+\mu_b)P_{0,L,0} = \mu_a P_{1,L-1,0} + \mu_c P_{0,L,1} \quad (4)$$

$$(\lambda(1+\gamma)+\mu_a+\mu_b)P_{K,L,0} = \lambda(1+\gamma)P_{K-1,L,0} + \mu_a P_{K+1,L-l,0} + \mu_c P_{K,L,1} \quad (5)$$

$$(\lambda(1+\gamma)+\mu_a+\mu_c)P_{k,0,m} = \lambda(1+\gamma)P_{K-1,0,M} + \mu_b P_{K,1,M-1} + \mu_c P_{K,0,M+1} \quad (6)$$

$$(\lambda(1+\gamma)+\mu_b+\mu_c)P_{0,l,m} = \mu_a P_{1,L-1,M} + \mu_b P_{0,L+1,M-1} + \mu_c P_{0,L,M+1} \quad (7)$$

$$(\lambda(1+\gamma)+\mu_a+\mu_b+\mu_c)P_{K,L,M} = \lambda(1+\gamma)P_{K-1,L,M} + \mu_a P_{K+1,L-1,M} + \mu_c P_{K,L,M+1} + \mu_b P_{K,L+1,M-1} \quad (8)$$

Moreover, the addition of entire probabilities must equal to one. i.e.,

$$\sum_K \sum_L \sum_M P_{K,L,M} = 1 \quad (9)$$

Burke's theorem is used in this instance to solve the governing equations mentioned above. The following is the theorem's statement:

V. BURKE'S THEOREM

Poisson arrival queue with single waiting queue with no departures, and exponentially distributed service times, the equilibrium distribution of the number of service accomplishments in a random time duration is equal as the arrival distribution, for every number of service providers.

To illustrate the outcome, Burke developed the supposition that the span of the expected inter-arrival $\frac{1}{\lambda(1+\gamma)}$, there are c service providers with exponentially distributed mean rate $\frac{1}{\mu}$ and $c\mu > \lambda(1+\gamma)$. By utilizing the conventions in its place of verifying the equilibrium distribution of the number of subscribers completing service between an arbitrary duration of span t follow poisson with rate $\lambda(1+\gamma)t$, Burke verified an equivalent output, that the time durations among successive service accomplishments are independent, exponentially distributed and it is identical as the inter-arrival times (visit [4]).

The process of entering (arriving) at T2, which is identical to the process of leaving T1

is also Poisson with parameter $\lambda(1+\gamma)$ from the theorem. In case of the service providers T_2 and T_3 same is detained. Hence, the queueing models are $M/M/1$ queueing system. so far $M/M/1$ queueing model is taken into account, now we govern the probability distribution for the three phase series queue by utilizing the Burke's theorem.

$P_K -$ Probability that k number of subscribers at the node T_1

$$= \left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^K \left[1 - \frac{\lambda(1+\gamma)}{\mu_a}\right] \tag{10}$$

$P_L -$ Probability that l number of subscribers at the node T_2

$$= \left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^L \left[1 - \frac{\lambda(1+\gamma)}{\mu_b}\right] \tag{11}$$

$P_M -$ Probability that m number of subscribers at the node T_3

$$= \left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^M \left[1 - \frac{\lambda(1+\gamma)}{\mu_c}\right] \tag{12}$$

At each node T_1, T_2, T_3 the number of subscribers are random variables which is independent, hence the probability of K amount of subscribers at node T_1 and L number of subscribers at node T_2 and m subscribers at node T_3 are jointly assumed by,

$$P_{K,L,M} = \left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^K \left[1 - \frac{\lambda(1+\gamma)}{\mu_a}\right] \left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^L \left[1 - \frac{\lambda(1+\gamma)}{\mu_b}\right] \left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^M \left[1 - \frac{\lambda(1+\gamma)}{\mu_c}\right] \tag{13}$$

where $K, L, M \geq 0$

VI. PERFORMANCE MEASURES

For analyzing the series queueing model performance, different performance measures are developed utilizing steady state probability distributions as-

(i) The expected number of subscribers in the system

$$L=\sum_{K} KP_K+\sum_{l} LP_L+\sum_{m} MP_M$$
$$=\sum_{K=0}^{\infty} K\left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^K\left[1-\frac{\lambda(1+\gamma)}{\mu_a}\right]+\sum_{L=0}^{\infty} L\left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^L\left[1-\frac{\lambda(1+\gamma)}{\mu_b}\right]$$
$$+\sum_{M=0}^{\infty} M\left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^M\left[1-\frac{\lambda(1+\gamma)}{\mu_c}\right]$$
$$=\frac{\lambda(1+\gamma)}{\mu_a-\lambda(1+\gamma)}+\frac{\lambda(1+\gamma)}{\mu_b-\lambda(1+\gamma)}+\frac{\lambda(1+\gamma)}{\mu_c-\lambda(1+\gamma)}$$

(ii) The expected waiting time of the subscribers in the system

$$W=\frac{1}{\lambda(1+\gamma)}L$$
$$=\frac{1}{\mu_a-\lambda(1+\gamma)}+\frac{1}{\mu_b-\lambda(1+\gamma)}+\frac{1}{\mu_c-\lambda(1+\gamma)}$$

(iii) The probability that the three service nodes are free

$$P_{0,0,0}=\left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_c}\right]$$
$$=\left[\frac{\mu_a-\lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\mu_b-\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\mu_c-\lambda(1+\gamma)}{\mu_c}\right]$$

(iv) The probability that K subscribers at node T_1 and no subscribers at T_2 and T_3

$$P_{K,0,0}=\left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^K\left[1-\frac{\lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_c}\right]$$
$$=\lambda(1+\gamma)^K\left[\frac{\mu_1-\lambda(1+\gamma)}{\mu_a^{K+1}}\right]\left[\frac{\mu_b-\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\mu_c-\lambda(1+\gamma)}{\mu_c}\right]$$

(v) The probability that no subscribers at nodes T_1 and T_3 and L number of subscribers at node T_2

$$P_{0,L,0}=\left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^L\left[1-\frac{\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_c}\right]$$
$$=\lambda(1+\gamma)^L\left[\frac{\mu_a-\lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\mu_b-\lambda(1+\gamma)}{\mu_b^{L+1}}\right]\left[\frac{\mu_c-\lambda(1+\gamma)}{\mu_3}\right]$$

(vi) The probability that zero subscribers at nodes T_1 and T_2 and M subscribers at node T_3

$$P_{0,0,M}=\left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^0\left[1-\frac{\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^M\left[1-\frac{\lambda(1+\gamma)}{\mu_c}\right]$$
$$=\lambda(1+\gamma)^M\left[\frac{\mu_a-\lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\mu_b-\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\mu_c-\lambda(1+\gamma)}{\mu_c^{M+1}}\right]$$

(vii) The Probability that K, M subscribers at nodes T_1 and T_2 respectively and no subscribers at node T_3

$$P_{K,0,M}=\lambda(1+\gamma)^{K+M}\left[\frac{\mu_a-\lambda(1+\gamma)}{\mu_a^{K+1}}\right]\left[\frac{\mu_b-\lambda(1+\gamma)}{\mu_b}\right]\left[\frac{\mu_c-\lambda(1+\gamma)}{\mu_c^{M+1}}\right]$$

(viii) The Probability that K, L, zero subscribers at nodes T, T_2T_3 respectively

$$P_{K,L,0}=\lambda(1+\gamma)^{K+l}\left[\frac{\mu_a-\lambda(1+\gamma)}{\mu_a^{K+1}}\right]\left[\frac{\mu_b-\lambda(1+\gamma)}{\mu_b^{L+1}}\right]\left[\frac{\mu_c-\lambda(1+\gamma)}{\mu_c}\right]$$

(ix) The Probability that 0, L, M subscribers at nodes T, T_2T_3 respectively

$$P_{0,L,M} = \lambda(1+\gamma)^{L+M}\left[\frac{\mu_a - \lambda(1+\gamma)}{\mu_a}\right]\left[\frac{\mu_b - \lambda(1+\gamma)}{\mu_b^{L+1}}\right]\left[\frac{\mu_c - \lambda(1+\gamma)}{\mu_c^{M+1}}\right]$$

(x) The Probability that K, L, M subscribers at nodes $T_1, T_2 T_3$ respectively

$$P_{K,L,M} = \lambda(1+\gamma)^{K+L+M}\left[\frac{\mu_a - \lambda(1+\gamma)}{\mu_a^{K+1}}\right]\left[\frac{\mu_b - \lambda(1+\gamma)}{\mu_b^{L+1}}\right]\left[\frac{\mu_c - \lambda(1+\gamma)}{\mu_c^{M+1}}\right]$$

(xi) At the nodes T_1, T_2 and T_3 the probability that the subscribers h,l is where h>K, and i>L and j>M is given by

$$P_{h>K,i>L,j>M} = \sum_{h=K+1}^{\infty} P_h \cdot \sum_{i=L+1}^{\infty} P_i \cdot \sum_{j=M+1}^{\infty} P_j$$
$$= \left[\frac{\lambda(1+\gamma)}{\mu_a}\right]^{K+1} \cdot \left[\frac{\lambda(1+\gamma)}{\mu_b}\right]^{L+1} \cdot \left[\frac{\lambda(1+\gamma)}{\mu_c}\right]^{M+1}$$
$$= \frac{\lambda(1+\gamma)^{k+l+m+3}}{\mu_a^{K+1} \cdot \mu_b^{L+1} \cdot \mu_c^{M+1}}$$

(xii) The expected number of subscribers in the queue

$$L_Q = \sum_{K=1}^{\infty} (K-1)P_K + \sum_{L=1}^{\infty} (i-1)P_i + \sum_{M=1}^{\infty} (M-1)P_M$$
$$= \frac{\lambda(1+\gamma)}{\mu_a} \cdot \frac{\lambda(1+\gamma)}{\mu_a - \lambda(1+\gamma)} + \frac{\lambda(1+\gamma)}{\mu_b} \cdot \frac{\lambda(1+\gamma)}{\mu_b - \lambda(1+\gamma)} + \frac{\lambda(1+\gamma)}{\mu_c} \cdot \frac{\lambda(1+\gamma)}{\mu_c - \lambda(1+\gamma)}$$
$$= (\lambda(1+\gamma))^2\left[\frac{1}{\mu_a(\mu_b - \lambda(1+\gamma))} + \frac{1}{\mu_b(\mu_b - \lambda(1+\gamma))} + \frac{1}{\mu_c(\mu_c - \lambda(1+\gamma))}\right]$$

(xiii) The expected waiting time of a subscribers in the queue

$$W_Q = \frac{L_q}{\lambda(1+\gamma)}$$
$$W_Q = \lambda(1+\gamma)\left[\frac{1}{\mu_a(\mu_a - \lambda(1+\gamma))} + \frac{1}{\mu_b(\mu_b - \lambda(1+\gamma))} + \frac{1}{\mu_c(\mu_c - \lambda(1+\gamma))}\right]$$

VII. NUMERICAL EXAMPLES

In this section, to formulate the series queue with 3-nodes T1, T2, T3. The steady-state probabilities that there are K number of subscribers at node T1, L-number of subscribers in T2 and M-subscribers in T3. States of the model are given by (K, L, M).The performance of the queuing system is examined numerically in relation to the parameters $\lambda(1+\gamma)$. Where γ =10% and 20%. Since Little's law values for the chosen parameter values are displayed in the table as a distinct column

Table1: *Represents the value of L and W when γ=0.0*

Λ	L	W	L =λ(1 + γ)W
1	0.869	0.869	0.869
1.1	0.9849	0.8954	0.9849
1.2	1.1080	0.9233	1.1080
1.3	1.2391	0.9531	1.2391
1.4	1.3790	0.9850	1.3790
1.5	1.5286	1.0190	1.5286

Table2: *Represents the value of L and W when γ=0.1*

Λ	L	W	L=λ(1 + γ)W
1	0.9849	0.8954	0.9849
1.1	1.1207	0.9262	1.1207
1.2	1.2663	0.9593	1.2663
1.3	1.4228	0.9950	1.4228
1.4	1.5914	1.0334	1.5914
1.5	1.7736	1.0745	1.7736

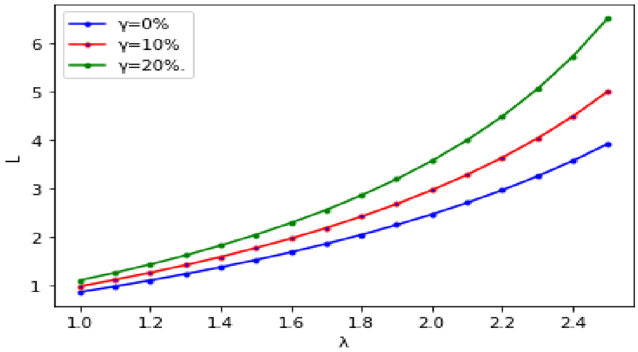

Figure 4: *plot against L and λ*

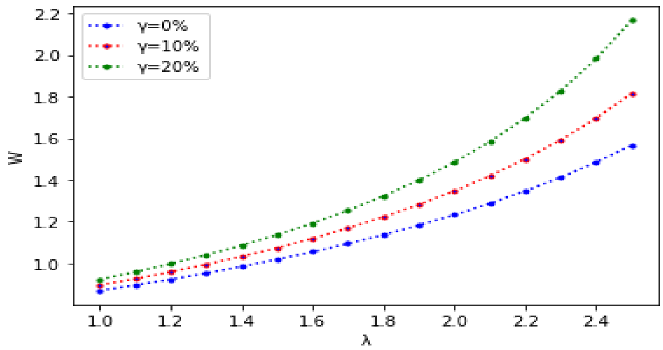

Figure5: *plot against W and λ*

VIII. RESULTS AND DISCUSSIONS

In table 1, the values of L and W increases when we increase the arrival rate λ with γ =0. Table 2, the values of L and W increases when we increase the arrival rate λ with γ =0.1. Table 3, the system of L and W increases when we increase the arrival rate λ with γ =0.2. Figure 4, When γ =0.2 (20%) we got maximum value of L than in table 1 and table 2. Figure 5, the results of W for different values of γ=0 to 0.2.

IX. CONCLUSIONS

In terms of operations management, planning, outlining, and execution, this study is quite helpful. Developing services for customers, and other areas. We increased the expected number of subscribers in the system. We developed the performance metrics by applying Burke's theorem. We enlarged the system size by using encouraged arrival discounts. Increasing the service subscribers will surely improve the company's revenue. We can further extend this article by increasing the number of servers and adding optimal cost to study the behavior of the series queues.

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