DOUBLE SAMPLING INSPECTION PLAN UNDER ZERO-ONE FAILURE SCHEME FOR GENERALIZED INVERTED EXPONENTIAL DISTRIBUTION

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Abstract

This article presents a double acceptance sampling plan for products whose lifetimes follow a generalized inverted exponential distribution. The plan uses a zero-one failure scheme, where a lot is accepted if there are no failures observed in the first sample, and it is rejected if more than one failure occurs. In cases where there is only one failure from the first sample, a second sample is drawn and tested for the same duration as the first sample. To ensure that the true median lifetime is longer than the specified lifetime at a given consumer's confidence level, the minimum sample sizes of the first and second samples are determined. The operating characteristics of the plan are analyzed for various ratios of the true median lifetime to the specified lifetime. Finally, an example is given to explain the results. The example shows how the double acceptance sampling plan can be used to determine the sample size and acceptance criteria for a product with a specified lifetime and a given consumer's confidence level. The results of the example demonstrate the effectiveness of the plan in ensuring that the true median lifetime of the product is longer than the specified lifetime at the desired level of confidence.

Keywords: Generalized inverted exponential distribution($GIED$), Double acceptance sampling plan, Operating Characteristics ($O.C.$), Producer's risk, Consumer's risk

1. Introduction

Statistical quality control involves the study of the quality of manufactured items, which is often characterized by their lifetimes. However, items produced under identical conditions may have variable lifetimes due to chance causes. To avoid issues with consumer acceptance, producers prioritize the quality of their products. However, it may not be feasible to inspect the entire lifetime of all products due to destruction during inspection and time/cost limitations. As such, a sampling inspection plan is recommended. However, both producers and consumers face risks when making acceptance/rejection decisions based on samples. If a sample from a lot is accepted, and several products have a mean or median lifetime less than the specified lifetime (a "bad" lot), this is known as consumer's risk. On the other hand, the probability that a "good" lot of products is rejected is known as the producer's risk. Increasing the sample size can minimize these risks, but it can also lead to higher costs and longer inspection times for producers. To minimize risks while considering time and cost constraints, an efficient acceptance sampling scheme should be used. Therefore, implementing a sampling inspection plan is recommended in this process, as both the producer and consumer are exposed to risks when deciding whether to accept or reject products. If a sample from a lot is accepted, and several products have a mean or median lifetime less than the specified lifetime (known as a "bad" lot), this represents consumer's risk. The probability of rejecting a "good" lot of products is known as the producer's risk. Increasing the sample size can be an effective way to minimize the risks faced by both the producer and consumer, but this approach may incur higher costs and longer inspection times for the producer. To minimize these risks while adhering to time and cost constraints, an efficient acceptance sampling scheme should be utilized.

A statistical method for product control known as acceptance sampling or sampling inspection as an alternative to 100% complete inspection was developed by [1]. An acceptance sampling plan for exponential distribution was the first time considered by [2]. The problem of acceptance sampling plans when the life test is truncated at a pre-assigned time under the gamma distribution by taking mean lifetime as the average lifetime of the product studied in detail by [3]. The acceptance sampling plans at various confidence levels for various values of the ratio of the prefixed experimental time to the specified mean lifetime under inverse Rayleigh distribution investigated by [4]. To use median lifetime as the average lifetime of the product for acceptance sampling plans under the generalized Birnbaum-Saunders distribution proposed by [7]. Truncated life tests for log-logistic distribution at various confidence levels for different values of the ratio of the pre-fixed experimental time to the specified average lifetime considered by [5]. Time truncated acceptance sampling plans for generalized exponential distribution by taking the median lifetime of the product as the preferred average lifetime of the product developed by [9]. An acceptance sampling plans based on truncated life tests in the Pareto distribution of the second kind provided by [6]. A double acceptance sampling plans based on truncated life tests for Marshall Olkin extended exponential distribution considered by [10], he also calculate the consumer's risk for the various choices of prefixed producer's risk. An acceptance sampling plan based on truncated life tests for exponentiated Frechet distribution developed by [11]. A comparison study of the three sampling plans namely single acceptance sampling plan, double acceptance sampling plan and group acceptance sampling plan provided by [14]. A new attribute sampling inspection plan and its applications has been discussed by [23]. An acceptance sampling plan based on truncated life tests for generalized inverse weibull distribution by assumming mean lifetime as the preferred average lifetime of the product developed [16]. The time truncated double acceptance sampling inspection plan for Maxwell distribution considered by [12]. A double acceptance sampling inspection plan for time truncated life tests based on the transmuted generalized inverse Weibull distribution provided by [17]. A double acceptance sampling inspection plan for time truncated life tests based on the transmuted new Weibull-Pareto distribution by taking median lifetime as the average lifetime of the product considered by [15]. An acceptance sampling plan based on the time truncated life tests for Sushila distribution by assumming mean lifetime as the average life of the product and also illustrate an example by taking real data application considered [18]. A single and double acceptance sampling plans for truncated life tests based on transmuted Rayleigh distribution by taking different values of the parameters developed by [21]. A double and group acceptance sampling plan for truncated life test based on inverse log-logistic distribution by taking mean lifetime as the average lifetime of the product compared by [22]. An improved acceptance sampling plans based on truncated life tests for Garima distribution developed by [19].

GIED is a very useful lifetime distribution given by [8] and it is also found to be very useful in the reliability testing. This distribution has non- constant hazard rate function, which is also unimodal and right positively skewed distribution.

The objective of this paper is to propose a double acceptance sampling plan for life tests assuming that the product's lifetime follows a GIED. The zero-one failure scheme is primarily considered, as it is generally more effective than the ordinary sampling scheme. Under this scheme, a lot is accepted if no failures are observed from the first sample, and it is rejected if there are more than one failures. If there is exactly one failure, a second sample is selected and tested for the same duration as the first sample. The zero-one failure scheme is particularly useful in situations where the cost of the product is high, and it is not feasible to bear the high cost of inspection. The minimum sample sizes for the first and second samples are determined at a specified consumer's confidence level. The operating characteristics are analyzed as a function of the ratio of the time to the specified median lifetime, and the minimum of such ratios is obtained to minimize the producer's risk at the specified level.

The model (GIED) is introduced in Section 2. The methodology used in double sampling plan is given in Section 3 and its operating characteristics(OC) function, Producer's risk and Consumer's risk are analyzed in Section 4, Section 4.1 and Section 4.2 respectievely. An example to explain the double sampling plan based on zero-one failure scheme are given in Section 5.

2. The Model

GIED is the generalization of the one parameter IED which was proposed by [8]. The various statistical properties and reliability estimation has also been studied in detail by [8].

A two parameter GIED has the following PDF and CDF as follows:

$$f(x) = \frac{\alpha\lambda}{x^2}\exp\left(\frac{-\lambda}{x}\right)\left(1-\exp\left(\frac{-\lambda}{x}\right)\right)^{\alpha-1}; \qquad x>0, \alpha>0, \lambda>0$$
and,
$$F(x) = 1-(1-exp\,(-\lambda/x))^{\alpha}. \tag{1}$$

Where, α is the shape parameter and λ is the scale parameter.
Let t_p be the p^{th} percentile of the GIED then it is given by

$$p = 1-\left(1-exp\left(-\frac{\lambda}{t_p}\right)\right)^{\alpha} \tag{2}$$

This implies that

$$t_p = \frac{-\lambda}{\ln(1-(1-p)^{1/\alpha})}$$

and median of the distribution is given by

$$m_d = \frac{-\lambda}{\ln(1-(0.5)^{1/\alpha})}. \tag{3}$$

For eliminating λ, we can write

$$\frac{t_p}{m_d} = \frac{ln(1-(0.5)^{1/\alpha})}{ln(1-(1-p)^{\frac{1}{\alpha}})},$$

$$p = 1-(1-\ exp(A))^{\alpha}. \tag{4}$$

Where,

$$A = \frac{ln(1-(0.5)^{1/\alpha})}{\frac{t_p}{m_d}}. \tag{5}$$

3. Methodology

Let us consider the median lifetime of a product as m_d, which is used to determine the quality of a submitted lot. A lot is considered to be of acceptable quality (i.e., good) if the null hypothesis $H_0 : m_d \geq m_d^0$ is supported by the data against the alternative hypothesis $H_1 : m_d < m_d^0$, where m_d^0 is a specified lifetime. The consumer's risk is used as the significance level for the test, denoted by $1-P^{\star}$, where $P^{\star}$ is the consumer's confidence level. To implement the double sampling plan, the following method is used: the first sample is drawn from the lot, and if no failures are observed in this sample, the lot is accepted. If more than one failure occurs, the lot is rejected. When only one failure occurs in the first sample, a second sample is drawn and tested for the same duration as the first sample. The minimum sample sizes of the first and second samples are determined to ensure that the true median lifetime is longer than the specified lifetime at the specified consumer's confidence level. The method is designed to minimize the producer's risk while keeping the time and cost of inspection within acceptable limits. Assume that the lifetime of a products can be represented by its median lifetime, denoted by m_d. We say that the submitted lot has a good quality (lot of acceptable quality) and the lot will be accepted if the data supports the following null hypothesis, $H_0 : m_d \geq m_d^0$ (m_d^0 is a specified lifetime) against the alternative hypothesis, $H_1 : m_d < m_d^0$. As the significance level for the test, the consumer's risk is used through $1 - P^\star$, where $P^\star$ is the consumer's confidence level. The method of double sampling plan is given as follows:

- To begin the inspection process, a random sample of size n_1 is drawn from the lot and put to the test. If the number of failures observed before a pre-fixed experiment time t is equal to or less than c_1, the lot is accepted. However, if the number of failures observed is greater than c_2, the experiment is terminated before t, and the lot is rejected. Here, it is important to note that c_1 must be less than c_2.
- A double sampling plan is used to make a decision about whether to accept or reject a lot based on two samples that have been inspected. If the lot has not been rejected or accepted, which means that the number of failures by time t is between $c_1 + 1$ and c_2, a second sample of size n_2 is drawn and tested during time t. The lot is deemed acceptable if no more than c_2 failures are observed from both samples. On the other hand, if there are more than c_2 failures, the lot is rejected. A sampling plan in which a decision about the acceptance or rejection of a lot is based on two samples that have been inspected is known as a double sampling plan.

Double sampling is utilized when a definite verdict regarding the approval or disapproval of a batch cannot be made based on a single sample. Typically, in a double sampling plan, the approval or disapproval of a batch is determined by evaluating two samples. If the initial sample is satisfactory, the batch is approved without requiring a second sample. Conversely, if the initial sample is unsatisfactory, the batch is rejected without necessitating a second sample. However, if the initial sample is inconclusive and there are doubts about its outcome, a second sample is taken and the approval or disapproval decision is made based on the evidence obtained from both samples.

Assuming that N lots of equal size are received from either the supplier or the final assembly line and examined individually, the acceptance probability can be calculated using the binomial distribution since the lot size is sufficiently large. The double acceptance sampling plan, which is based on a zero-one failure scheme, determines whether the lot is accepted or rejected.

$$P_a = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \left[\sum_{j=0}^{c_2-i} \binom{n_2}{j} p^j (1-p)^{n_2-j} \right]. \tag{6}$$

Where, p is given in (4)

In the context of the zero-one failure scheme for double acceptance sampling, we are specifically concerned with the case where $c_1 = 0$ and $c_2 = 1$. In this acceptance sampling plan, the consumer prefers a smaller number of accepted lots. The acceptance probability of the lot in the zero-one failure scheme can be expressed as follows:

$$P_a = (1-p)^{n_1} + n_1 p (1-p)^{n_1-1} (1-p)^{n_2}. \tag{7}$$

Therefore, the minimum required sample sizes n_1 and n_2 ensuring $m_d \geq m_d^0$ at the consumer confidence level P^* can be found as follows:

$$(1-p)^{n_1} + n_1 p (1-p)^{n_1-1} (1-p)^{n_2} \leq 1 - P^*. \tag{8}$$

There will be infinite many solutions satisfying Eq(8). Ultimately our goal is to minimize the average sample number (ASN) under the constraint that $n_2 \leq n_1$.

The ASN for double sampling plan based on zero-one failure scheme is given by

$$ASN = n_1 P_1 + (n_1 + n_2)(1 - P_1).$$

Where, P_1 is probability of acceptance or rejection of the lot based on the first sample is given by

$P_1 = 1 - \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i}.$

Table 1: *Minimum sample size required for the double acceptance sampling plan with shape parameter $\alpha = 1$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.33	(5,3)	(7,5)	(8,8)	(12,9)
0.942	0.479	(3,2)	(4,4)	(5,5)	(8,4)
1.257	0.576	(3,1)	(3,3)	(4,3)	(6,3)
1.571	0.643	(2,2)	(3,2)	(4,2)	(5,3)
2.356	0.745	(2,1)	(2,2)	(3,2)	(4,2)
3.142	0.802	(2,1)	(2,2)	(3,1)	(3,3)

Table 2: *Minimum sample size required for the double acceptance sampling plan with shape parameter $\alpha = 2$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.2630	(6,5)	(9,6)	(11,8)	(16,10)
0.942	0.4694	(3,3)	(5,2)	(5,5)	(8,4)
1.257	0.6112	(2,2)	(3,3)	(4,2)	(5,5)
1.571	0.7059	(2,1)	(3,1)	(3,2)	(4,3)
2.356	0.8350	(2,1)	(2,1)	(2,2)	(3,2)
3.142	0.8954	(1,1)	(2,1)	(2,1)	(3,1)

Table 3: *Minimum sample size required for the double acceptance sampling plan with shape parameter $\alpha = 4$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.1976	(8,7)	(12,10)	(15,11)	(22,14)
0.942	0.4583	(3,3)	(5,3)	(6,3)	(8,5)
1.257	0.6515	(2,2)	(3,2)	(4,2)	(5,3)
1.571	0.7738	(2,1)	(2,2)	(3,1)	(4,2)
2.356	0.9139	(1,1)	(2,1)	(2,1)	(2,2)
3.142	0.9615	(1,1)	(1,1)	(2,1)	(2,1)

Table 4: *Minimum sample size required for the double acceptance sampling plan with shape parameter $\alpha = 6$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.1638	(10,8)	(15,11)	(18,15)	(27,18)
0.942	0.4513	(3,3)	(5,3)	(6,3)	(8,6)
1.257	0.6769	(2,1)	(3,2)	(3,3)	(5,2)
1.571	0.8134	(2,1)	(2,1)	(2,2)	(3,2)
2.356	0.9487	(1,1)	(1,1)	(2,1)	(2,1)
3.142	0.9832	(1,1)	(1,1)	(1,1)	(2,1)

RT&A, No 4(80)
Volume 19, December, 2024

4. OC function

The operating characteristic function of the sampling plan provides the probability of accepting the lot of incoming lot quality p. For the above acceptance sampling plan this probability is given by

$$\begin{aligned} OC(p) &= Prob(\text{accepting the lot of the incoming lot quality}) \\ &= (1-p)^{n_1} + n_1 p(1-p)^{n_1-1}(1-p)^{n_2}. \end{aligned} \tag{9}$$

Table 5: *OC function of the incoming lot quality with shape parameter $\alpha = 1$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.33	0.2350	0.0888	0.0471	0.0095
0.942	0.479	0.2473	0.0936	0.0452	0.0084
1.257	0.576	0.2079	0.0999	0.0457	0.0094
1.571	0.643	0.1860	0.0768	0.0312	0.0082
2.356	0.745	0.1619	0.0897	0.0260	0.0074
3.142	0.802	0.1021	0.0517	0.0264	0.0085

Table 6: *O.C. function of the incoming lot quality with shape parameter $\alpha = 2$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.2630	0.2348	0.0972	0.0467	0.0096
0.942	0.4694	0.2086	0.0944	0.0499	0.0098
1.257	0.6112	0.2230	0.0750	0.0446	0.0095
1.571	0.7059	0.2087	0.0793	0.0413	0.0093
2.356	0.8350	0.0727	0.0727	0.0347	0.0063
3.142	0.8953	0.1983	0.0306	0.0306	0.0042

Table 7: *O.C. function of the incoming lot quality with shape parameter $\alpha = 4$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.1976	0.2443	0.0945	0.0489	0.0098
0.942	0.4583	0.2231	0.0780	0.0457	0.0098
1.257	0.6515	0.1766	0.0711	0.0281	0.0072
1.571	0.7738	0.1304	0.0691	0.0385	0.0045
2.356	0.9139	0.1648	0.0210	0.0210	0.0086
3.142	0.9615	0.0755	0.0755	0.0043	0.0043

Table 8: *O.C. function of the incoming lot quality with shape parameter $\alpha = 6$*

t_p/m_d	p	P^*			
		0.75	0.90	0.95	0.99
0.628	0.1638	0.2455	0.0965	0.0496	0.0097
0.942	0.4513	0.2326	0.0835	0.0495	0.0097
1.257	0.6769	0.2458	0.0559	0.0409	0.0074
1.571	0.8134	0.0914	0.0914	0.0454	0.0094
2.356	0.9487	0.0999	0.0999	0.0076	0.0076
3.142	0.9832	0.0333	0.0333	0.0333	0.00083

4.1. Producer's risk

The probability of rejecting a lot of acceptance quality level (AQL) p_1 is called producer's risk. It is given for the double sampling inspection plan based on zero-one failure scheme as follows:

$$\begin{aligned} PR(p) &= Prob(\text{ rejecting a lot}) \\ &= 1 - Prob(\text{accepting a lot of the quality } p_1) \\ &= 1 - (1-p_1)^{n_1} - n_1 p_1 (1-p_1)^{n_1-1}(1-p_1)^{n_2}. \end{aligned} \tag{10}$$

Table 9: *Producer's risk of the ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 1$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.1704	0.3020	0.4048	0.5717
0.10	0.20	0.942	0.0742	0.1526	0.2158	0.3185
0.10	0.25	1.257	0.0523	0.0939	0.1313	0.2103
0.05	0.15	1.571	0.0118	0.0204	0.0324	0.0516
0.05	0.20	2.356	0.0073	0.0118	0.0204	0.0307
0.05	0.25	3.142	0.0073	0.0118	0.0140	0.0266

Table 10: *Producer's risk of the ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 2$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.2594	0.4067	0.5211	0.6998
0.10	0.20	0.942	0.0939	0.1438	0.2158	0.3185
0.10	0.25	1.257	0.0442	0.0939	0.1077	0.2158
0.05	0.15	1.571	0.0073	0.0140	0.0204	0.0385
0.05	0.20	2.356	0.0073	0.0073	0.0118	0.0204
0.05	0.25	3.142	0.0025	0.0073	0.0073	0.0140

Table 11: *Producer's risk of the ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 4$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.3865	0.5863	0.6864	0.8465
0.10	0.20	0.942	0.0939	0.1704	0.2103	0.3436
0.10	0.25	1.257	0.0442	0.0742	0.1077	0.1704
0.05	0.15	1.571	0.0073	0.0118	0.0140	0.03074
0.05	0.20	2.356	0.0025	0.0073	0.0073	0.0118
0.05	0.25	3.142	0.0025	0.0025	0.0073	0.0073

Table 12: *Producer's risk of the ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 6$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.4845	0.6864	0.7881	0.9157
0.10	0.20	0.942	0.0939	0.1704	0.2103	0.3662
0.10	0.25	1.257	0.0280	0.0742	0.0939	0.1438
0.05	0.15	1.571	0.0073	0.0073	0.0118	0.0204
0.05	0.20	2.356	0.0025	0.0025	0.0073	0.0073
0.05	0.25	3.142	0.0025	0.0025	0.0025	0.0073

4.2. Consumer's risk

The probability of accepting the lot of the rejecting quality level (RQL) p_2 is called consumer's risk. The consumer's risk for the double acceptance sampling plan based on zero-one failure scheme is given by

$$c(p) = (1-p_2)^{n_1} + n_1 p_2 (1-p_2)^{n_1-1}(1-p_2)^{n_2}. \tag{11}$$

Where, p be the percentage defective in the lot that is lot quality(p), p_1 be the acceptance quality level of lot quality p, p_2 be the rejecting quality level of lot quality p.
Also, P_1 be the probability of accepting or rejecting the lot on the basis of the first sample and similar argument can be given for P_2.

Table 13: *Consumer's risk of ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 1$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.6610	0.4696	0.3773	0.2120
0.10	0.20	0.942	0.2103	0.5774	0.4619	0.3052
0.10	0.25	1.257	0.7207	0.5998	0.4944	0.3281
0.05	0.15	1.571	0.9067	0.8490	0.7838	0.6841
0.05	0.20	2.356	0.8960	0.8448	0.7578	0.6717
0.05	0.25	3.142	0.8906	0.7734	0.7207	0.5999

Table 14: *Consumer's risk of ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 2$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.5543	0.37034	0.2559	0.1155
0.10	0.20	0.942	0.7086	0.5898	0.4619	0.3052
0.10	0.25	1.257	0.7734	0.5999	0.5537	0.3312
0.05	0.15	1.571	0.9393	0.8905	0.8490	0.7483
0.05	0.20	2.356	0.896	0.896	0.8448	0.7578
0.05	0.25	3.142	0.9375	0.8437	0.8437	0.7383

Table 15: *Consumer's risk of ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 4$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.3958	0.2015	0.1260	0.0392
0.10	0.20	0.942	0.7086	0.5374	0.4635	0.2777
0.10	0.25	1.257	0.7734	0.6592	0.5537	0.4042
0.05	0.15	1.571	0.9393	0.9067	0.8905	0.7882
0.05	0.20	2.356	0.96	0.896	0.896	0.8448
0.05	0.25	3.142	0.9375	0.9375	0.8437	0.8437

Table 16: *Consumer's risk of ($n_1, n_2,\ t_p/m_d$) with shape parameter $\alpha = 6$*

AQL(p_1)	RQL(p_2)	t_p/m_d	p^*			
			0.75	0.90	0.95	0.99
0.10	0.15	0.628	0.2915	0.1260	0.0685	0.0156
0.10	0.20	0.942	0.7086	0.5374	0.4635	0.2557
0.10	0.25	1.257	0.8437	0.6592	0.5999	0.4598
0.05	0.15	1.571	0.9393	0.9393	0.9067	0.8490
0.05	0.20	2.356	0.9600	0.9600	0.8960	0.8960
0.05	0.25	3.142	0.9375	0.9375	0.9375	0.8437

5. Description of tables with examples

The choices of $t_p/m_d = 0.628, 0.942, 1.257, 1.571, 2.356, 3.142$ are consistent with [3], [5], [7].
Table 1 indicates that with a constant termination ratio (t_p/m_d), fixed proportion defective (p), and a consistent value of the shape parameter, there exists a clear relationship between the consumer's confidence level and the sample size; as the confidence level increases, the sample size also increases. This identical pattern holds true from Table 2 to Table 4, considering diverse values of the shape parameter α. Table 5 demonstrates that under a constant termination ratio (t_p/m_d), fixed proportion defective (p), and a given shape parameter value, an increase in the consumer's confidence level leads to a decrease in the Operating Characteristic (O.C.) function value. This consistent trend is also evident from Table 6 to Table 8 for varying values of the shape parameter α. Table 9 shows that under a constant termination ratio (t_p/m_d), shape parameter, and given AQL (p_1) and RQL (p_2) values, an increase in the consumer's confidence level leads to a reduction in the Producer's risk. This trend is consistent across Table 10 to Table 12 when considering different values of the shape parameter α.
Table 13 demonstrates how, with a constant termination ratio (t_p/m_d) as well as fixed AQL (p_1) and RQL (p_2) values, an increase in Consumer's confidence level leads to a decrease in Consumer's risk. This trend is consistent across Table 14 to Table 16 for different values of the shape parameter α.
Example:- The first table determines the minimum sample size required from a lot in order to make a decision about accepting or rejecting it. For instance, in analyzing the reliability of laptops using a double acceptance sampling plan under the zero-one failure scheme, the failure times in days (denoted by T) are used to represent the lifetimes of the laptops from the start of their operation. It is assumed that T follows a generalized inverted exponential distribution.

Assuming a specified median life of 1000 days and a testing time of 942 days, the manufacturer would like to determine if the median life of the laptops is at least 1000 days with a confidence level of 0.99. The experiment will end after 942 days using the zero-one failure scheme of the double sampling plan, which refers to a ratio of 0.942. From the information provided in Table 1, the minimum sample sizes required are $n_1 = 8$ and $n_2 = 4$. To start, eight items are monitored for 942 days, and the lot is accepted if there are no failures during the experiment. If more than one failure occurs, the lot is rejected. Next, a second sample of four items is selected from the lot and tested for 942 days, during which only one failure occurred.

The laptop has a specified median life of 1000 days, and the manufacturer wants to determine whether the median life is at least 1000 days with 99% confidence. The experiment will use a double sampling plan with a zero-one failure scheme, ending at 942 days. The ratio of testing time to specified median life is 0.942. According to Table 1, the minimum required sample sizes are $n_1 = 8$ and $n_2 = 4$. To conduct the experiment, the first step is to monitor eight items for 942 days. If no failures occur during this time, the lot is accepted. If there are one or more failures, the lot is rejected. If the lot is accepted, a second sample of size four is selected from the lot and tested for 942 days. If only one failure occurs during this time, the manufacturer can conclude with 99% confidence that the median life of the laptop is at least 1000 days.

6. Conclusion

This article presents a novel double acceptance sampling plan for the zero-one failure scheme based on the median life of products with a fixed tenure life. The plan is developed for the GIED and assumes that the life test is truncated with the end of the product's lifetime. The goal of the plan is to ensure that the product has a better life than the specified one, and it considers the percentile ratio of lifetimes to determine the design parameters of the double sampling plan, namely, $(n_1, n_2,\ t_p/m_d)$. The study shows that the zero-one failure scheme is superior to the ordinary sampling plan in terms of the required sample size for inspection. This work can be extended for various lifetime models under different sampling plan.

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