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QUADRASOPHIC FUZZY MATRIX AND ITS APPLICATION

G. Aruna[1], J. Jesintha Rosline[2]

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[1,2] PG and Research Department of Mathematics, Auxilium College (Autonomous) ,
Affiliated to Thiruvalluvar University, Vellore Dist., Tamil Nadu, India
anu9117@gmail.com[1] , jesi.simple@gmail.com[2]

Abstract

Quadrasophic Fuzzy Set is one of the generalizations of Fuzzy set theory. In this artifact, a definition of the Quadrasophic Fuzzy Algebra and its characteristics are provided. The definition of a Quadrasophic Fuzzy Matrix is explored with the aid of Quadrasophic Fuzzy Algebra. The binary operators of Fuzzy Matrices are used to describe various kinds and specific operations on Quadrasophic Fuzzy Matrices. The theorems and results of Quadrasophic Fuzzy Matrix are demonstrated with pertinent examples and proofs. Additionally, the illustration of the identification of paddy illnesses is analyzed with the tool of Quadrasophic Fuzzy Matrix in the decision-making process.

Keywords: Quadrasophic Fuzzy Set, Quadrasophic Fuzzy Algebra, Quadrasophic Fuzzy Matrix, Operations, Plant illness identification

1. Introduction

The ability to recognize both the advantages and disadvantages of a problem is crucial in problem-solving. In recent years, we have considered other elements while making judgements, like the environment and peer pressure. The bipolar fuzzy set addresses both the problems in positive and negative aspects. However, the environment's impact on the problem is crucial to manage the unclear scenario. The current fuzzy set extension is insufficient to account for the environment's influence or the rate at which the issue is becoming more prevalent. With the unique characteristics of positive reluctant membership and negative reluctant membership functions, QFS facilitates solving such issues.

One of the generalizations of fuzzy set theory is the Quadrasophic Fuzzy Set (QFS) [1]. Many industries, including medicine, business, decision-making, agriculture, etc., rely heavily on QFS. Any sort of work has both a positive and a negative aspect.These aspects might change due to influential variables, producing different kinds of outcomes. QFS considers the impact of influential factors as the reluctant positive side, i.e., the partial positive side, and the reluctant negative side [1]. Let's take an example where someone is driving from City A to City B. Positive membership indicates that the traveller has reached City B; negative membership indicates that there has been a delay in reaching the destination because of traffic or road construction; these variations in positive and negative membership values are therefore regarded as positive and negative reluctant memberships. The QFS quantifies the value of positive membership in the interval [0,1], negative membership in the interval [-1,0], negative reluctant ranges in [-0.5,0], and postive reluctant ranges in [0,0.5] [2].

In numerous distinct domains, matrices are significant. The traditional matrix is insufficient to cover unpredictable circumstances. The ambiguous situations make it possible to create the idea of a fuzzy matrix to depict the actual situation. Thomason discussed the convergence of power in a fuzzy matrix in 1977 [3]. Fuzzy matrices were first conceptualized by Hashimoto. Several authors established and validated numerous results, including determinant, trace, Nil potency, adjoint matrices, adjacent matrices, Idem potency, features of transitive and closure, and power series convergence in fuzzy matrices as well as in their different expansions [4], [5], [6].

Cho conducted research on the regularity characteristics and rankings of fuzzy matrices in 1999 [7]. Pal applied numerous matrix concepts to the fuzzy sets. Pal investigated and established Interval-valued Intuitionistic fuzzy matrices as well as the Intuitionistic fuzzy determinant [5], [8]. The notion of a matrix was applied to triangular fuzzy numbers by A.K. Shyamal et al. [9]. They also developed triangular fuzzy matrices and addressed their specific types of properties. Pal (2016) developed four binary fuzzy operators, expanded the idea of fuzzy matrices with fuzzy rows and columns, and applied them to image processing [10]. In 2016, Mondal and Pal investigated the eigen vectors and eigen values of bipolar fuzzy matrices [11]. In 2019, Pal and Sanjib defined and introduced the concept of bipolar fuzzy matrices, their power convergence, and their applications[12]. Dogra and Pal introduced the picture fuzzy matrix with the determinant and adjoint of its unique restricted square picture fuzzy matrices [13]. Fuzzy matrices were created and used in a variety of fields to handle uncertain situations. Numerous authors, including Sanchez, Pal, Muthuraji, Nagoor Gani, and Meenakshi, have examined and applied various results on fuzzy intuitionistic matrix, fuzzy interval valued matrix, triangular fuzzy matrix, bipolar fuzzy matrix, picture fuzzy matrix, neutrosophic fuzzy number, and fuzzy soft matrix in the field of decision making [14],[15], [16], [17], [18]. Inspired by these inventions, we define the Quadrasophic Fuzzy Matrix (QFM) in this artifact. Also, we explore the application of Quadrasophic Fuzzy Matrix in the detection of plant illnesses.

In this artifact, Section 2 provides an overview of QFS's fundamental definitions and functions. In Section 3, we defined the operations of QFS, and Section 4 presented the introduction to Quadrasophic Fuzzy Algebra. The Quadrasophic Fuzzy Matrix is defined in Section 5 and includes relevant examples along with its operations and propositions. In Section 6, the identification of plant diseases serves as an example of the use of the Quadrasophic Fuzzy Matrix in decision-making. The conclusion and the scope of future investigation potential are presented in Section 7.

2. Preliminaries

Quadrasophic fuzzy set [1]: The Quadrasophic fuzzy set q on the set U is defined as

$$q = \{(x, \eta_q(x), \lambda_{\eta_q}(x), \lambda_{\mu_q}(x), \mu_q(x)) \,|x \in U\}$$

where the degree of positive membership grade is $\mu_q(x) : U \to [0,1]$, the degree of negative membership is $\eta_q(x) : U \to [-1,0]$ the degree of restricted positive membership is $\lambda_{\mu_q}(x) : U \to [0,0.5]$ the degree of restricted negative membership is $\lambda_{\eta_q}(x) : U \to [-0.5,0]$ And the condition follows: $-1 \leq \mu_q(x) + \eta_q(x) \leq 1$, $-0.5 \leq \lambda_q \leq 0.5$ and $0 \leq \mu_q^2 + \eta_q^2 + \lambda_q^2 \leq 3$ for all $x \in U$, such that $\lambda_q =$ Length of $(\lambda_{\mu_q}, \lambda_{\eta_q})$.

Intersection operation of QFS [2]: Let q_a, $q_b \in$ QFS, then for all $x \in X$ the intersection of q_a and q_b is represented as:

$$q_a \cap q_b = (\eta_{q_a}(x) \vee \eta_{q_b}(x), \lambda_{\eta_{qa}}(x) \vee \lambda_{\eta_{q_b}}(x), \lambda_{\mu_{qa}}(x) \wedge \lambda_{\mu_{q_b}}(x), \mu_{q_a}(x) \wedge \mu_{q_b}(x))$$

Union operation of QFS [2]: Let q_a, $q_b \in$ QFS, then for all $x \in X$ the union of q_a and q_b is represented as:

$$q_a \cup q_b = (\eta_{q_a}(x) \wedge \eta_{q_b}(x), \lambda_{\eta_{qa}}(x) \wedge \lambda_{\eta_{q_b}}(x), \lambda_{\mu_{qa}}(x) \vee \lambda_{\mu_{q_b}}(x), \mu_{q_a}(x) \vee \mu_{q_b}(x))$$

Complement[2]: Let $q \in$ QFS, then the complement of q is described as $q^c = (-1-\eta_q, -0.5-\lambda_{\eta_q}, 0.5-\lambda_{\mu_q}, 1-\mu_q)$.

Score- valued function [2]: The score- valued function $score(q)$ of QFS is represented as $score(q) = \frac{\mu_q(x)+\lambda_{\mu_q}(x)+\eta_q(x)+\lambda_{\eta_q}(x)}{3}$, for all $score(q) \in [-1,1]$.

The $\ominus$ operator: [6] For $u, v \in [0,1]$, the $\ominus$ operator can be defined as :

$$u \ominus v = \begin{cases} u, & \text{if } u > v \\ 0, & \text{if u} \leq \text{v.} \end{cases}$$

3. Certain Operations on Quadrasophic Fuzzy Sets

Operations on Quadrasophic Fuzzy Sets: Let $q_1 = (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1})$ and $q_2 = (-\eta_{q_2}, -\lambda_{\eta_{q_2}}, \lambda_{\mu_{q_2}}, \mu_{q_2})$ be two Quadrasophic Fuzzy sets in the set Q_F.

Disjunction:
$$\begin{aligned} q_1 + q_2 =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) + (-\eta_{q_2}, -\lambda_{\eta_{q_2}}, \lambda_{\mu_{q_2}}, \mu_{q_2}) \\ &= \{-max(\eta_{q_1}, \eta_{q_2}), -max(\lambda_{\eta_{q_1}}, \lambda_{\eta_{q_2}}), \\ &max(\lambda_{\mu_{q_1}}, \lambda_{\mu_{q_2}}), max(\mu_{q_1}, \mu_{q_2})\} \\ &= \{-(\eta_{q_1} \vee \eta_{q_2}), -(\lambda_{\eta_{q_1}} \vee \lambda_{\eta_{q_2}}), (\lambda_{\mu_{q_1}} \vee \lambda_{\mu_{q_2}}), (\mu_{q_1} \vee \mu_{q_2})\} \end{aligned}$$

Parallel conjunction:
$$\begin{aligned} q_1 \cdot q_2 =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) + (-\eta_{q_2}, -\lambda_{\eta_{q_2}}, \lambda_{\mu_{q_2}}, \mu_{q_2}) \\ &= \{-min(\eta_{q_1}, \eta_{q_2}), -min(\lambda_{\eta_{q_1}}, \lambda_{\eta_{q_2}}), \\ &min(\lambda_{\mu_{q_1}}, \lambda_{\mu_{q_2}}), min(\mu_{q_1}, \mu_{q_2})\} \\ &= \{-(\eta_{q_1} \wedge \eta_{q_2}), -(\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_2}}), \\ &(\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_2}}), (\mu_{q_1} \wedge \mu_{q_2})\} \end{aligned}$$

Serial conjunction:
$$\begin{aligned} q_1 \otimes q_2 =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) + (-\eta_{q_2}, -\lambda_{\eta_{q_2}}, \lambda_{\mu_{q_2}}, \mu_{q_2}) \\ &= \{-((\eta_{q_1} \wedge \mu_{q_2}) \vee (\mu_{q_1} \wedge \eta_{q_2}))\}, \\ &\{-((\lambda_{\eta_{q_1}} \wedge \lambda_{\mu_{q_2}}) \vee (\lambda_{\mu_{q_1}} \wedge \lambda_{\eta_{q_2}}))\}, \\ &\{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_2}}) \vee (\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_2}})\}, \\ &\{-((\eta_{q_1} \wedge \eta_{q_2}) \vee (\mu_{q_1} \wedge \mu_{q_2}))\} \end{aligned}$$

Negation of q_1:
$$\begin{aligned} -q1 &= -(-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) \\ &= (-\mu_{q_1}, -\lambda_{\mu_{q_1}}, \lambda_{\eta_{q_1}}, \eta_{q_1}) \end{aligned}$$

Complement of q_1:
$$\begin{aligned} \neg q1 &= \neg(-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) \\ &= (-1+\eta_{q_1}, -0.5+\lambda_{\eta_{q_1}}, 0.5-\lambda_{\mu_{q_1}}, 1-\mu_{q_1}) \end{aligned}$$

QFS $\ominus$ operator:

$$q_1 \ominus q_2 = \begin{cases} -\eta_{q_1}, & \text{for } -(\eta_{q_1} > \eta_{q_2}) \\ -\lambda_{\eta_{q_1}}, & \text{for } -(\lambda_{\eta_{q_1}} > \lambda_{\eta_{q_2}}) \\ \lambda_{\mu_{q_1}}, & \text{for } (\lambda_{\mu_{q_1}} > \lambda_{\mu_{q_2}}) \\ \mu_{q_1}, & \text{for } (\mu_{q_1} > \mu_{q_2}) \\ 0, & \text{otherwise.} \end{cases}$$

Zero element of QFS: The zero element of QFS is represented by 0_q and is described as $0_q = (0,0,0,0)$.
Unit element of QFS: The unit element of QFS is represented by i_q and is described as $i_q = (-1,-0.5,0.5,1)$.

4. Quadrasophic Fuzzy Algebra

A Quadrasophic Fuzzy Algebra is a mathematical structure (QF, +,.) with two binary operations (+ and .) defined on QFS that fulfill the following principle.

Theorem 1. Consider $q_1 = (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1})$ $q_2 = (-\eta_{q_2}, -\lambda_{\eta_{q_2}}, \lambda_{\mu_{q_2}}, \mu_{q_2})$ $q_3 = (-\eta_{q_3}, -\lambda_{\eta_{q_3}}, \lambda_{\mu_{q_3}}, \mu_{q_3})$ belongs to QFS. Then the results as follows:
i. Law of Idempotent: $q_1 + q_1 = q_1, q_1.q_1 = q_1$
ii. Law of Commutativity: $q_1 + q_2 = q_2 + q_1, q_1.q_2 = q_2.q_1$
iii. Law of Associativity: $q_1 + (q_2 + q_3) = (q_1 + q_2) + q_3, q_1.(q_2.q_3) = (q_1.q_2).q_3$
iv. Law of Absorption: $q_1 + (q_1.q_2) = q_1, q_1.(q_1 + q_2) = q_1$
v. Law of Distributivity: $q_1.(q_2 + q_3) = (q_1.q_2) + (q_1.q_3), q_1 + (q_2.q_3) = (q_1 + q_2).(q_1 + q_3)$
vi. Universal Law: $q_1 + 0_q = q_1, q_1 + i_q = i_q, q_1.0_q = 0_q, q_1.i_q = q_1$

Proof. *i.* $q_1 + q_1 = (-\{\eta_{q_1} \vee \eta_{q_1},\}, -\{\lambda_{\eta_{q_1}} \vee \lambda_{\eta_{q_1}}\}, \{\lambda_{\mu_{q_1}} \vee \lambda_{\mu_{q_1}}\}, \{\mu_{q_1} \vee \mu_{q_1}\})$
$= (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1})$
$= q_1$
Similarly, $q_1 \cdot q_1 = (-\{\eta_{q_1} \wedge \eta_{q_1},\}, -\{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_1}}\}, \{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_1}}\}, \{\mu_{q_1} \wedge \mu_{q_1}\})$
$= (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1})$
$= q_1$

ii. $q_1 + q_2 = (-\{\eta_{q_1} \vee \eta_{q_2},\}, -\{\lambda_{\eta_{q_1}} \vee \lambda_{\eta_{q_2}}\}, \{\lambda_{\mu_{q_1}} \vee \lambda_{\mu_{q_2}}\}, \{\mu_{q_1} \vee \mu_{q_2}\})$
$= (-\eta_{q_1}, -\lambda_{\eta_{q_2}}, \lambda_{\mu_{q_1}}, \mu_{q_2})$
$= (-\{\eta_{q_2} \vee \eta_{q_1},\}, -\{\lambda_{\eta_{q_2}} \vee \lambda_{\eta_{q_1}}\}, \{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_1}}\}, \{\mu_{q_2} \vee \mu_{q_1}\})$
$= (-\eta_{q_2}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_2}}, \mu_{q_1})$
$= q_2 + q_1$
Similarly, $q_1 \cdot q_2 = q_2 \cdot q_1$.

$$\begin{aligned} \textit{iii. } q_1 + (q_2 + q_3) =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) + (-\{\eta_{q_2} \vee \eta_{q_3},\}, -\{\lambda_{\eta_{q_2}} \vee \lambda_{\eta_{q_3}}\}, \\ & \{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_3}}\}, \{\mu_{q_2} \vee \mu_{q_3}\}) \\ &= -max(\eta_{q_1}, \{\eta_{q_2} \vee \eta_{q_3}\}), -max(\lambda_{\eta_{q_1}}, \{\lambda_{\eta_{q_2}} \vee \lambda_{\eta_{q_3}}\}), \\ & max(\lambda_{\mu_{q_1}}, \{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_3}}\}), max(\mu_{q_1}, \{\mu_{q_2} \vee \mu_{q_3}\}) \\ &= -(\eta_{q_1} \vee \{\eta_{q_2} \vee \eta_{q_3}\}), -(\lambda_{\eta_{q_1}} \vee \{\lambda_{\eta_{q_2}} \vee \lambda_{\eta_{q_3}}\}), \\ & (\lambda_{\mu_{q_1}} \vee \{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_3}}\}), (\mu_{q_1} \vee \{\mu_{q_2} \vee \mu_{q_3}\}) \end{aligned}$$

$$\begin{aligned} (q_1 + q_2) + q_3 =& (-\{\eta_{q_1} \vee \eta_{q_2},\}, -\{\lambda_{\eta_{q_1}} \vee \lambda_{\eta_{q_2}}\}, \{\lambda_{\mu_{q_1}} \vee \lambda_{\mu_{q_2}}\}, \{\mu_{q_1} \vee \mu_{q_2}\}) \\ & + (-\eta_{q_3}, -\lambda_{\eta_{q_3}}, \lambda_{\mu_{q_3}}, \mu_{q_3}) \\ &= -max(\{\eta_{q_1} \vee \eta_{q_2}\}, \eta_{q_3}), -max(\{\lambda_{\eta_{q_1}} \vee \lambda_{\eta_{q_2}}\}, \lambda_{\eta_{q_3}}), \\ & max(\{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_3}}\}, \lambda_{\mu_{q_1}}), max(\mu_{q_1}, \{\mu_{q_2} \vee \mu_{q_3}\}) \\ &= -(\eta_{q_1} \vee \{\eta_{q_2} \vee \eta_{q_3}\}), -(\lambda_{\eta_{q_1}} \vee \{\lambda_{\eta_{q_2}} \vee \lambda_{\eta_{q_3}}\}), \\ & (\lambda_{\mu_{q_1}} \vee \{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_3}}\}), (\mu_{q_1} \vee \{\mu_{q_2} \vee \mu_{q_3}\}) \end{aligned}$$

Hence, $q_1 + (q_2 + q_3) = (q_1 + q_2) + q_3$
In similar way, $q_1 \cdot (q_2 \cdot q_3) = (q_1 \cdot q_2) \cdot q_3$

$$\begin{aligned}
iv.\ q_1 + (q_1 \cdot q_2) =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) + (-\{\eta_{q_1} \wedge \eta_{q_2},\}, -\{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_2}}\}, \\
& \{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_2}}\}, \{\mu_{q_1} \wedge \mu_{q_2}\}) \\
&= -(\eta_{q_1} \vee \{\eta_{q_1} \wedge \eta_{q_2}\}), -(\lambda_{\eta_{q_1}} \vee \{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_2}}\}), \\
& (\lambda_{\mu_{q_1}} \vee \{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_2}}\}), (\mu_{q_1} \vee \{\mu_{q_1} \wedge \mu_{q_2}\}) \\
&= (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) = q_1
\end{aligned}$$

Similarly, $q_1.(q_1 + q_2) = q_1$

$$\begin{aligned}
v.\ q_1 \cdot (q_2 + q_3) =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) \cdot (-\{\eta_{q_2} \vee \eta_{q_3},\}, -\{\lambda_{\eta_{q_2}} \vee \lambda_{\eta_{q_3}}\}, \\
& \{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_3}}\}, \{\mu_{q_2} \vee \mu_{q_3}\}) \\
&= -(\eta_{q_1} \wedge \{\eta_{q_2} \vee \eta_{q_3}\}), -(\lambda_{\eta_{q_1}} \wedge \{\lambda_{\eta_{q_2}} \vee \lambda_{\eta_{q_3}}\}), \\
& (\lambda_{\mu_{q_1}} \wedge \{\lambda_{\mu_{q_2}} \vee \lambda_{\mu_{q_3}}\}), (\mu_{q_1} \wedge \{\mu_{q_2} \vee \mu_{q_3}\}) \\
&= -(\{\eta_{q_1} \wedge \eta_{q_2}\} \vee \{\eta_{q_1} \wedge \eta_{q_3}\}), (\{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_2}}\} \vee \{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_3}}\}), \\
& (\{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_2}}\} \vee \{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_3}}\}), (\{\mu_{q_1} \wedge \mu_{q_2}\} \vee \{\mu_{q_1} \wedge \mu_{q_3}\}) \\
(q_1 \cdot q_2) + (q_1 \cdot q_3) =& (-\{\eta_{q_1} \wedge \eta_{q_2}\}, -\{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_2}}\}, \{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_2}}\}, \{\mu_{q_1} \wedge \mu_{q_2}\}) \\
& + (-\{\eta_{q_1} \wedge \eta_{q_3}\}, -\{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_3}}\}, \{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_3}}\}, \{\mu_{q_1} \wedge \mu_{q_3}\}) \\
&= -(\{\eta_{q_1} \wedge \eta_{q_2}\} \vee \{\eta_{q_1} \wedge \eta_{q_3}\}), -(\{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_2}}\} \vee \{\lambda_{\eta_{q_1}} \wedge \lambda_{\eta_{q_3}}\}), \\
& (\{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_2}}\} \vee \{\lambda_{\mu_{q_1}} \wedge \lambda_{\mu_{q_3}}\}), (\{\mu_{q_1} \wedge \mu_{q_2}\} \vee \{\mu_{q_1} \wedge \mu_{q_3}\})
\end{aligned}$$

Thus, $q_1 \cdot (q_2 + q_3) = (q_1 \cdot q_2) + (q_1 \cdot q_3)$
Similarly, $q_1 + (q_2 \cdot q_3) = (q_1 + q_2) \cdot (q_1 + q_3)$

$$\begin{aligned}
vi.\ q_1 + 0_q =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) + (0, 0, 0, 0) \\
&= (-\{\eta_{q_1} \vee 0\}, -\{\lambda_{\eta_{q_1}} \vee 0\}, \{\lambda_{\mu_{q_1}} \vee 0\}, \{\mu_{q_1} \vee 0\}) \\
&= (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) \\
&= q_1 \\
q_1 + i_q =& (-\eta_{q_1}, -\lambda_{\eta_{q_1}}, \lambda_{\mu_{q_1}}, \mu_{q_1}) + (-1, -0.5, 0.5, 1) \\
&= (-\{\eta_{q_1} \vee -1\}, -\{\lambda_{\eta_{q_1}} \vee -0.5\}, \{\lambda_{\mu_{q_1}} \vee 0.5\}, \{\mu_{q_1} \vee 1\}) \\
&= (-1, -0.5, 0.5, 1) \\
&= i_q
\end{aligned}$$

Similarly, we can prove $q_1 \cdot 0_q = 0_q$ and $q_1.i_q = q_1$. ■ Thus, $(QFS, +, \cdot)$ is Quadrasophic Fuzzy Algebra (QFA).

Example 2: Let us consider three Quadrasophic Fuzzy Sets q_1=$(-0.4, -0.1, 0.3, 0.5)$, $q_2 =$

$(-0.5, -0.3, 0.4, 0.7)$ and $q_3 = (-0.7, -0.4, 0.2, 0.6)$

$$\begin{aligned}
i.q_1 + (q_1 \cdot q_2) &= q_1 + (-0.4, -0.1, 0.3, 0.5) = (-0.4, -0.1, 0.3, 0.5) \\
&= q_1 \\
ii.q_1 \cdot (q_1 + q_2) &= q_1.(-0.5, -0.3, 0.4, 0.7) \\
&= (-0.4, -0.1, 0.3, 0.5) \\
&= q_1. \\
iii.\, q_1 + (q_2 \cdot q_3) &= q_1 + (-0.5, -0.3, 0.2, 0.6) \\
&= (-0.5, -0.3, 0.3, 0.6) \\
iv.(q_1 + q_2) \cdot (q_1 + q_3) &= (-0.5, -0.3, 0.4, 0.7) \cdot (-0.7, -0.4, 0.3, 0.6) \\
&= (-0.5, -0.3, 0.3, 0.6)
\end{aligned}$$

Hence, $q_1 + (q_2 \cdot q_3) = (q_1 + q_2) \cdot (q_1 + q_3)$

5. Quadrasophic Fuzzy Matrix

Let $Q_{r\times s}$ be a Quadrasophic Fuzzy Matrix, where

$$Q_{r\times s} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1s} \\ q_{21} & q_{22} & \cdots & q_{2s} \\ \vdots & & \ddots & \vdots \\ q_{r1} & q_{r2} & \cdots & q_{rs} \end{bmatrix}$$

Here, $q_{ij} = (-\eta_{q_{ij}}, -\lambda_{\eta_{q_{ij}}}, \lambda_{\mu_{q_{ij}}}, \mu_{q_{ij}})$, $(\eta_{q_{ij}}, \mu_{q_{ij}}) \in [0,1]$ represents the positive and negative grade, and $(\lambda_{\eta_{q_{ij}}}, \lambda_{\mu_{q_{ij}}}) \in [0, 0.5]$ represents the restricted positive and restricted negative grade of $q_i j$. A Quadrasophic Fuzzy Matrix (QFM) is the matrix defined over the Quadrasophic Fuzzy Algebra.

Zero Quadrasophic Matrix: The zero Quadrasophic Fuzzy Matrix 0_{q_s} of order $s \times s$ is the matrix where all the elements are $0_q = (0, 0, 0, 0)$.

Identity Quadrasophic Matrix: The identity Quadrasophic Matrix I_{q_s} of order $s \times s$ is the matrix where all the diagonal entries are $i_q = (-1, -0.5, 0.5, 1)$ and all the other entries are $0_q = (0, 0, 0, 0)$.

Remark 1. M_{rs} indicates the set of all QFM's of order $r \times s$, while M_s indicates the set of square QFM's of order $s \times s$.

5.1. Operations on Quadrasophic Fuzzy Matrix

Binary opertors of QFM: Let $Q_1 = (q_{1ij})$ and $Q_2 = (q_{2ij})$ be two QFMs of order $r \times s$. Then the following operations as follows:

$$\begin{aligned}
Q_1 + Q_2 &= (q_{1ij} + q_{2ij})_{r\times s} \\
&= \{-max(\eta_{q_{1ij}}, \eta_{q_{2ij}}), -max(\lambda_{\eta_{q_{1ij}}}, \lambda_{\eta_{q_{2ij}}}), \\
&\quad max(\lambda_{\mu_{q_{1ij}}}, \lambda_{\mu_{q_{2ij}}}), max(\mu_{q_{1ij}}, \mu_{q_{2ij}})\} \\
Q_1 \cdot Q_2 &= (q_{1ij} \cdot q_{2ij})_{r\times s} \\
&= \{-min(\eta_{q_{1ij}}, \eta_{q_{2ij}}), -min(\lambda_{\eta_{q_{1ij}}}, \lambda_{\eta_{q_{2ij}}}), \\
&\quad min(\lambda_{\mu_{q_{1ij}}}, \lambda_{\mu_{q_{2ij}}}), min(\mu_{q_{1ij}}, \mu_{q_{2ij}})\}
\end{aligned}$$

$\odot$ and $\otimes$ operators of QFM: Let $Q_1 = (q_{1ij})_{r\times s}$ and $Q_2 = (q_{2ij})_{s\times t}$ be two QFMs. Then the following operations as follows:

$$Q_1 \odot Q_2 = (\sum_{l=1}^{s} q_{1il} \cdot q_{2lj})_{r\times t}$$
$$= (-max_{l=1}^{s}(min\{\eta_{q_{1il}}, \eta_{q_{2lj}}\}), (-max_{l=1}^{s}(min\{\lambda_{\eta_{q_{1il}}}, \lambda_{\eta_{q_{2lj}}}\}),$$
$$max_{l=1}^{s}(min\{\lambda_{\mu_{q_{1il}}}, \lambda_{\mu_{q_{2lj}}}\}), max_{l=1}^{s}(min\{\mu_{q_{1il}}, \mu_{q_{2lj}}\}))$$

$$Q_1 \otimes Q_2 = (\prod_{l=1}^{s} q_{1il} + q_{2lj})_{r\times t}$$
$$= (-min_{l=1}^{s}(max\{\eta_{q_{1il}}, \eta_{q_{2lj}}\}), (-min_{l=1}^{s}(max\{\lambda_{\eta_{q_{1il}}}, \lambda_{\eta_{q_{2lj}}}\}),$$
$$min_{l=1}^{s}(max\{\lambda_{\mu_{q_{1il}}}, \lambda_{\mu_{q_{2lj}}}\}), min_{l=1}^{s}(max\{\mu_{q_{1il}}, \mu_{q_{2lj}}\}))$$

Theorem 2. Let Q_1, Q_2, and Q_3 be three Quadrasophic Fuzzy matrices, and then we have the following results:
i. $Q_1 + Q_2 = Q_2 + Q_1$, $Q_1 \cdot Q_2 = Q_2 \cdot Q_1$
ii. $Q_1 + (Q_2 + Q_3) = (Q_1 + Q_2) + Q_3$, $Q_1 \cdot (Q_2 \cdot Q_3) = (Q_1 \cdot Q_2) \cdot Q_3$
iii. $Q_1 \cdot (Q_2 + Q_3) = (Q_1.Q_2) + (Q_1 \cdot Q_3)$, $Q_1 + (Q_2 \cdot Q_3) = (Q_1 + Q_2) \cdot (Q_1 + Q_3)$
iv. $Q_1 + 0_q = 0_q + Q_1 = Q_1$, $Q_1 \cdot 0_q = 0_q \cdot Q_1 = 0_q$, where 0_q is the zero QFM.

Proof. To prove the results, let us consider the following example:

$$Q_1 = \begin{bmatrix} (-0.3, -0.1, 0.3, 0.5) & (-0.4, -0.4, 0.5, 0.7) \\ (-0.2, -0.1, 0.4, 0.7) & (-0.7, -0.3, 0.4, 0.8) \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} (-0.4, -0.3, 0.4, 0.7) & (-0.6, -0.2, 0.5, 0.8) \\ (-0.6, -0.2, 0.3, 0.9) & (-0.7, -0.4, 0.3, 0.5) \end{bmatrix}$$

$$\textit{(i.)}\ Q_1 + Q_2 = \begin{bmatrix} \{-max(0.3,0.4), -max(0.1,0.3), & \{-max(0.4,0.6), -max(0.4,0.2), \\ max(0.3,0.4), max(0.5,0.7)\} & max(0.5,0.5), max(0.7,0.8)\} \\ \{-max(0.2,0.6), -max(0.1,0.2), & \{-max(0.7,0.7), -max(0.3,0.4), \\ max(0.4,0.3), max(0.7,0.9)\} & max(0.4,0.3), max(0.8,0.5)\} \end{bmatrix}$$

$$Q_1 + Q_2 = \begin{bmatrix} (-0.4, -0.3, 0.4, 0.7) & (-0.6, -0.4, 0.5, 0.8) \\ (-0.6, -0.2, 0.4, 0.9) & (-0.7, -0.4, 0.4, 0.8) \end{bmatrix}$$

$$Q_1 + Q_2 = \begin{bmatrix} \{-max(0.4,0.3), -max(0.3,0.1), & \{-max(0.6,0.4), -max(0.2,0.4), \\ max(0.4,0.3), max(0.7,0.5)\} & max(0.5,0.5), max(0.8,0.7)\} \\ \{-max(0.2,0.6), -max(0.1,0.2), & \{-max(0.7,0.7), -max(0.4,0.3), \\ max(0.4,0.3), max(0.7,0.9)\} & max(0.3,0.4), max(0.5,0.8)\} \end{bmatrix}$$

$$Q_2 + Q_1 = \begin{bmatrix} (-0.4, -0.3, 0.4, 0.7) & (-0.6, -0.4, 0.5, 0.8) \\ (-0.6, -0.2, 0.4, 0.9) & (-0.7, -0.4, 0.4, 0.8) \end{bmatrix}$$

Hence, $Q_1 + Q_2 = Q_2 + Q_1$

$$Q_1 \cdot Q_2 = \begin{bmatrix} \{-min(0.3,0.4), -min(0.1,0.3), & \{-min(0.4,0.6), -min(0.4,0.2), \\ min(0.3,0.4), min(0.5,0.7)\} & min(0.5,0.5), min(0.7,0.8)\} \\ \{-min(0.2,0.6), -min(0.1,0.2), & \{-min(0.7,0.7), -min(0.3,0.4), \\ min(0.4,0.3), min(0.7,0.9)\} & min(0.4,0.3), min(0.8,0.5)\} \end{bmatrix}$$

$$Q_1 \cdot Q_2 = \begin{bmatrix} (-0.3,-0.1,0.3,0.5) & (-0.4,-0.3,0.5,0.7) \\ (-0.2,-0.1,0.3,0.7) & (-0.7,-0.3,0.3,0.5)] \end{bmatrix}$$

$$Q_2 \cdot Q_1 = \begin{bmatrix} \{-min(0.4,0.3), -min(0.3,0.1), & \{-min(0.6,0.4), -min(0.4,0.3), \\ min(0.4,0.3), min(0.7,0.5)\} & min(0.3,0.4), min(0.5,0.8)\} \\ \{-min(0.6,0.2), -min(0.2,0.1), & \{-min(0.7,0.7), -min(0.4,0.3), \\ min(0.3,0.4), min(0.9,0.7)\} & min(0.3,0.4), min(0.5,0.8)\} \end{bmatrix}$$

$$Q_2 \cdot Q_1 = \begin{bmatrix} (-0.3,-0.1,0.3,0.5) & (-0.4,-0.3,0.5,0.7) \\ (-0.2,-0.1,0.3,0.7) & (-0.7,-0.3,0.3,0.5)] \end{bmatrix}$$

$\implies Q_1 \cdot Q_2 = Q_2 \cdot Q_1$,
Thus, (i.) is proved.
(ii.) The proof is obvious.
(iii.)

$$Q_1 \cdot (Q_2 + Q_3) = \begin{bmatrix} (-0.3,-0.1,0.3,0.5) & (-0.4,-0.3,0.5,0.7) \\ (-0.2,-0.1,0.3,0.7) & (-0.7,-0.3,0.4,0.8) \end{bmatrix}$$

$$(Q_1 \cdot Q_2) + (Q_1 \cdot Q_3) = \begin{bmatrix} (-0.3,-0.1, & (-0.4,-0.3, \\ 0.3,0.5) & 0.5,0.7) \\ (-0.2,-0.1, & (-0.7,-0.3, \\ 0.3,0.7) & 0.3,0.5) \end{bmatrix} + \begin{bmatrix} (-0.3,-0.1, & (-0.4,-0.3, \\ 0.3,0.5) & 0.2,0.7) \\ (-0.2,-0.1, & (-0.7,-0.3, \\ 0.2,0.5) & 0.4,0.8) \end{bmatrix}$$

$$(Q_1 \cdot Q_2) + (Q_1 \cdot Q_3) = \begin{bmatrix} (-0.3,-0.1,0.3,0.5) & (-0.4,-0.3,0.5,0.7) \\ (-0.2,-0.1,0.3,0.7) & (-0.7,-0.3,0.4,0.8) \end{bmatrix}$$

Hence, $Q_1 \cdot (Q_2 + Q_3) = (Q_1 \cdot Q_2) + (Q_1 \cdot Q_3)$.

$$Consider, Q_1 + (Q_2 \cdot Q_3) = \begin{bmatrix} (-0.4,-0.3,0.3,0.6) & (-0.6,-0.4,0.5,0.7) \\ (-0.5,-0.2,0.4,0.7) & (-0.7,-0.4,0.4,0.8) \end{bmatrix}$$

$$(Q_1 + Q_2) \cdot (Q_1 + Q_3) = \begin{bmatrix} (-0.4,-0.3, & (-0.6,-0.4, \\ 0.4,0.7) & 0.5,0.8) \\ (-0.6,-0.2, & (-0.7,-0.4, \\ 0.4,0.9) & 0.4,0.8) \end{bmatrix} + \begin{bmatrix} (-0.7,-0.4, & (-0.6,-0.4, \\ 0.3,0.6) & 0.5,0.7) \\ (-0.5,-0.3, & (-0.8,-0.4, \\ 0.4,0.7) & 0.4,0.9) \end{bmatrix}$$

$$(Q_1 + Q_2) \cdot (Q_1 + Q_3) = \begin{bmatrix} (-0.4,-0.3,0.3,0.6) & (-0.6,-0.4,0.5,0.7) \\ (-0.5,-0.2,0.4,0.7) & (-0.7,-0.4,0.4,0.8) \end{bmatrix}$$

Hence, $Q_1 + (Q_2 \cdot Q_3) = (Q_1 + Q_2) \cdot (Q_1 + Q_3)$ Thus, (iii.) is proved.

$$(iv).Q_1 + 0_q = \begin{bmatrix} \{-max(0.3,0), -max(0.1,0), & \{-max(0.4,0), -max(0.4,0), \\ max(0.3,0), max(0.5,0)\} & max(0.5,0), max(0.7,0)\} \\ \{-max(0.2,0), -max(0.1,0), & \{-max(0.7,0), -max(0.3,0), \\ max(0.4,0), max(0.7,0)\} & max(0.4,0), max(0.8,0)\} \end{bmatrix}$$

$$= \begin{bmatrix} (-0.3,0.1,0.3,0.5) & (-0.4,-0.4,0.5,0.7) \\ (-0.2,-0.1,0.4,0.7) & (-0.7,-0.3,0.4,0.8) \end{bmatrix} = Q_1$$

$$0_q + Q_1 = \begin{bmatrix} (-0.3, 0.1, 0.3, 0.5) & (-0.4, -0.4, 0.5, 0.7) \\ (-0.2, -0.1, 0.4, 0.7) & (-0.7, -0.3, 0.4, 0.8) \end{bmatrix} = Q_1$$

$$Consider,\ Q_1 \cdot 0_q = \begin{bmatrix} \{-min(0.3,0), -min(0.1,0), & \{-min(0.4,0), -min(0.4,0), \\ min(0.3,0), min(0.5,0)\} & min(0.5,0), min(0.7,0)\} \\ \{-min(0.2,0), -min(0.1,0), & \{-min(0.7,0), -min(0.3,0), \\ min(0.4,0), min(0.7,0)\} & min(0.4,0), min(0.8,0)\} \end{bmatrix}$$

$$= \begin{bmatrix} (0,0,0,0) & (0,0,0,0) \\ (0,0,0,0) & (0,0,0,0) \end{bmatrix} = 0_q$$

Hence, $Q_1 \cdot 0_q = 0_q \cdot Q_1 = 0_q$
Thus, (i.), (ii.) , (iii.) and (iv.) are easily verified when Q_1, Q_2, Q_3 are of same order. ■

Theorem 3. Let Q_1, Q_2, and Q_3 be three Quadrasophic Fuzzy matrices, and then we have the following results:
i. $Q_1 \odot (Q_2 \odot Q_3) = (Q_1 \odot Q_2) \odot Q_3$
ii. $Q_1 \otimes (Q_2 \otimes Q_3) = (Q_1 \otimes Q_2) \otimes Q_3$
iii. $Q_1 \odot i_q = i_q \odot Q_1 = i_q$
iv. $Q_1 \otimes i_q = i_q \otimes Q_1 = i_q$
v. $Q_1 \odot Q_2 \neq Q_2 \odot Q_1$
vi. $Q_1 \otimes Q_2 \neq Q_2 \otimes Q_1$
vii. $Q_1 \odot (Q_2 + Q_3) \neq (Q_1 \odot Q_2) + (Q_1 \otimes Q_3)$
viii. $Q_1 \otimes (Q_2 \cdot Q_3) \neq (Q_1 \otimes Q_2) \cdot (Q_1 \otimes Q_3)$

Proof. By using the definitions (i.),(ii.), (iii.) and (iv.) are easily verified.
(v.) Let $Q_1 = (q_{1ij})_{r\times s}$ and $Q_2 = (q_{2ij})_{s\times t}$ be two QFMs. Then $Q_1 \odot Q_2 = (q_{12ij})_{r\times t}$ exist. But $Q_2 \odot Q_1$ does not exist. Suppose Q_1 and Q_2 are square matrices. Then by using previous example we can easily verify the result.

$$Q_1 \odot Q_2 = \begin{bmatrix} (-0.4, -0.2, 0.3, 0.7) & (-0.4, -0.4, 0.3, 0.5) \\ (-0.6, -0.2, 0.4, 0.7) & (-0.7, -0.3, 0.4, 0.7) \end{bmatrix}$$

$$Q_2 \odot Q_1 = \begin{bmatrix} (-0.3, -0.1, 0.4, 0.7) & (-0.6, -0.3, 0.4, 0.8) \\ (-0.3, -0.1, 0.3, 0.5) & (-0.7, -0.3, 0.3, 0.7) \end{bmatrix}$$

Hence, $Q_1 \odot Q_2 \neq Q_2 \odot Q_1$
In similar way, (vi.) $Q_1 \otimes Q_2 \neq Q_2 \otimes Q_1$ inequality holds.
(vii.) By computing,

$$Q_1 \odot (Q_2 + Q_3) = \begin{bmatrix} (-0.4, -0.2, 0.3, 0.7) & (-0.4, -0.4, 0.4, 0.7) \\ (-0.6, -0.2, 0.4, 0.7) & (-0.7, -0.3, 0.4, 0.7) \end{bmatrix}$$

$$(Q_1 \odot Q_2) + (Q_1 \odot Q_3) = \begin{bmatrix} (-0.4, -0.2, & (-0.4, -0.4, \\ 0.3, 0.7) & 0.3, 0.5) \\ (-0.6, -0.2, & (-0.7, -0.3, \\ 0.4, 0.7) & 0.4, 0.7) \end{bmatrix} + \begin{bmatrix} (-0.4, -0.3, & (-0.4, -0.4, \\ 0.3, 0.5) & 0.4, 0.7) \\ (-0.5, -0.3, & (-0.7, -0.3, \\ 0.3, 0.6) & 0.4, 0.8) \end{bmatrix}$$

$$= \begin{bmatrix} (-0.4, -0.3, 0.3, 0.7) & (-0.4, -0.4, 0.4, 0.7) \\ (-0.5, -0.3, 0.4, 0.8) & (-0.7, -0.3, 0.4, 0.8) \end{bmatrix}$$

Hence, $Q_1 \odot (Q_2 + Q_3) \neq (Q_1 \odot Q_2) + (Q_1 \odot Q_3)$
Similarly, (viii.) $Q_1 \otimes (Q_2 \cdot Q_3) \neq (Q_1 \otimes Q_2) \cdot (Q_1 \otimes Q_3)$. holds. ■

6. Identification of Plant diseases by employing a Quadrasophic Fuzzy Matrix

The implementation of the Fuzzy Matrix framework uses several techniques to diagnose a wide range of uncertain problems. A Fuzzy Matrix is a crucial tool for decision-making analysis. In India, the primary factor influencing employment and economic standing is agriculture. Approximately 60% of India's rural community works in agriculture [19]. Moreover, the backbone of the Indian economy is agriculture [19]. The damage that natural catastrophes, plant diseases, and climate change have caused to agriculture. To increase the pace of agricultural production, we might focus on plant diseases among these elements. Insects, bacteria, viruses, fungi, and protozoa can all have an impact on plants. In Tamil Nadu, the most important crops farmed are rice, oilseeds, pulses, sugarcane, cotton, and millets. Tamil Nadu is the third-largest producer of paddy in India among these crops. The main staple food in Tamil Nadu is paddy.

In this section, a new ranking mechanism for the Quadrasophic Fuzzy Matrix is proposed and applied to the detection of agricultural plant disease.

Let us consider four paddy plants $\rho = \{p_1, p_2, p_3, p_4\}$ that are afflicted with bacterial infections, with indications represented by $\delta = \{i_1, i_2, i_3, i_4\}$ are presented in Table 1. Let $\kappa = \{l_1, l_2, l_3, l_4\}$ indicates the collection of paddy illnesses.

Table 1: *Notations of paddy illness and indicators*

Notation	Description
i_1	Spindle shaped spots with brown margin and grey centre.
i_2	yellowing of leaves and wilting
i_3	Irregular greyish brown water -soaked lesions on flag leaf sheath
i_4	Oval or cylindrical dark brown spots with a yellow halo
l_1	Rice blast
l_2	Bacterial leaf blight of rice
l_3	Sheath rot of rice
l_4	Rice brown spot

Procedure:

Step 1: Construct the Quadrasophic Fuzzy Matrix Q_1, illustrating the association between indicators and illnesses of the paddy, and Q_2, which illustrates the association between illnesses and paddy.

$$Q_1 = \begin{bmatrix} (-0.7,-0.3,0.2,0.6) & (-0.6,-0.4,0.3,0.8) & (-0.5,-0.2,0.3,0.8) & (-0.6,-0.2,0.4,0.9) \\ (-0.5,-0.2,0.3,0.7) & (-0.7,-0.3,0.2,0.5) & (-0.7,-0.3,0.2,0.8) & (-0.9,-0.3,0.4,0.6) \\ (-0.6,-0.3,0.4,0.8) & (-0.8,-0.4,0.3,0.7) & (-0.3,-0.2,0.3,0.7) & (-0.7,-0.3,0.4,0.9) \\ (-0.9,-0.4,0.3,0.7) & (-0.3,-0.2,0.4,0.8) & (-0.7,-0.3,0.4,0.9) & (-0.7,-0.3,0.3,0.9) \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} (-0.6,-0.3,0.3,0.7) & (-0.4,-0.2,0.3,0.6) & (-0.3,-0.1,0.2,0.5) & (-0.6,-0.2,0.3,0.8) \\ (-0.8,-0.4,0.2,0.4) & (-0.9,-0.4,0.3,0.7) & (-0.3,-0.1,0.4,0.7) & (-0.5,-0.3,0.4,0.9) \\ (-0.9,-0.3,0.2,0.5) & (-0.7,-0.2,0.2,0.6) & (-0.3,-0.2,0.4,0.7) & (-0.6,-0.3,0.3,0.8) \\ (-0.3,-0.1,0.2,0.4) & (-0.2,-0.1,0.4,0.8) & (-0.3,-0.2,0.4,0.9) & (-0.4,-0.2,0.5,0.9) \end{bmatrix}$$

Step 2: Generate the complement matrices Q_1^c and Q_2^c.

$$Q_1^c = \begin{bmatrix} (-0.3,-0.2,0.3,0.4) & (-0.4,-0.1,0.2,0.2) & (-0.5,-0.3,0.2,0.5) & (-0.4,-0.3,0.1,0.1) \\ (-0.5,-0.3,0.2,0.3) & (-0.3,-0.2,0.3,0.5) & (-0.3,-0.2,0.3,0.2) & (-0.1,-0.2,0.1,0.4) \\ (-0.4,-0.2,0.1,0.2) & (-0.2,-0.1,0.2,0.3) & (-0.7,-0.3,0.2,0.3) & (-0.3,-0.2,0.1,0.1) \\ (-0.1,-0.1,0.2,0.3) & (-0.7,-0.3,0.1,0.2) & (-0.3,-0.2,0.1,0.1) & (-0.3,-0.2,0.2,0.4) \end{bmatrix}$$

$$Q_2^c = \begin{bmatrix} (-0.4,-0.2,0.2,0.3) & (-0.6,-0.3,0.2,0.4) & (-0.7,-0.4,0.3,0.5) & (-0.4,-0.3,0.2,0.2) \\ (-0.2,-0.1,0.3,0.6) & (-0.1,-0.1,0.2,0.3) & (-0.7,-0.4,0.1,0.3) & (-0.5,-0.2,0.1,0.1) \\ (-0.1,-0.2,0.3,0.5) & (-0.3,-0.3,0.3,0.4) & (-0.7,-0.3,0.1,0.3) & (-0.4,-0.2,0.2,0.2) \\ (-0.7,-0.4,0.3,0.6) & (-0.8,-0.4,0.1,0.2) & (-0.7,-0.3,0.1,0.1) & (-0.6,-0.3,0.0,0.1) \end{bmatrix}$$

Step 3: Construct $Q_3 = Q_2 \otimes Q_1$, which describes the relationship between Q_1 and Q_2.

$$Q_3 = \begin{bmatrix} (-0.5,-0.2,0.3,0.7) & (-0.6,-0.2,0.3,0.6) & (-0.3,-0.2,0.3,0.7) & (-0.6,-0.3,0.3,0.6) \\ (-0.6,-0.3,0.2,0.6) & (-0.5,-0.3,0.3,0.7) & (-0.3,-0.2,0.3,0.5) & (-0.7,-0.3,0.4,0.7) \\ (-0.6,-0.2,0.2,0.6) & (-0.6,-0.3,0.2,0.6) & (-0.3,-0.2,0.2,0.5) & (-0.7,-0.3,0.3,0.6) \\ (-0.5,-0.2,0.2,0.6) & (-0.4,-0.2,0.3,0.8) & (-0.3,-0.2,0.3,0.5) & (-0.6,-0.2,0.4,0.8) \end{bmatrix}$$

Step 4: Construct $Q_4 = Q_2^c \otimes Q_1^c$, which provides the relation between Q_1^c and Q_2^c.

$$Q_4 = \begin{bmatrix} (-0.4,-0.2,0.2,0.5) & (-0.4,-0.2,0.2,0.2) & (-0.4,-0.3,0.2,0.2) & (-0.4,-0.3,0.2,0.3) \\ (-0.3,-0.2,0.1,0.3) & (-0.3,-0.1,0.1,0.2) & (-0.3,-0.2,0.1,0.1) & (-0.1,-0.2,0.1,0.3) \\ (-0.3,-0.2,0.1,0.3) & (-0.3,-0.2,0.2,0.2) & (-0.3,-0.2,0.2,0.2) & (-0.3,-0.2,0.1,0.3) \\ (-0.6,-0.3,0.1,0.2) & (-0.7,-0.3,0.1,0.2) & (-0.6,-0.3,0.1,0.1) & (-0.6,-0.3,0.1,0.2) \end{bmatrix}$$

Step 5: By using the definition of QFS $\ominus$ operator, compute $Q_3 \ominus Q_4$.

$$Q_5 = (Q_3 \ominus Q_4) = \begin{bmatrix} \begin{matrix}(-0.5,0,\\0.3,0.7)\end{matrix} & \begin{matrix}(-0.6,0,\\0.3,0.6)\end{matrix} & \begin{matrix}(0,0,\\0,0.7)\end{matrix} & \begin{matrix}(-0.6,0,\\0.3,0.6)\end{matrix} \\ \begin{matrix}(-0.6,-0.3,\\0.2,0.6)\end{matrix} & \begin{matrix}(-0.5,-0.3,\\0.3,0.7)\end{matrix} & \begin{matrix}(0,0,\\0.3,0.5)\end{matrix} & \begin{matrix}(-0.7,-0.3,\\0.4,0.7)\end{matrix} \\ \begin{matrix}(-0.6,0,\\0.2,0.6)\end{matrix} & \begin{matrix}(-0.6,-0.3,\\0,0.6)\end{matrix} & \begin{matrix}(0,0,\\0,0.5)\end{matrix} & \begin{matrix}(-0.7,-0.3,\\0.3,0.6)\end{matrix} \\ \begin{matrix}(0,0,\\0.2,0.6)\end{matrix} & \begin{matrix}(0,0,\\0.3,0.8)\end{matrix} & \begin{matrix}(0,0,\\0.3,0.5)\end{matrix} & \begin{matrix}(0,0,\\0.4,0.8)\end{matrix} \end{bmatrix}$$

Step 6: By using the score function, compute the scores of Q_5.

$$Score(Q_5) = \begin{bmatrix} 0.166 & 0.1 & 0.233 & 0.1 \\ -0.03 & 0.066 & 0.266 & 0.033 \\ 0 & -0.1 & 0.166 & -0.033 \\ 0.266 & 0.366 & 0.266 & 0.4 \end{bmatrix}$$

The score values indicated the phase of plant disease. The impact of plant illnesses is identified by the lowest value.
1. i_2 and i_4 illnesses may have an impact on p_1.The impact of illnesses on p_1 can be mitigated by applying appropriate manure or fertilizer, as every value falls within the restricted positive range.
2. p_2 is influenced by i_1's illness. Due to its negative restricted range, the condition of p_2 can be effectively managed to prevent the plant from rotting.
3. p_3 is affected by illnesses i_2 and i_4. Because of its negatively restricted range, p_3 can be most effectively regulated in order to prevent the plant from rotting. However, yield has not increased.
4. The illnesses i_1 and i_3 afflict p_4, with each value falling within a restricted positive range, allowing for the application of appropriate manure or fertilizer to prevent all the illnesses.
Hence, Quadrasophic Fuzzy Matrix is a powerful tool for estimating uncertainty in agriculture, aiding in the identification of illness stages in targeted plant and improving production levels.

7. Conclusion

Certain operations of the Quadrasophic Fuzzy Set are defined in this artifact. The properties of Quadrasophic Fuzzy Algebra are defined. The development of the Quadrasophic Fuzzy Matrix and its properties are facilitated by the Quadrasophic Fuzzy Algebra. Propositions and theorems of the Quadrasophic Fuzzy Matrix are explained through relevant cases. Further, a new ranking technique utilizing the Quadrasophic Fuzzy Matrix tool is proposed and implemented to identify paddy ailments in the farming industry. The conceptual framework of the Quadrasophic Fuzzy Matrix can be used in decision-making, business statistics, medicine, and operations research. The forthcoming investigation will explore the features of the Quadrasophic Fuzzy Matrix and their uses in various disciplines.

References

[1] Aruna, G., J. Jesintha Rosline., and A. Anthoni Amali (2023). Quadrasophic Fuzzy Sets and its Applications in OCD - Decision Making Analysis. *Tuijin Jishu/Journal of Propulsion Technology* ISSN:1001-4055, Vol. 44 No. 4: 3241-3258.

[2] Aruna, G., and J. Jesintha Rosline.(2023). PROPERTIES OF QUADRASOPHIC FUZZY SET AND ITS APPLICATIONS. *Reliability: Theory & Applications* 18, no. 4 (76) : 208-220.

[3] Thomason, Michael G.(1977). Convergence of powers of a fuzzy matrix. *Journal of mathematical analysis and applications* 57, no. 2: 476-480.

[4] Mondal, Sanjib, and Madhumangal Pal.(2016). Rank of interval-valued fuzzy matrices.*Afrika Matematika* 27 : 97-114.

[5] Pal, M. (2001). Intuitionistic fuzzy determinant. *V. U. J. Phys Sci*7: 87-93.

[6] Shyamal, Amiya K., and Madhumangal Pal. (2004). Two new operators on fuzzy matrices. *Journal of Applied Mathematics and Computing* 15: 91-107.

[7] Cho, Han Hyuk.(1999). Regular fuzzy matrices and fuzzy equations. *Fuzzy sets and systems* 105, no. 3 : 445-451.

[8] Pal, Madhumangal. (2015). Interval-valued fuzzy matrices with interval-valued fuzzy rows and columns. *Fuzzy Information and Engineering* 7, no. 3 : 335-368.

[9] Shyamal, Amiya K., and Madhumangal Pal.(2007). Triangular fuzzy matrices. *Iranian Journal of Fuzzy Systems* 4(1): 75-87.

[10] Pal, Madhumangal.(2016).Fuzzy matrices with fuzzy rows and columns. *Journal of Intelligent & Fuzzy Systems* 30, no. 1 : 561-573.

[11] Mondal, Sanjib, and Madhumangal Pal.(2016). Similarity relations, eigenvalues and eigenvectors of bipolar fuzzy matrix. *Journal of Intelligent & Fuzzy Systems* 30, no. 4 : 2297-2307.

[12] Pal, Madhumangal, and Sanjib Mondal.(2019). Bipolar fuzzy matrices. *Soft computing* 23, no. 20 : 9885-9897.

[13] Dogra, Shovan, and Madhumangal Pal.(2020) Picture fuzzy matrix and its application.*Soft Computing* 24, no. 13: 9413-9428.

[14] GANI, A. NAGOOR, M. AFFROSE BEGUM, and P. MURUGANANTHAM.(2023). AN APPLICATION OF TRIANGULAR FUZZY NUMBER MATRICES WITH TRIPLET OPERATOR IN MEDICAL DIAGNOSIS. .*Advances and Applications in Mathematical Sciences* 22,no. 8: 1715-1720.

[15] Khan, Susanta Kumar, and Madhumangal Pal.(2014). Interval-valued intuitionistic fuzzy matrices. *arXiv preprint* arXiv:1404.6949. https://doi.org/10.48550/arXiv.1404.6949

[16] Meenakshi, A. R., and M. Kaliraja.(2011). An application of interval valued fuzzy matrices in medical diagnosis. *International Journal of Mathematical Analysis* 5, no. 36 : 1791-1802.

[17] Muthuraji, T., and P. Punitha Elizabeth.(2023). Application of bipolar fuzzy matrices in Flood Damage.*In 2023 Fifth International Conference on Electrical, Computer and Communication Technologies (ICECCT)*, IEEE.:pp. 1-5.

[18] Sanchez, Elie.(1979). Inverses of fuzzy relations. Application to possibility distributions and medical diagnosis.*Fuzzy sets and systems* 2, no. 1: 75-86.

[19] https://en.wikipedia.org/wiki/Agriculture_in_India