

AN IMPROVED ESTIMATOR OF FINITE POPULATION MEAN UNDER RANKED SET SAMPLING

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Abstract

To obtain reliable estimates of population parameters, data that is sampled for estimation must accurately represent the underlying population. Sampled data that is representative of the underlying population depends also on the sampling technique that was used in obtaining them. This is very important since sampling bias could lead to over or under estimation of parameters. Ranked Set Sampling is considered to be a better alternative to the classical sampling designs in obtaining such data. Ranked Set Sampling is designed to minimize the number of measured observations required to achieve a desired precision in making inferences, and thus it is more economical to use for the purposes of estimation, compared to the classical sampling designs. This is also an added advantage in cases where it is difficult to obtain data. Many estimators have been developed recently for the estimation of finite population mean under ranked set sampling. This paper aims to improve estimation by modifying an existing estimator using a simple linear combination of the known population mean, square root of the known coefficient of variation, and the known median of an auxiliary variable. The theoretical properties of the proposed estimator, such as the bias and mean squared error were derived up to the first order of approximation, using Taylor's expansion. The bias, mean squared error, absolute relative bias, and the relative efficiency were used as means of evaluation and comparison between the proposed modified estimator and its competitors. The R software was used to aid computations. Empirical applications to real data showed that the proposed modified estimator is superior to the competing estimators that were compared since it has least bias, the least mean squared error, the least absolute relative bias, and the highest relative efficiency in all sample sizes that were considered. The bias and mean squared error of the modified estimator under Ranked Set Sampling was found to be smaller than those of the existing estimators that were compared. Hence it is more efficient and capable of providing reliable estimates than the existing estimators that were compared and so we recommend that it should be used in survey estimations.

Keywords: Ranked Set Sampling, Ratio Estimator, Bias, Mean Squared Error, Auxiliary Variable

1. INTRODUCTION

Over the years, researchers have been preoccupied with the development of new estimators for finite heterogeneous population mean with the aim of reducing the associated bias and MSE of existing estimators to the barest minimum [1, 5, 9, 16]. For reliable estimates, data that is employed for estimation must be representative of the underlying population. Sampled data that is representative of the underlying population depends also on the sampling method [18]. Sampling bias could lead to over or under estimation of population parameters. Consequently,

one field of interest currently has been in the area of identifying designs that generate representative samples for super populations. Estimation of finite population mean has been based disproportionately on the classical sampling designs, especially simple random sampling (SRS). However, the SRS procedure as noted by [3] is incapable of generating representative samples for certain populations. The consequence, as noted by [10] is that, a specific sample which is not truly representative of the underlying population can possibly be included for estimation, and that can lead to unreliable estimates. Therefore, to improve accuracy and precision in the estimation of finite population mean, sampling procedures which do not suffer such weaknesses as the SRS must be considered. Among the sampling methods, the Ranked Set Sampling (RSS) technique is a good alternative to SRS for obtaining data that are truly representative of the population under study [2]. The goal of RSS is to collect observations that are more likely to span the full range of values in the population and therefore produces more representative samples than SRS [10]. RSS was first introduced by [12] and was used to estimate pasture yield. RSS was introduced for circumstances where difficulty exist in taking actual measurements for sample units. [17] established the statistical methodology for RSS.

The procedure for obtaining a ranked set sample is briefly outlined by the following steps:

1. Randomly select a sample of size m^2 from the targeted population.
2. Distribute the m^2 selected units in m sets, each of size m .
3. Rank the units within each set with respect to the attribute of interest, using the judgement of an expert or by the aid of an auxiliary variable that is correlated with the study variable.
4. Select the i^{th} ranked unit from the i^{th} set for actual measurement of the attribute of interest, in the order $i = 1, 2, 3, \dots, m$.
5. Repeat steps i to iv for r cycles if it is desired to obtain a sample of size, $n = mr$.

RSS is preferred when mechanisms are readily available for ranking a set of sample units, whether by the use of an auxiliary variable, or by the use of the judgement of an expert. [7] proved that the ranked set sample mean is an unbiased estimator for population mean, even in cases of imperfect ranking. [10] adduced that an auxiliary variable, X could be used to rank any variable under study, Y in cases where judgement ranking of Y is difficult. Consequently, a lot of estimators have been developed under RSS, employing a variety of auxiliary variables for ranking.

[16] introduced the classical ratio estimator under RSS. Several other authors have since extended the work of [16], employing a variety of auxiliary variables. [11] suggested a modified ratio estimator for population mean under RSS utilizing the quartile deviations and the known mean of an auxiliary variable. [4] proposed a generalized ratio estimator for population mean under RSS using the known population mean of an auxiliary variable and some pre-assigned constants. [13] suggested a modified ratio-cum-product estimator for finite population mean under RSS using the known population information on the mean, the coefficients of variation and of kurtosis of an auxiliary variable under RSS. [15] proposed a ratio-type estimator for population mean under RSS, using the known population mean and quartiles of an auxiliary variable. [8] proposed a ratio-type estimator under RSS based on known population mean and population deciles of an auxiliary variable. [9] proposed a generalised ratio-type estimator based on RSS, employing known parameters of the population such as the coefficients of variation, kurtosis and skewness as well as the mean of the auxiliary variable. [14] suggested a ratio type estimator of population mean based on RSS employing the known coefficient of variation, known median, as well as the known population mean of the auxiliary variable. These estimators were more efficient and superior to their competitors. Notwithstanding, the existing estimators wield significant biases and are fraught with large mean squared errors. Therefore, this study sought to improve estimation by modifying an existing estimator of finite population mean that was based on RSS.

2. REVIEW OF EXISTING ESTIMATORS

Suppose the study variable Y and the auxiliary variable, X are positively correlated. Then [16] expressed the classical ratio estimator of population mean under RSS as

$$\bar{y}_{R,RSS} = \bar{y}_{[n]} \left(\frac{\bar{X}}{\bar{x}_{(n)}} \right) \quad (2.1)$$

where

$$Bias(\bar{y}_{R,RSS}) = \bar{Y} \left[\theta \left(C_x^2 - \rho C_x C_y \right) - \left(W_{x(i)}^2 - W_{yx(i)} \right) \right] \quad (2.2)$$

$$MSE(\bar{y}_{R,RSS}) = \bar{Y}^2 \left[\theta \left(C_x^2 - 2\rho C_x C_y + C_y^2 \right) - \left(W_{x(i)}^2 - 2W_{yx(i)} + W_{y[i]}^2 \right) \right] \quad (2.3)$$

Through out of this study,

$$W_{x(i)}^2 = \frac{1}{r} \sum_{i=1}^m \left(\frac{\mu_{x(i)} - \bar{X}}{m\bar{X}} \right)^2, W_{y[i]}^2 = \frac{1}{r} \sum_{i=1}^m \left(\frac{\mu_{y[i]} - \bar{Y}}{m\bar{Y}} \right)^2, W_{yx(i)} = \sum_{i=1}^m \frac{(\mu_{y[i]} - \bar{Y})(\mu_{x(i)} - \bar{X})}{m^2 r \bar{Y} \bar{X}}, \theta = \frac{1}{m r}, C_x^2 = \frac{s_x^2}{\bar{X}^2}$$

and $C_y^2 = \frac{s_y^2}{\bar{Y}^2}$.

[13] modified the classical ratio estimator of population mean under RSS respectively using C_x and $\beta_2(x)$ as

$$\bar{y}_{M1,RSS} = \bar{y}_{[n]} \left[\frac{\bar{X} + C_x}{\bar{x}_{(n)} + C_x} \right] \quad (2.4)$$

and

$$\bar{y}_{M2,RSS} = \bar{y}_{[n]} \left[\frac{\bar{X} C_x + \beta_2(x)}{\bar{x}_{(n)} C_x + \beta_2(x)} \right]. \quad (2.5)$$

with the respective biases

$$B(\bar{y}_{M1,RSS}) = \bar{Y} \left[\theta \left(\varphi^2 C_x^2 - \varphi \rho C_x C_y \right) - \left(\varphi^2 W_{x(i)}^2 - \varphi W_{yx(i)} \right) \right] \quad (2.6)$$

$$B(\bar{y}_{M2,RSS}) = \bar{Y} \left[\theta \left(v^2 C_x^2 - v \rho C_x C_y \right) - \left(v^2 W_{x(i)}^2 - v W_{yx(i)} \right) \right] \quad (2.7)$$

and the respective MSEs

$$MSE(\bar{y}_{M1,RSS}) = \bar{Y}^2 \left[\theta \left(\varphi^2 C_x^2 + C_y^2 - 2\varphi \rho C_x C_y \right) - \left(\varphi^2 W_{x(i)}^2 + W_{y[i]}^2 - 2\varphi W_{yx(i)} \right) \right] \quad (2.8)$$

$$MSE(\bar{y}_{M2,RSS}) = \bar{Y}^2 \left[\theta \left(v^2 C_x^2 + C_y^2 - 2v \rho C_x C_y \right) - \left(v^2 W_{x(i)}^2 + W_{y[i]}^2 - 2v W_{yx(i)} \right) \right] \quad (2.9)$$

where

$$\varphi = \frac{\bar{X}}{\bar{X} + C_x}$$

$$v = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_2(x)}$$

[14] used the coefficient of variation (C_x) and the median of an auxiliary variable (M_d) to propose a ratio-type estimator of population mean as

$$\bar{y}_{P,RSS} = \bar{y}_{[n]} \left[\frac{\bar{X} + C_x M_d}{\bar{x}_{(n)} + C_x M_d} \right]. \quad (2.10)$$

with the respective bias and MSE as

$$B(\bar{y}_{P,RSS}) = \bar{Y} \left[\theta \left(\lambda^2 C_x^2 - \lambda \rho C_x C_y \right) - \left(\lambda^2 W_{x(i)}^2 - \lambda W_{yx(i)} \right) \right] \quad (2.11)$$

and

$$MSE(\bar{y}_{P,RSS}) = \bar{Y}^2 \left[\theta \left(\lambda^2 C_x^2 + C_y^2 - 2\lambda \rho C_x C_y \right) - \left(\lambda^2 W_{x(i)}^2 + W_{y[i]}^2 + 2\lambda W_{yx(i)} \right) \right] \quad (2.12)$$

where

$$\lambda = \frac{\bar{X}}{\bar{X} + C_x M_d}$$

3. THE PROPOSED ESTIMATOR

Motivated by [14], this study proposes modified ratio-type estimator for finite population mean under Ranked Set Sampling, which utilizes the coefficient of variation (C_x) and the median (M_d) of the employed auxiliary variable as

$$\bar{y}_{B,RSS} = \bar{y}_{[n]} \left[\frac{\bar{X} + M_d \sqrt{C_x}}{\bar{x}_{(n)} + M_d \sqrt{C_x}} \right]. \quad (3.1)$$

Using large sample properties, the following assumptions are made: $\bar{y}_{[n]} = \bar{Y}(1 + e_0)$ and $\bar{x}_{(n)} = \bar{X}(1 + e_1)$, where $E(e_0) = E(e_1) = 0$.

Therefore, equation (3.1) evolves as

$$\begin{aligned} \bar{y}_{B,RSS} &= \bar{y}_{[n]} \left[\frac{\bar{X} + M_d \sqrt{C_x}}{\bar{X}(1 + e_1) + M_d \sqrt{C_x}} \right] \\ &= \bar{y}_{[n]} \left[\frac{\bar{X} + M_d \sqrt{C_x}}{\bar{X} + M_d \sqrt{C_x} + \bar{X}e_1} \right] \\ &= \bar{y}_{[n]} \left[\frac{1}{1 + \left(\frac{\bar{X}}{\bar{X} + M_d \sqrt{C_x}} \right) e_1} \right] \\ &= \bar{y}_{[n]} \left(\frac{1}{1 + \omega e_1} \right) \end{aligned}$$

where

$$\omega = \frac{\bar{X}}{\bar{X} + M_d \sqrt{C_x}}.$$

Now,

$$\begin{aligned} \bar{y}_{B,RSS} &= \bar{y}_{[n]} \left(\frac{1}{1 + \omega e_1} \right) \\ &= \bar{Y}(1 + e_0) (1 + \omega e_1)^{-1}. \end{aligned}$$

Assuming $|\omega e_1| < 1$ and using Taylor's expansion to the second order,

$$\begin{aligned} \bar{y}_{B,RSS} &= \bar{Y}(1 + e_0) \left(1 - \omega e_1 + \omega^2 e_1^2 + \dots \right) \\ &= \bar{Y} \left(1 - \omega e_1 + \omega^2 e_1^2 + e_0 - \omega e_0 e_1 + \dots \right). \end{aligned}$$

Therefore the bias of the proposed estimator is obtained as

$$B(\bar{y}_{B,RSS}) = \bar{Y} \left[\omega^2 E(e_1^2) - \omega E(e_0 e_1) \right]$$

$$\begin{aligned} \Rightarrow B(\bar{y}_{B,RSS}) &= \bar{Y} \left[\omega^2 (\theta C_x^2 - W_{x(i)}^2) - \omega (\theta \rho C_x C_y - W_{yx(i)}) \right] \\ &= \bar{Y} \left[\theta (\omega^2 C_x^2 - \omega \rho C_x C_y) - (\omega^2 W_{x(i)}^2 - \omega W_{yx(i)}) \right] \end{aligned} \quad (3.2)$$

The mean squared error of the proposed estimator is obtained as

$$\begin{aligned} MSE(\bar{y}_{B,RSS}) &= \bar{Y}^2 \left[\omega^2 E(e_1^2) - 2\omega E(e_0 e_1) + E(e_0^2) \right] \\ \Rightarrow MSE(\bar{y}_{B,RSS}) &= \bar{Y}^2 \left[\omega^2 (\theta C_x^2 - W_{x(i)}^2) + (\theta C_y^2 - W_{y[i]}^2) - 2\omega (\theta \rho C_x C_y - W_{yx(i)}) \right] \\ &= \bar{Y}^2 \left[\theta (\omega^2 C_x^2 + C_y^2 - 2\omega \rho C_x C_y) - (\omega^2 W_{x(i)}^2 + W_{y[i]}^2 - 2\omega W_{yx(i)}) \right] \end{aligned} \quad (3.3)$$

4. EFFICIENCY COMPARISON

The proposed estimator, $\bar{y}_{B,RSS}$ was compared to the RSS estimators of [13] and that of [14]. The proposed estimator, $\bar{y}_{B,RSS}$ is more efficient than the estimator of [14] if

$$\begin{aligned} MSE(\bar{y}_{B,RSS}) &< MSE(\bar{y}_{P,RSS}) \\ \Rightarrow \omega^2 (\theta C_x^2 - W_{x(i)}^2) - 2\omega (\theta \rho C_x C_y - W_{yx(i)}) &< \lambda^2 (\theta C_x^2 - W_{x(i)}^2) - 2\lambda (\theta \rho C_x C_y - W_{yx(i)}) \\ \Rightarrow (\omega^2 - \lambda^2) (\theta C_x^2 - W_{x(i)}^2) &< 2(\omega - \lambda) (\theta \rho C_x C_y - W_{yx(i)}) \end{aligned}$$

Hence, provided $\omega < \lambda$, the proposed estimator is more efficient than the estimator of [14] if

$$\rho < \frac{(\omega + \lambda) (\theta C_x^2 - W_{x(i)}^2) + 2W_{yx(i)}}{2\theta C_x C_y} \quad (4.1)$$

where ρ is the correlation coefficient between the auxiliary variable X and the study variable Y . Let $P = \theta C_x^2 - W_{x(i)}^2$, $Q = 2W_{yx(i)}$ and $R = 2\theta C_x C_y$. Then the proposed estimator, $\bar{y}_{B,RSS}$ is respectively more efficient than $\bar{y}_{M1,RSS}$ and $\bar{y}_{M2,RSS}$ if

$$\rho < [P(\omega + \varphi) + Q] / R \text{ and } \rho < [P(\omega + v) + Q] / R,$$

where

$$\omega = \frac{\bar{X}}{\bar{X} + M_d \sqrt{C_x}}, \quad \varphi = \frac{\bar{X}}{\bar{X} + C_x}, \quad v = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_2(x)}.$$

5. EMPIRICAL APPLICATION

The dataset that was used for evaluating the estimators is taken from page 34 of [6] and a general description is given below.

X : Weekly family income.

Y : Weekly family expenditure.

Objective: To estimate mean weekly family expenditure.

$N = 33$, $\bar{X} = 72.5454$, $\bar{Y} = 27.4909$, $\rho = 0.2521$, $M_d = 69$, $\beta_2(x) = 2.1429$, $C_x = 0.1436$, $C_y = 0.3629$

The ARB of the various proposed estimators were obtained by the formula

$$ARB = \left| \frac{Bias(\bar{y}_i)}{Bias(\bar{y}_{R,RSS})} \right|, \quad (5.1)$$

where $i = (M1,RSS), (M2,RSS), (P,RSS), (B,RSS)$.

The Percent Relative Efficiency (PRE) of an estimator \bar{y}_i compared to the classical ratio estimator \bar{y}_R of [5], was obtained by

$$PRE = \frac{MSE(\bar{y}_R)}{MSE(\bar{y}_i)} \times 100, \tag{5.2}$$

where $i = (M1,RSS), (M2,RSS), (P,RSS), (B,RSS)$.

Six ranked set sample sizes were considered with the data for different set sizes m and the corresponding number of cycles r and the results displayed in Tables 1 to 6. For each case, corresponding values of $W_{x(i)}^2, W_{y[i]}^2$ and $W_{yx(i)}$ were determined for the sample size $n = m \times r$.

For sample size $n = 9$, where $m = 3$ and $r = 3$, then $W_{x(i)}^2 = 0.0012, W_{y[i]}^2 = 0.0064, W_{yx(i)} = 0.0028$ and the corresponding performance of the various estimators is displayed in Table 1.

Table 1: $m=3, r=3$

Estimator	Bias	MSE	ARB	PRE
$\bar{y}_{R, RSS}$	0.0668	9.0725	1.0000	100.0
$\bar{y}_{M1, RSS}$	0.0667	9.0653	0.9985	100.1
$\bar{y}_{M2, RSS}$	0.0562	8.6695	0.8413	104.6
$\bar{y}_{P, RSS}$	0.0556	8.6428	0.8323	105.0
$\bar{y}_{B, RSS}$	0.0433	8.1567	0.6482	111.2

If $n = 12$ where $m = 3, r = 4$, then $W_{x(i)}^2 = 0.0009, W_{y[i]}^2 = 0.0050, W_{yx(i)} = 0.0021$ and the performance of the various estimators is displayed in Table 2.

Table 2: $m=3, r=4$

Estimator	Bias	MSE	ARB	PRE
$\bar{y}_{R, RSS}$	0.0501	6.6533	1.0000	100.0
$\bar{y}_{M1, RSS}$	0.0450	6.6478	0.8982	100.1
$\bar{y}_{M2, RSS}$	0.0418	6.3319	0.8343	105.1
$\bar{y}_{P, RSS}$	0.0417	6.3310	0.8323	105.1
$\bar{y}_{B, RSS}$	0.0325	5.9664	0.6487	111.5

If $n = 15$ where $m = 3, r = 5$, then $W_{x(i)}^2 = 0.0007, W_{y[i]}^2 = 0.0040, W_{yx(i)} = 0.0017$ and the performance of the various estimators is displayed in Table 3.

Table 3: $m=3, r=5$

Estimator	Bias	MSE	ARB	PRE
$\bar{y}_{R, RSS}$	0.0412	5.3680	1.0000	100.0
$\bar{y}_{M1, RSS}$	0.0411	5.3635	0.9976	100.1
$\bar{y}_{M2, RSS}$	0.0346	5.1103	0.8398	105.0
$\bar{y}_{P, RSS}$	0.0343	5.1031	0.8325	105.2
$\bar{y}_{B, RSS}$	0.0267	4.8055	0.6481	111.7

If $n = 16$ where $m = 4, r = 4$, then $W_{x(i)}^2 = 0.0008, W_{y[i]}^2 = 0.0044, W_{yx(i)} = 0.0019$ and the performance of the various estimators is displayed in Table 4.

Table 4: $m=4, r=4$

Estimator	Bias	MSE	ARB	PRE
$\bar{y}_{R, RSS}$	0.0431	4.8955	1.0000	100.0
$\bar{y}_{M1, RSS}$	0.0429	4.8908	0.9954	100.1
$\bar{y}_{M2, RSS}$	0.0367	4.5020	0.8515	108.7
$\bar{y}_{P, RSS}$	0.0364	4.5020	0.8445	108.7
$\bar{y}_{B, RSS}$	0.0291	4.2936	0.6752	114.1

For $n = 20$ where $m = 4, r = 5$, then $W_{x(i)}^2 = 0.0006, W_{y[i]}^2 = 0.0036, W_{yx(i)} = 0.0015$ and the performance of the various estimators is displayed in Table 5.

Table 5: $m=4, r=5$

Estimator	Bias	MSE	ARB	PRE
$\bar{y}_{R, RSS}$	0.0350	3.8559	1.0000	100.0
$\bar{y}_{M1, RSS}$	0.0349	3.8521	0.9971	100.1
$\bar{y}_{M2, RSS}$	0.0310	3.6368	0.8857	106.0
$\bar{y}_{P, RSS}$	0.0296	3.6292	0.8457	106.3
$\bar{y}_{B, RSS}$	0.0234	3.3685	0.6686	114.5

If $n = 25$ where $m = 5, r = 5$, then $W_{x(i)}^2 = 0.0006, W_{y[i]}^2 = 0.0033, W_{yx(i)} = 0.0013$ and the performance of the various estimators is displayed in Table 6.

Table 6: $m=5, r=5$

Estimator	Bias	MSE	ARB	PRE
$\bar{y}_{R, RSS}$	0.0275	2.8278	1.0000	100.0
$\bar{y}_{M1, RSS}$	0.0274	2.8248	0.9963	100.1
$\bar{y}_{M2, RSS}$	0.0239	2.6750	0.8691	105.7
$\bar{y}_{P, RSS}$	0.0235	2.6487	0.8545	106.8
$\bar{y}_{B, RSS}$	0.0190	2.4400	0.6909	115.9

6. CONCLUSION

The study modified the ratio estimator of [14] and derived the theoretical properties of the modified estimator up to order $O(n^{-1})$. The modified estimator was compared to the all the RSS estimators that were considered by [14] using the classical RSS ratio estimator of [16] as the basis of comparison. Ranked Set Samples various sizes were considered to test the performance of the various estimators and all sizes, the proposed modified estimator had the least bias and MSE. Compared to the classical RSS ratio estimator of [16], the efficiency of the proposed modified estimator ranged from 11% to 16% whilst the efficiency of the estimator of [14] ranged from 5% to 8%. The bias of the proposed modified estimator was also the least in all the sample combinations that were considered. This study therefore recommend the use of improved estimator for estimation since it can provide more efficient and more accurate estimates, compared to its competitors.

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Data Availability: The data that was used in this study is available within the paper.

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