RT&A, No 3 (79)
Volume 19, September 2024

MODIFIED GROUP RUNS CONTROL CHART FOR MONITORING PROCESS DISPERSION

Chandrakant G. Gardi and Vikas B. Ghute[1]

•

Department of Statistics
Punyashlok Ahilyadevi Holkar Solapur University, Solapur (M.S.), India
[1]Corresponding Author: Email- vbghute_stats@rediffmail.com

Abstract

Due to a rise in competitiveness, it has become an intense concern to the manufacturers to monitor process dispersion to avoid low quality production. To ensure quality production, the control chart that gives early detection of change in the dispersion is always encouraged. Researchers have suggested various control charts based on different estimators of process dispersion. Recently, many synthetic control charts based on such estimators are put forth by researchers to effectively monitor the dispersion in the process. Modified Group Runs (MGR) control chart is an extension of synthetic charts with further enhancement in the detection ability. In this paper, we propose a MGR control chart based on Downton's estimator (D). Comparison of MGR control chart with synthetic chart based on estimator D reveals the enhanced performance of MGR-D chart.
Keywords: Control chart, Process dispersion, Downton estimator, Modified group runs.

I. Introduction

For optimizing the cost and resources constraints for the manufacturing processes, it is vital to monitor the changes in the process location and/or dispersion. Control chart is a prominently used statistical process control tool to identify any change in the process parameters. Shewhart [1] proposed the idea of control charts for monitoring the process, in which certain thresholds are set for a test statistic and process is said to be in control as long as test statistic falls within these threshold values. Bourke [2]) introduced the Conforming Run Length (CRL) chart for qualitative data. Wu and Spedding [3] introduced a synthetic control chart for monitoring process location as an enhancement to the Shewhart-type control chart, which includes $\overline{X}$ chart and CRL chart. When the dispersion of the underlying process is of interest, dispersion control charts are utilized. Huang and Chen [4] extended the application of synthetic charts to monitor process dispersion by introducing the synthetic S chart, which combines S and CRL chart. Chen and Huang [5] constructed synthetic R chart, which comprises of R and CRL chart. Synthetic charts have been observed to surpass Shewhart-type control charts in performance. The development of synthetic control charts is well documented by Davis and Woodall [6], Ghute and Shirke [7], Ghute and Shirke [8], Ghute and Shirke [9]. A detailed overview of synthetic charts is given by Rakitzis et al. [10].

Klein [11] proposed runs rule control chart to enhance the effectiveness of Shewhart control chart. These two approaches, synthetic as well as runs rule charts, are superior to Shewhart-type charts. Combining these two approaches, Gadre and Rattihalli [12] constructed Group Runs (GR) control chart for monitoring the process mean. GR control charts are demonstrated to be more effective than synthetic charts. Gadre and Kakade [13] proposed a nonparametric GR chart to detect shifts in the process median. Gadre and Rattihalli [14] introduced the Modified Group Runs scheme for detecting changes in the mean of a normally distributed process. The MGR chart is found to be more efficient than Shewhart, synthetic, and GR charts. The development of GR and MGR control charts is well documented by Gadre and Kakade [15], Rakitzis et al. [10], Khilare and Shirke [16], Ghadge and Ghute [17].

Researchers have also sought to enhance the effectiveness of control charts by considering alternative charting statistics. Abbasi and Miller [18] constructed a chart based on statistic D to monitor the process variability. The statistic D is proposed by Downton [19] as an unbiased estimator of standard deviation for normally distributed process. It has been demonstrated that the D chart is as effective as the Shewhart S chart in identifying changes in the standard deviation of a normally distributed process. Gardi and Ghute [20] constructed D chart using runs rule, whereas Rajmanya and Ghute [21] developed synthetic D chart as a combination of D chart and CRL chart, which enhanced the performance of D chart. This paper attempts to improve the performance of synthetic D chart by using the idea of Modified Group Runs (MGR-D chart).

The rest of the paper is structured as follows: The D chart and CRL chart are discussed in section 2. In section 3, MGR-D chart is proposed for monitoring the process dispersion. The performance evaluation and comparison of proposed chart with synthetic D chat is made in section 4. Conclusions are given in the section 5.

II. The D chart

For normally distributed process, Downton [19] proposed an unbiased estimator D of process standard deviation σ. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Then Downton's statistic D is given as

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[i - \frac{1}{2}(n+1)\right] X_{(i)} \tag{1}$$

where $X_{(i)}$ is the $i^{th}(i = 1,2, \ldots, n)$ order statistic for the given sample. Abbasi and Miler [18] developed Shewhart-type control chart based on the statistic D and revealed that the D-chart is as effective as Shewhart S chart in discovering a dispersion change. The probabiity limits for D-chart were obtained using the distribution of $Z = {}^{D}/_{\sigma}$. The Upper and Lower Control Limit (UCL and LCL) for the D chart is given as

$$\left.\begin{aligned} UCL &= Z_{1-\alpha}\overline{D} \text{ with } P(Z \geq Z_{1-\alpha}) = 1-\alpha \\ LCL &= Z_{\alpha}\overline{D} \text{ with } P(Z < Z_{\alpha}) = \alpha \end{aligned}\right\} \tag{2}$$

Where Z_α is α^{th} quantile point of the distribution of Z and α is Type-I error probability, specified in advance. To monitor process dispersion, D-values are plotted on the chart. Decision regarding out-of-control signal is taken based on whether a D-value goes beyond UCL, for the case of detecting positive shift or whether D-value is smaller than LCL, for the case of detecting negative shift.

We assume that the process parameters μ and σ^2 are known, that is, $\mu = \mu_0$ and $\sigma^2 = \sigma_0^2$ are in-control mean and variance of the process respectively. Let $\sigma_1 = \delta\sigma_0$ $(0 < \delta \neq 1)$ be the shifted value of standard deviation, with a shift of size δ. In order to detect a positive (negative) sift, that is, $\delta > 1$ $(\delta < 1)$ in the process standard deviation, an upper control limit $k^+\sigma_0$ (a lower control limit $k^-\sigma_0$) of D-chart is required, and a signal is given if $D > k^+\sigma_0$ $(D < k^-\sigma_0)$. The average run length (ARL) is the average number of D samples required to identify a shift in σ of the D-chart. It is calculated as

$$ARL_D(\delta) = \frac{1}{P(\delta)} \tag{3}$$

where $P(\delta)$ is the probability of detecting a shift δ in the process standard deviation.
For $\delta > 1$,

$$P(\delta) = P(D > k^{+}\sigma_0 \mid \sigma = \delta\sigma_0) = P\left(Z > {k^{+}}/{\delta}\right) = 1 - F(k^{+}/\delta) \quad (4)$$

For $\delta < 1$,

$$P(\delta) = P(D < k^{-}\sigma_0 \mid \sigma = \delta\sigma_0) = P\left(Z < {k^{-}}/{\delta}\right) = F\left({k^{-}}/{\delta}\right) \quad (5)$$

Where $F(.)$ denotes the cumulative distribution function.

Conforming Run Length (CRL) chart

For monitoring attribute characteristic, the conforming run length (CRL) chart is used, which detects shift in the fraction nonconforming p. The random variable CRL is defined as the number of conforming units between two successive nonconforming units including ending nonconforming unit. The distribution of CRL is as follows.

$$F(CRL) = 1 - (1-p)^{CRL}, CRL = 1,2,3, \dots \quad (6)$$

where p is the probability of the nonconforming unit. If practitioner is only concerned with detection of an increase in p, the lower control limit (L) of the CRL chart serves the purpose. It is given by

$$L = \frac{\ln(1-\alpha_{CRL})}{\ln(1-p_0)} \quad (7)$$

Where $\alpha_{CRL} = 1 - (1-p_0)^L = F_{p_0}(L)$ is the type-I error probability of the CRL chart and p_0 is the in-control fraction nonconforming. If a sample CRL is not greater than L, it indicates the increase in fraction nonconforming and an out-of-control signal is issued.

The ARL of such CRL chart is given by

$$ARL_{CRL} = \frac{1}{F_{CRL}(L)} = \frac{1}{1-(1-p)^L} \quad (8)$$

III. Modified Group Runs Control Chart

In this section, we present the design of modified group runs control chart based on Downton estimator D. Modified group runs control chart is the integration of D-chart and extended version of CRL chart. The D chart has only upper control limit (UCL) and CRL chart has two limits, namely the warning limit L_1 and the lower limit L_2. Let Y_r be the r^{th}group based CRL, then modified group runs control chart declares the process as out-of-control if $Y_1 \le L_1$ or for some r $(> 1), Y_r \le L_1$ and $Y_{r+1} \le L_2$.

The steps for implementing the MGR-D chart are as follows:

1. Fix the UCL for the D chart and L_1 and L_2 of the CRL chart.
2. Select a subgroup of n items at each inspection point j and compute the chart statistic, say D_j
3. If $D_j \le UCL$, then the subgroup is considered 'conforming' and control flow returns to step 2. Else the subgroup is considered 'nonconforming' and control flow goes to the next step.
4. Check the number $(CRL_i, i = 1,2, \dots)$ of subgroups between the current and previous nonconforming groups.
5. If $CRL_1 \le L_2$ or for some $i > 1, CRL_i \le L_1$ and $CRL_{i+1} \le L_2$, for $i = 2,3, \dots$ for the first time, the process is thought to be out-of-control, and control flow moves to the next step. Else flow returns to step 2.
6. Signal the out-of-control state.
7. An assignable cause should be identified and corrective action should be taken to remove it.

Let the expected number of subgroups needed to identify a shift of magnitude δ in process dispersion be $ARL_{MGR}(\delta)$. Following Gadre and Rattihalli (2004), the performance measure $ARL_{MGR}(\delta)$ for the MGR-D chart can be expressed as follows:

For increase in the process dispersion, that is, when $\delta > 1$,

$$ARL_{MGR}(\delta) = \frac{1}{P(\delta)} \times \frac{[1+Q(\delta)^{L_2} - Q(\delta)^{L_1}]}{[1-Q(\delta)^{L_1}][1-Q(\delta)^{L_2}]} \quad (9)$$

Where $Q(\delta) = 1 - P(\delta)$ and $P(\delta)$ is given in equation (4) when $\delta > 1$and is given in equation (5) when $\delta < 1$.

For constructing MGR-D chart, the below ARL model is used.

$$\text{Minimize } ARL_{MGR}(\delta)$$
$$\text{Subject to } ARL_{MGR}(0) \geq \tau$$

where τ is the minimum required value of $ARL_{MGR}(0)$.

Optimal Design Procedure

The optimal design procedure for MGR-D chart is given below:

1. Specify subgroup size n, shift size δ^* and in-control ARL as ARL_0.
2. Initialize L_1 as 1.
3. Initialize L_2 as 1.
4. Obtain $P(0)$ by solving Equation (9) numerically. From the obtained value of $P(0)$, obtain k^+ (or k^-) using Equation (4) (or Equation (5)).
5. From the current values of L_1, L_2 and k^+ (or k^-), obtain $P(0)$ from Equation (4) (or Equation (5)). Then compute $ARL_{MGR}(\delta)$ using Equation (9).
6. If $ARL_{MGR}(\delta)$ is reduced, then increase L_1 by 1 and go back to step 4. Else go to the next step.
7. Mark the minimum $ARL_{MGR}(\delta)$ for current combination of L_1, L_2. If currently marked $ARL_{MGR}(\delta)$ for (L_1, L_2) is less than previously marked $ARL_{MGR}(\delta)$, present pair of (L_1, L_2) is the optimum value. Else increase L_2 by 1, initialize L_1 as 1 and go back to step 4.

IV. Performance Evaluation

In this section the performance of proposed MGR-D chart is evaluated and is also compared with synthetic D chart. The comparison is made based on the Average Run Length (ARL). The underlying process is assumed to have normal distribution with mean 0 and variance 1. Since, the exact distribution of D statistic is not, we used simulation approach to obtain quantiles of distribution of D. We considered three different subgroup sizes as $n = 5,8,10$. For each subgroup size, 50000 subgroups were simulated. Based on D statistic of these 50000 subgroups, required quantile points of the distribution of D were determined. This process was repeated 500 times and average of these 500 quantiles was considered for obtaining control limits of the proposed chart. For fair comparison, all charts are fabricated such that in-control ARL remains the same as 200. The optimal design parameters L_1, L_2 and k^+(or k^-) of proposed MGR-D chart are obtained for a pre-determined shift of size $\delta = 1.2$ ($\delta = 0.8$) for positive (negative) shift in process deviation. The L_1 is initiated as 1, and L_2 is gradually incremented by 1, for each combination of (L_1, L_2), ARL is determined. Once the minimum ARL is achieved, the combination (L_1, L_2) is noted and L_1 is incremented by 1 and same process is repeated. If the minimum ARL for present combination of (L_1, L_2) is greater than that for the earlier combination of (L_1, L_2), the earlier combination of (L_1, L_2) is considered. That is, the combination of (L_1, L_2) at which ARL attains its minimum across L_1 as well as L_2. Table 1 gives a demonstration for obtaining the optimal parameters for $n = 10$ for detecting positive shift. Here, the minimum ARL is 4.235941 and is attained at $(L_1 = 1, L_2 = 13)$.

Once the limits are set ensuring in-control ARL to be 200, out-of-control ARLs are determined for various shift sizes. For positive shift in the process dispersion, the shift sizes considered are $\delta = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 2.0$ and that for negative shift in the process dispersion are $\delta = 0.9, 0.8, 0.7, 0.6, 0.5, 0.1$ The ARL values are determined using simulation of size 50000 for various sample sizes as well as for different subgroup sizes. Table 2 presents these ARL values as well as ARL values for synthetic D chart for positive shift in process dispersion, whereas Table 3 provides ARLs for negative shift. Since out-of-control ARL values of MGR-D chart are smaller than that of the synthetic-D chart for all considered shifts in the process standard deviation and considered subgroup sizes, the proposed MGR-D chart is very superior to synthetic D chart for shifts in either direction as well as for different subgroup sizes.

Table 1: *Optimal Parameters for MGR-D chart.*

L_1	L_2	k^+	ARL	L_1	L_2	k^+	ARL
1	1	1.229106	11.51437	2	1	1.257766	11.71673
1	2	1.271325	8.132786	2	2	1.29923	8.422839
1	3	1.295223	6.644562	2	3	1.32252	7.002282
1	4	1.311976	5.79848	2	4	1.338762	6.204549
1	5	1.324845	5.264146	2	5	1.351146	5.696124
1	6	1.335386	4.913096	2	6	1.361216	5.359928
1	7	1.344217	4.66926	2	7	1.369726	5.130402
1	8	1.351881	4.502848	2	8	1.37713	4.97412
1	9	1.358633	4.38984	2	9	1.38362	4.866527
1	10	1.364715	4.316068	2	10	1.389399	4.794161
1	11	1.370216	4.269155	2	11	1.394621	4.74751
1	12	1.37528	4.244542	2	12	1.399379	4.721099
1	**13**	**1.379935**	**4.235941**	2	13	1.403793	4.710871
1	14	1.384273	4.239508	2	14	1.40785	4.711723

Table 2: *ARL comparison for positive shift in process dispersion.*

Shift (δ)	$n = 5$		$n = 8$		$n = 10$	
	Synthetic D chart	MGR D Chart	Synthetic D chart	MGR D Chart	Synthetic D chart	MGR D Chart
	$L = 17$ $k^+ = 1.843$	$L_1 = 1$ $L_2 = 24$ $k^+ = 1.647$	$L = 12$ $k^+ = 1.595$	$L_1 = 1$ $L_2 = 16$ $k^+ = 1.448$	$L = 12$ $k^+ = 1.519$	$L_1 = 1$ $L_2 = 13$ $k^+ = 1.380$
1.0	200 (2.371)	199.498 (1.99014)	200 (2.366)	199.864 (1.74324)	201 (2.388)	199.787 (1.63627)
1.1	43.92 (0.560)	22.1446 (0.27269)	32.80 (0.422)	16.271 ((0.17797)	28.61 (0.366)	14.3427 (0.14795)
1.2	15.89 (0.202)	7.23788 (0.05486)	10.75 (0.132)	5.07486 (0.03393)	8.66 (0.103)	4.23606 (0.02686)
1.3	8.34 (0.093)	4.45104 (0.01932)	5.27 (0.057)	3.09928 (0.01272)	4.41 (0.045)	2.61958 (0.00995)
1.4	5.38 (0.055)	3.32406 (0.01248)	3.44 (0.032)	2.32074 (0.00785)	2.89 (0.024)	1.97842 (0.00617)
1.5	3.92 (0.036)	2.68268 (0.00952)	2.56 (0.028)	1.89742 (0.00585)	2.16 (0.016)	1.62993 (0.0045)
2.0	1.79 (0.012)	1.51418 (0.00395)	1.31 (0.007)	1.1828 (0.00208)	1.18 (0.005)	1.10386 (0.00151)

Table 3: *ARL comparison for negative shift in process dispersion.*

Shift (δ)	n = 5		n = 8		n = 10	
	Synthetic D chart	MGR-D Chart	Synthetic D chart	MGR-D Chart	Synthetic D chart	MGR-D Chart
	$L = 5$ $k^{-} = 0.396$	$L_1 = 1$ $L_2 = 36$ $k^{+} = 0.423$	$L = 7$ $k^{+} = 0.516$	$L_1 = 1$ $L_2 = 18$ $k^{+} = 0.587$	$L = 6$ $k^{+} = 0.5757$	$L_1 = 1$ $L_2 = 13$ $k^{+} = 0.648$
1.0	200 (2.433)	200.11 (2.254)	200 (2.292)	201.67 (1.829)	200 (2.228)	199.9 (1.674)
0.9	68.14 (0.840)	54.32 (0.73)	67.11 (0.808)	32.74 (0.367)	55.55 (0.651)	26.31 (0.271)
0.8	30.20 (0.382)	16.33 (0.194)	22.28 (0.291)	7.57 (0.067)	15.47 (0.198)	5.47 (0.042)
0.7	15.98 (0.201)	7.56 (0.045)	8.00 (0.096)	3.46 (0.014)	5.19 (0.061)	2.54 (0.01)
0.6	10.07 (0.120)	4.51 (0.018)	3.32 (0.033)	2.05 (0.007)	2.28 (0.019)	1.55 (0.004)
0.5	6.99 (0.076)	2.78 (0.01)	1.78 (0.012)	1.34 (0.003)	1.35 (0.007)	1.13 (0.002)
0.1	2.78 (0.023)	1.00 (0)	1.00 (0)	1.00 (0)	1.00 (0)	1.00 (0)

V. Conclusions

In this paper, MGR-D control chart is proposed for monitoring changes in the process dispersion of normally distributed process. The proposed chart is based on Downton's statistic D and is an integration of D chart and an extended version of CRL chart. The ARL comparison highlights that the proposed MGR-D chart performs better than the synthetic D chart.

References

[1] Shewhart, W. A. (1931). Economic control of quality of manufactured product. Reprinted by the American Society for Quality Control in 1980.

[2] Bourke, P. D. (1991). Detecting a shift in fraction nonconforming using run-length control charts with 100% inspection. Journal of Quality Technology, 23(3), 225-238.

[3] Wu, Z. and Spedding, T. A. (2000). A synthetic control chart for detecting small shifts in the process mean. Journal of Quality Technology, 32(1), 32-38.

[4] Huang, H. J. and Chen, F. L. (2005). A synthetic control chart for monitoring process dispersion with sample standard deviation. Computers and Industrial Engineering, 49(2), 221-240.

[5] Chen, F. L. and Huang, H. J. (2005). A synthetic control chart for monitoring process dispersion with sample range. The International Journal of Advanced Manufacturing Technology, 26, 842-851.

[6] Davis, R. B., and Woodall, W. H. (2002). Evaluating and improving the synthetic control chart. Journal of Quality Technology, 34(2), 200-208.

[7] Ghute, V. B. and Shirke, D. T. (2007). Joint monitoring of multivariate process using synthetic control charts, International Journal of Statistics and Management System, 2(1-2), 129-141.

[8] Ghute, V. B., and Shirke, D. T. (2008a). A multivariate synthetic control chart for monitoring process mean vector. Communications in Statistics-Theory and Methods, 37(13), 2136-2148.

[9] Ghute, V. B. and Shirke, D. T. (2008b). A multivariate synthetic control chart for process dispersion. Quality Technology and Quantitative Management, 5(3), 271-288.

[10] Rakitzis, A. C., Chakraborti, S., Shongwe, S. C., Graham, M. A., andKhoo, M. B. C. (2019). An overview of synthetic-type control charts: Techniques and methodology. Quality and Reliability Engineering International, 35(7), 2081-2096.

[11] Klein, M. (2000). Two alternatives to the Shewhart X control chart. Journal of Quality Technology, 32(4), 427-431.

[12] Gadre, M.P. and Rattihalli, R.N. (2004). A group runs control chart for detecting shifts in the process mean. Economic Quality Control, 19, 29-43.

[13] Gadre, M. P.and Kakade, V. C. (2014). A nonparametric group runs control chart to detect shift in the process median. IAPQR Transactions, 39(1), 29-53.

[14] Gadre, M. P. and R. N. Rattihalli (2006) Modified Group Runs Control Charts to Detect Increases in Fraction Non Conforming and Shifts in the Process Mean, Communications in Statistics - Simulation and Computation, 35:1, 225-240, DOI: 10.1080/03610910500416256

[15] Gadre, M. P. and Kakade, V. C. (2016). Some group runs based multivariate control charts for monitoring the process mean vector. Open Journal of Statistics, 6(6), 1098-1109.

[16] Khilare, S. K., and Shirke, D. T. (2023). A nonparametric group runs control chart for location using sign statistic. Thailand Statistician, 21(1), 165-179.

[17] Ghadge, O.D. and Ghute, V. B. (2023). Group runs and modified group runs control charts for monitoring linear regression profiles. Reliability: Theory & Applications, 18 (4 (76)), 513-524.

[18] Abbasi, S. A. and Miller, A. (2011). D chart: An efficient alternative to monitor process dispersion. In Proceedings of the World Congress on Engineering and Computer Science (Vol. 2, pp. 19-21).

[19] Downton, F. (1966). Linear estimates with polynomial coefficients. Biometrika, 53(1/2), 129-141.

[20] Gardi, C. and Ghute, V. (2023). A new control chart for process dispersion based on ranked set sampling. Reliability: Theory & Applications, 18 (2 (73)), 154-166. doi: 10.24412/1932-2321-2023-273-154-166

[21] Rajmanya, S. V. and Ghute, V. B. (2014). A synthetic control chart for monitoring process variability. Quality and Reliability Engineering International, 30(8), 1301-1309.