FUZZY VARIABLE LINEAR PROGRAMMING PROBLEMS USING A FUZZY DUAL SIMPLEX ALGORITHM

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Abstract

In modern research, several brilliant minds investigate linear programming problems involving fuzzy variable quantities. Many researchers have turned to linear programming by fuzzy variables to address this problem. Various fuzzy simplex approaches have been developed, using ranking functions to handle fuzzy numbers. Results from this research suggest that linear ranking functions can provide a straightforward interpretation of problems involving linear programming by fuzzy variable quantities. To solve these types of problems, the Fuzzy Dual Simplex Tableau method is often applied, which proves useful for sensitivity analysis when modifications are made to the activity vectors of the fundamental columns. In this study, a numerical case is presented to demonstrate the potential benefits of this approach for future technologies.

Keywords: Linear programming by fuzzy variable quantity, fuzzy numbers, ranking functions, fuzzy simplex and duality algorithms, and trapezoidal fuzzy numbers.

I. Introduction

The fuzzy set model has proven useful in several areas, including simulation, artificial intelligence, control systems and organisational skills, scientific modelling, operations research, and manufacturing applications. The results of fuzzy mathematical programming problems have been the subject of extensive theoretical and computational research in recent years. On a more global scale, this concept was initially developed by [2] and presented the first-ever construction of fuzzy linear programming (FLP). Since then, various authors have investigated different FLP challenges and proposed various approaches to addressing them [3-6]. To address issues with fuzzy linear programming, several authors have turned to the method of fuzzy-number comparison. Methods that employ the idea of comparing fuzzy numbers with rank-order functions [2, 3, 6] have proven to be the most useful. Researchers have developed ranking functions to suit their purposes, but it is possible that these proposals are not necessarily based on industry-standard practices Fuzzy mathematical programming is reviewed in [7]. While there is a wealth of literature on modelling, publications on duality still need to be made available, leading to issues with a linear program in a fuzzy setting [7,8, 9] suggesting a primal-dual strategy utilizing linear ranking functions for resolving linear programming with fuzzy variable quantities. Based on the duality outcomes published [4], this research provides an innovative fuzzy dual simplex strategy for solving fuzzy number linear programming problems. This method will benefit sensitivity analysis when the recent set of primary vectors rejection extended method is fundamental after difference due to changes in the elementary column. We base our method for solving FNLP problems on the linear rank functions initially introduced by [11] and exploited by in [4-6]. They point out that while trapezoidal fuzzy numbers were used to construct the procedure described in this research, linear ranking functions are only some of the options available.

Introduced a duality model to address issues with FVLP [4]. This inquiry emphasizes the problems associated with FVLPs. We, therefore, analyze the initial reactions to the theory and determine how people respond to the contrast presented by fuzzy numbers using a linear ranking function. Furthermore, a fuzzy elementary achievable resolution is suggested for FVLP problems and related optimality requirements, and a fuzzy dual simplex technique is projected to be useful in tackling FVLP concerns.

The procedures to be followed in writing this paper include providing an overview of the basic concepts of fuzzy established models in Part 2, presenting a brief overview of linear programming using fuzzy numbers, discussing the issue and definition of the consistent dual dilemma in Section II&III, advancing a fuzzy dual technique based on the fuzzy simplex tableau for solving linear programming problems using fuzzy numbers in section IV, and presenting the final results in Section V.

II. Preliminaries and fundamental concepts

The fundamental terms and symbols of the fuzzy established model are briefly covered (taken from [4]).

I. Definition Assume ® as a collective set. $\bar{a}$ are termed as fuzzy set of X if $\bar{a}$ is a set of well-organized sets $\bar{a}$ = {$((x, \mu_{\bar{a}}(x))/\ x \in$ ®}, wherever $\mu_{\bar{a}}(x)$ is relationship function corresponding to $\bar{a}$, $\mu_{\bar{a}}(x) \in [0,1]$.

II. Definition The α-level related to a fuzzy set $\bar{a}$ are termed for a normal set $[\bar{a}]_\alpha$ for whom the Maximum power of its relationship function tops the level α, $[\bar{a}]_\alpha = \{x \in$ ® $| \mu_{\bar{a}}(x) \geq \alpha\}$.

III. Definition Support for fuzzy set $\bar{a}$ $\in$ ® for $\mu_{\bar{a}}(x)$ are positive, supp $\bar{a}$ = {$x \in$ ® $| \mu_{\bar{a}}(x) > 0)$ } .

IV. Definition Fuzzy set $\bar{a}$ will be referred to as convex only in the case of $x, y \in$ ®, $\rho \in [0,1]$, $\mu_{\bar{a}}(\rho x + (1-\rho)y) \geq \min\{\mu_{\bar{a}}(x), \mu_{\bar{a}}(y)\}$.

V. Definition A LR-type fixed fuzzy number [1] can be represented by $\bar{a} = (a^L, a^U, \alpha, \beta)_{LR}$, only for

$$\mu_{\bar{a}}(x) = \begin{cases} L\left(\frac{a^L - x}{\alpha}\right) & for\ a^L - \alpha \leq x \leq a^L \\ 1 & for\ a^L \leq x \leq a^U \\ R\left(\frac{x - a^L}{\beta}\right) & for\ a^U \leq x \leq a^U + \beta \\ 0 & else \end{cases} \tag{1}$$

$$\mu_{\bar{a}}(x) = \begin{cases} \frac{x-(a^L-\alpha)}{\alpha} & for\ a^L - \alpha \le x \le a^L \\ 1 & for\ a^L \le x \le a^U \\ \frac{(a^L-\beta)-x}{\beta} & for\ a^U \le x \le a^U + \beta \\ 0 & else \end{cases} \tag{2}$$

I. Remark in case of $a^L =, a^U = a$ in TRFN $\bar{a} = (a^L, a^U, \alpha, \beta)_{LR}$, a three-cornered fuzzy number (TNF) can be accomplished, and denoted by $\bar{a} = (a, \alpha, \beta)$.
Let $\bar{a} = (a^L, a^U, \alpha, \beta)$ and $\bar{a} = (b^L, b^U, \gamma, \theta)$

Define,

$$x > 0, x \epsilon ® ;\ \ x\bar{a} = (xa^L, xa^U, x\alpha, x\beta)$$
$$x < 0, x \epsilon ® ;\ \ x\bar{a} = (xa^U, xa^L, -x\beta, -x\alpha)$$
$$\bar{a} + \bar{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$$

I. Ranking functions

When faced with a fuzzy linear programming problem, one effective strategy is to make use of ranking functions that convey the impression of contrast between fuzzy numbers [4-7]). It is possible to take a functional approach to the set of F® components by constructing a ranking function ®:F® —®

$$\bar{a} \ge \bar{b}\ \ if\ and\ only\ if\ ®(\bar{a}) \ge ®(\bar{b}) \tag{3}$$
$$\bar{a} > \bar{b}\ \ if\ and\ only\ if\ ®(\bar{a}) > ®(\bar{b}) \tag{4}$$
$$\bar{a} \cong \bar{b}\ \ if\ and\ only\ if\ ®(\bar{a}) = ®(\bar{b}) \tag{5}$$
$$\bar{a}\ and\ \bar{b}\ \ \epsilon\ F®, \bar{a} \le \bar{b} \leftrightarrow\ \bar{b} \le \bar{a}$$
$$®(k\bar{a} + \bar{b}) = k®(\bar{a}) + ®(\bar{b}) \tag{6}$$
$$\bar{a}\ and\ \bar{b}\ \epsilon F®,\ \ k \in ®.$$
$$®(\bar{a}) = c_L a^L + c_U a^U + c_\alpha \alpha + c_\beta \beta \tag{7}$$

Wherever $\bar{a} = (a^L, a^U, \alpha, \beta)$, and $c_L, c_U, c_\alpha, c_\beta$ is a set of permanent numbers, minimum one is non-zero. However, only the computation of a a tiny portion of the available ranking functions for TFFVs for the sake of illustration. Listed below are second rank function proposed by [11].

$$Y_2(\bar{a}) = \frac{1}{2}\int_0^1 \{inf(\bar{a})_\alpha + sup(\bar{a})_\alpha\} d\alpha$$
$$Y_2(\bar{a}) = \frac{1}{2}\left\{a^L + a^U + \frac{\beta - \alpha}{2}\right\} \tag{8}$$

In their seminal work [6] the two authors developed a ranking function that looks like this:

$$CM_1^\rho(\bar{a}) = \int_0^1 \{\rho\ inf(\bar{a})_\alpha + (1-\rho) sup(\bar{a})_\alpha\} d\alpha$$
$$CM_1^\rho(\bar{a}) = a^L + \rho\left\{(a^U - a^L) + \frac{\alpha+\beta}{2}\right\} - \frac{\alpha}{2} \tag{9}$$

This is a representation of a 2nd rank function proposed by [6]

$$CM_2^\rho(\bar{a}) = \int_0^1 \alpha\{\rho\ inf(\bar{a})_\alpha + (1-\rho) sup(\bar{a})_\alpha\} d\alpha$$

$$CM_2^\rho(\bar{a}) = a^L + \rho\left\{(a^U - a^L) + \frac{\alpha+\beta}{3}\right\} - \frac{\alpha}{3} \tag{10}$$

Fuzzy numbers in the context of probability assumption were ranked by [7] To grow this, assume there are two fuzzy numbers, $\bar{a}$, $\bar{b}$. So, by expansion value given by Zadeh, crisp $x \le y$

$$®(\bar{a} \le \bar{b}) = \sup_{x \le y}\{\min(\mu_{\bar{a}}(x), \mu_{\bar{a}}(y))\}$$

The truthfulness worth $®(\bar{a} \leq \bar{b})$ which is additionally termed as ranking the option as the domination of term $\bar{b}$ *on* $\bar{a}$ and is symbolized as P $(\bar{a} \leq \bar{b})$. Next, describe $\bar{a} \leq \bar{b} \leftrightarrow$ P $(\bar{a} \leq \bar{b}) \leq$ P $(\bar{a} \leq \bar{a})$.

Conclusion: When ranking fuzzy variables, we adopt the first class since any FVLP illustrations can be transformed into a crisp linear programming illustration by employing a linear ranking algorithm. For FLP resolution, we can also use existing dual simplex techniques, which we can apply in this context.

III. Linear Programming with Fuzzy Variables

I. Definition According to research [5] following descriptions of FVLPs are correct:

$$\begin{aligned} &\text{Min } \bar{z} \cong c\bar{x} \\ &\text{s.t. } A\bar{x} \geq \bar{b} \\ &\bar{x} \geq 0 \\ &c \in ®^n, \bar{x} \in (F(®))^n, \mathrm{A} \in ®^{\mathrm{m+n}}, \bar{b} \in (F(®))^m. \end{aligned} \tag{11}$$

II. Definition: A vector $\bar{x} \in (F(®))^n$ provides practical result for Equivalence (11) $\leftrightarrow \bar{x}$ fulfils the problem's restraints.

III. Definition: An Variable result $\bar{x}_*$ such as optimum result of Equation (5) only for, ∀ such as possible results $\bar{x}$ of the Equation (11), $c\,\overline{x}_* \leq c\bar{x}$.

IV. Definition: The FVLP problem:

$$\begin{aligned} &\text{Min } \bar{z} = c\bar{x} \\ &\text{s.t . } A\bar{x} = \bar{b} \\ &\bar{x} \geq 0 \end{aligned} \tag{12}$$

where the problem's limitations are those specified by the (6)

V. Definition: Let $A = [a_{(i,j)}]_{m \times n}$ Assume $rank(A) = m$. Divider $A\ as\ [B\ N]$, where $B, m \times n$ is Plural. Clearly stated that $rank(B) = m$.assume y_J as result of $By = a_j$

$$\bar{x}_B = (x_{B_1}, x_{B_2}, \ldots \ldots x_{B_m},)' \cong B^{-1}\bar{b}, \bar{x}_N = \bar{0} \tag{13}$$

is a solution of $A\bar{x} = \bar{b}$. $\bar{X} = (\bar{x}_B', \bar{x}_N')'$. If $\bar{x}_B \geq \bar{0}, \bar{z} = c_B\bar{x}_B$, where $C_B = (C_{B_1}, C_{B_2}, \ldots \ldots C_{B_m})$ The need for conformity with non-elementary variables has $\bar{x}_j$, $1 \leq j \leq n, j \neq B_i$, $i = 1, 2, \ldots \ldots \ldots \ldots m$,

$$z_j = C_B y_j = C_B B^{-1} a_j .$$

In this section, we detail some of the most salient findings concerning the development of a workable solution, the existence of unbounded conditions, and the presence of optimality situations [5]

I. Theorem in case we obtain an fuzzy elementary viable result related with an fuzzy objective value $\bar{z} \ni z_k - c_k > 0$ for some non-primary variable $\overline{x}_k$ and $y_k \leq 0$, then it is probable to attain an original fuzzy elementary feasible result consisting of an initial fuzzy objective value $\bar{\bar{z}}$ *and* $\bar{\bar{z}} \leq \bar{z}$.

II. Theorem A result for illustration with a fuzzy elementary feasible is $z_k - c_k < 0$ with a few complex variable quantity $\bar{x}_k$ and $y_k \leq 0$, then the solution to the illustration (Equation 10) is limitless.

III. Theorem Optimality restrictions. If an elementary solution $\overline{x}_B = B^{-1}\bar{b}, \bar{x}_N = \bar{0}$ are possible for Equation (10), $z_j - c_j$ for the whole of $j, 1 \leq j \leq n$

I. Duality

VI. Definition At (Mahdavi-Amiri and Nasseri, 2007), the FVLP illustrations' dual (equation 9) is described in the following,

$$\begin{aligned} &\text{Max } \bar{z} = w\bar{b} \\ &\text{s.t . } wA \leq c \end{aligned} \tag{14}$$

$w \geq 0$

$w = (w_1, w_2, \dots . w_m) \in ®^n$, includes a crisp variable quantity conforming to the restrictions of Equation (9).

Without going into proofs, we will list several important results about the FVLP problem and its dual, the DFVLP problem, as proven by [4]

IV. Theorem If x_0, w_0 act as practical results for FVLP, DFVLP illustrations, correspondingly, $\bar{c}\, x_0 \geq w_0 \bar{b}$.

I. Corollary If x_0, w_0 act as practical results for FVLP, DFVLP illustrations, correspondingly, and $\bar{c}\, x_0 \cong w_0 \bar{b}$,. x_0 and w_0 are optimal solutions.

II. Corollary If any DFVLP problems are boundless, therefore, additional issue has zero practical result.

V. Theorem In case of FVLP, DFVLP illustrations can be solved optimally, then the two problems can be solved optimally.

IV. A Fuzzy Dual Simplex Method

I. Primal optimality and dual feasibility

FNLP

$$\text{Min } \bar{z} \cong \bar{c}x \tag{15}$$

Subject to restrictions $\bar{A}x \geq \bar{b}$

$x \geq 0$

Where $A = \gamma(\bar{a}_{(i,j)})$ and $b = \gamma(\bar{b}) = \gamma(\bar{b}_i)$, All $i = 1,2, \dots \dots , m$ *and* $j = 1,2 \dots \dots . n$.

nonnegative slack variables s_i is $i = 1,2, \dots \dots , m$ We can reformat (16) as follows to account for the ith constraint.

$$\begin{aligned} &\text{Min } \bar{z} \cong \bar{c}x + \bar{0}s \\ &\text{Subject to restrictions } Ax - s_i = b \\ &x \geq 0, s \geq 0 \\ &\text{Where } s = (s_1, s_2, \dots \dots s_m)' \end{aligned} \tag{16}$$

Define $\bar{x} \epsilon ®^{n+m}$ and $\bar{\bar{c}}\, \epsilon \left(F(®)\right)^{n+m}$ as

$$\bar{x}_j = \begin{cases} x_j, & j=1,2,\dots\dots\dots n \\ s_{j-n}, & j = n+1, \dots \dots , n+m, \end{cases} \tag{17}$$

$$\bar{\bar{c}}_j = \begin{cases} \bar{\bar{c}}_j, & j=1,2,\dots\dots\dots n \\ \bar{0}, & j = n+1, \dots \dots , n+m, \end{cases} \tag{18}$$

Let's pretend that there is a simple answer to (17): $\bar{x}_B = B^{-1}b$, let $z_j = \bar{c}_B B^{-1} a_j$ where $\bar{c}_B = \left(\gamma(\bar{c})\right)_B = \left(\gamma(\bar{c})\right)_{B_1}, \dots \dots . . \left(\gamma(\bar{c})\right)_{B_m}$. and a_j as j^{th} line as constant matrix $[A, I] = [\gamma(\bar{A}) \quad I]$.

Table 4.1, wherever $(\bar{x}_B)_r$ as $x^{r\,th}$ elementary variable, $y_j = B^{-1}a_j$, and $\gamma(\bar{z}_j - \bar{c}_j) = \gamma(\bar{c}_B)B^{-1}a_j - \gamma(\bar{c}_j) = \gamma(\bar{c}_B)y_j - \gamma(\bar{c}_j)$ is the real number consistent to $\bar{z}_j - \bar{c}_j \cong \bar{c}_B y_j - \bar{c}_j$. Supposing as $j = 1, \dots \dots , n + m$,thus, we get $\bar{z}_j - \bar{\bar{c}}_j \leq \bar{0}$ $w = \bar{c}_B B^{-1}$, where $w = (w_1, w_1, \dots \dots w_m)$. for $j = 1, \dots \dots , n + m$ we have $\bar{z}_j - \bar{c}_j \leq 0$ for $j = 1, \dots \dots , n$,$wa_j - c_j \leq 0$. then, $w \geq 0$ $\bar{z}_j - \bar{c}_j \leq 0$ for $j = 1, \dots \dots , n + m \rightarrow wA \leq c$ and $w \geq 0$, where $w = \bar{c}_B B^{-1}$. the simple FVLP problem.

$$\bar{c}\bar{\bar{x}} = \bar{c}_B \bar{\bar{b}} = \bar{c}_B B^{-1} \bar{b} = w\bar{b} = w\gamma(\bar{b})$$

II. Corollary proves x and was optimal results as the FVL, DFVLP illustrations, correspondingly.

Table 1: *An example of a fuzzy dual practical simplex tableau*

Basis	$\bar{x}_1$	...	$\bar{x}_k$	....	$\bar{x}_{n+m}$	$\gamma(R.H.S)$
$\gamma(\bar{z})$	$\gamma(\bar{z}_1 - \bar{\bar{c}}_1)$	...	$\gamma(\bar{z}_k - \bar{\bar{c}}_k)$	...	$\gamma(\bar{z}_{n+m} - \bar{\bar{c}}_{n+m})$	$\gamma(\bar{\bar{c}}_B b)$
$\bar{z}$	$(\bar{z}_1 - \bar{\bar{c}}_1)$	...	$\gamma(\bar{z}_k - \bar{\bar{c}}_k)$	...	$(\bar{z}_{n+m} - \bar{\bar{c}}_{n+m})$	$(\bar{\bar{c}}_B b)$
$(\bar{x}_B)_1$	$y_{(1,1)}$	...	$y_{(1,k)}$	...	$y_{(1,n+m)}$	$\gamma(\bar{\bar{b}}_1)$
$\vdots$	$\vdots$					$\vdots$
$(\bar{x}_B)_r$	$y_{(r,1)}$	.....	$y_{(r,k)}$	.....	$y_{(r,n+m)}$	$\gamma(\bar{\bar{b}}_2)$
$\vdots$	$\vdots$					$\vdots$
$(\bar{x}_B)_m$	$y_{(m,1)}$	.....	$y_{(m,k)}$	.....	$y_{(m,n+m)}$	$\gamma(\bar{\bar{b}}_m)$

I. Lemma If there is a r such that in a fuzzy dual viable simplex tableau, then.
$\gamma(\bar{\bar{b}}_r) < 0$, as well as a non-primitive index number j exists $\ni$ $y_{(r,j)} < 0$, the objective value corresponding to fuzzy dual feasible tableau will not decrease when the turning column k is set to y ((r,k)).

Proof. A criteria for picking an complex variable for the input the base is required to guarantee for news fuzzy simplex tableau retains dual viability, that the newest objective value is not reducing. Let's pretend, for the moment, that column k is the crucial one. We obtain the following new zero rows after turning on the pivot y ((r,k)):

$$\left(\bar{z}_j - \bar{\bar{c}}_j\right)_{New} = \left(\bar{z}_j - \bar{\bar{c}}_j\right) - \frac{(\bar{z}_k - \bar{\bar{c}}_k)}{y_{(r,k)}} y_{(r,j)}, j = 1, \ldots\ldots, n+m, \quad j \neq B_i \tag{19}$$

$$\left(\bar{z}_j - \bar{\bar{c}}_j\right)_{New} \leq \bar{0}\,, j \neq B_i \tag{20}$$

Which, using (19), results in

$$\frac{(\bar{z}_k - \bar{\bar{c}}_k)}{y_{(r,k)}} \leq \frac{(\bar{z}_k - \bar{\bar{c}}_k)}{y_{(r,j)}}, \text{ for all } j \neq B_i \tag{21}$$

To satisfy (20), it is sufficient to let.

$$\frac{\gamma(\bar{z}_k - \bar{\bar{c}}_k)}{y_{(r,k)}} = \min_{j \neq B_i} \left\{ \frac{\gamma(\bar{z}_k - \bar{\bar{c}}_k)}{y_{(r,j)}}\ y_{(r,j)} < 0 \right\} \tag{22}$$

$$\left(w\gamma(\bar{b})\right)_{New} = w\gamma(\bar{b}) - \frac{\gamma(\bar{z}_k - \bar{\bar{c}}_k)}{y_{(r,k)}} \gamma(\bar{b}_r) \geq w\gamma(\bar{b}),$$

$$\frac{\gamma(\bar{z}_k - \bar{\bar{c}}_k)}{y_{(r,k)}} \gamma(\bar{b}_r) \leq 0.$$

I. Algorithm The fuzzy dual simplex method

- Assigned a base B to FNLP illustration for
  $\bar{z}_j - \bar{\bar{c}}_j \leq \bar{0}\ \ \forall\ \ j$
- If $\bar{\bar{b}} \geq 0$, if the current solution is best, then do nothing; if not, pick row r from the pivot table using $\bar{\bar{b}}_r < 0$ (that is, r so that $(\bar{\bar{b}}_r) < 0$.
- If $y_{(r,j)} \geq 0$ for all j, if the minimal ratio test fails (the FNLP problem is infeasible), then stop; otherwise, choose column k as the pivot.

$$\frac{\gamma(\bar{z}_k-\bar{c}_k)}{y_{(r,k)}} = \min_{j\neq B_i}\left\{\frac{\gamma(\bar{z}_k-\bar{c}_k)}{y_{(r,j)}}\ y_{(r,j)} < 0\right\}.$$

- Pivot on $y_{(r,k)}$ and proceed to (2).

I. Remark In (2), r so that is one possible proposition for a special of r.

$$\gamma(\bar{\bar{b}}_r) = \min_{1\leq i\leq m}\{\gamma(\bar{\bar{b}}_i)\}.$$

I. Numerical example

We examine the seeing instance, which may be explained using Maleki's method, to further our understanding of the aforesaid strategy [5, 7]. Please take note that the linear ranking function on F(®) is modelled after the one proposed by Yager in this section.

Minimum $z \cong (1,3,1,1)x_1 + (2,4,1,1)x_1(3,5,1,1)x_3$

Subject to restrictions
$$\begin{cases}\left(0,2,\frac{1}{2},\frac{1}{2}\right)x_1 + (1,3,1,1)x_1\left(0,2,\frac{1}{2},\frac{1}{2}\right)x_3 \geq (2,4,1,1)\\ (1,3,1,1)x_1 - \left(0,2,\frac{1}{2},\frac{1}{2}\right)x_2 + (2,4,1,1)x_3 \geq (3,5,1,1)\\ x_1,x_2,x_3 \geq 0\end{cases}$$

The fuzzy coefficient matrix is ranked using the Yager function $\bar{A}$ as well as fuzzy right-hand-cross vector $\bar{b}$. Therefore, establishing slack of variable quantity x_4 and x_5 the problem decreases to:

Minimum $z \cong (1,3,1,1)x_1 + (2,4,1,1)x_1(3,5,1,1)x_3$

Subject to restrictions
$$\begin{cases}x_1 + 2x_2 + x_3 - x_4 = 3\\ 2x_1 - x_2 + 3x_3 - x_5 = 4\\ x_1,x_2,x_3,x_4,x_5 \geq 0\end{cases}$$

Now, x_1 is an incoming variable, and x_5 is an exit variable quantity. Then by rotating on $y_{21} = -2$, we attain the next tableau as

Table 2: *The first iteration*

Basic	x_1	x_2	x_3	x_4	x_5	R.H. S
$\gamma(\bar{z})$	0	-4	-1	0	-1	4
$\bar{z}$	$\bar{0}$	$(-4,-4,\frac{3}{2},\frac{3}{2})$	$(-3,1,2,2)$	$\bar{0}$	$(-1,-1,0,0)$	$(0,0,0,0)$
x_4	0	$-\frac{5}{2}$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1
x_1	1	$-\frac{1}{2}$	$\frac{3}{2}$	0	$-\frac{1}{2}$	2

Table 3: *The second iteration*

Basic	x_1	x_2	x_3	x_4	x_5	R.H.S
$\gamma(\bar{z})$	0	0	$-\frac{9}{5}$	$-\frac{8}{5}$	$-\frac{1}{5}$	$\frac{28}{5}$
$\bar{z}$	$\bar{0}$	$\bar{0}$	$\left(-\frac{31}{5},\frac{13}{5},\frac{18}{5},\frac{18}{5}\right)$	$\left(-\frac{8}{5},-\frac{8}{5},0,0\right)$	$\left(-\frac{1}{5},-\frac{1}{5},0,0\right)$	$\left(\frac{11}{5},9,\frac{17}{5},\frac{17}{5}\right)$
x_4	0	1	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
x_1	1	0	$\frac{7}{5}$	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{11}{5}$

To clarify, x_2 ,an input variable x_4 as an output variable in this scenario. Updated data can be seen in Table 3 below.

Hence the optimum solution attained by the fuzzy dual simplex is $x_1 = \frac{11}{5}$, $x_2 = \frac{2}{5}$, also the fuzzy optimum estimate of the objective function is $\overline{z} \cong \left(\frac{11}{5}, 9, \frac{17}{5}, \frac{17}{5}\right)$.

V. Conclusion

In this study, we introduce the idea of a duality outcome in fuzzy variable linear programming problems using fuzzy dual simplex algorithms with generic linear ranking functions on fuzzy variables. In particular, we emphasize the proposed method for directly addressing fuzzy variable linear programming problems with the assistance of linear ranking functions. This method is applicable in linear programming and will prove useful in sensitivity analysis for basic column activity vectors. We use the fuzzy dual simplex approach to perform dual pivots and reach feasibility. Interesting studies will be possible in the future when the variables are also fuzzy integers.

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