SINGLE AND DOUBLE ACCEPTANCE SAMPLING PLAN FOR TRUNCATED LIFE TESTS BASED ON GAMMA LINDLEY DISTRIBUTION

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Abstract

For time-truncated life tests, this work defines single acceptance and double acceptance sampling plans assuming that the product's lifespan follows the Gamma Lindley distribution. The minimum sample size needed in a single acceptance sampling plan for lot approval is calculated for a range of parameter combinations and a fixed test termination time. This ensures the given average product life and the corresponding number of failures. Operational characteristic and producer risk values are also tabulated for these parameter values. Using a double acceptance sampling plan, the best first and second samples are obtained to ensure that the products specified average with a certain level of customer trust. Finally, under the same conditions, the minimum sample size obtained using these strategies are compared with other acceptance sampling plans.

Keywords: Acceptance sampling plan, consumer's risk, producer's risk, time truncated life test, Gamma Lindley distribution.

I. Introduction

A decision is made based on the quality of the samples after samples are removed from the lot and examined to determine whether the lot should be accepted. When inspecting the entire lot, it is usually not a good idea because it takes a long time and is very expensive. So, in order to minimize both producer and consumer risk, decisions are made, inspections are carried out, and samples are taken from the lot. The term acceptance sampling plan based on time truncated life tests refers to the fact that engineers typically incorporate the time parameter into the plan. There exist multiple methods for executing acceptance sampling plans that include time-truncated life tests. The two most widely used plans among the many that are available are the single acceptance and double acceptance sampling plans (SASP and DASP). Because of this, the two acceptance sampling plans are the primary subject of this work.

It is known as a SASP when only one sample is selected for decision-making out of the entire lot. This plan involves running the test for a predetermined amount of time and counting the number of failures that occur during that time. For a lot to be accepted, the number of failures observed at the end of the predetermined time must be less than the acceptance number; if it is greater than the acceptance number, the lot is rejected. In order to determine the consumer risk for an acceptance number and a fixed time t, it is therefore necessary to determine the minimum sample size required to guarantee the average life of the product at different probability values. A minimum sample size must be determined for both samples in a DASP, where the acceptance numbers for the first and second samples are fixed at zero and one, respectively. Numerous researchers have examined the SASP and DASP for a range of lifetime models whose lives follow a specific probability distribution. Among the works are those by Mahendra Saha et. Al. [1] who created SASP and DASP for time-truncated life tests based on transmuted Rayleigh distribution (TRD), Amjad and Amer [2] who created an SASP based on time-truncated life tests for an exponentiated Frechet distribution; Sridhar Babu et al., [3] who examined a DASP for an exponentiated Frechet distribution with known shape parameters; Lio et. Al. [4] examined SASP for time-truncated life tests for Birnbaum-Saunders distribution for percentiles; Min and Gui [5] investigated SASP based on time-truncated life tests for Burr type X distribution; Balakrishnan et al., [6] investigated the generalized Birnbaum-Saunders model's SASP based on time-truncated life tests; Amjad and Obeidat [7] created an SASP using time-truncated life tests based on the Tsallis-q-exponential distribution. Further, Sriramachandran [8] studied sampling plan for truncated life test for Logistic family of distributions.

In order to analyse the lifetime data, Lindley [9] introduced a probability distribution that alternates between the gamma and exponential distributions in the appropriate ratio. The Lindley distribution has not been investigated since exponential and gamma distributions are applied independently in numerous statistical domains and are the subject of extensive research by researchers. Ghitany et al., [10] was the first to study the properties of Lindley distribution, which led to its popularity in data modelling. Numerous authors have also applied it to a variety of disciplines. Shanker et al., [11] also transformed the one parameter Lindley distribution into the two parameter Lindley distribution. SASP was created by Wu [12] for two parameter Lindley distribution, Deniz [13] described the discrete form of Lindley distribution and stated its properties, Ghitany et al [14] stated the two-parameter weighted Lindley distribution and its applications, Sihem [15] and Zeghdoudi [16] defined the Gamma Lindley distribution and studied its properties. The article's goal is to create single acceptance and double acceptance sampling plans that take into account the Gamma Lindley distribution (GLD) for the average life of the product being tested. For a given consumer risk probability, the minimum sample size needed to guarantee a given average life is obtained is known as the single acceptance sampling plan. For this plan, the production risk and the OC values are also obtained. The zero-one model is used in the double acceptance sampling plan to determine the minimum sample size for the first and second samples for different probabilities in order to guarantee the consumer risk. The structure of this article is as follows: Section 2 elaborates on the GLD. The average life of the product is assumed to follow GLD in Section 3 when designing and deriving the SASP for time-truncated life tests. In Section 4, a DASP for time-truncated life tests is designed and derived with the assumption that the product lifetime follows GLD. The minimum sample size required to be inspected under the SASP is presented along with the OC value and producer risk. In Section 5, the plan proposed in this article is compared with other suitable acceptance sampling plan based on the minimum sample size to be inspected. Section 6 submits the conclusions from this article.

II. Gamma Lindley Distribution Model

In lifetime data modelling, Lindley distribution is the special case of two parameter Lindley distribution (TPLD) studied by [11]. The probability density function (pdf) of TPLD is stated as:

$$f(\epsilon;\tau,\omega)=\frac{\tau(\omega+\tau\epsilon)}{\omega+1}e^{-\tau\epsilon} \tag{1}$$

Hence, the corresponding cumulative distribution function (cdf) and the hazard rate function (hrf) of TPLD are stated as:

$$F(\epsilon;\tau,\omega)=1-\frac{1+\omega+\tau\epsilon}{\omega+1}e^{-\tau\epsilon} \tag{2}$$

$$h(\epsilon;\tau,\omega)=\frac{\tau(\omega+\tau\epsilon)}{1+\omega+\tau\epsilon},\ \epsilon>0,\tau>0,\omega>-1,\omega>-\tau\epsilon \tag{3}$$

[16] mixes the gamma distribution with two parameter Lindley distribution and formed a new distribution called the GLD and studied all its properties.
The pdf of GLD is

$$f(\varepsilon;\tau,\omega)=\begin{cases}\frac{\tau^2((\omega+\omega\tau-\tau)\epsilon+1)}{\omega(\tau+1)}e^{-\tau\varepsilon}, \epsilon,\tau,\omega>0\\ 0,\ \ otherwise\end{cases} \tag{4}$$

The corresponding cdf and hrf are defined as

$$F(\epsilon;\tau,\omega)=1-\frac{((\tau\omega+\omega-\tau)(\tau\epsilon+1)+\tau)}{\omega(\tau+1)}e^{-\tau\epsilon} \tag{5}$$

$$h(\varepsilon;\tau,\omega)=\frac{\tau^2((\omega+\omega\tau-\tau)\epsilon+1)}{((\tau\omega+\omega-\tau)(\tau\epsilon+1)+\tau)} \tag{6}$$

Table 1: *Minimum sample size for different parameter value*

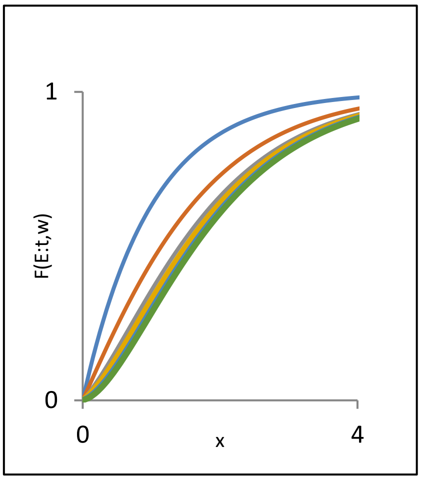
Parameter value	P*	c	$\frac{\varepsilon}{\upsilon_0}$				
			0.628	0.942	1.257	2.356	3.141
$\tau=0.5$, $\omega=1$	0.95	0	6	4	3	2	1
		1	10	7	5	3	2
		2	14	9	7	4	4
	0.99	0	9	6	4	2	2
		1	14	9	7	4	3
		2	18	11	9	5	4
$\tau=1$, $\omega=1$	0.95	0	6	4	3	2	1
		1	10	6	5	3	3
		2	13	9	7	4	4
	0.99	0	9	6	4	2	2
		1	13	9	7	4	3
		2	17	11	9	5	4
$\tau=1$, $\omega=0.5$	0.95	0	5	4	3	2	1
		1	9	6	5	3	3
		2	12	8	7	4	4
	0.99	0	8	5	4	2	2
		1	12	8	6	4	3
		2	15	11	8	5	5

The expected value and variance of GLD are derived as

$$E(x)=\frac{2\omega(1+\tau)-\tau}{\tau\omega(1+\tau)} \tag{7}$$

$$Var(x) = \frac{-(-2\omega\tau+\tau)^2+(2+6\tau)\omega^2-2\omega(\omega\tau-3\omega\tau^2+2\tau^2)}{\omega^2\tau^2(1+\tau)^2} \tag{8}$$

The cdf, pdf and hrf of GLDs for different parameter values are presented in Figure 1 and 2.



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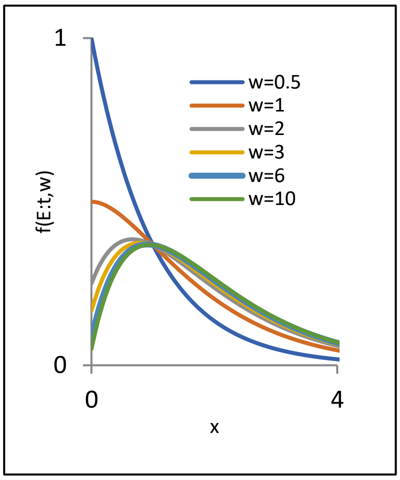

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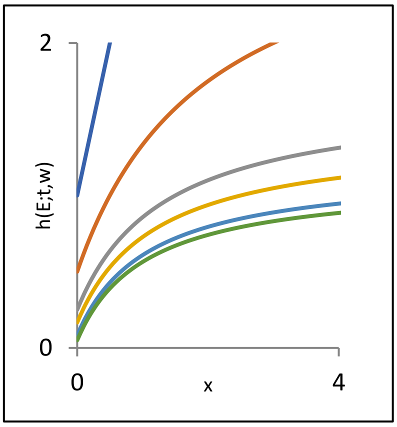

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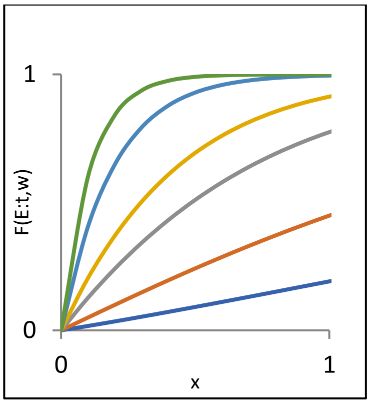
Figure 1: (a) *cdf, (b) pdf and (c) hrf of GLDs with parameters* $\tau = 1$ *and* $\omega = 0.5, 1, 2, 3, 6$ *and* 10.

III. Design of Single Acceptance Sampling Plan (SASP) Based on Mean

When creating the SASP with time-truncated life tests, it was assumed that the products being studied would have an average lifespan of υ. The above assumption is tested using the hypothesis test, where the product's precise average life, denoted by υ_0 and H0: $\upsilon \geq \upsilon_0$, alternative hypothesis H1: $\upsilon < \upsilon_0$. The test's level of significance is defined as ρ^* represents the consumer's confidence level, the value $1 - \rho^*$ is called the level of significance for the test. The application of the binomial distribution in this case is noteworthy because the sample size is noticeably large. As proposed by the plan; to locate the minimum sample size, we have to iterate the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^* \tag{9}$$

where $\rho = F(\epsilon; \tau, \omega)$ as in (5).



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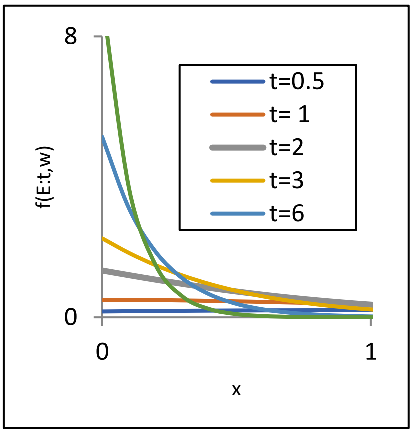

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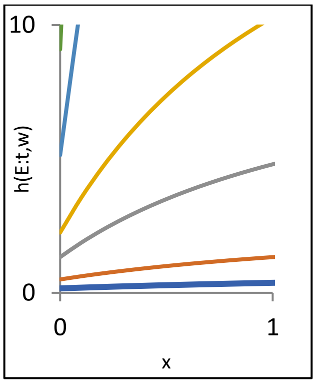

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Figure 2: (a) *cdf, (b) pdf and (c) hrf of GLDs with parameters* $\omega = 1$ *and* $\tau = 0.5, 1, 2, 3, 6$ *and* 10

Minimum sample size for ε/υ_0= (0.628, 0.942, 1.257,2.356 and 3.141), ρ^*= 0.95 and 0.99 and for the parameters $\tau = 0.5, \omega = 1, \tau = 1, \omega = 1$ *and* $\tau = 1, \omega = 0.5$ are calculated and tabulated in the Table 1. The OC values of the plan give the probability of acceptance of the lot and it is stated as

$$L(\mathrm{p}) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{10}$$

where $\rho = F(\epsilon; \tau, \omega)$ in (5). For the fixed acceptance number i.e. *c=2* the OC values are calculated and the values are tabulated in the Table 2 for the parameters $\tau = 0.5,\ \omega = 1,\ \tau = 1, \omega = 1$ *and* $\tau = 1, \omega = 0.5$ respectively. Next, it is necessary to reduce the producer's risk and we should guarantee that rejection of good lot is minimum. By fixing the PR as 95%, the value of ϵ/υ_0 are obtained for the plan. The value obtained is the least number for which $\rho = F(\epsilon; \tau, \omega)$as in (5) satisfies the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 0.95 \tag{11}$$

The least values of ϵ/υ_0which satisfies the equation (11) are determined and presented for the proposed plan and presented in Table 3.

Table 2: *OC values for the parameter value when c=2*

Parameter value	P*	$\frac{\varepsilon}{\upsilon_0}$	n	$\frac{\upsilon}{\upsilon_0}$ 2	4	6	8	10	12
$\tau = 0.5, \omega = 1$	0.95	0.628	14	0.445	0.855	0.947	0.976	0.987	0.992
		0.942	9	0.449	0.864	0.952	0.979	0.989	0.993
		1.257	7	0.411	0.850	0.948	0.977	0.988	0.993
		2.356	4	0.391	0.843	0.947	0.977	0.989	0.993
		3.141	4	0.181	0.696	0.884	0.947	0.973	0.984
	0.99	0.628	18	0.269	0.754	0.902	0.952	0.974	0.984
		0.942	11	0.300	0.785	0.919	0.962	0.979	0.988
		1.257	9	0.224	0.736	0.898	0.952	0.974	0.985
		2.356	5	0.196	0.716	0.893	0.951	0.974	0.985
		3.141	4	0.181	0.696	0.884	0.947	0.973	0.984
$\tau = 1, \omega = 1$	0.95	0.628	13	0.401	0.812	0.924	0.963	0.979	0.987
		0.942	9	0.380	0.805	0.922	0.962	0.979	0.987
		1.257	7	0.363	0.799	0.920	0.961	0.978	0.987
		2.356	4	0.387	0.816	0.929	0.967	0.982	0.989
		3.141	4	0.191	0.671	0.859	0.929	0.960	0.976
	0.99	0.628	17	0.218	0.681	0.857	0.926	0.957	0.973
		0.942	11	0.236	0.704	0.871	0.934	0.962	0.977
		1.257	9	0.185	0.660	0.849	0.922	0.955	0.972
		2.356	5	0.193	0.676	0.860	0.930	0.960	0.976
		3.141	4	0.191	0.671	0.859	0.930	0.960	0.976
$\tau = 1, \omega = 0.5$	0.95	0.628	12	0.333	0.752	0.891	0.943	0.967	0.979
		0.942	8	0.368	0.775	0.902	0.950	0.971	0.982
		1.257	7	0.285	0.715	0.870	0.932	0.960	0.975
		2.356	4	0.362	0.765	0.896	0.946	0.969	0.980
		3.141	4	0.193	0.619	0.812	0.896	0.937	0.960
	0.99	0.628	15	0.188	0.625	0.818	0.901	0.941	0.962
		0.942	11	0.155	0.585	0.793	0.885	0.931	0.951
		1.257	8	0.193	0.629	0.821	0.902	0.942	0.962
		2.356	5	0.174	0.602	0.803	0.891	0.935	0.957
		3.141	5	0.064	0.418	0.669	0.803	0.875	0.916

IV. Design of Double Acceptance Sampling Plan (DASP)

DASP, or two-stage acceptance sampling, is the preferred method for providing greater protection to producers and consumers. Because a second sample is tested before a final decision is made on the lot, producers are given double protection, as the name suggests. As a result, it diminishes producer risk and offers complete protection to producers. The plan's parameters are as follows: the testing time *t*, the first sample size *(n1)*, its acceptance number *(c1)*, and the second sample size *(n2)*. In order to approve the lot, the sample must substantiate the hypothesis that the sample mean exceeds the given mean. The lot will be turned down otherwise. The consumer's risk is now fixed at no more than *(1-P*)*, where *P** is the confidence level.

Then probability of acceptance of the lot is

$$PA = \sum_{i=0}^{c1} \binom{n1}{i} p^i (1-p)^{n1-i} + \sum_{x=c1+i}^{c2} \binom{n1}{x} p^x (1-p)^{n1-x} \sum_{j=0}^{c2} \binom{n2}{j} p^j (1-p)^{n2-j} \tag{12}$$

where p is defined in Equation (5) and depends on ratio $\frac{\varepsilon}{v_0}$. As we are considering only zero-one failure form i.e., *c1 =0 and c2 =1*, the above Equation (13) for the considered GLD is given by

Table 3: *Minimum ratio when producer's risk=95%*

Parameter value	P*	c	$\frac{\varepsilon}{v_0}$ 0.628	0.942	1.257	2.356	3.141
$\tau = 0.5, \omega = 1$	0.95	0	41.49	41.82	42.17	53.46	37.04
		1	9.94	10.44	9.91	10.87	9.03
		2	6.12	5.90	6.08	6.12	8.15
	0.99	0	61.90	62.23	55.80	53.46	71.
		1	13.89	13.42	13.93	14.77	14.49
		2	7.85	7.22	7.87	7.93	8.15
$\tau = 1, \omega = 1$	0.95	0	55.33	55.44	55.60	69.76	47.04
		1	12.80	11.21	12.29	12.91	17.22
		2	7.11	7.19	7.25	6.86	9.15
	0.99	0	82.88	82.99	73.98	69.76	93.00
		1	16.78	17.20	17.63	18.00	17.22
		2	9.43	8.93	9.59	9.14	9.15
$\tau = 1, \omega = 0.5$	0.95	0	61.22	73.46	73.52	91.86	61.24
		1	14.99	14.51	15.81	16.20	21.60
		2	8.42	8.00	9.12	8.24	10.99
	0.99	0	97.95	91.83	98.02	91.86	122.47
		1	20.30	19.83	19.37	22.94	21.60
		2	10.73	11.47	10.67	11.23	14.97

$$PA = (1-p)^{n1}[1 + n_1 p (1-p)^{n_2 - 1}] \tag{13}$$

where p is given by the equation (5). Our aim is to find the minimum sample size for the plan, for this we have to minimize the following equation:

$$(1-p)^{n1}[1 + n_1 p (1-p)^{n_2-1}] \leq 1 - P^* \tag{14}$$

Now, for the given consumer's confidence level P*, the minimum sample size for both the samples *n1 and n2*, which ensure $v \geq v_0$, can be found by the solution of the following optimization problem, given as:

$$\begin{aligned} &Min\ ASN = n_1 + n_1 n_2 p (1-p)^{n1-1} \\ &subject\ to\ (1-p)^{n1}[1 + n_1 p (1-p)^{n_2-1}] \leq 1 - P^*, \\ &1 \leq n_2 \leq n_1 \\ &n_1, n_2\ are\ \text{integers} \end{aligned} \tag{15}$$

While solving the above optimization problem, it provides many solutions for both *n1 and n2*. We take the solution which minimizes our objective function i.e. our ASN as our best solution. Minimum sample size obtained for P* = 0.90, 0.95 and 0.99 and $\frac{\varepsilon}{\upsilon_0}$ = 0.628, 0.942, 1.257 and 2.356 are presented in Table 4.

V. Comparison of GLD With Other Models

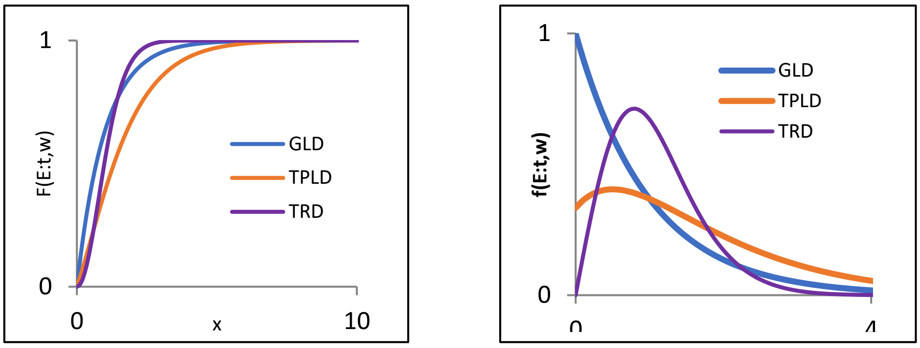
Planning an acceptance sampling plan with a minimum sample size is considered more efficient because it will cut down on the amount of time and money spent on inspection, as desired by quality engineers. This section compares and provides the following Figures 3 and 4 for the GLD's CDF and PDF, as well as the two-parameter Lindley distribution (TPLD) and transmuted Rayleigh distribution (TRD). In addition, the following tables 11 and 12 list the minimal sample sizes that were obtained from this distribution under comparable conditions. The following observations about the sample size n are found in Table 1 to 3.

- There is little variation in sample size n value when τ increases from 0.5 to 1, keeping $\omega = 1$.
- There is a marginal shift in the sample size n value when $\tau = 1$ and ω increases from 0.5 to 1
- Table 1 show that an increase in the value of c corresponds to an increase in the value of n.
- Table 4 indicates that the minimum sample size in a DASP when, $\tau = 1$, $\omega = 0.5$.
- When comparing GLD to TPLD and TRD, Table 5 indicates that the former requires the smallest sample size.
- According to Table 6, the GLD for DASP requires the smallest sample size when compared to TPLD and TRD.

Table 4: *Minimum sample size for plan*

Parameter value	P*	$\frac{\varepsilon}{\upsilon_0}$							
		0.628		0.942		1.257		2.356	
		(n_1, n_2)	ASN	(n_1, n_2)	ASN	(n_1, n_2)	ASN	(n_1, n_2)	ASN
$\tau = 0.5$, $\omega = 1$	0.90	(6,3)	6.55	(4,2)	4.35	(3,1)	3.18	(2,1)	2.13
	0.95	(7,4)	7.51	(4,3)	4.53	(3,2)	3.37	(2,1)	2.13
	0.99	(10,5)	10.19	(6,3)	6.14	(4,3)	4.53	(3,1)	3.18
$\tau = 1$, $\omega = 1$	0.90	(5,4)	5.92	(4,1)	4.17	(3,1)	3.18	(2,1)	2.13
	0.95	(6,5)	6.79	(4,3)	4.53	(3,2)	3.37	(2,1)	2.13
	0.99	(9,5)	9.22	(6,3)	6.14	(4,3)	4.53	(3,1)	3.18
$\tau = 1$, $\omega = 0.5$	0.90	(5,2)	5.38	(4,1)	4.14	(3,1)	3.17	(2,1)	2.17
	0.95	(6,3)	6.36	(4,2)	4.29	(3,2)	3.35	(2,1)	2.17
	0.99	(8,5)	8.23	(5,4)	5.25	(4,3)	4.20	(3,1)	3.02

Therefore, if quality engineers wish to check the lot with a minimum sample size, the GLD of the SASP and DASP are commendable.

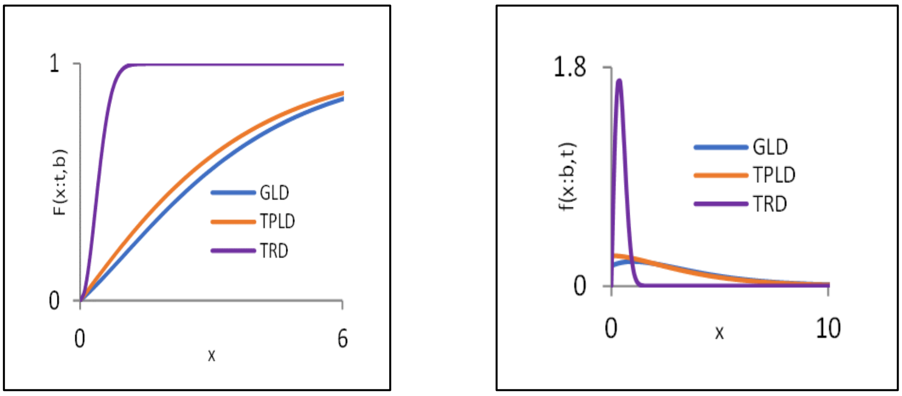


(a) (b)

Figure 3: Comparison of (a) cdf and (b) pdf of GLD, TPLD and TRD when $\tau = 1$ *and* $\omega = 0.5$

Table 5: *Comparison of single acceptance sampling plan when* $\tau = 0.5, \omega = 1$ *, P*=0.99 and c=0*

Distributions	$\frac{\varepsilon}{\upsilon_0}$		
	0.628	0.942	1.257
GLD	9	6	4
TPLD	12	7	5
TRD	16	8	3



(a) (b)

Figure 4: Comparison of (a) cdf and (b) pdf of GLD, TPLD and TRD when $\tau = 0.5$ *and* $\omega = 1$

Table 6: *Comparison of double acceptance sampling plan when* $\tau = 0.5, \omega = 1$ *, P*=0.99 and* c_1*=0 and* c_2*=1*

Distributions	$\frac{\varepsilon}{\upsilon_0}$		
	0.628	0.942	1.257
GLD	(10,5)	(6,3)	(4,3)
TPLD	(12,6)	(7,4)	(5,3)
TRD	(16,17)	(8,4)	(3,2)

VI. Conclusion

Within this paper, time-truncated life tests based on GLD were suggested for SASP and DASP. We consider the zero-one model for the DASP. Different GLD parameter values are used to calculate the minimum sample size needed to ensure a given average product life, the minimum producer's risk, and the OC values. Tables for both plans display the results that were obtained. It was discovered that the suggested plan required the smallest sample size for both single and double acceptance sampling plans when it came to minimum sample size comparisons with TPLD and TRD. Based on how this suggested plan is used, future work may expand the scope to include additional plans.

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