EXPLORING LENGTH BIASED QUASI SUJA DISTRIBUTION: PROPERTIES AND APPLICATIONS

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Abstract

This paper introduces a new statistical distribution called length biased quasi suja distribution (LBQS). It explores its properties, including moments, moment generating function(MGF), characteristic function(CF), harmonic mean, reliability, hazard rate and reverse hazard rate. Order statistics of the above distribution is obtained. Furthermore, the paper also examines various entropy which measures the randomness of system, like Renyi entropy and Tsalli's entropy. It also evaluates Bonferroni and Lorenz curves which are useful in measuring the inequality. It also discusses parameter estimation techniques specifically maximum likelihood estimation and likelihood ratio testing. Moreover, a simulation study has been conducted to demonstrate how well the distribution would perform in real-life situation. The validity of the distribution is also demonstrated with real-world data example of failure data, highlighting its potential for practical applications in data analysis.

Key words: Length biased quasi suja distribution, Moments, Entropy, Estimation, Simulation

1. Introduction

The weighted distributions are applied in various research areas related to biomedicine, reliability, ecology and branching processes. A number of continuous distributions are used like Weibull, lindley, exponential, lognormal and gamma for modelling this type of data. If x is the original observation with its pdf f(x), then in case of any bias in sampling appropriate weighted function, say w(x) which is a function of random variable will be introduced to model the situation. This concept of weighted distributions was given by Fisher [7] to model the ascertainment bias. Later Rao [13] developed this concept in a unified manner while modelling the statistical data when the standard distributions were not appropriate to record these observations with equal probabilities. As a result, weighted models were formulated in such situations to record the observations according to some weighted function. The weighted distribution reduces to length biased distribution when the weight function considers only the length of the units. The concept of length biased sampling was first introduced by Cox [5]. Weighted distributions are applied in various research areas related to reliability, biomedicine, ecology and branching processes. Dey et al [6] discussed weighted exponential distribution with its properties and different methods of estimation. Kilany [12] have obtained the weighted version of lomax distribution. Ahmad et al [1] have obtained the length biased weighted version of lomax distribution with properties and applications. Khan et al. [11] discussed the weighted modified weibull distribution. Rather and Subramanian [17] discussed the characterization and estimation of length biased weighted generalized uniform distribution. Recently Rather and Subramanian [19] also discussed on weighted

Sushila distribution with properties and applications. Ganaie R.A et. al [9] studied about how Uma distribution is applicable in engineering sciences. Saraja V D et al. [20] explored the length biased Tornumonkpe distribution, their properties, estimations and practical applications. Rashid Ganaie A et.al [16] formulated exponentiated ADYA distribution and studied their properties and applications. A new generalization of Akshaya distribution with applications in engineering science was studied by Rather and Subramanian [18]. Rather and Ozel [14] modelled Weighted Power Lindley distribution and its application on Life time data.

Recently, Shanker [21] proposed a one parameter distribution suja distribution and studied its statistical properties, estimation of parameter using method of moment and method of maximum likelihood and applications to some real lifetime data and observed that Suja distribution gives much closer fit than several one parameter lifetime distributions. Recently, Al Omari and Alsmairan [2] obtained length-biased Suja distribution and studied its statistical properties and applications. Al-Omari et al [3] proposed power length-biased suja distribution and discussed its properties and applications. Alsmairan I. K [4] derived weighted suja distribution and discussed its statistical properties and applications to ball bearings data in safety engineering. Todoka et al [22] have studied on the cdf of various modifications of suja distribution and discussed their applications in the field of analysis of computer- virus propagation and debugging theory.

In this paper, we proposed length biased quasi suja distribution. The quasi suja distribution, introduced by Shanker et al in [15], is a recently obtained two-parameter model for extreme right skewed data which contains suja distribution as particular case designed for various applications in engineering and medical sciences. Validity and significance of proposed model in modelling lifetime data was better than quasi suja distribution, exponential distribution, Lindley and erlang truncated exponential distribution.

The paper is classified into following sections: Section 2 defines the proposed length biased quasi suja distribution and reliability. Some structural properties are discussed in Section 3. The likelihood ratio test is given in Section 4. Then, Renyi and Tsalli's entropy measures of the LBQS distribution are obtained in Section 5. Section 6 describes the method of obtaining order statistics. Income distribution curve and estimation of parameters is discussed in section 7 and 8 respectively. Simulation study is shown in section 9. Finally, fitted the distribution to the real-life data and found to be fitting good compared to various other models.

2. Length biased quasi suja distribution (LBQS)

2.1 Density and cumulative density functions

The probability density function (pdf) and cumulative distribution function (cdf) of quasi suja distribution with parameters α and θ is defined by

$$f(x;\theta,\alpha)=\frac{\theta^4}{\alpha\theta^3+24}\left(\alpha+\theta x^4\right)e^{-\theta x} \qquad x>0,\ \theta>0,\alpha>0 \tag{1}$$

$$F(x;\theta,\alpha)=1-\left[1+\frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+24\theta x}{\alpha\theta^3+24}\right]e^{-\theta x} \qquad x>0,\ \theta>0,\alpha>0$$

Suppose X is a non -negative random variable with pdf f(x). Let w(x) be the non-negative weight function, then the pdf of the weighted random variable X_w is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))},\ x>0$$

where w(x) is a non-negative weight function and $E(w(x)) = \int w(x) f(x) dx$

For different weighted models, we have different choices of the weight function w(x). when w(x) = x^c, the resulting distribution is termed as weighted distribution. In this paper, we are finding the length biased version of quasi suja distribution, so we will take c = 1 in weights x^c, in order to get the length biased quasi suja distribution and its probability density function (pdf) is given by:

$$f_1(x) = \frac{x f(x)}{E(x)} \tag{2}$$

using (1) we get

$$E(x) = \int_0^\infty x f(x)\, dx$$

$$= \frac{\alpha\theta^3 + 120}{\theta(\alpha\theta^3 + 24)} \tag{3}$$

substituting equations (1) and (3) in (2) we obtain the density function of length biased quasi suja distribution as follows

$$f_1(x;\theta,\alpha) = \frac{\theta^5 x(\alpha + \theta x^4) e^{-\theta x}}{\alpha\theta^3 + 120} \qquad x > 0, \theta > 0 \tag{4}$$

and the cumulative distribution function (cdf) of LBQS distribution is obtained by

$$F_1(x) = \int_0^x f_1(x;\alpha,\theta) dx$$

$$= \int_0^x \frac{\theta^5 x(\alpha + \theta x^4) e^{-\theta x}}{\alpha\theta^3 + 120} dx$$

on simplification, the cdf of LBQS distribution is given by

$$F_1(x;\theta,\alpha) = \frac{\alpha\theta^3 \gamma(2,\theta x) + \gamma(6,\theta x)}{\alpha\theta^3 + 120} \tag{5}$$

Graphs for the pdf and cdf of the LBQS distribution for several values of parameters are showed in Fig. 1 and Fig. 2.

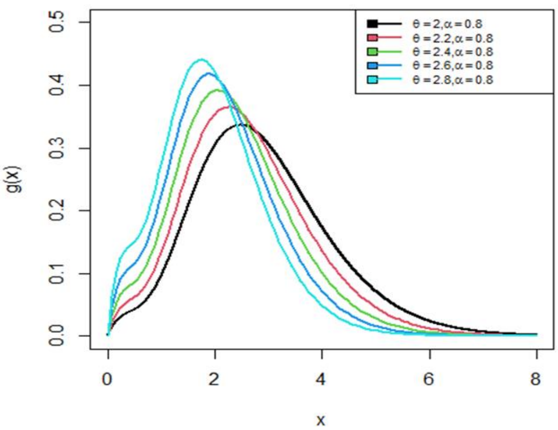

Fig.1: *pdf plot of LBQS distribution*

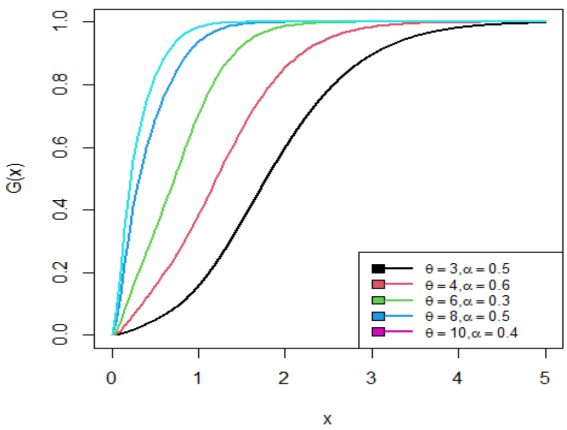

Fig.2: *cdf plot of LBQS distribution*

2.2 Survival, Hazard and Reversed hazard functions

In this section, we discuss about the survival function, hazard and reverse hazard functions of the LBQS distribution.
The survival function or the reliability function of the LBQS distribution is given by

$$S(x) = 1 - F_1(x)$$

$$S(x) = 1 - \left(\frac{\alpha\theta^3\gamma(2,\theta x) + \gamma(6,\theta x)}{\alpha\theta^3 + 120} \right)$$

$$= \frac{\alpha\theta^3(1-\gamma(2,\theta x)) + (120 - \gamma(6,\theta x))}{\alpha\theta^3 + 120}$$

The hazard function is also known as the hazard rate function, instantaneous failure rate or force of mortality and is given for LBQS distribution as

$$h(x) = \frac{f_1(x)}{S(x)}$$

$$= \frac{x\theta^5(\alpha + \theta x^4)e^{-\theta x}}{\alpha\theta^3(1-\gamma(2,\theta x)) + (120 - \gamma(6,\theta x))}$$

The reverse hazard function of the LBQS distribution is given by

$$h_{r1}(x) = \frac{f_1(x)}{F_1(x)}$$

$$= \frac{x\theta^5(\alpha + \theta x^4)e^{-\theta x}}{\alpha\theta^3\gamma(2,\theta x) + \gamma(6,\theta x)}$$

Fig. 3 and Fig. 4 represent graphs for the Survival function and Hazard rate function respectively of the LBQS distribution for several values of parameters.

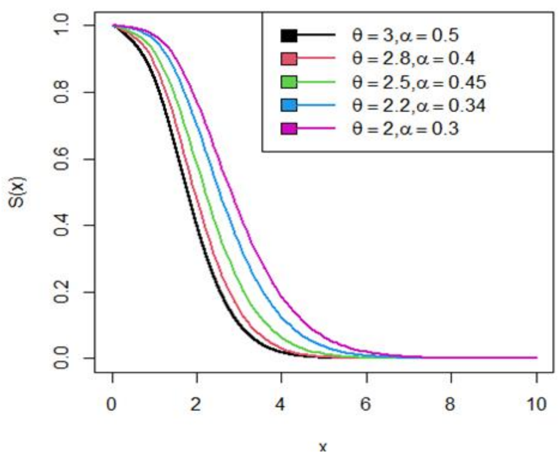

Fig.3: Survival plot of LBQS distribution

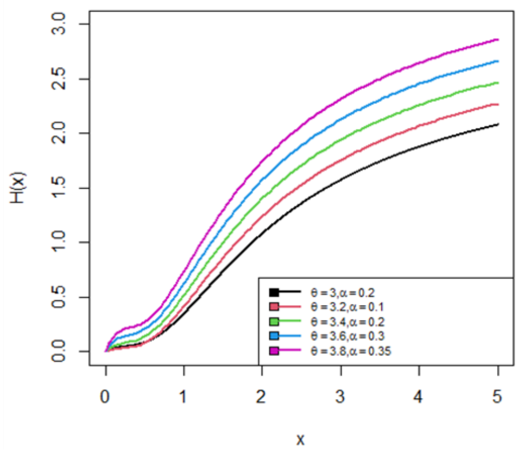

Fig.4: Hazard rate plot of LBQS distribution

3. Structural Properties

In this section, we investigate various structural properties of the LBQS distribution. Let X denotes the random variable of LBQS distribution with parameters α, θ and then its r^{th} order moment $E(X^r)$ about origin is given by

$$E(X^r) = \mu_r^{'} = \int_0^\infty x^r f_1(x)dx$$

$$= \int_0^\infty \frac{\theta^5 x^{r+1}\left(\alpha + \theta x^4\right)e^{-\theta x}}{\alpha\theta^3 + 120}dx$$

After simplifying the expression, we get

$$E(X^r) = \frac{\alpha\theta^3\Gamma(2+r) + \Gamma(6+r)}{\theta^r\left(\alpha\theta^3 + 120\right)} \tag{6}$$

Putting r=1, we get the expected value of LBQS distribution as follows

$$E(X) = \frac{2\alpha\theta^3 + 720}{\theta\left(\alpha\theta^3 + 120\right)}$$

and putting r=2, we get the second moment as follows

$$E(X^2) = \frac{6\alpha\theta^3 + 5040}{\theta^2\left(\alpha\theta^3 + 120\right)}$$

therefore, the variance of LBQS distribution is given by

$$V(X) = \frac{6\alpha\theta^3 + 5040}{\theta^2\left(\alpha\theta^3 + 120\right)} - \left(\frac{2\alpha\theta^3 + 720}{\theta\left(\alpha\theta^3 + 120\right)}\right)^2$$

3.1 Harmonic mean

The harmonic mean of LBQS distributed random variable X can be written as

$$H = E\left(\frac{1}{X}\right) = \int_0^\infty \frac{1}{x} f_1(x)dx$$

$$= \int_0^\infty \frac{\theta^5\left(\alpha + \theta x^4\right)e^{-\theta x}}{\alpha\theta^3 + 120}dx$$

after simplifying the expression, we get

$$H = \frac{\alpha\theta^3 + 24}{\theta\left(\alpha\theta^3 + 120\right)}$$

3.2 Moment generating function (MGF) and Characteristic function

Let X have a LBQS distribution, then the MGF of X is obtained as

$$M_X(t) = E(e^{tx}) = \int e^{tx} f_1(x)dx$$

According to Taylor's series, we obtain

$$M_X(t) = E(e^{tx}) = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \ldots\ldots\right) f_1(x)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!} E(X^j)$$

$$= \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_1(x)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\left(\frac{\alpha\theta^3\Gamma(j+2) + \Gamma(j+6)}{\theta^j\left(\alpha\theta^3 + 120\right)}\right)$$

Similarly, we obtained characteristic function of the LBQS distribution as follows

$$\phi_X(t) = M_X(it) = \sum_{j=0}^\infty \frac{(it)^j}{j!}\left(\frac{\alpha\theta^3\Gamma(j+2) + \Gamma(j+6)}{\theta^j\left(\alpha\theta^3 + 120\right)}\right)$$

4. Likelihood Ratio Test

Suppose X be a random sample from the LBQS distribution. We use the following hypothesis

H_o: $f(x)=f\ (x;\alpha,\theta)$ against H_1: $f(x)=f_1(x;\alpha,\theta)$

to test whether the random sample of size n comes from the quasi suja (QS) distribution or the LBQS distribution. The test statistic used is

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{f_l(x;\alpha,\theta)}{f(x;\alpha,\theta)}$$

$$= \left(\frac{\left(\alpha\theta^3 + 24\right)\theta}{\alpha\theta^3 + 120} \right)^n \prod_{i=1}^{n} x_i$$

We reject null hypothesis, if

$$\Delta > k$$

$$\left(\frac{\left(\alpha\theta^3 + 24\right)\theta}{\alpha\theta^3 + 120} \right)^n \prod_{i=1}^{n} x_i > k$$

$$\prod_{i=1}^{n} x_i > k\left(\frac{\alpha\theta^3 + 120}{\left(\alpha\theta^3 + 24\right)\theta} \right)^n$$

$$\text{or } \Delta^* = \prod_{i=1}^{n} x_i > k^* \text{ where } k^* = k\left(\frac{\alpha\theta^3 + 120}{\left(\alpha\theta^3 + 24\right)\theta} \right)^n$$

5. Entropy Measures

The idea of entropy is important in various areas such as probability and statistics, physics, communication theory and economics. Entropy measures quantify the diversity, uncertainty, or randomness of a system. Entropy of a random variable X is a measure of variation of the uncertainty.

5.1 Renyi Entropy

The Renyi entropy is important in ecology and statistics as index of diversity. It was proposed by Renyi. The Renyi entropy of order β for a random variable X is given by

$$e(\beta) = \frac{1}{1-\beta} \log\left(\int_0^\infty f^\beta(x)dx \right)$$

Where $\beta > 0$ and $\beta \neq 1$. So, we get

$$e_1(\beta) = \frac{1}{1-\beta} \log \int_0^\infty \left(\frac{\theta^5 x\left(\alpha + \theta x^4\right) e^{-\theta x}}{\alpha\theta^3 + 120} \right)^\beta dx$$

$$= \frac{1}{1-\beta} \log\left(\left(\frac{\theta^5}{\alpha\theta^3 + 120} \right)^\beta \int_0^\beta \left(x e^{-\theta x} \left(\alpha + \theta x^4\right) \right)^\beta dx \right)$$

After simplification we get

$$= \frac{1}{1-\beta} \log\left(\left(\frac{\theta^5}{\alpha\theta^3 + 120} \right)^\beta \sum_{j=0}^{\infty} {}^\beta C_j \alpha^{\beta-j} \theta^j \frac{\Gamma(\beta + 4j + 1)}{(\theta\beta)^{\beta+4j+1}} \right)$$

5.2 Tsalli's Entropy

$$S_\lambda = \frac{1}{\lambda-1}\left(1-\int_0^\infty f^\lambda(x)dx\right)$$

$$= \frac{1}{\lambda-1}\left(1-\int_0^\infty \left(\frac{\theta^5 x\left(\alpha+\theta x^4\right)e^{-\theta x}}{\alpha\theta^3+120}\right)^\lambda dx\right)$$

$$= \frac{1}{\lambda-1}\left(1-\left(\frac{\theta^5}{\alpha\theta^3+120}\right)^\lambda \sum_{j=0}^\infty {}^\lambda C_j \alpha^{\lambda-j}\theta^j \frac{\Gamma(\lambda+4j+1)}{(\theta\lambda)^{\lambda+4j+1}}\right)$$

6. Order Statistics

Consider $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics of a random sample $X_1, X_2, \ldots, X_n$ drawn from the continuous population with pdf f(x) and cdf $F_x(x)$, then the pdf of r^{th} order statistic $X_{(r)}$ is given by

$$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left[F_X(x)\right]^{r-1}\left[1-F_X(x)\right]^{n-r} \tag{7}$$

Using Equations (4) and (5) in Equation (7), the pdf of r^{th} order statistic $X_{(r)}$ of the LBQS distribution is given by

$$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{\theta^5 x\left(\alpha+\theta x^4\right)e^{-\theta x}}{\alpha\theta^3+120}\right)\times\left[\frac{\alpha\theta^3\gamma(2,\theta x)+\gamma(6,\theta x)}{\alpha\theta^3+120}\right]^{r-1}$$
$$\times\left[1-\frac{\alpha\theta^3\gamma(2,\theta x)+\gamma(6,\theta x)}{\alpha\theta^3+120}\right]^{n-r}$$

pdf of higher order statistics is given by

$$f_{X(n)}(x) = n\left(\frac{\theta^5 x\left(\alpha+\theta x^4\right)e^{-\theta x}}{\alpha\theta^3+120}\right)\times\left[\frac{\alpha\theta^3\gamma(2,\theta x)+\gamma(6,\theta x)}{\alpha\theta^3+120}\right]^{n-1}$$

pdf of first order statistics is given by

$$f_{X(1)}(x) = n\left(\frac{\theta^5 x\left(\alpha+\theta x^4\right)e^{-\theta x}}{\alpha\theta^3+120}\right)\times\left[1-\frac{\alpha\theta^3\gamma(2,\theta x)+\gamma(6,\theta x)}{\alpha\theta^3+120}\right]^{n-1}$$

7. Income Distribution Curve

The bonferroni and the Lorenz curves are not only used in economics in order to study the income and poverty, but it is also being used in other fields like reliability, medicine, insurance and demography. The bonferroni and lorenz curves are given by

$$B(p) = \frac{1}{p\mu_1'}\int_0^q xf(x)dx$$

and

$$L(p) = pB(p) = \frac{1}{\mu_1'}\int_0^q xf(x)dx$$

From (6)

$$\mu_1' = \frac{2\alpha\theta^3 + 720}{\theta(\alpha\theta^3 + 120)}$$

and $q = F^{-1}(p)$. Then, we have

$$B(p) = \frac{1}{p\mu_1'}\int_0^q \frac{\theta^5 x^2(\alpha + \theta x^4)e^{-\theta x}}{\alpha\theta^3 + 120}dx$$

after simplification, we get

$$B(p) = \frac{\alpha\theta^3\gamma(3,\theta q) + \gamma(7,\theta q)}{p(2\alpha\theta^3 + 720)}$$

similarly, lorenz curve is obtained as

$$L(p) = \frac{\alpha\theta^3\gamma(3,\theta q) + \gamma(7,\theta q)}{(2\alpha\theta^3 + 720)}$$

8. Estimation

In this section, we will discuss the maximum likelihood estimators (MLE's) of the parameters of the LBQS distribution. Consider X_1, X_2, ..., Xn be the random sample of size n from the LBQS distribution, then the likelihood function is given by

$$L(x;\alpha,\theta) = \left(\frac{\theta^5}{\alpha\theta^3 + 120}\right)^n \prod_{i=1}^n \left(x_i(\alpha + \theta x_i^4)e^{-\theta x_i}\right)$$

The log likelihood is obtained as

$$\log L = 5n\log\theta - n\log(\alpha\theta^3 + 120) + \sum_{i=1}^n \log x_i + \sum_{i=1}^{i=n}\log(\alpha + \theta x_i^4) - n\theta\sum_{i=1}^n x_i \quad (8)$$

Differentiating (8) w.r.t θ and α we get the following likelihood equations

$$\frac{\partial \log L}{\partial\theta} = \frac{5n}{\theta} - \frac{n}{\alpha\theta^3 + 120}(3\alpha\theta^2) + \sum_{i=1}^n \frac{x_i^4}{\alpha + \theta x_i^4} - n\sum_{i=1}^n x_i = 0$$

$$\frac{\partial \log L}{\partial\alpha} = \frac{-n\theta^3}{\alpha\theta^3 + 120} + \sum_{i=1}^n \frac{1}{\alpha + \theta x_i^4} = 0$$

Because of the complicated form of the likelihood equations, algebraically it is very difficult to solve the system of nonlinear equations. Therefore, we use R-software for estimating the required parameters.

9. Simulation Study

In this section, a simulation study is conducted to examine the performance of maximum likelihood estimators of the LBQS distributions using R-software. We generated random number for different sample sizes and different parameter values. We examined the mean estimates, biases, mean square errors and variances of the MLE's. The simulation results for different parameter values of LBQS distribution is presented in table 1. It reveals from the table that as the sample size increases, biases, MSEs and variances of the MLE's of the parameters become smaller respectively. Fig. 5, Fig. 6, Fig.7, Fig. 8, Fig.9, Fig.10, Fig.11 and Fig. 12 shows histogram graphs of simulated data.

Table 1: *The biases variances and MSEs of LBQS distribution for different parameter values*

n	**θ=2**			**α=8**		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
20	-0.1663918	0.07275417	0.1004404	9.1904	113.4698	197.9333
30	-0.02764637	0.09921434	0.09997866	4.44103	79.39125	99.114
50	-0.07826967	0.04104919	0.04717534	5.231117	48.07655	75.44114
80	-0.0005882063	0.0192557	0.01925604	1.946187	26.27957	30.06722
100	-0.06645069	0.0126943	0.01710999	2.637763	18.40008	25.35787
200	0.0317005	0.00394192	0.004946842	-0.8056873	7.551154	8.200286
300	-0.003812567	0.004438482	0.004453018	0.7725432	4.295084	4.891907
	θ=3			**α=0.8**		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
20	0.1077014	0.1079198	0.1195194	0.10619	0.7403965	0.7516728
30	0.05250122	0.08043579	0.08319217	-0.095104	0.3896097	0.3896097
50	0.01232364	0.07130409	0.07145596	-0.1934582	0.256577	0.2940031
80	0.07141307	0.03529702	0.04039685	-0.1773731	0.2315595	0.2630207
100	0.03355058	0.01024581	0.01137146	0.01653806	0.05744206	0.05771557
200	-0.02356278	0.01017962	0.01073482	0.009685886	0.05365214	0.05374595
300	0.01235768	0.00395297	0.004105682	-0.03921203	0.0146893	0.01622688
	θ=6			**α=0.3**		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
20	-0.2409736	0.8117087	0.869777	0.4400109	0.6451856	0.8387952
30	0.2179345	0.4019304	0.4494258	-0.03356909	0.04080886	0.04193574
50	0.2340156	0.3274527	0.382216	0.00622955	0.03390483	0.03394364
80	0.15726	0.08666046	0.1113912	-0.02508142	0.01842258	0.01905166
100	-0.1231709	0.06968093	0.08485199	0.02899184	0.01667758	0.01751811
200	0.07676134	0.06580578	0.07169809	0.0002268879	0.007942669	0.007942721
300	0.023855	0.0234289	0.02399796	0.009857188	0.005338747	0.005435911
	θ=8			**α=0.5**		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
20	1.798051	6.027411	9.260398	0.01659618	0.3433603	0.3436357
30	0.5363827	2.157112	2.444818	0.04257179	0.2365076	0.23832

50	0.7529427	1.423766	1.990688	-0.1185865	0.09495143	0.1090142
80	0.6645098	0.729458	1.171031	-0.1246661	0.04489143	0.06043306
100	0.1758446	0.4423283	0.4732496	-0.02355057	0.05949109	0.06004572
200	0.1146044	0.1470872	0.1602214	-0.03848292	0.01408208	0.01556301
300	0.1408959	0.06484056	0.08469221	-0.04973074	0.007839619	0.01031277

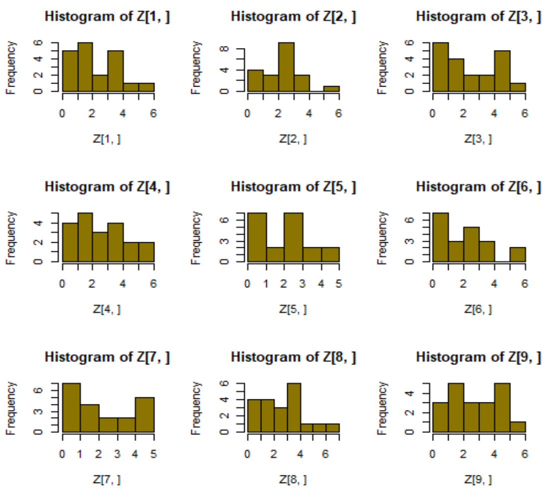

Fig.5: *Simulation histogram when n=20, θ=2, α=8*

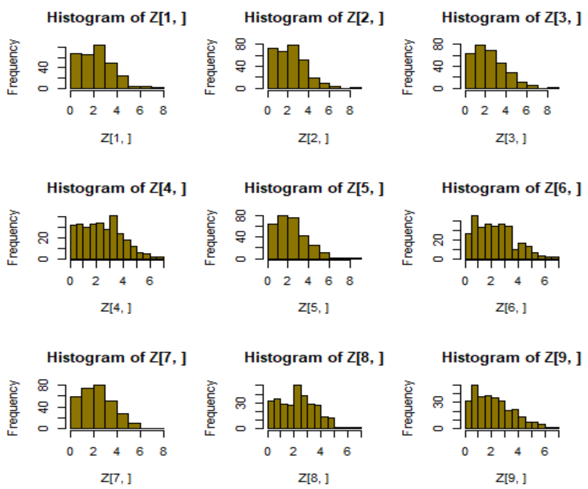

Fig.6: *Simulation histogram when n=300, θ=2, α=8*

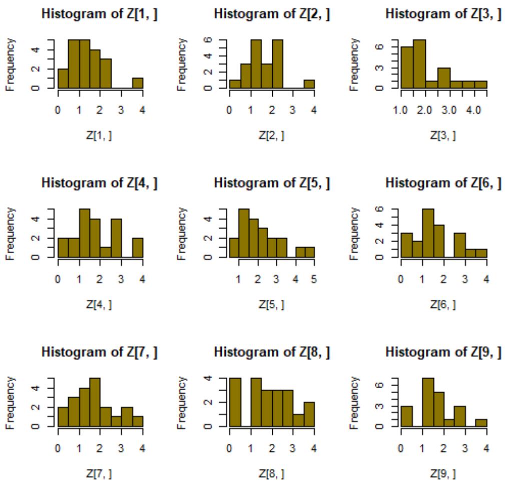

Fig.7: *Simulation histogram when n=20, θ=3, α=0.8*

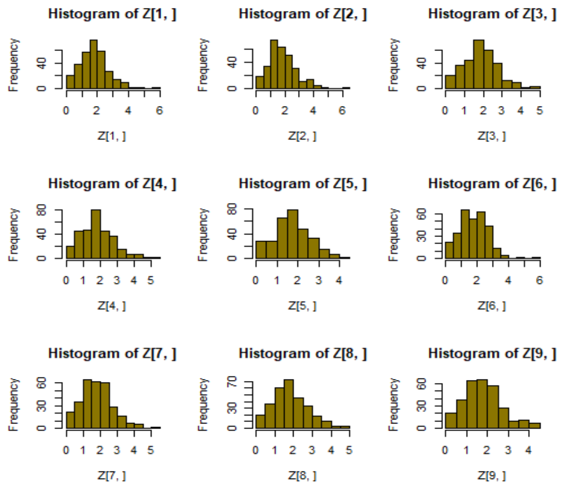

Fig. 8: *Simulation histogram when n=300, θ=3, α=0.8*

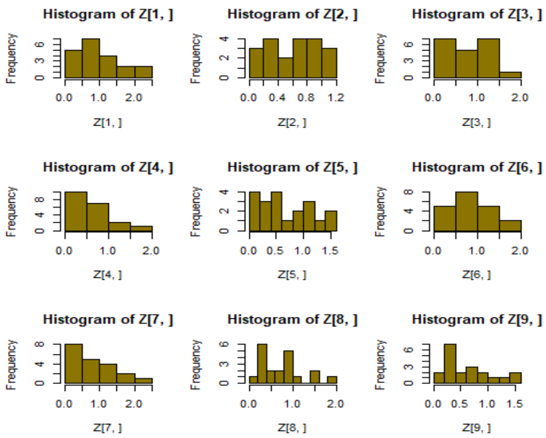

Fig.9: *Simulation histogram when n=20, θ=6, α=0.3*

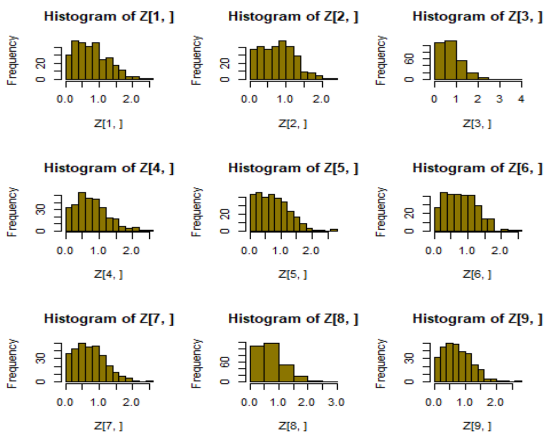

Fig. 10: Simulation histogram when n=300, θ=6, α=0.3

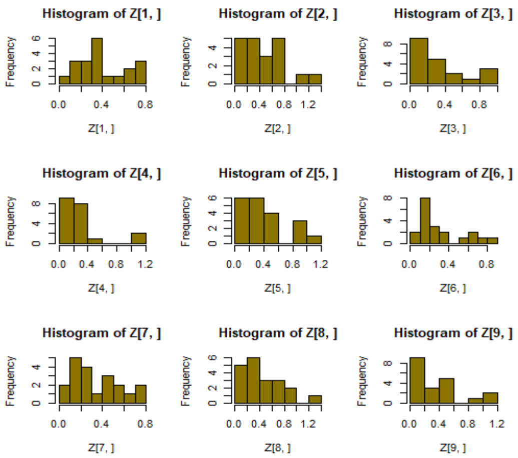

Fig.11: *Simulation histogram when n=20, θ=8, α=0.5*

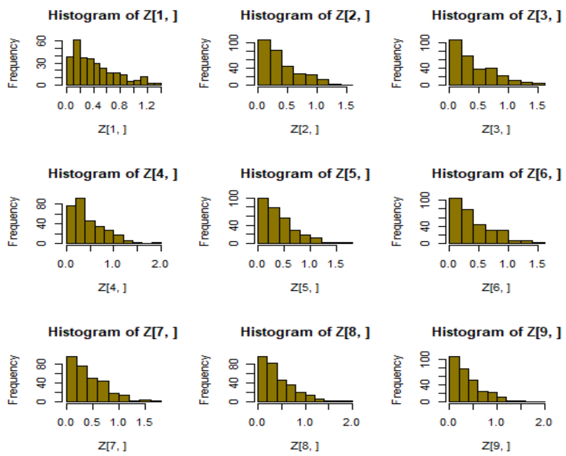

Fig.12: *Simulation histogram when n=300, θ=8, α=0.5*

10. Application

In this section we fitted LBQS distribution on a real-life time data set and compared the model with various distributions namely, Quasi Suja (QS), Exponential, Lindley, Erlang truncated exponential (ETE) distribution.

The data set is the period between failures for three repairable objects. This data set is provided in Hassan A.S [10] and explored later by Gadde S. R [8]

1.43, 1.23, 1.46, 0.11, 0.94, 0.30, 0.71, 4.36, 1.82, 0.77, 0.40, 2.37, 2.63, 1.74, 0.63, 1.49, 4.73, 1.23, 3.46, 2.23, 1.24, 2.46, 0.45, 1.97, 0.59, 0.70, 1.86, 0.74, 1.06, 1.17.

In order to compare LBQS distribution with the above-mentioned distributions, we consider the criteria like Bayesian information criterion (BIC), Akaike information criterion (AIC), Akaike information criterion corrected (AICC) and -2 logL. Better distribution is said to be the one which has lower values of AIC, BIC, AICC and -2logL. These criteria can be calculated by using the following formulae.

$$AIC = 2k - 2\log L,\ \ BIC = k\log n - 2\log L,\ \ AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

k is the number of parameters; n is the sample size and -2logL is the maximized value of log likelihood

Data set	Distributions	Parameters	MLE (Standard error)	-2logL	AIC	BIC	AICC
1	LBQS	α	3.06130 (2.37265)	79.26209	83.26209	86.06448	83.706
		θ	1.29691 (.16734)				
	QS	α	0.29151 (0.29806)	86.74344	90.74344	93.54584	91.632
		θ	2.72816 (0.34681)				
	ETE	β	1.02149 (78.67718)	86.01075	90.01075	92.81315	90.455
		θ	1.00673 (133.7592)				
	Exponential	θ	0.64822 (0.11835)	86.01075	88.01075	89.41195	88.153
	Lindley	θ	0.9762395 (0.1345043)	83.09456	85.09456	86.49576	85.238

function. They are calculated for above mentioned data set and showed in table 2.

Table 2: *Parameter estimations and goodness of fit test statistics*

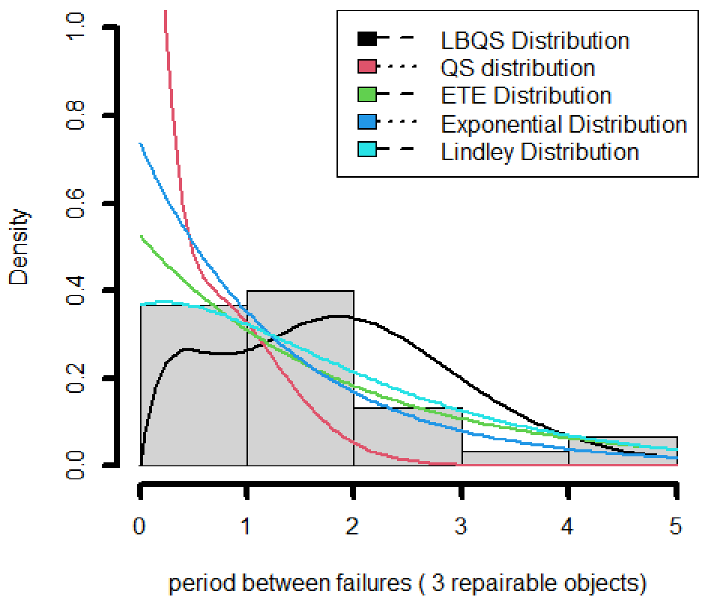

Fig. 13: *Density curve of data set*

From table 2 and density curve of data shown in Fig.13, it is evident that the LBQS distribution leads to better fit than the Quasi Suja, Exponential, Lindley, ETE distributions.

11. Conclusion

In this paper, a new modification of quasi suja distribution is executed namely length biased quasi suja distribution with two parameters and its different statistical properties are discussed and investigated. The distribution is generated by using the length biased technique and taking the two-parameter quasi suja distribution as the base distribution. The parameters of the executed distribution are obtained by using the maximum likelihood estimator. Finally, the usefulness of newly introduced distribution is discussed by applying the to real life data set and the result of the data set witnessed that the length biased quasi suja distribution fits better than the quasi suja, exponential, erlang truncated exponential, and lindley distributions.

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