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OPTIMIZING INVENTORY CONTROL THROUGH A GRADIENT-BASED MULTILEVEL APPROACH IN THE FACE OF DEMAND AND LEAD TIME UNCERTAINTIES

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Abstract

Systems of two-level assembly with unknown timing of leads are taken into consideration while arranging supplies. Probably, the final product's demand and its deadline are known. When all required parts are on hand, each level's assembly process gets underway. To address these problems, we have developed a model for the control of inventories for an uncapitatedwarehousing space in a manufacturing plant with unpredictable demand and lead times. The goal is to choose orders in a way that minimizes the overall system's cost. We present a multilevel optimization model including a rotating horizon that utilizes gradients to handle unknown lead time and demand, irrespective of the distributions at the core of them. Furthermore, a precise algorithm is created to solve the model. In a case study, we compare our approach with the current model. Our computational results indicate that while the new gradient-based multi-level optimization model nearly continuously yields the least expensive overall across all parameter settings. These models' performances are either systematically worse or extremely sensitive to cost parameters (holding cost, shortfall cost, etc.).

Keywords: multilevel optimization model; rolling horizon;uncertain demand; uncertain lead time.

1. Introduction

Supply chain management is a top priority for businesses in the modern global marketplace because of the fierce competition and elevated customer expectations. A special focus is on supply chain network architecture because it is recognized that efficient management of the supply chain a critical role in lowering costs and improving service levels. A skilled network design approach can significantly reduce expenses for a company by as much as 60% [1-3]. Historically, the planning and placement of facilities have been considered a strategic choice in supply chain network architecture. That being said, sub-optimality may result from the traditional method of making tactical inventory decisions after site selections. Inventory costs are heavily impacted by strategic location choices, highlighting the necessity of incorporating inventory concerns into strategic network design models [4-6]. As a result, there has been a significant push in recent years for the creation of inventory models that integrate tactical and strategic decisions. Several reasons, including changes in consumer needs and the arrival of raw materials, cause uncertainty to become a ubiquitous feature in supply chain networks. Three main areas of uncertainty are identified: customers, manufacturing, and suppliers. Unexpected expenses can arise from supplier uncertainty, which introduces unpredictability in lead time, and customer uncertainty, which shows up as variances in order time or quantity. To improve overall operational efficiency and optimize supply chains, businesses must acknowledge and manage these uncertainties [7-8]. This research presents a novel Model of inventory control that considers demand variability and lead time uncertainty. Stochastic programming is accepted as a useful technique when the randomness's probabilistic description is provided; but, in practice, this information is not always available. The suggested model attempts to strike a balance between the curse of dimensionality and solution resilience when addressing multi-period decision-making situations with uncertainty [9–10]. The inventory control model is intended for usage in a manufacturing facility warehouse, where a single product is produced from an ordered part. Even though in reality several items are manufactured using different parts, in some situations it is appropriate to assume that there is only one product, particularly when production lines are independent and separate for different products. The goal of the study is to specify an order strategy that reduces system expenses [11–12]. The paper analyzes two ambiguous parameters: lead time and demand with unknown likelihood distributions and assumes that these quantities are independent random variables within given intervals. The assumption is consistent with empirical observations of dynamic lead time and demand dynamics, where trends in the past are not indicative of the future. The model also allows for shortages, which have a backlog that is entirely unfilled. To minimize the entire rate, which includes order, inventory holding, and shortfall expenses, the goal is to ascertain the timing and size of orders. Because demand is unpredictable at each stage and lead times are realized only after orders are placed, the intrinsic dimensionality curse affects multi-stage decision-making problems. [13–14]. This study presents three distinct contributions. Unlike earlier models, it first takes supply and demand uncertainties into account. Second, it presents a novel multilevel inventory control optimization model that approximates the multi-stage computational tractability decision-making problem. Thirdly, the work creates a precise algorithm for the multilevel optimization framework based on gradients, which makes it possible to explore the scenario space and worst-case situations with efficiency. The following is the arrangement of the paper's succeeding sections: Section 2 discuss about existing relevant works. Section 3 goes into a thorough discussion of the problem formulation and algorithm. Experimental results and sensitivity analysis are detailed in Section 4. Lastly, Section 5 concludes the paper, summarizing key findings.

2. Literature survey

This literature survey delves into various topics related to inventory control, including lost sales inventory systems, perishable inventory systems, and supply chain management. These papers can provide valuable insights into the challenges and solutions related to inventory control and supply chain management, which can help in developing an effective inventory control policy for the given research problem. Hansen et al. [15] presented a perishable product inventory control policy for business-to-consumer retail that takes lead time and demand volatility into account. To reduce expenses associated with stockouts and excess inventory, the study concentrated on handling perishable inventory concerns. The established replenishment approach provided efficient management of perishable inventory by balancing holding costs against lost sales expenses. The idea outperformed traditional approaches, as shown by mathematical models, offering shops a useful resolution. Dey et al. [2] investigated controlled lead time and adaptability in astute supply chain management, highlighting the advantages of shortening lead time for diverse elements. Instead of using projected formulae for total costs, the study presented an accurate overall cost calculation that took backorder relationships and on-hand inventory into account. The study's use of marginal value analysis showed that the total expense of the supply chain is convex in terms of lead time and volatility. An intelligent manufacturing procedure that addresses stochastic demand and variable production rates was presented by the researcher. It was validated by numerical examples and classical optimization, and the model validation was further strengthened by sensitivity analysis and graphical representations. Zhang et al. [16] presented a learning algorithm that takes into account missed sales, positive lead times, and suppressed demand for a single-product inventory system assessment regularly. It tackled the problem of adaptive inventory ordering depending primarily on previous sales information. A random cycle-updating rule with essential components including withheld on-hand inventory and estimate of double-phase cycle gradients was introduced by the nonparametric simulated cycle-update policy. The study demonstrated efficacy in managing intricate system dynamics by establishing a square root convergence rate as a lower constraint for learning algorithms through regret analysis. The discovered methods reduced the cost differences between practical learning algorithms and clairvoyance benchmarks by enabling adaptive inventory decisions based on historical sales. Das et al. [17] addressed inventory control with partial backlog, price-dependent demand, and preservation technology applied to non-instantaneously decaying commodities. With the inclusion of a Weibull distribution with three parameters for deterioration and preservation, the model took into account the effects of preservation technology, price-dependent demand, and deterioration. The extremely nonlinear optimization issues were solved using quantum-behaved particle swarm optimization (QPSO) techniques. By comparing findings with several QPSO variations, numerical examples were used to validate the proposed model. Sensitivity analysis looked into how changing a parameter would affect the best course of action. Sarkar et al. [18] discussed a collaborative advertising strategy for supply chain management in ambiguous circumstances. Equations [34–37] restrictions were used to examine the model's goal. Equation (24) provided the overall cost of the supply chain under a cooperative advertising collaboration policy with ambiguous conditions. Equation (15) provided the supplier's total cost after modeling each of the separate charges related to the supplier under the cooperative advertising collaboration. While creating the model, the paper took into account a few suppositions. Transchel et al. [19] addressed considering lead time unpredictability and service level limitations, inventory management and supply planning are implemented for perishable goods. A dynamic inventory control policy was implemented, taking into account a specified service level and an unpredictable lead time for replenishments. With a first-in-first-out (FIFO) inventory system and a non-stationary demand process, the study concentrated on a business-to-business (B2B) setting. Through simulation-based optimization, the authors addressed the influence on service levels and waste rates while offering analytical insights into the ideal replacement quantity under lead-time uncertainty. Goli et al. [20] addressed the arc-routing problem of sustainable periodic garbage pickup. To solve the garbage collection problem, the research presented a hybrid multi-objective optimization strategy. The issue was formulated as an arc-routing problem in the paper, which made use of multi-objective optimization to take into account numerous objectives at once and the creation of a hybrid algorithm to discover the best possible answers. Our goal is to critically assess how these studies contribute to inventory control advancements, particularly focusing on their effectiveness in handling uncertainty and several other issues.

3. Methodology

3.1. Problem statement

We examine a manufacturing facility's vacant warehouse for a single item. Both the lead time and the demand are ambiguous. To reduce the costs associated with orders, inventory, and shortages,Choices are made across an arbitrary distinct time frame. We execute the assumption that the management has complete knowledge of the level of demand and inventories at the moment shortfall, and order arrival status before deciding which order to place during that time. We also assume that the shortfall is fully backlogged, and order volume and demand arrive at the start of the time frame for decisions. The present period is designated as period 1 for modeling reasons, and a finite planning horizon n {1, 2... T} is enforced. The model has been resolved using updated data for every decision-making window, and alone the sequence choice for the present period is carried out when using the rolling horizon method of this model 105's answer [23–24] The planning horizon of the $P(\tau)$ decision-making model spans the time from τ to $\tau + T - 1$ .Following the resolution of the model $P(\tau)$ for decision-making, the policy of order was established, We separate the horizon of planning decision into two sections: The choice reached in the first era, τ, and in the second subsequent periods, $(\tau + 1, ..., \tau + T - 1)$. The order guidelines for period τ are put into effect, and τ is raised by one are repeated after updating the model's initial settings for the upcoming planning horizon. As a result, the choice about periods $(\tau + 1 ... \tau + T - 1)$ may need to be rescheduled for the following planning span.

The planned horizon measure T has a significant impact on how accurate the previously mentioned planning model is. Because $T \geq 3$ are multi-phase models of decision-making that are free from the well-acknowledged dimensionality curse. Infamously difficult to solve from the standpoint of computational tractability. However, from an application standpoint, models with this little planning horizon are essentially blind and could produce overly narrow-minded results.

3.2. Deterministic model

Consider an examination of a condensed form of the inventory control model, which takes lead time and demand into account to be known and constant for every period. As a result, a Model for deterministic single-stage optimization is all that remains of the multi-phase issue requiring decision-making. It is important to note that a set of binary parameters $\delta_{k,t} \forall\ k,\ t$, which indicates whether or not the order placed in period k arrives by period t, constitutes the random lead time. For instance, if an order placed during period 3 has a lead time of 4, then $\delta_{3,4} = \delta_{3,5} = \delta_{3,6} = 0$ and $\delta_{3,t} = 1, \forall t \in \{7, 8, ... T\}$.The model for deterministic inventory management is presented in *(1a)–(1d)*. The model aims to reduce the overall cost along the horizon of planning. The inventory holding cost, shortfall cost, and both fixed and variable order costs are the four cost terms in (*1a*), in that order. After period t, the inventory level is determined by equation (*1b*). The quantity of deficiency at period t, the total number of ordered products that reach by time t, the initial inventory at period 0, and the entire quantity of the demand that is satisfied among periods 1 and t consist of the four terms listed on Constraint (*1b*)'s right side.By requiring the ordering of at least one item within that time frame, constraint (*1c*) guarantees the imposition of a fixed order fee. Constraint (*1d*) defines the decision variable supports. article amsmath

$$\min \quad \zeta = (c\mu) \sum_{t=1}^{T} q_t + f \sum_{t=1}^{T} v_t + h \sum_{t=1}^{T} I_t + p \sum_{t=1}^{T} g_t \tag{1a}$$

$$\text{s.t.} \quad I_t = I_0 + \sum_{k=1-K}^{t-1} \mu q_k \widehat{\delta}_{k,t} + g_t - \sum_{i=1}^{t} \widehat{d}_i \quad t \in \{1, 2, \ldots, T\} \tag{1b}$$

$$q_t \leq M v_t \quad t \in \{1, 2, \ldots, T\} \tag{1c}$$

$$q_t, I_t, g_t \in \mathbb{Z}^+; \quad v_t \in \{0, 1\} \quad t \in \{1, 2, \ldots, T\} \tag{1d}$$

3.3. Proposed method: Gradient computation in the approximated problem

The estimated problem for multilevel optimization is asymptotically convergent; hence we suggest using a projected gradient approach. The formula for $\nabla_{x_1} \widetilde{F}_1(x_1)$ is derived in this section, and the projected gradient method's local and global convergenceis verified [18-24].

(i)Gradient of the objective function in the approximated problem

A formula for computing is given by the following theorem $\nabla_{x_1} \widetilde{F}_1(x_1)$.

Theorem 1: Formula for (gradients in n-level optimization problems). The expression for the gradient $\nabla_{x_1}\widetilde{F}_1(x_1)$is as follows. article amsmath

$$\begin{aligned}
\nabla_{x_1}\widetilde{F}_1(x_1) &= \nabla_{x_1} f_1 s(x_1, x_2^{(T_2)}, \ldots, x_n^{(T_n)}) \\
&\quad + \sum_{i=2}^{n} Z_i \nabla_{x_1} f_1(x_1, x_2^{(T_2)}, \ldots, x_n^{(T_n)}), \\
Z_i &= \sum_{t=1}^{T_i} \left(\sum_{j=2}^{i-1} Z_j C_{ij}^{(t)} + B_i^{(t)} \right) \prod_{s=t+1}^{T_t} A_i^{(s)}, \\
A_i^{(t)} &= \nabla_{x_i} \Phi_i^{(t)}(x_1, x_2^{(T_2)}, \ldots, x_{i-1}^{(T_{i-1})}, x_i^{(t-1)}), \\
B_i^{(t)} &= \nabla_{x_1} \Phi_i^{(t)}(x_1, x_2^{(T_2)}, \ldots, x_{i-1}^{(T_{i-1})}, x_i^{(t-1)}), \\
C_{ij}^{(t)} &= \nabla_{x_j} \Phi_i^{(t)}(x_1, x_2^{(T_2)}, \ldots, x_{i-1}^{(T_{i-1})}, x_i^{(t-1)})
\end{aligned}$$

For any $i = 2, \ldots, n; t = 1, \ldots, T_i;$and j $= 2, \ldots, i-1$, wherever we define $\prod_{s=t+1}^{T_t} A_i^{(s)} := A_i^{(t+1)} A_i^{(t+2)} \ldots A_i^{(T_i)}$ for $t < T_i$and $\prod_{s=T_{i+1}}^{T_i} A_i^{(s)} = I$.

We consider computing $\nabla_{x_1}\widetilde{F}_1(x_1)$ using **Theorem 1**. Observe that computing is simple for us $Z_2 = \sum_{t=1}^{T_2} B_2^{(t)} \prod_{s=t+1}^{T_2} A_2^{(s)}$. For i $= 3, \ldots, n$, when we have$Z_2, \ldots, Z_{i-1}$, we can calculate Z_i. We present a computation technique that $\nabla_{x_1}\widehat{F}_1(x_1)$ by computing $Z_2, \ldots, Z_n$ in this sequence within Algorithm 1 article amsmath algorithm algpseudocode

Algorithm 1: Calculation of $\nabla_{x_1}\widehat{F}_1(x_1)$.

Input: x_1 = existing first-level variable value. $\{x_i^{(0)}\}_{i=2}^{n}$ = the lower-level iteration's initial values.
Output: The exact value of $\nabla_{x_1}\bar{F}_1(x_1)$.

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Output: The exact value of $\nabla_{x_1}\bar{F}_1(x_1)$. $g := (0, \ldots, 0)^\top$.
for i := $2, \ldots,$ n do
$Z_i := O$.
for t := $1, \ldots, T_i$do
$x_i^{(t)} := \Phi_i^{(t)}\left(x_1, x_2^{(T_2)}, \ldots, x_{i-1}^{(T_{1-1})}, x_i^{(t-1)}\right)$.
$\widehat{B}_i^{(t)} := \sum_{l=2}^{i-1} Z_l C_{\text{il}}^{(t)} + B_i^{(t)}$.
$Z_i := Z_i A_i^{(t)} + \bar{B}_i^{(t)}$.
for i $= 2, \ldots,$ n do
$g := g + Z_i \nabla_{x_1} f_1$.
$g := g + \nabla_{x_1} f_1$.
return g

For $i = 2, \ldots, n$and t $= 1, \ldots, T_i, \Phi_i^{(t)}$, It is mentioned in Algorithm 1's fifth line is the revised formula that takes the gradient into account.$\nabla_{x_4}\bar{F}_i(x_1, \ldots, x_i)$ of the ith function of the level objective. $\nabla_{x_i}\widetilde{F}_i(x_1, \ldots, x_i)$is calculated by utilizing Algorithm 1 on the $(n-i+1)$- a level optimization issue using objective functions $\widetilde{F}_i, \ldots, \bar{F}_n$. Therefore, in the computation, call Algorithm 1 recursively of $\phi_i^{(t)}$, we can compute $\nabla_{x_1}\widehat{F}_1(x_1)$.

3.4 Complexity of the gradient computation

By repeatedly running Algorithm 1, we examine the computational complexity of $\nabla_{x_1}\widehat{F}_1(x_1)$. An asymptotic large symbol for O notation is $O(*)$ in the following theorem.

Theorem 2. Let c_i and $s_i$be the complexity of time and space, respectively, for computing$\nabla_{x_1}\widetilde{F}_i(x_i)$. To calculate$\nabla_{x_1}\widehat{F}_i(x_1, \ldots, x_i)$, we recursively call the algorithm $\Phi_i^{(t_i)}$ based on $\nabla_{x_1}\widetilde{F}_i(x_i)$. Moreover, we use the following assumptions:

- The intricacy of space and time in assessing $\Phi_i^{(t_i)}$ are $O(c_i)$and $O(s_i)$.
- The time and space complexity of Algorithm 1 pales in contrast to the for loops in lines 2–9. of$\nabla_{x_1} f_1$ and $\nabla_{x_1} f_1$ for $i = 1, \ldots, n$ are reduced in terms of order.
- Then, the whole intricacy of time c_1 and space intricacy s$_1$ for calculating can $\nabla_{x_1} \widehat{F}_i (x_1, \ldots, x_i)$ be expressed as

$$c_1 = O\left(p^n n! c_n \prod_{i=1}^{n-1} (T_{i+1} d_i)\right),$$
$$s_1 = O(q^n s_n).$$

Correspondingly, for a fixed $p, q > 1$.

3.4. Global convergence of the projected gradient method

Here, we investigate using the projected gradient approach to solve the problem. In this method,the revised point is projected on S_1in each iteration and computes the gradient vector using Algorithm [1]. We may determine when all lower-level updates are made using the steepest descent method, the Lipschitz continuity of the objective function's gradient. Therefore, by choosing a short enough step size, we may ensure both local and global projected gradient method convergence.

Theorem 3. Assume that $\Phi_i^{(t)}\left(x_1, \ldots, x_{i-1}, x_i^{(t-1)}\right) = x_i^{(t-1)} - \alpha_i^{(t-1)} \nabla_{x_1} \widetilde{F}_i \left(x_1, \ldots, x_{i-1}, x_i^{(t-1)}\right)$ for all $i = 2, \ldots, n$ and $t_i = 1, \ldots, T_i$where parameters for $\alpha_i^{(t-1)}$ and $x_i^{(0)}$ are provided for each i and t. Let us assume that $\nabla_{x_j} f_i$is limited and Lipschitz continuous for all $i = 1, \ldots, n$and $j = 1, \ldots, n$; furthermore, $\nabla_{x_1} \Phi_i^{(t)}, \nabla_{x_1} \Phi_i^{(t)}$, and $\nabla_{x_j} \Phi_i^{(t)}$are bounded and Lipschitz continuous for all i $= 2, \ldots, n$; $j = 2, \ldots, i-1; t = 1, \ldots, T_i$.. In such cases, $\nabla_{x_1} \widehat{F}_1$ is Lipschitz continuous with$\nabla_{(x_1)}$.

This theorem is proven; see Supplementary material A.5. Assume the same premises as Theorem 3 in the following corollary. Assume that the set $S_1$is compact and convex. Assume that L is the Lipschitz constant for $\nabla_{x_1} \widehat{F}_1$. In the case of Problem (6), a sequence $\left\{x_1^{(t)}\right\}$ produced by the projected gradient technique with a compact, steady-step size (e.g., smaller than $2/L$) converges to a stationary point at a convergence rate via a convergent subsequence starting at any initial point of $O(1/\sqrt{t})$.

Proof. The gradient of the problem's L-Lipschitz is the objective function continuous, according to **Theorem 3.** The gradient mapping related to $\widetilde{F}_1$, the indicator function of S_1, and the fixed step magnitude $\alpha_1^{(t)}$with fulfilling $0 < \alpha_1^{(t)} < 2/L$for all t should be represented by G : $int\left(dom\left(\widetilde{F}_1\right)\right) \to \mathbb{R}^{d_1}$Keep in mind that €'G(x_1)€=0 only in the event that x_1 is a stationary point. With $\bar{x}_1$as a limit point of $\left\{x_1^{(t)}\right\}$., we obtain $\min_{s=0}^{t} \parallel G\left(x_1^{(s)}\right) \parallel \leq O(1/\sqrt{t})$ and $\parallel G(\bar{x}_1) \parallel = 0$.

4. Computational experiments

To evaluate and contrast the capabilities of novel gradient based multilevel optimization model with existing stochastic programming, and pessimistic decision-making models, we ran an experiment. The five patterns of demand utilized in the research are based on actual request information from the Census Bureau of the United States Department of Commerce and the Federal Reserve Bank of St. We carried a total of five sets of tests for each of the six models and five cases, with *h/p* = {0.1, 0.3, 0.5, 0.7, 0.9} for *h* = 5 and *c* = 1. Since the ratio of *h/p* is important in inventory control models. Given the importance of the *h/p* ratio in models of inventory control, we ran a total of five sets of experiments for each of the six models and five occurrences, with *h/p* = {0.1, 0.3, 0.5, 0.7, and 0.9} when h = 5 and c = 1. We took the demand data some of which are expressed in millions of dollars and created random lead-time estimates. We divided the values by the price of one unit to translate them to the number of units and assure uniformity. It's also believed that period 1 will not see any orders placed, but that period 1's initial inventory will be enough to meet demand for the first two periods. It is significant to highlight that the computer system using MATLAB (R2022a) software was able to generate the numerical results and tables following the specifications.

Simulation results

According to simulation data, for various *h/p* ratios, the gradient-based multilevel model has a lower total cost on average less optimistic than other models of decision-making, such as the stochastic programming model. We performed five instances of a sensitivity analysis using various cost parameter choices for each model, and the results are displayed in Figure 1. The combination of the cost factors h and p is shown in this figure. One instance is represented by each graph column, and the first row displays the pattern of demand for every case. The cost parameter configurations for various *h/p* ratios are shown in the following graph rows. The difference between each model and the ideal model is shown bythe bars' vertical axis, which is computed by dividing each model's answer by the perfect model's solution minus one. Consequently, a smaller value implies the model's output is more similar to the perfect model. For instance, in the graph where *h/p* = 0.5, the multilevel optimization model's solution is 1.39e7, whereas the perfect model's solution is 0.85e7. Consequently, the multilevel model's value of the bar chart on this graph is (1:39e7/0:85e7) - 1 = 64%. Overall parameter settings and instances, the average difference between our model and the ideal model is 71.4%, 84.7%, and 79.5%, respectively.

To demonstrate the extent to which the multilevel model's total cost is superior to or inferior to other models, we also provide the numerical experiment's results from a different angle. The multilevel model's relative performance is assessed using the ratio R = 100 in comparison to the stochastic programming and pessimistic techniques for making decisions. The expression "multi" denotes the average overall price of the multilevel model, whereas "mdl" represents the average overall expense of the pessimistic models or stochastic programming, across five instances. In Figure 2, we conspire and display the outcomesof the overall cost-performance ratio. The multilevel model performs better than the comparison model if there is a good performance ratio; hence, a greater percentage indicates an improved comparative performance of the multilevel model. For example, the multilevel model's average overall cost is 10.6% lower than that of the stochastic programming and pessimistic models when *h/p* = 0.3. For example, for *h/p* = 0.3, the multilevel model's average total cost beats the probabilistic and stochastic programming models by 2.6% and 10.6%, respectively. When *h/p* is raised, the pessimistic model performs worse. By projectingit usually has a larger inventory level to minimize lead times and future needs; hence, raising h/p raises the model's overall cost.

Several performance metrics in use now have different service levels. We examined the single service level—the multilevel, pessimistic models, and stochastic programming fill rate. The proportion of client orders that are fulfilled right away from available stock is known as the fill rate. Reducing the h/p ratio generally improves it. Figure 3 shows the average percentage of fills of five instances for each model. The pessimistic model's fill rates are 97% in all possible combos of h and p. The fill rates of the stochastic programming model and the multilevel optimization model are nearly equal at 98% when the expense of a shortage is extremely significant, that is, *h/p* = 0.1. Nevertheless, the multilevel optimization model's fill rate decreases by only 2%, while the stochastic programming model's fill rate falls to 85% when the shortage cost is reduced.

While the multilevel optimization model responds to adjustments to shortages cost extra subtly and effectively than the stochastic model, figure 3 shows the outcomes suggest that the multilevel optimization model is not the stochastic model, sensitive to scarcity cost. We divided the overall cost into the costs associated with shortages and inventory holding to explain this finding. Table 1 summarizes the percentage shifts in shortage and inventory levels for each of the five cases when the h/p ratio rises from 0.1 to 0.9. Percentages that are positive or negative

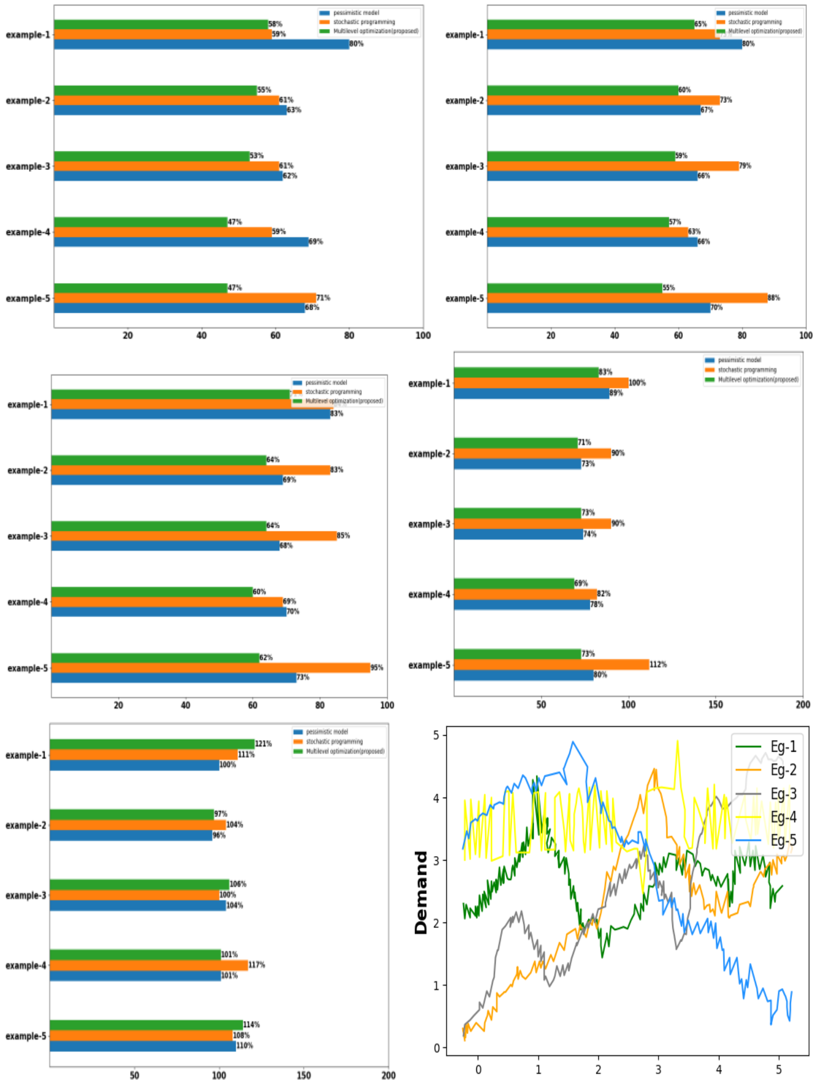

Figure 1: *The comparison of multilevel optimization, stochastic programming, Pessimistic models under* h/p= *{0.9, 0.7, 0.5, 0.3, 0.1}*

denote growth or decrease, accordingly. The multilevel model decreased the average inventory level each period (and the related inventory cost) by 22% to get the benefits of the decreased shortfall cost when the shortage cost decreased (the *h/p* ratio increased from 0.1 to 0.9) in Example 1 in Table 1. The fill rate decreased by 2% as a result, and the average shortage level rose by 160% (from 0.34 to 0.90 units each period). However, because of the sharp decline in shortfall cost per unit, the shortfall cost was drastically reduced by 71%. However, there was a more notable reaction from the stochastic model to the lower shortage cost. 51% less inventory meant a 14% lower fillrate, an increase in the scarcity level of 961% (from 0.35 to 3.70 units each period), and an 18% increase in the cost of shortages. These adjustments result in a 36% and 31% reduction in the multilevel optimization model's total amount of inventory held and shortfall cost, respectively, and the stochastic programming model. The average percentages for five cases are reported in the final two rows of Table 1.

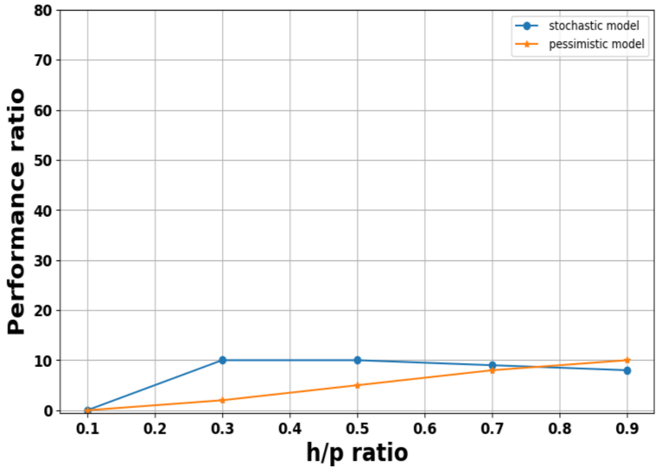

Figure 2: *Retaining and scarcity costs' effects on the multi-level model's (multi) relative performance ratio concerning other models (model). * (mdl - multi)/mdl is the performance ratio, or R = 100.*

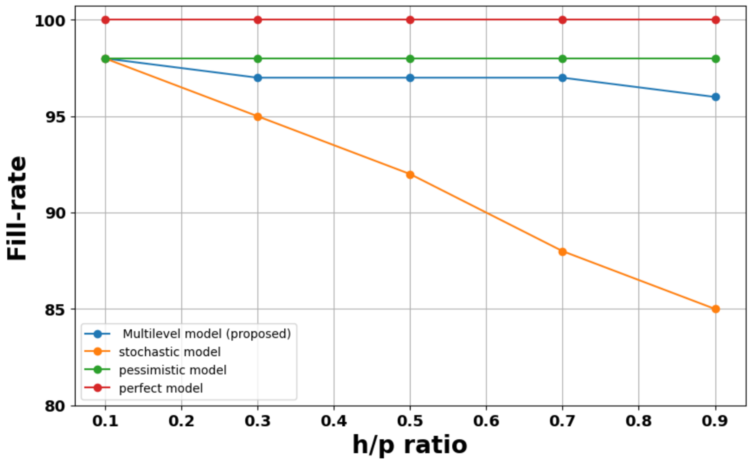

Figure 3: *Effects of holding and shortage expenses on fill rate*

Table 1: *Inventory, shortage level, cost, and fill rate change when the h/p ratio rises from 0.1 to 0.9.*

Example	Type	Inventory Cost	Shortage Cost	Inventory Level	Shortage Level	Fill-rate	Total Inventory and Shortage Cost
1	Multi-level	-22%	-71%	-22%	160%	-2%	-36%
	Stochastic	-51%	18%	-51%	961%	-14%	-31%
2	Multi-level	-14%	-69%	-14%	175%	-2%	-27%
	Stochastic	-50%	26%	-50%	1032%	-15%	-27%
3	Multi-level	-14%	-75%	-14%	126%	-2%	-32%
	Stochastic	-47%	28%	-47%	1056%	-14%	-25%
4	Multi-level	-26%	-67%	-26%	200%	-3%	-35%
	Stochastic	-41%	-19%	-41%	631%	-13%	-32%
5	Multi-level	-17%	-83%	-17%	51%	-1%	-38%
	Stochastic	-30%	-4%	-30%	762%	-9%	-22%
Average	Multi-level	-19%	-73%	-19%	143%	-2%	-34%
	Stochastic	-44%	10%	-44%	888%	-13%	-27%

In conclusion, the thorough comparison of the multilevel model with other methods shows its reliable performance, resilience, and sophisticated reaction to changing circumstances. These results further our knowledge of efficient inventory control techniques and offer practitioners and researchers insightful information for enhancing supply chain management in practical settings.

5. Conclusion

In this research, we suggest an innovative method for dealing with unpredictability in a manufacturing facility that places fresh orders in response to demand. There is a backlog of shortages, and the lead time and demand are unreliable metrics. Selecting orders in a way that minimizes the overall cost is the goal. Three new insights are added to the literature by this work. Initially, we consider two distinct types of uncertainty arising from lead time and demand. The majority of previously suggested models concentrated on one of these two, but they are still highly ambiguous due to the interactions between the two sources and the other. Second, as a trade-off between computational tractability and accurately representing the multiple-phase decision-making process under uncertainty, we suggest a multilevel Inventory control problem optimization model. Third, we design a precise algorithm for the multilevel optimization model that effectively searches in the worst instance, without counting all possible outcomes in the vast scenario space using Bender's decomposition foundation. The results imply that in reaction to the variety of cost factors, the multilevel optimization model operates more adaptable. The performance of the model of stochastic programming is primarily dependent on distributional knowledge and historical data; however, it attempts to strike a trade-off between holding costs and shortage. Concerning the cost parameters, the suggested multilevel optimization model automatically modifies its optimal ordering methods to produce the lowest (or nearly the lowest) total cost across all parameter configurations. Furthermore, the outcomes demonstrate that in terms of fill rate and total cost, the multilevel optimization model performs better than the stochastic programming model under various cost parameter values. The moderate model is nearly always in the middle of the results of the pessimistic and optimistic models, which are dependent on the cost factors. On the other hand, the multilevel optimization model finds the lowest (or nearly the lowest) total expense for every parameter configuration by automatically modifying its optimal ordering methods based on the cost parameters. There are various limitations to this study that point to potential areas for future investigation. In the suggested model, for instance, a single item created from a single component is assumed. If this supposition were to be relaxed, a more intricate model reflecting the ambiguity and interdependency of several components on the supply and demand sides would be necessary. Moreover, the decision-maker may choose to ship particular components or goods as a batch to reduce transportation costs by incorporating fixed and variable transportation costs into the model.

References

[1] Belanche, D., Casal, L. V., &Flavin, C. (2021). Frontline robots in tourism and hospitality: service enhancement or cost reduction?. *Electronic Markets*, *31*(3), 477-492.

[2] Dey, B. K., Bhuniya, S., &Sarkar, B. (2021). Involvement of controllable lead time and variable demand for a smart manufacturing system under a supply chain management. *Electronic Markets Expert Systems with Applications*, *184*, 115464.

[3] Babagolzadeh, M., Shrestha, A., Abbasi, B., Zhang, Y., Woodhead, A.,& Zhang, A. (2020). Sustainable cold supply chain management under demand uncertainty and carbon tax regulation. *Transportation Research Part D: Transport and Environment*, *80*, 102245.

[4] Golpira, H. (2020). Optimal integration of the facility location problem into the multi-project multi-supplier multi-resource Construction Supply Chain network design under the vendor managed inventory strategy *Expert Systems with Applications*, *139*, 112841.

[5] Ye, Y., Jiao, W., & Yan, H. (2020). Managing relief inventories responding to natural disasters: Gaps between practice and literature. *Production and Operations Management*, *29*(4), 807-832.

[6] De Giovanni, P. (2021). Smart Supply Chains with vendor managed inventory, coordination, and environmental performance. *European Journal of Operational Research*, *292*(2), 515-531.

[7] Kim, T., Kim, Y. W., & Cho, H. (2020). Dynamic production scheduling model under due date uncertainty in precast concrete construction. *Journal of Cleaner Production*, *257*, 120527.

[8] Alzoubi, H. M., Elrehail, H., Hanaysha, J. R., Al-Gasaymeh, A., & Al-Adaileh, R. (2022). The role of supply chain integration and agile practices in improving lead time during the COVID-19 crisis. *International Journal of Service Science, Management,Engineering, and Technology (IJSSMET)*, *13*(1), 1-11.

[9] Li, X., Uysal, A. S., &Mulvey, J. M. (2022). Multi-period portfolio optimization using model predictive control with mean-variance and risk parity frameworks. *European Journal of Operational Research*, *299*(3), 1158-1176.

[10] Lan, G., Lee, S., & Zhou, Y. (2020). Communication-efficient algorithms for decentralized and stochastic optimization. *Mathematical Programming*, *180*(1-2), 237-284.

[11] Lyu, Z., Lin, P., Guo, D., & Huang, G. Q. (2020). Towards zero-warehousing smart manufacturing from zero-inventory just-in-time production. *Robotics and Computer-Integrated Manufacturing*, *64*, 101932.

[12] Liu, A., Zhu, Q., Xu, L., Lu, Q., & Fan, Y. (2021). Sustainable supply chain management for perishable products in emerging markets: An integrated location-inventory-routing model. *Transportation Research Part E: Logistics and Transportation Review*, *150*, 102319.

[13] Ponte, B., Framinan, J. M., Cannella, S., & Dominguez, R. (2020). Quantifying the Bullwhip Effect in closed-loop supply chains: The interplay of information transparencies, return rates, and lead times. *International Journal of Production Economics*, *230*, 107798.

[14] Ghadge, A., Er, M., Ivanov, D., &Chaudhuri, A. (2022). Visualisation of ripple effect in supply chains under long-term, simultaneous disruptions: a system dynamics approach. *International Journal of Production Research*, *60*(20), 6173-6186.

[15] Hansen, O., Transchel, S., & Friedrich, H. (2023). Replenishment strategies for lost sales inventory systems of perishables under demand and lead time uncertainty. *European Journal of Operational Research*, *308*(2), 661-675.

[16] Zhang, H., Chao, X., & Shi, C. (2020). Closing the gap: A learning algorithm for lost-sales inventory systems with lead times. *Management Science*, *66*(5), 1962-1980.

[17] Das, S. C., Zidan, A. M., Manna, A. K., Shaikh, A. A., &Bhunia, A. K. (2020). An application of preservation technology in inventory control system with price dependent demand and partial backlogging. *Alexandria Engineering Journal*, *59*(3), 1359-1369.

[18] Sarkar, B., Omair, M., & Kim, N. (2020). A cooperative advertising collaboration policy in supply chain management under uncertain conditions. *Applied Soft Computing*, *88*, 105948.

[19] Transchel, S., & Hansen, O. (2019). Supply planning and inventory control of perishable products under lead-time uncertainty and service level constraints. *Procedia Manufacturing*, *39*, 1666-1672.

[20] Goli, A., Tirkolaee, E. B., & Weber, G. W. (2020). A perishable product sustainable supply chain network design problem with lead time and customer satisfaction using a hybrid whale-genetic algorithm. *Logistics operations and management for recycling and reuse*, 99-124.

[21] Zaneti, L. A., Arias, N. B., de Almeida, M. C., & Rider, M. J. (2022). Sustainable charging schedule of electric buses in a University Campus: A rolling horizon approach. *Renewable and Sustainable Energy Reviews*, *161*, 112276.

[22] Cuisinier, ‰.,Lemaire, P., Penz, B., Ruby, A., &Bourasseau, C. (2022). Sustainable charging schedule of electric buses in a New rolling horizon optimization approaches to balance short-term and long-term decisions: An application to energy planning. *Energy*, *245*, 122773.

[23] Wunnava, A., Naik, M. K., Panda, R., Jena, B., & Abraham, A. (2020). An adaptive Harris hawks optimization technique for two dimensional grey gradient based multilevel image thresholding. *Applied Soft Computing*, *95*, 106526.

[24] Ahmadianfar, I., Bozorg-Haddad, O., & Chu, X. (2020). Gradient-based optimizer: A new metaheuristic optimization algorithm. *Information Sciences*, *540*, 131-159.