STRATEGIES FOR REPLACEMENT IN WORKFORCE SCHEDULING RELIABILITY MODELS

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Abstract

It is a typical occurrence to replace some industrial equipment or components, such as electronic chips, bulbs, etc. It deals with the ideas of dependability theory, in which the likelihood of an equipment malfunctioning instantly is calculated assuming that it has operated normally for a given amount of time, 't'. In reliability, it's known as the hazard rate. The rate of hazard may be rising, falling, or staying the same. However, replacement tactics are employed to maintain output. Basically, two distinct approaches are employed. 1. Replacing a broken item; 2. Replacing items on a regular basis. Since it is a preventive measure to maintain production, the effective administration of maintaining system functionality depends on the application of both reliability concepts in addition to replacement theory. One may envision a similar issue with the personnel system as well. To determine the best manpower policies in this situation, replacement methods and dependability theory can also be coupled. This theory's application to labor systems is examined, and appropriate methods for employee replacement and advancement are covered in order to ensure the system's successful upkeep. In order to obtain the best fit, manpower planning is a dynamic process that controls the movement of workers into, through, and out of the organization. In order to determine appropriate manpower replacement policies for promotion and replacement of personnel for the successful maintenance of the system, this study discusses the application of dependability theory and the renewal process.

Keywords: Degradation problem, Life time, Replacing, Reliability model, Normal Distribution, Hazard Rate.

I. Introduction

Manpower planning evaluates the organization's present staffing and skill levels, connects these factors to the market demand for its goods, and offers options to align manpower resources with projected demand. It is a dynamic process that controls the movement of workers into, through, and out of the company in order to find the best fit. The methodical process of using available labour in accordance with the demands of the nation's many industries is known as manpower planning. As a result, manpower models are created while accounting for the diverse real-world scenarios. Manpower refers to a group of individuals who have obtained a specific ability or expertise to perform a specific kind of work. Developing appropriate economic policies and applying scientific approaches to economic problems are crucial components of effective administration. To determine the best workforce policy, replacement strategies and reliability theory can be coupled. This theory's applicability to labour systems is examined, and appropriate methods for employee replacement and advancement are covered in order to ensure the system's continued viability. Robinson [18] has examined the application of replacement techniques to workforce planning.

The grade I, also known as the training grade, and the grade II, also known as the grade in the organization, are conceptualized as the two divisions that are taken into consideration in this study. In stage I, the cost of an individual replacement is higher than in step II. When an individual in stage II departs the organization, the void is filled by someone in stage I. If any wastage in stage I as a result of exits and transfers from stage I to stage II, after which training grade recruiting is completed. Planning for manpower must take into account the overall business strategy as well as factors that affect employment, such as technology, competition, government laws and regulations, and shifting social norms and expectations. Plans that guarantee important concerns are addressed, suitable measures are performed, and the process is maintained will be effective; this will give the plan credibility and instill a feeling of pride in those who carry it out. When analyzing the planning of human resources, statistical techniques are a vital resource. Predicting demand may involve looking at productivity changes, technological changes, market forces and trends and the corporate strategies. Predicting supply involves knowledge of the current manpower stocks and looking at future recruitment, wastage, working conditions, promotion policies and labour market trends. Closing the gap means examining training, remuneration career planning, redundancies and further consideration of all the factors under the other headings.

II. Methods

I. Model

The current model is designed with two compartments: Grade I, sometimes known as the training grade, and Grade II, which denotes a specific working location or formal hierarchy position. In stage I, the cost of an individual replacement is higher than in step II. When an individual in stage II departs the organization, the void is filled by someone in stage I. Recruitment for training grade is carried out if there is any waste in stage I as a result of exit and transfer from stage I to stage II. It can be seen that the force of separation, sometimes referred to as the hazard rate, is dependent on the total period of service. It may be observed that the force of separation or the propensity of the individual to leave the organization depends upon completed length of service and it is defined as the probability that a person leaves in a small time interval $(\xi,\ \xi+\delta\xi)$ having served a period of t and $\lambda(\xi) = \frac{f(\xi)}{G(\xi)}$ Under these assumptions it is proposed to determine the size of grade I or the number of persons in the training grade. The mean time to promotion is also to be determined.

In any organization there is loss of manpower, which arises due to the leaving of the personnel from the organization. The rate at which the loss of manpower arises due to the leaving process is very important. If the rate of leaving can be accurately measured then the appropriate policies for the number of persons to be recruited at various time points in the future can be decided. In the study of leaving process the completed length of service (CLS) distribution plays an important role. Hence an appropriate statistical model assuming different probability distributions of the random variable should be fitted to the data obtained from any organization. Several distributions have been suggested for this purpose. Lawless [15] have suggested a model for the

CLS distribution Chien-Yu Peng [11] has suggested a model for the CLS distribution. Initially exponential distribution has been suggested for the CLS distribution. The density function is $f(\xi) = \lambda e - \lambda\xi$ where λ is called the loss intensity. Tweedie [17] has suggested that the parameter of the distribution λ be treated as a random variable whose distribution is $H(\lambda)$. Therefore the CLS distribution in the Silcock form is given by

$$f(\xi) = \int \lambda e^{-\lambda\xi} dH(\lambda), \quad (\xi \geq 0)$$

The mixed exponential distribution has been used by Bartholomew and Forbes [2]. The concept of change of distribution is discussed in Chien-Yu Peng [11]. Meeker [16] has used this concept in shock model and cumulative damage process, to estimate the expected time to cross the threshold, where the threshold random variable Y undergoes a change in the distribution itself. In inventory theory, the probabilistic demand can undergo change in the very distribution itself after a change point. This concept is used in manpower planning problems. Assuming that the wastages in successive decision epochs are correlated random variables, the expected time to recruitment is derived and in doing so the results of Gurland [8] are used. In addition to this assumption it is also assumed that the random variable which denotes the threshold level for wastages is one which satisfies the so called Setting the Clock Back to Zero (SCBZ) property, due to Jerry Lawless [14]. The expression for mean and variance of the expected time to recruitment are derived using the shock model approach by Esary Marshall and Proschan [12].

The term "Manpower planning" defined by Walker [9], "Manpower planning refers to the rather complex task of forecasting and planning for the right numbers and at the right kinds of people at the right places and the right times to perform activities that will benefit both the organization and the individuals in it". According to Bartholomew [1], "Manpower planning is concerned, in aggregate terms, with matching jobs to people. In the broadest sense, manpower planning is concerned with matching the supply of people available for employment with the jobs available". According to Grinold and Marshall [8 & 12], Manpower planning must be an ancient art, since manpower problems have existed for centuries. People, jobs, time and money are the basic ingredients of a manpower system. A decision-maker must be aware of the interactions among these four ingredients in order to formulate and evaluate manpower policy. Manpower planning within an organization has the basic purpose of producing the correct numbers of correct type of people in the correct jobs at appropriate times. System constraints do not often allow for perfect matching of the people to jobs. A more realistic view of manpower planning is that it avoids having too many of the wrong types of people in the wrong jobs too frequently. For a detailed study of the subject in this direction can be seen in Bartholomew [4], Bartholomew and Forbes [6], Nikulin et al. [10], Butler [7].

The reliability $R(\xi)$ of the device is given by

$$R(\xi) = \sum_{k=0}^{\infty} P_k V_k(\xi)$$

where $V_k(\xi)$ is the probability that k damages are caused during (0, t]. The above model has been considered by Esary, Marshall and Proschan [12] with the underlying process generating the shocks as Poisson. Gurland [8] has shown that the characteristic function ϕ(λ1,λ2…λn) of the joint distribution of any n random variables from a sequence {Xn} of constantly correlated, exchangeable random variables each following the exponential distribution with p.d.f $f(x) = \left(\frac{1}{a}\right) e - \frac{x}{a}, \ a > 0, \ 0 < x < \infty$) such that the correlation coefficient R between any X_i and X_j, i≠j (independent of i and j) is given by

We suppose that all individuals have the same completed length of service (CLS) distribution, with density function f(ξ), where t denotes the time which has elapsed since the person in question joined the organization. Thus f(ξ) δt is the probability that a man leaves with t period of service in (ξ, ξ +δ ξ). The survivor function G(ξ) gives the probability that a person remains in the organization for at least time t.

Hence

$$G(x) = \int_{\xi}^{\infty} f(\omega)d\omega \tag{1}$$

We define the further quantity λ(ξ), known as the 'force of separation at length of service ξ', as follows. λ(ξ) δ ξ is the probability that an individual who has been in the organization for a length of time 'ξ', leaves in the interval (ξ, ξ +δ ξ). It is easily shown that

$$\lambda(\xi) = \frac{f(\xi)}{G(\xi)}, for\ all\ \xi \geq 0 \tag{2}$$

Since the input and output of each grade must balance, we have for Grade II

$$N_1 P = N_2 W W_2 \tag{3}$$

Moreover, in equilibrium the expected input to the system over unit time is N/μ, where μ the mean length of completed service. Thus for grade I, we have,

$$\frac{N}{\mu} = N_1(P + W_1) = N_1 W_1 + N_2 W_2 \tag{4}$$

In general W_1, W_2 and P are functions of time. It shows that P, W_1, W_2 tend to equilibrium values which are independent of the age of the system. Let μ_1 be the average time spent in Grade I. Then in equilibrium the expected number of vacancies occurring per unit time in this Grade is N_1/μ_1. These can be caused either by promotion or losses, and hence

$$\frac{N_1}{\mu_1} = N_1(P + W_1) \tag{5}$$

$$\frac{1}{\mu_1} = P + W_1 \tag{6}$$

It follows that equation (4) and equation (6) that

$$\frac{N}{\mu_1} = \frac{N_1}{\mu_1} \tag{7}$$

II. Promotion by Seniority

Let $a(\xi/T)$ denote the age distribution of the system at time T given that the system was established at $(T = 0)$. Thus $a(\xi/T)\delta T$ is the probability that an individual chosen at random at time T has length of service in $(\xi, \xi + \delta\xi)$. Thus

$a(\xi/T)\delta T$ = P{ individual joined in $(T - \xi, T - \xi + \delta\xi)$ and remained for time t}

$$= h(\xi - T)\delta T\ G(\xi)$$

Where h(ξ) is the renewal density for the whole system, i.e. h(ξ)δ ξ is the probability of loss in (ξ, ξ +δ ξ). Hence $a\left(\frac{\xi}{T}\right) = h(T - \xi)h(\xi)$. This is true for ξ <T, when ξ =T, We have

a(T/T) = P{Original member of organization is still there at time T}

= G (T)

Now,

$$\lim_{T\to\infty} h(T) = 1/\mu \tag{8}$$

Whatever the form of the CLS distribution.

Also

$$\lim_{T\to\infty} G(T) = 0 \tag{9}$$

.

Thus

$$\lim_{T\to\infty} a\left(\frac{T}{T}\right) = G(\xi)/\mu \tag{10}$$

Now, since a loss from Grade II is replaced by the most senior member of Grade I, it follows that at any time every individual in Grade II has length of service at least as long as any individual in Grade I and hence that there exists some threshold value ti such that all individuals with length of service less than t1 are in Grade I, where t1 is a random variable. But if the grade sizes are large their expected value can be found from the approximate formula,

$$\int_{t_1}^{\infty} a(\xi)d\xi = N_2/(N_1 + N_2) \tag{11}$$

The expected number of promotions per unit time will be the proportion of new recruits whose services to the threshold length of service t_1 is

$$N_1 P = G(\xi_1)N/\mu \tag{12}$$

III. Promotion at Random

Let $F_1(\xi)$ be the probability that an individual remains in the system for a time t without being promoted. Let $n\xi$ be the number of promotions in $(0, \xi)$, then

$F_1(\xi)$ = Prob{Individual not promoted in (0, ξ) / doesn't

leave in (0, ξ)} . Prob {doesn't leave in (0, ξ)}

$$= \left(1 - \frac{1}{N_1}\right)^{n\xi} G(\xi)$$

Now expected value of n ξ is N_1P_1. Thus, as a first approximation we can take

$$F_1(\xi) = G(\xi)^{\left(1-\frac{1}{N_1}\right)^{N_\xi P_\xi}} \quad as \quad N_1 \to \infty, we\ have$$

$$F_1(\xi) = G(\xi)e^{-P_\xi}$$

It follows that the average time spent in Grade I

$$\mu_1 = \int_0^{\infty} F_1(\xi)d\xi = \int_0^{\infty} G(\xi)e^{-P_\xi}\, d\xi$$

But from (7), $\mu_1 = N_1\mu/N$, Thus

$$1/\mu \int_0^{\infty} G(\xi)e^{-P_\xi} d\xi = N_1/N \tag{13}$$

IV. The Mean Time to Promotion

Let μ_L be the average length of time spent in Grade I, and let μ_P be the average length of time spent in Grade I by those who are eventually promoted to Grade II. As before, let μ_1 be the average sojourn time in Grade I. (This can be terminated by promotion on leaving). Let us consider the problem of choosing N_1 so that μp has some predetermined value. The two promotion rules are considered separately.

V. Promotion by Seniority

In this case μp is equivalent to the average value of ξ_1 introduced earlier, and hence equations (11) and (12) hold.

Thus we have

$$\int_{\mu_P}^{\infty} a(\xi)d\xi = N_2/N \tag{14}$$

$$1/\mu \int_{\mu_P}^{\infty} G(\xi)d\xi = N_2/N \tag{15}$$

Also $N_1P = G(\mu_P)\frac{N}{\mu}$ Equation (15) gives $R = N_1/N_2$ as a function of μ_p and hence knowing the size N_2 of the organization, we can determine N_1 for any specified value of μ_P.

III. Results

I. Numerical Study

Case (i)

CLS is taken to be a exponential distribution,

$$f(\xi) = \lambda e^{-\lambda\xi}; \qquad \lambda > 0, \xi > 0$$

Then $G(\xi) = e^{-\lambda\xi}$ and $\mu = 1/\lambda$

From equation (15) implies

$$\frac{1}{\lambda(1+R)} = \int_{\mu_p}^{\infty} e^{-\lambda\xi}\, dt \text{ , then}$$

$$R = \left(e^{-\lambda\mu_p} - 1\right) \tag{16}$$

The values of R for various values of μ_P and λ are given in Table 1.

Case (ii)

CLS is taken to be mixed exponential distribution

$$f(\xi) = x\lambda e^{-\lambda_t\xi} + (1-x)\lambda_2 e^{-\lambda_2\xi};\ \ 0 < x < 1;\ \lambda_1, \lambda_2 > 0\ ; \xi \geq 0$$

Then

$$G(t) = xe^{-\lambda_1\xi} + (1-x)e^{-\lambda_2\xi}$$

$$\mu = \frac{x}{\lambda_1} + (1-x)/\lambda_2 \tag{17}$$

From equation (15) implies

$$R = \frac{\frac{x\left(1-e^{\lambda_1\mu_p}\right)}{\lambda_1} + \left((1-x)(1-e^{\lambda_2\mu_p})\right)/\lambda_2}{\frac{x\left(e^{-\lambda_1\mu_p}\right)}{\lambda_1} + \frac{(1-x)(1-e^{\lambda_2\mu_p})}{\lambda_2}} \tag{18}$$

The values of R corresponding to various values of μ_p for the mixed Exponential CLS distribution with parameters $x = 0.4, \lambda_1 = 0.2\ and\ \lambda_2 = 2.0$ so that $\mu = 2.3$ are given in table .2.

Case (iii)

Now the CLS taken to be a Pearsonian Type XI distribution (Silcock's form) and the behavior of R with respect to μ_p, γ and c is studied.

$$f(t) = \frac{\gamma}{c}\left(1 + \frac{\xi}{c}\right)^{-(\gamma-1)} \tag{19}$$

$$G(t) = (1 + \tfrac{\xi}{c})^{-\gamma}; \qquad \xi \geq 0, \gamma > 1, c > 0 \tag{20}$$

$$\mu = \frac{c}{\gamma - 1}$$

From equation (15) implies

$$\frac{N_2}{N} = \frac{1}{\mu}\xi \tag{21}$$

$$\frac{N_2}{N} = \frac{1}{\mu}\int_{\mu_0}^{\infty}(1 + \xi/c)^{-\gamma} d\xi \tag{22}$$

$$R = \left[\left(\frac{c}{\mu_p + c}\right)^{1-\gamma} - 1\right] \tag{23}$$

Then, the value of R for the various values of μ_p, γ and $c = 3$ is given in Table 3.

Table 1: *CLS as Exponential Distribution*

μ_p / λ	0.5	1.0	1.5	2.0	2.5	3.0
0.2	0.111	0.212	0.315	0.492	0.651	0.821
0.4	0.221	0.491	0.822	1.232	1.722	2.322
0.8	0.351	0.823	1.453	2.331	3.483	5.053
1.0	0.451	1.231	2.322	3.952	6.381	10.023
1.2	0.652	1.723	3.482	6.391	11.182	19.092
1.4	0.823	2.322	5.051	10.022	19.081	35.601
1.6	1.011	3.064	7.172	15.442	32.122	65.693
1.8	1.232	3.951	10.021	23.533	53.592	120.511
1.9	1.461	5.052	13.882	35.601	89.021	220.412
2.0	1.721	6.392	19.092	53.602	147.412	402.432

Table 2: *CLS as Mixed Exponential Distribution*

μ_p	0.5	1.0	1.5	2.0	2.5	3.0
R	0.1196	0.3919	0.6109	0.8128	1.0519	1.3016

Table 3: *CLS as Pearsonian Type XI Distribution*

γ μ	1.5	2.0	2.5	3.0	3.5
2	0.29109	0.66667	1.15117	1.77718	2.58611
4	0.52175	1.33333	2.56412	4.44444	7.31615
6	0.71321	2.00010	4.19612	7.99999	14.58815
8	0.91148	2.66677	6.02111	12.44444	24.74412
10	1.08116	3.33333	8.02015	17.77718	38.08911

It is discovered that in the case of the mixed exponential distribution, it is preferable to have fewer recruits at the training grade in order to get promoted sooner. Since a tiny percentage of trainees would survive to be promoted in the case of CLS as a Pearsonian Type XI distribution, a high training grade would be necessary. The analysis of data pertaining to completed length of service will be helpful in large organizations with different hierarchical grades to predict the size or number of individuals to be trained at each grade, ensuring that the organization's regular operations are not disrupted by the waste of personnel in the various grades. This is because it is feasible to anticipate the amount of leavers for varying CLS lengths.

IV. Discussion

Table 1 shows values of R calculated for various values of λ and μp. The values of R required becomes large when $\lambda\mu_p > 1$, and then increases rapidly as $(\lambda\mu_p)$ increases. This is what one would intuitively expect, since $\lambda\mu_p > 1$ implies that $\mu_p > 1/\lambda$, i.e. the average time to promotion is greater than the overall mean length of completed service. Thus only a very small proportion of trainees would survive to promotion and hence a large training grade would be required. Table 2 shows the values of R corresponding to various values of μ_p and the particular distribution is fitted to manpower data, it is normally found that x takes a value near ½ and that λ_2 is about ten times the value of λ_1, and hence producing a fairly skew distribution. Then the mixed exponential CLS distribution with parameters $\lambda_1 = 0.2$, $\lambda_1 = 2.0$, $x = 0.4$, so that $\mu = 2.3$.

Table 3 shows values of R calculated for various values of γ and μ_p. The values of R becomes large when γ, μp increase. The average time to promotion is greater than the overall mean length of completed service. Thus only a very small proportion of trainees would survive to promotion and hence a large training grade would be required. The mean duration of finished services is less than the average time to promotion. Because of the extremely low rate of trainee survival and promotion, a high training grade would be necessary.

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