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RELIABILITY SAMPLING PLAN FOR GENERALIZED INVERTED EXPONENTIALLY DISTRIBUTED UNDER PROGRESSIVE TYPE-II CENSORED DATA

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Abstract

This article aims to explore a sampling strategy designed to assess the reliability of products that exhibit lifetimes following a GIED. Considered sampling approach has been specially constructed for a Type-II progressive censoring scheme, which includes binomial removals as part of its methodology. Its core objectives is to find out acceptance constant and the optimum sample size. To facilitate practical implementation, the article presents a tabulated form of the sampling plan for the selected specification, as per the considered censoring scheme. To validate the dependability and precision of the suggested sampling approach, we perform a Monte Carlo experiment under various scenarios.

Keyword *: Generalized Inverted Exponential Distribution (GIED); OC-Curve; Reliability Sampling Plan; Sinulation; Progressive Censoring.*

1. Introduction

In life testing and reliability studies, direct observation of the exact lifetime of a specific event of interest for all tested units is often impractical. This situation arises in various scenarios, such as clinical trials and in engineering where individuals may remain alive or disease-free beyond the study period. To streamline costs and time, some units may be randomly withdrawn from the experiment, resulting in censored data. It is essential to assess the effect of censoring on reliability and determine whether it provides meaningful information or not.

In the field of statistics, a variation of the exponential distribution, termed the one-parameter inverse exponential or inverted exponential distribution (IED), has been advanced. This distribution exhibits an inverted bathtub hazard rate. The utilization of IED in survival analysis has been advocated by several researchers, as exemplified by [22] and [23]. A two-parameter extension of the inverted exponential distribution (IED), called the generalized inverted exponential distribution (GIED), was proposed by [24] and demonstrated that GIED fits real datasets better than IED, based on K-S statistics and likelihood ratio tests. Furthermore, [25] conducted a study on reliability estimation based on progressive Type-II censored samples under the classical paradigm. A common way to evaluate the quality of a product is to check if it meets certain specifications related to its reliability and lifetime. Some standard sampling plans, such as $MIL-STD-414$ and $MIL-STD-105$ as discussed by [1], can be used to compare the results with predefined criteria. However, these plans may not be suitable for situations where observing all failures is too expensive or time-consuming, especially for products with high reliability. In such cases,

censored tests are often used. Censoring is one of the main feature of lifetime study or reliability study. Censoring desirably or undesirably occurred in the experiment. There are several type of censoring schemes discussed by [2] and [9]. Now a days practitioners and researchers have advocated for a versatile censoring scheme known as progressive Type-II censoring. Progressive Type-II censoring is a method of reliability sampling that involves removing a certain number of units that have not failed at each failure time. This method can reduce the cost and time of testing, but it also introduces some challenges in the analysis.

The progressive Type-II censoring scheme is an extension of Type-II censoring, which incorporates the removal of units from a life-test at predetermined or random inspection times. In this scheme, out of the initial total of (n) units simultaneously placed on a life test, only (m) units are fully observed, while the remaining $(n-m)$ units are withdrawn from the experiment at different time points. Some of the researchers who have developed reliability sampling plans with progressive Type-II censoring are [6], [8] and [7]. A reliability sampling plan by [8] focused on the exponential distribution, while [7] considered the Log-normal and Weibull distributions. Also, [10], [12], and [11] also studied the exponential, Weibull, and Log-normal distributions, respectively, but with different assumptions on the number of units removed at each failure. A comprehensive review of progressive Type-II censoring and its applications provided by [9]. Progressive Type-II censoring is a complex process that requires careful planning and analysis.

An example of the application of progressive censoring in evaluating the performance of electronic components provided by [13]. In such cases, certain test units may require removal due to factors like excessive heat, resulting in situations that fall under the purview of Type-II PCR. Furthermore, [16] conducted extensive investigations into issues related to parameter estimation and the expected duration of experiments under Type-II PCR censoring. In numerous practical scenarios, managing removals presents a formidable challenge, rendering the assumption of fixed and known removals impractical. In acknowledgment of this constraint, [3] advocates the adoption of random removals, emphasizing its practical viability. Therefore, the implementation of Type-II censoring with random removals becomes a more pragmatic choice. In this approach, a suitable distribution, such as the binomial distribution, can be employed to model the removal pattern. To the best of our knowledge, the utilization of reliability sampling plans for Type-II censoring with binomial removals has not been previously documented. In current study, our primary focus lies in the development of a reliability sampling plan for the GIED under Type-II progressive censoring with random removals. This entails that the number of removals at each failure is subject to a binomial distribution. In Section 2, we will introduce our proposed model and establish the maximum likelihood estimators (MLEs) for the model parameters. In Section 3, we present the Operating Characteristic (O.C.) curve, providing insights into the performance of our sampling plan. Section 4 delves into an in-depth examination of the sampling plan's design. Finally, in Section 5, we offer our concluding remarks and provide a succinct summary of the key findings derived from this study.

2. Methods

A generalisation of the one parameter IED is a two parameter GIED having PDF and CDF as follows:

$$\begin{aligned} \xi(t) &= \frac{\nu\eta}{t^2}\exp\left(\frac{-\eta}{t}\right)\left(1-\exp\left(\frac{-\eta}{t}\right)\right)^{\nu-1}; \qquad t>0,\nu>0,\eta>0. \\ \Xi(t) &= 1-\left(1-exp\left(-\frac{\eta}{t}\right)\right)^{\nu} \qquad\qquad ; \qquad t>0,\nu>0,\eta>0. \end{aligned} \tag{1}$$

Where, ν is the shape parameter and η is the scale parameter. Let t_p be the p^{th} percentile of the GIED, it is given by,

$$p = 1-\left(1-exp\left(-\frac{\eta}{t_p}\right)\right)^{\nu}. \tag{2}$$

On simplification, we get

$$t_p = \frac{-\eta}{\ln(1-(1-p)^{1/\nu})}.$$

and median of the distribution is given by

$$m_d = \frac{-\eta}{\ln(1-(0.5)^{1/\nu})}.$$

the reliability function is given by

$$S(t) = \left(1 - exp\left(-\frac{\eta}{t}\right)\right)^{\nu}; \qquad t \geq 0, (\eta, \nu) > 0.$$

The failure rate function of the GIED(η, ν) is given by

$$h(t) = \frac{f(t)}{S(t)} = \frac{\nu\eta}{t^2}\frac{e^{-\frac{\eta}{t}}}{1-e^{-\frac{\eta}{t}}}; \qquad t \geq 0, (\eta, \nu) > 0.$$

For simplicity point of view, let us make a transformation $Z = ln(t)$.

$$\psi(z) = \nu\eta \exp(-z - \eta\exp(-z))\left(1 - \exp\left(-\eta\exp(-\eta\exp(-z))\right)\right)^{\nu-1} ; z > 0, \nu > 0, \eta > 0. \tag{3}$$

and its distribution function is given by

$$\Psi(z) = 1 - \left(1 - exp\left(-\eta\exp(-z)\right)\right)^{\nu}; z > 0, \nu > 0, \eta > 0. \tag{4}$$

Let's consider the following transformations: $\mu = \ln\nu$ and $\sigma = \frac{1}{\eta}$. It simplifies our analysis to work with the model represented by equation 3. Now, we have a set of m ordered log-failure times, denoted as $Z_1 < Z_2 < \ldots < Z_m$, selected from a pool of n items. The value of m is pre-determined, indicating the number of failures that occur before the testing concludes.

At the i^{th} failure event, a random removal of r_i items takes place from the testing pool. The number of items removed, r_i, follows a binomial distribution characterized by parameters $(n-m)$ and removal probability (p_r). In the context of a Type II progressive censoring (Type II PCR), we define the likelihood function as follows:

$$L(t;\mu,\sigma) = L_1(t;\mu,\sigma)P_R.$$

where,

$$L_1(t;\mu,\sigma) = \prod_{i=1}^{m}\psi(z_i)(1-\Psi(z_i))^{r_i}.$$

and

$$\begin{aligned} P_R &= P(R_{m-1} = r_{m-1}|R_{m-2} = r_{m-2}, R_{m-3} = r_{m-3}...R_1 = r_1) \\ \times P(R_{m-2} &= r_{m-2}|R_{m-3} = r_{m-3}, R_{m-4} = r_{m-4}...R_1 = r_1)...P(R_2 = r_2|R_1 = r_1). \\ &= \frac{(n-m)!}{\prod_{i=1}^{m} r_i!(n-m-\sum_{j=1}^{m-1})!}p_r^{\sum_{j=1}^{m-1} r_j}(1-p_r)^{(m-1)(n-1)-\sum_{j=1}^{m-1}(m-j)r_j}. \end{aligned}$$

Where, $C_i = n - \sum_{j=1}^{i-1}(r_j + 1)$. To obtain the maximum likelihood estimators (MLEs) of μ and σ, the likelihood have been maximized at MLEs of the parameters, for more details see [28] and [27]. In this type of censoring scheme, the number of failures is predetermined before the experiment begins. The experiment is terminated once the desired number of failures is observed. Assuming that the number of failures is fixed as m, we denote t_i as the time at the i^{th} removal, and r_i as the number of the random removals of the i^{th} component.

3. OC curve

The OC curve is a tool used to evaluate the effectiveness of a sampling plan. It does so by charting the likelihood of accepting a lot against the proportion of non-conforming items within that lot. This evaluation relies on the principles of asymptotic distribution theory.

In the context of this evaluation, we utilize the following equations:

First, we have:

$$\frac{T' - (\mu - k\sigma)}{\sqrt{AsVar[T']}} \sim N(0,1)$$

The standardized variate is expressed as:

$$W = \frac{(T' - (\mu - k\sigma))\sqrt{n}}{\sqrt{V}}$$

To construct the OC curve, which represents the probability of accepting a lot, denoted as $L(p)$, we use the following equation:

$$L(p) = Pr[T' \geq L'] = 1 - \Phi\left[\frac{\sigma(u_p + k_1)\sqrt{n}}{\sqrt{V}}\right]$$

In this equation, u_p stands for the quantile of the standard logistic distribution that corresponds to the given proportion of non-conforming items, denoted as p. $\Phi(\cdot)$ represents the standard normal distribution function.

Therefore, to determine an optimal sampling plan for specific points on the OC curve, denoted as $(p_\alpha, 1-\alpha)$ and (p_β, β), the following equations need to be solved for the variables k and n:

$$\begin{aligned} z_\alpha - \frac{\sigma(u_{p_\alpha} + k_1)\sqrt{n}}{\sqrt{V}} &= 0 \\ z_{1-\beta} - \frac{\sigma(u_{p_\beta} + k_1)\sqrt{n}}{\sqrt{V}} &= 0 \end{aligned} \tag{5}$$

where, u_{p_α} and u_{p_β} denote the quantiles of the standard normal distribution and z_α and z_β denotes the quantiles of the log-life distribution. Thus on solving equation (5), we get

$$k = \frac{z_{p_\alpha} u_{1-\beta} - z_{p_\beta} u_\alpha}{u_\alpha - u_{1-\beta}} \tag{6}$$

and

$$n = \left(\frac{u_\alpha - u_{1-\beta}}{z_{p_\alpha} - z_{p_\beta}}\right)^2 \left(\frac{\sigma^2}{n}\left(\gamma_{11}(n, p_r, f_c) - 2k\gamma_{12}(n, p_r, f_c) + \gamma_{22}(n, p_r, f_c)\right)\right) \tag{7}$$

4. Layout of sampling plan

4.1. Sampling Plan

In our study, we adopt the methodology originally proposed by [17] to evaluate the acceptability of a batch. Specifically, we concentrate on variable sampling plans with one-sided specification limits. Let's consider a lot of size n randomly drawn from a larger population. The log-lifetimes of the items in this lot follow a distribution characterized by Equation (3). This distribution is defined by a set of unknown parameters, denoted as ν and η. We seek to obtain the maximum likelihood estimators for these parameters, denoted as ν and η.

In this context, we have a lot with a proportion of non-conforming items, denoted as p_0 (where $p_0 \leq p_\alpha$), which is considered acceptable and should be approved with a probability of

at least $(1-\alpha)$. Here, p_α represents the proportion of non-conforming items that corresponds to the desired probability of acceptance, denoted as $(1-\alpha)$ on the operating characteristic (OC) curve. We define L' as the quantile of the $\Psi(\cdot)$ given in the equation (4) that corresponds to the proportion of non-conforming items for the chosen probability of acceptance $(1-\alpha)$. This is calculated as $L' = \Psi^{-1}(p_\alpha)$. The decision to accept or reject the lot hinges on the comparison of the estimate $(\hat{\mu} - k\hat{\sigma})$ with the value of L'. If $(\hat{\mu} - k\hat{\sigma})$ is greater than or equal to L', the lot is accepted; otherwise, it is rejected. The acceptance constant, denoted as k, is a pivotal factor in making this decision.

The key focus is on specifying the optimal sample size (n) and the pertinent acceptance constant (k) within the framework of the proposed censoring scheme. One can note that, the distribution of the variable $(\hat{\mu} - k\hat{\sigma})$ will be $AN\left((\mu - k\sigma), \frac{\sigma^2}{n}(\gamma_{11}(n, p_r, f_c) - 2k\gamma_{12}(n, p_r, f_c) + \gamma_{22}(n, p_r, f_c))\right)$. Here, $\gamma_{11}(n, p_r, f_c)$, $\gamma_{12}(n, p_r, f_c)$, and $\gamma_{22}(n, p_r, f_c)$ are elements of the asymptotic dispersion matrix. You can find detailed expressions for these in the Appendix provided by [26]. Here, p_r represents the removal probability, and f_c denotes the censoring fraction. Choose two points, $(p_\alpha, 1-\alpha)$ and (p_β, β), on the OC curve suggested by [7]. To calculate these points, we use the formulas: $y_\tau = \Psi^{-1}(\tau) = -\ln\left(-\frac{1}{\eta}\ln\left(1-(1-U)^{\frac{1}{\nu}}\right)\right)$ and $u_\alpha = \Phi^{-1}(\alpha)$, where $\Psi(\cdot)$ and $\Phi(\cdot)$ are the cumulative distribution functions (CDF) of the log-GIED and the standard normal distribution, respectively. To determine the acceptance constant (k) and the sample size (n) for a given pair of points, $(p_\alpha, 1-\alpha)$ and (p_β, β), on the OC curve, along with specified censoring fraction (f_c) and removal probability (p_r), we solve equations (5).

In Tables 1 and 2, we present the results for various removal probabilities ($p_r = 0.1$, 0.3, and 0.5) and censoring fractions ($f_c = 0.2$, 0.3, 0.4, 0.5, 0.6, and 0.7). The selection of values for p_α and p_β aligns with the criteria set by MIL-STD-105D. Additionally, we include results for a limit case of standard Type-II censoring, where $p_r = 0.00001$, allowing for a comparison with the findings of [7]. For the computation of the terms $\gamma_{11}(n, p_r, f_c)$, $\gamma_{12}(n, p_r, f_c)$, and $\gamma_{22}(n, p_r, f_c)$, we employ a Monte Carlo simulation, generating progressive Type-II censored samples initially.

Specifically, we calculate the moments based on 2000 simulations, assessing the average values of the terms $\gamma_{11}(n, p_r, f_c)$, $\gamma_{12}(n, p_r, f_c)$, and $\gamma_{22}(n, p_r, f_c)$ for various values of n. The outcomes are detailed in Table 1 and Table 2 for $\beta = 0.05$ and $\beta = 0.10$, respectively. The results reveal that, when maintaining a constant p_r, the optimum value of n decreases as f_c declines, irrespective of the acceptance constant (k). A lower f_c implies lesser dropouts, resulting in fewer accurate lifetime observations. Consequently, a larger sample size is necessary to compensate for the loss of information when assessing lot acceptability.

On the other hand, when the censoring fraction f_c is held constant, and the same acceptance constant k is used, the sample size does not exhibit a consistent pattern with respect to the removal probability. This discrepancy arises because removal shifts the observations toward the tail of the lifetime distribution, improving the accuracy of lifetime parameter estimation but leading to a loss of information due to dropouts. However, an excessive number of dropouts early in the process diminishes this advantage. Hence, for higher values of p_r, the sample size increases as p_r, such as $p_r = 0.5$, increases. Conversely, for low to moderate values of p_r, the sample size decreases as p_r increases due to the impact of a significant number of dropouts.

Table 1: *Type-II PCR reliability sampling plan for p_α and p_β to match with $MIL-STD-105D$ for $1-\alpha = 0.95, \beta = 0.10$.*

		n						
p_α	p_β	$f_r \rightarrow$ 0.7	0.6	0.5	0.4	0.3	0.2	k
		$p_r = 0.1$						
0.00041	0.01840	14	10	9	9	8	8	1.1352
0.00284	0.03110	24	19	18	17	16	16	0.9834
0.00654	0.04260	32	27	26	25	24	23	0.8981
0.01090	0.05350	40	34	33	31	30	29	0.8372
0.02090	0.07420	53	47	45	43	41	39	0.7482
0.03190	0.09420	65	58	55	53	50	48	0.6816
		$p_r = 0.3$						
0.00041	0.01840	14	12	11	10	9	8	1.1352
0.00284	0.03110	26	23	21	19	18	17	0.9834
0.00654	0.04260	36	33	30	28	25	24	0.8981
0.01090	0.05350	45	41	37	35	32	30	0.8372
0.02090	0.07420	61	56	51	47	44	41	0.7482
0.03190	0.09420	74	68	63	58	54	50	0.6816
		$p_r = 0.5$						
0.00041	0.01840	16	13	12	10	9	9	1.1352
0.00284	0.03110	29	26	23	20	18	17	0.9834
0.00654	0.04260	40	36	32	29	26	24	0.8981
0.01090	0.05350	50	45	40	36	33	31	0.8372
0.02090	0.07420	68	61	55	50	45	42	0.7482
0.03190	0.09420	83	75	67	61	56	51	0.6816
		$p_r = 0.0001$						
0.00041	0.01840	50	15	9	8	8	7	1.1352
0.00284	0.03110	66	23	17	16	15	15	0.9834
0.00654	0.04260	75	30	24	23	23	22	0.8981
0.01090	0.05350	81	37	31	30	29	28	0.8372
0.02090	0.07420	91	49	44	42	41	39	0.7482
0.03190	0.09420	99	61	56	54	51	49	0.6816

4.2. Simulated sampling plan

It is worth noting that in the discussion of the distribution of $(\hat{\mu} - k\hat{\sigma})$, asymptotic distribution theory is applied, and the derived sampling plans are based on this approximation. However, it is essential to investigate the finite sample behavior of these sampling plans by conducting a Monte Carlo simulations to assess the true probability of acceptance.

In this research, we employ a Monte Carlo simulation to compare the expected probability of acceptance with the actual probability when a designed sampling plan is put into practice within a specific censoring framework. We investigate various scenarios, incorporating removal probabilities (p_r) of 0.1, 0.3, and 0.5, as well as censoring fractions (f_c) of 0.3, 0.5, and 0.7. Additionally, we consider fixed producer's and consumer's risk (ν, β) settings at (5%, 10%) and (5%, 5%). For each combination of these parameters, we conduct 2000 Monte Carlo simulations to provide precise estimates of the probability of acceptance, denoted as $\widehat{L(p)}$.

To obtain estimates for the parameters k and $\widehat{L(p)}$, we employ the bias-corrected maximum likelihood estimators (MLEs), as detailed by [9]. The obtained results are presented in Table 3 through Table 11, covering various removal probabilities p_r and diverse levels of censoring proportions.

Table 2: *Progressive Type-II reliability sampling plan with random removals aligned with the requirements of* $MIL-STD-105D$ *for* $1-\alpha=0.95$ *and* $\beta=0.05$.

		n						
p_α	p_β	$f_r \rightarrow$ 0.7	0.6	0.5	0.4	0.3	0.2	k
		$p_r = 0.1$						
0.00041	0.01840	19	31	41	50	67	82	1.1664
0.00284	0.03110	13	25	35	43	60	74	1.0066
0.00654	0.04260	12	23	33	41	57	70	0.9180
0.01090	0.05350	11	22	31	40	54	67	0.8553
0.02090	0.07420	11	21	30	37	51	63	0.7641
0.03190	0.09420	10	20	28	36	49	61	0.6962
		$p_r = 0.3$						
0.00041	0.01840	19	33	46	57	76	93	1.1664
0.00284	0.03110	16	29	41	52	70	86	1.0066
0.00654	0.04260	14	26	37	47	64	78	0.9180
0.01090	0.05350	13	25	35	44	60	74	0.8553
0.02090	0.07420	11	22	32	40	54	67	0.7641
0.03190	0.09420	11	21	29	37	51	63	0.6962
		$p_r = 0.5$						
0.00041	0.01840	20	37	52	64	86	106	1.1664
0.00284	0.03110	17	32	45	56	76	93	1.0066
0.00654	0.04260	15	29	40	51	69	85	0.9180
0.01090	0.05350	13	26	37	46	63	77	0.8553
0.02090	0.07420	12	23	33	42	57	70	0.7641
0.03190	0.09420	11	21	30	38	52	64	0.6962
		$p_r = 0.0001$						
0.00041	0.01840	79	101	112	118	128	135	1.1664
0.00284	0.03110	21	32	41	49	64	79	1.0066
0.00654	0.04260	11	21	30	39	55	71	0.9180
0.01090	0.05350	10	19	28	36	52	66	0.8553
0.02090	0.07420	10	19	28	36	51	64	0.7641
0.03190	0.09420	9	19	27	35	48	60	0.6962

Table 3: *Simulated probabilities of acceptance for GIED with $p_r = 0.1$ and 70% censoring.*

	Sampling plan			Probability of acceptance
$p_r = 0.1$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	15	4	1.1352	0.5278
	24	7	0.9834	0.3812
	32	9	0.8981	0.3025
	39	11	0.8372	0.2403
	52	15	0.7482	0.1518
	64	19	0.6815	0.0833
$\alpha = 0.05, \beta = 0.05$	18	5	1.1664	0.6235
	30	9	1.0066	0.4398
	40	12	0.9180	0.3424
	49	14	0.8553	0.2751
	66	19	0.7641	0.1797
	81	24	0.6962	0.1083

Table 4: *Simulated probabilities of acceptance for GIED with $p_r = 0.1$ and 50% censoring. .*

	Sampling plan			Probability of acceptance
$p_r = 0.1$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	10	5	1.1352	0.5284
	19	9	0.9834	0.3869
	26	13	0.8981	0.2988
	33	16	0.8372	0.2393
	46	23	0.7482	0.1520
	57	28	0.6815	0.0842
$\alpha = 0.05, \beta = 0.05$	12	6	1.1664	0.6150
	23	11	1.0066	0.4357
	33	16	0.9180	0.3416
	42	21	0.8553	0.2755
	58	29	0.7641	0.1803
	71	35	0.6962	0.1084

In Table 3, with a constant removal probability p_r, varying consumer's risk (β) and fixed producer's risk (ν), and fixed censoring proportion f_c, as the sample size n rises, the corresponding value of the number of the failures m also rises. However, the probability of acceptance $\widehat{L(p)}$ diminishes. Similar pattern have been observed from Table 3 to Table 11 for the different values of the censoring proportion f_c. From the Table 3 and Table 4, one can study the effect of the change of censoring proportion f_c. Here, with a constant removal probability p_r, consumer's risk (β) and fixed producer's risk (ν), size of the sample n decreases as the censoring proportion f_c decreases. A similar patterns has been observed for the rest of the tables for different values of the censoring fraction f_c and removal probability p_r, so one can conclude the same in general.

5. Conclusion

Our study has delved into the challenges and intricacies of Type-II Progressive Censoring (Type-II PCR), a common practical scenario where the number of removals is uncertain. Our primary focus has been the development of optimum reliability sampling plans for the GIED lifetime distribution within the framework of Type-II PCR. We have rigorously examined a range of scenarios involving removal probabilities and censoring fractions, shedding light on their influence on these sampling plans.

Table 5: *Simulated probabilities of acceptance for GIED with $p_r = 0.1$ and 30% censoring. .*

	Sampling plan			Probability of acceptance
$p_r = 0.1$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	8	5	1.1352	0.4955
	17	11	0.9834	0.3853
	24	16	0.8981	0.3014
	30	21	0.8372	0.2399
	41	28	0.7482	0.1508
	51	35	0.6815	0.0838
$\alpha = 0.05, \beta = 0.05$	11	7	1.1664	0.6245
	21	14	0.9745	0.4376
	30	21	0.9180	0.3421
	38	26	0.8553	0.2755
	52	36	0.7641	0.1798
	64	44	0.6962	0.1086

Table 6: *Simulated probabilities of acceptance for GIED with $p_r = 0.3$ and 70% censoring..*

	Sampling plan			Probability of acceptance
$p_r = 0.3$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	14	4	1.1352	0.5104
	26	7	0.9834	0.3833
	36	10	0.8981	0.3012
	45	13	0.8372	0.2415
	60	18	0.7482	0.1513
	74	22	0.6815	0.0840
$\alpha = 0.05, \beta = 0.05$	18	5	1.1664	0.6163
	33	9	1.0066	0.4396
	46	13	0.9180	0.3444
	56	16	0.8553	0.2750
	76	22	0.7641	0.1800
	93	27	0.6962	0.1089

Table 7: *Simulated probabilities of acceptance for GIED with $p_r = 0.3$ and 50% censoring. .*

	Sampling plan			Probability of acceptance
$p_r = 0.3$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	11	5	1.1352	0.5438
	21	10	0.9834	0.3973
	30	15	0.8981	0.3130
	37	18	0.8372	0.2485
	51	25	0.7482	0.1590
	63	31	0.6815	0.0907
$\alpha = 0.05, \beta = 0.05$	14	7	1.1664	0.6225
	27	13	1.0065	0.4410
	38	19	0.9180	0.3436
	47	23	0.8553	0.2746
	65	32	0.7641	0.1803
	80	40	0.6962	0.1092

Table 8: *Simulated probabilities of acceptance for GIED with $p_r = 0.3$ and 30% censoring.*

	Sampling plan			Probability of acceptance
$p_r = 0.3$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	9	6	1.1352	0.5106
	18	12	0.9834	0.3838
	25	17	0.8981	0.2983
	32	22	0.8372	0.2400
	44	30	0.7482	0.1518
	54	37	0.6815	0.0837
$\alpha = 0.05, \beta = 0.05$	12	8	1.1664	0.6311
	23	16	1.0066	0.4418
	32	22	0.9180	0.3421
	40	28	0.8553	0.2742
	55	38	0.7641	0.1794
	68	47	0.6962	0.1087

Table 9: *Simulated probabilities of acceptance for GIED with $p_r = 0.5$ and 70% censoring..*

	Sampling plan			Probabilities of acceptance
$p_r = 0.5$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	16	4	1.1352	0.5240
	29	8	0.9834	0.3841
	40	12	0.8981	0.3006
	50	15	0.8372	0.2407
	67	20	0.7482	0.1510
	83	24	0.6815	0.0842
$\alpha = 0.05, \beta = 0.05$	20	6	1.1664	0.6224
	37	11	1.0066	0.4414
	51	15	0.9180	0.3436
	63	18	0.8553	0.2756
	85	25	0.7641	0.1799
	104	31	0.6962	0.1088

Table 10: *Simulated probabilities of acceptance for GIED with $p_r = 0.5$ and 50% censoring.*

	Sampling plan			Probability of acceptance
$p_r = 0.5$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	12	,6	1.1352	0.5217
	23	11	0.9834	0.3845
	32	16	0.8981	0.2995
	41	20	0.8372	0.2415
	55	27	0.7482	0.1508
	68	34	0.6815	0.0836
$\alpha = 0.05, \beta = 0.05$	15	7	1.1664	0.6213
	29	14	1.0066	0.4402
	41	20	0.9180	0.3434
	51	25	0.8553	0.2750
	70	35	0.7641	0.1800
	86	43	0.6962	0.1088

Table 11: *Simulated probabilities of acceptance for GIED with $p_r = 0.5$ and 30% censoring.*

	Sampling plan			Probability of acceptance
$p_r = 0.5$	n	m	k	$\widehat{L(p)}$
$\alpha = 0.05, \beta = 0.10$	10	7	1.1352	0.5259
	19	13	0.9833	0.3834
	27	18	0.8981	0.3010
	34	23	0.8372	0.2405
	46	32	0.7482	0.1507
	57	39	0.6815	0.0836
$\alpha = 0.05, \beta = 0.05$	12	8	1.1664	0.6128
	24	16	1.0066	0.4396
	34	23	0.9180	0.3427
	43	30	0.8553	0.2759
	58	40	0.7641	0.1791
	72	50	0.6962	0.1088

Our key findings, as evident in the parameters of sample size (n) and the acceptance constant (k), underscore the crucial role of an increasing censoring fraction (f_c) in necessitating a larger sample size. Generally, the optimal sample size (n) exhibits stability across varying removal probabilities (p_r). Nonetheless, it is of paramount importance to highlight the pivotal role played by the removal probability (p_r) in shaping the overall test duration. [16] has convincingly demonstrated that an escalation in the removal probability (p_r) leads to a significant extension of the test duration. In such cases, an increased sample size becomes imperative to effectively mitigate the extended testing period. These insights emphasize the practical significance of our research in addressing real-world challenges related to reliability testing under Type-II PCR.

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