# ENHANCED METHODS UNDER EXPONENTIAL DISTRIBUTION CONCERN WITH EWMA AND DEWMA METHODS

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**Abstract**

*Statistical Process Control plays a crucial part in improving the quality and lowering the fluctuation in the production process environment. In SPC, the most popularly used methods are Shewhart control chart techniques and EWMA techniques which distinguish itself for its quick identification of minute process deviations, which makes it an essential tool for guaranteeing product. EWMA methods detect variances in quality of the product as well as services, measure process mean shifts with control charts, and track manufacturing process parameters to find deviations and make necessary adjustments. The exponential distribution was employed in this study because it may reflect vast and bulk production in everyday life. Exponentially distributed data, evaluate it alongside the EWMA function. This paper's objective is to study the impact of EWMA & DEWMA parameters within the EWMA control chart's performance using exponential distribution. Further, A few tables are provided with suitable illustrations that can be available with parameters with the help of these findings. The study also examines how the EWMA parameter affects the shape of the distribution.*

**Keywords:** Average run length, Exponential distribution, EWMA control Chart, Statistical Process Control.

## 1. Introduction

Walter A. Shewhart created Statistical process control in the (SPC) early 1920s at Bell Laboratories. To keep an eye on industrial activities, SPC uses technology that evaluates and regulates quality. SPC is a technique that is commonly used to discover production-line flaws and verify that the final product meets defined quality requirements. SPC is commonly used in manufacturing or production processes to assess how reliably a product functions under its design specification parameters. SPC helps to improve product quality, eliminate process variation, maintain regulatory and customer requirements, and reduce scrap, waste, defects, and reworks. Shewhart pioneered the control chart and the theory of a statistical control condition in 1920. A control chart can be used to observe how a process develops over time. Control charts are utilized to ensure quality regularly. The basic limitation of the memory-less charting system is its inability to track minute changes in process parameters. The exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) are the two finest frequent types of memory-based control charts. They integrate historical and present data to discover minor and reduce changes in process parameters. The name Geometric weighted moving average was adapted to reflect the fact that EWMA charts are constructed using exponential smoothing, for additional coverage for the EWMA, see [6] and [10] developed the double exponential moving average (DEMA) to reduce the temporal delay caused by traditional moving averages. Moving averages, on the other hand, are prominent for their long lag. The double exponential moving average (DEMA) reacts by producing a more efficient averaging approach.

## 2. Exponential Weighted Moving Averages (EWMA)

The EWMA chart is an instance of a control chart used in Statistical Quality Control (SQC) for tracking variables or attribute-type data through an entire period of outcome from the tracked enterprise or industrial process. In contrast to other control charts, the EWMA chart estimates the EWMA of all historical sample averages. Although the EWMA chart relies on the normal distribution, it is also fairly robust in considering the existence of non-normally distributed quality parameters. There is a chart shift which lets quality attributes that are more accurately reflected through the Poisson distribution. The graphic just monitors the process mean; assessing the process variability implies an additional approach. Unlike a conventional moving average, which allocates equal weight to all data points within a specific period, EWMA offers higher importance to newly acquired observations while dropping the weight for previous observations exponentially. EWMA differs from a simple moving average because every data point in the time interval is given the same significance. EWMA can be especially beneficial in finance and time series analysis. Compared to a normal moving average, it gives a better representation of the data. The responsiveness might be transformed by improving the $\lambda$ value. Calculating the EWMA over a shorter time period (smaller in size $\lambda$) causes it to be more responsive to recent changes. It is also utilized in quality control and process improvement, as recent observations provide a better indication of a process's present status.

Enhanced the Shewhart control chart's power by integrating the EWMA statistic [14]. The exponentially weighted moving average concepts which is develops [8] . The properties and enhancements of EWMA was developed by [9] . The EWMA control charts are utilized by [4] to monitor an analytical procedure. The features of the exponential EWMA chart are developed with parameter estimates in [12]. Improving EWMA chart performance [1]. Using an exponential type estimator of mean, [13] functioned in a hybrid exponentially weighted moving average (HEWMA) control chart. A nonparametric HEWMA-p control chart is developed by [2] to adjust for variance in monitoring procedures. The maximum EWMA and DEWMA charts based on auxiliary information are compared with sampling intervals for process mean and variance in [7]. The EWMA charts are more sensitized to tiny fluctuations into the process mean, exceeding ordinary $\bar{X}$ charts to detect tiny shifts. The main difference is the smoothing effect resulting from exponential weighting, which reduces the influence of outliers or random fluctuations and emphasizes the underlying trend in the process. What distinguishes EWMA is its dynamic mobility in adapting to process scenarios, particularly adapting promptly to fluctuations in the mean by prioritizing recent data. This versatility is crucial in dynamic corporate environments where procedures undergo continual alteration. The rapid detection of errors in EWMA charts is a game changer, allowing for prompt intervention and correction before processes fall out of control or create out-of-spec goods. EWMA charts excel at managing auto-correlated data, outperforming traditional control charts in accurately illustrating process states by understanding autocorrelation trends via their weighted averaging strategy. Continuous monitoring capabilities boost EWMA's value by allowing real-time assessment of process performance for long-term stability. The convenience of perception adds the last feather to its gap, with expanding patterns or departures from the midline indicating probable alterations into the process mean.

Before designing the EWMA control chart, a researcher must pick two parameters:

- The 1st parameter act as the amount of weight given to the most current rational subgroup mean. The criteria $0 < \lambda \leq 1$ could be met, although determining the "proper" value is subjective depending on specific events.
- The 2nd parameter L act as a multiple of a rational subgroup standard deviation and specifies the control limitations. During alignment with other control charts, L is often set at 3. However, for smaller amounts of $\lambda$, L may need to be reduced significantly.

Instead of directly charting rational subgroup averages, the EWMA chart estimates consecutive observations $z_i$ by analyzing the rational subgroup average, using the running average of all past observations, $z_i - 1$, using the carefully determined weight, $\lambda$. It states that the EWMA is

$$z_i = \lambda x_i + (1-\lambda) z_{(i-1)}; i = 1, 2, 3, \ldots, n. \tag{1}$$

In which $X_i$ represents present measure value, $\lambda$ represents the smoothing constant that controls the depth of the memory EWMA, $\lambda$ must fulfill $0 < \lambda \leq 1$, $z_i$ signifies the present EWMA represents a EWMA statistic observed in the past measurement. (Needed the first sample at i =1) is the procedure aim, therefore $z_0 = \mu_0$. In some cases the average of preliminary information is applied as the EWMA's initial value, therefore $z_0 = \bar{x}$. To show that the EWMA $z_i$ is a weighted average of the early sample means, we may add $z_{(i-1)}$ to the right side of equation (1) to get

$$z_i = \lambda x_i + (1-\lambda)[1 - \lambda x_{(i-1)} + (1-\lambda) z_{(i-2)}]$$

$$z_i = \lambda x_i + \lambda(1-\lambda) x_{(i-1)} + (1-\lambda)^2 z_{(i-2)}$$

Trying to displace reclusively for $z_{(i-j)}$, j=1,2,3,... t,we get

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{(i-j)} + (1-\lambda)^i z_0 \tag{2}$$

The weights $\lambda(1-\lambda)^j$ diminishes geometrically with the age of the sample mean. Moreover, the weights add to unity, given that

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j = \lambda \frac{(1-(1-\lambda)^i)}{(1-(1-\lambda))} = 1 - (1-\lambda)^i$$

When the observations $x_i$ are random variables that are independent at variance ($\sigma^2$), the variance of $z_i$

$$\sigma_{z_i}^2 = \sigma^2 \left( \frac{\lambda}{2-\lambda} \right) [1 - (1-\lambda)^{2i}] \tag{3}$$

As a consequence, the EWMA control chart might be generated by arranging $z_i$ against the sample number i (or time). The EWMA control chart's centerline and control limits are displayed below: Control chart for EWMA:

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \tag{4}$$

$$CL = \mu_0 \tag{5}$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \tag{6}$$

In equations (4) and (6), the factor L represents the breadth of the control limit and it explore the possibility of the values L and $\lambda$ subsequently. Approach unity i as gets larger, the $(1-\lambda)^{2i}$ gets very close to zero and the equation (4), (5) and (6) preformed as the value of $(1-\lambda)^{2i}$ approaches 0, the equation is rearranged as follows:

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{7}$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{8}$$

Nonetheless, for tiny values of i and highly advise applying specific control limits in equations (4) and (6). By doing this, the control chart's ability to identify an off-target process as soon as the EWMA is activated will be greatly increased.

## 3. Double exponential weighted moving averages (DEWMA)

In 1994 Patrick G. Mulloy introduced the DEMA, often known as the DEWMA scheme, as an extension of classic EWMA concepts. They demonstrated that the DEWMA scheme beats the Shewhart scheme in tiny to moderate changes and has similar qualities for anticipating variation into the process mean to the EWMA control scheme. The DEWMA, likewise referred to by the Holt-Winters exponential smoothing, is widely used in forecasting, particularly in scenarios that require adapting to changing trends. The DEWMA has evolved to accommodate many variations and is now widely used in time series forecasting. Its applications range from banking to demand estimation, handling inventory, and environmental monitoring, all of which require accurate projections based on past data for decision-making. The DEWMA's adaptability and historical performance make it an invaluable resource for analysts & practitioners looking for reliable forecasting tools in a variety of fields.
The DEWMA is an indicator of trend designed to decrease noise in price charts employed by technical traders. It also tries to eliminate the lag time inherent in classic moving averages. A DEWMA variation was presented by [16], who also showed that it worked better than the EWMA system in detecting small mean shifts. Several researchers concluded that the DEWMA scheme is superior to the old EWMA system. The examination of the DEWMA control chart is presented in [11]. Using repetitive sampling, [3] develops the new DEWMA control chart. A Comparative Analysis of the EWMA and DEWMA is developed by [5]. The New Neutrosophic Double and Triple Exponentially Weighted Moving Average Control Charts are developed by [15]. The DEWMA is useful in financial and time series research because it may capture trends and produce smoother forecasts than regular moving averages. It is especially beneficial when working with data that has non-constant volatility. The DEWMA emphasizes recent observations by giving them larger weights, while simultaneously taking into account the trend using a second smoothing parameter. This dual-weighting strategy increases its responsiveness to changes in the underlying data pattern.

## 4. Exponential distribution

In the Poisson point process, the exponential distribution describes the probability distribution of the time among events. The exponential distribution is considered a version of the exponential distribution. Furthermore, the exponential distribution is the continuous equivalent of the geometric distribution. The exponential distribution was

$$f(x) = \lambda e^{(-\lambda x)}, x \geq 0 \tag{9}$$

Here $\lambda > 0$ is a constant.

$$F(a) = \int_0^a \lambda e^{(-\lambda x)}, a \geq 0 \tag{10}$$

The exponential distribution is a commonly used time-to-failure model in reliability engineering. An exponential distribution is a continuous probability distribution that is frequently used in statistics and probability to represent the amount of time that will pass before a particular event occurs. Events happen continuously, independently, and at a set average pace during this process. One important property of the exponential distribution is that it requires less memory. It is possible for the exponential random variable to have fewer large values or more small ones. As a result, a customer's total grocery shop spending on a single visit follows an exponential curve.

The exponential distribution is a probability distribution that defines the time between occurrences in a process that happens at a constant pace regardless of how long it has been since the last event. It's frequently utilized in process control and reliability testing. The exponential distribution is frequently used in process control to describe the duration between successive occurrences or failures, such as system breakdowns, manufacturing problems, or client arrivals in a queue. A key feature of this distribution is its lack of memory, which means that the chance of an event occurring in the following time interval remains constant regardless of the amount of time since the last occurrence. This Memorylessness property is especially useful in process management since it suggests a consistent chance of failure or event occurrence in the next instant, whatever the amount of time has passed since the previous event. The simulation settings are shown in Table 1.

**Table 1:** *Simulation Setting*

| Simulation setting | Value | Simulation setting | Value |
|---|---|---|---|
| The size of the Sample | 1000 | Replication | 100 |
| Distribution | Exponential | Alpha level | 0.95 |
| Rate parameter (?) | 0.2 | Statistical Software (execution) | Excel |
| Confidence Level | 0.05 | Statistical software (validation) | R |

The following Figure 1 depicts some shape of the exponential distribution

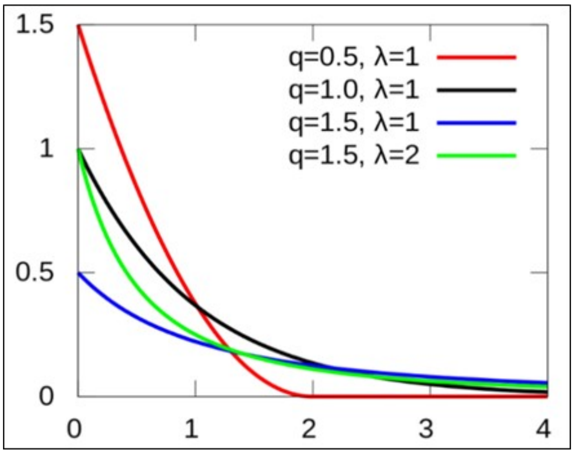



**Figure 1:** *Simulation Setting*

Figure 2 shows that the simulation has three steps

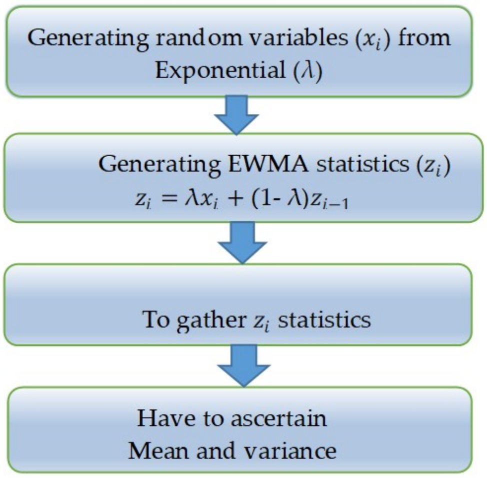



**Figure 2:** *Simulation Steps*

The following Figure 3 illustrates the Monte Carlo simulation using MS-Excel in detail

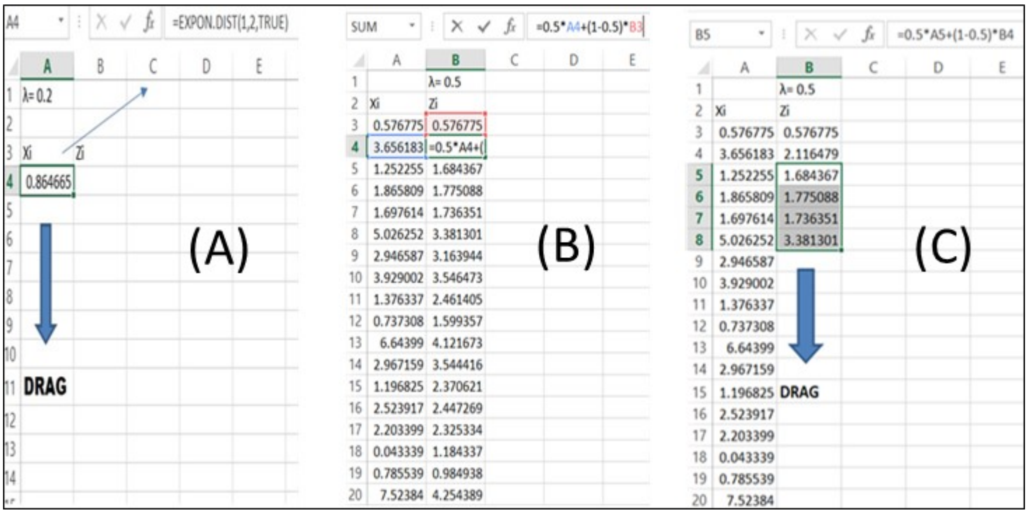



**Figure 3:** *Monte Carlo simulation with MS-Excel in detail*

## 5. Experimental approach

In 1946 During World War II, John von Neumann and Stainlaw Ulam pioneered the Monte Carlo simulation to aid in decision-making in unpredictable situations. Given that randomness lies at the heart of modeling, comparable approaches, and roulette games, the name of the Monte Carlo comes from the famous city with casinos. All the three phases of the simulation are as follows, as seen in Figure 2. To create a sequence of EWMA statistic, a first set of random variates is used. When creating the EWMA statistic (i.e., $z_1$), a random number variates is pulled, and this variate has the similar value of the variates that come from the exponential distribution (i.e., $x_1$)$z_2$ is then calculated using $z_1$ and the second produced random variates, $x_2$, as indicated by equation (1). Upto until the last random variates are formed, the procedure keeps going. This graphic is important since it explains the simulation stages and the mathematical portion of the EWMA estimations. Table 1 explains the simulated setup. This study applies Monte Carlo simulation using software. The exponential distribution was used to create random variates using software. The inverse value that is cumulative density of the exponential distribution was found using the Excel code "Exponential. INV (probability; $\lambda$)". As long as random probabilities are fed through the code, "Exponential. INV (probability; $\lambda$)" will produce random variates based on the given Exponential distribution. The probability distributed random probabilities must be uniform in order to provide non-disordered results because of the Nature of probabilities, that they're scattered over the field [0, 1]. Thus, these are evenly distributed randomly probabilities across the domain [0, 1] are produced using the Excel code Rand ( )". And adding "Rand ( )" in the place of "Probability," it is possible to archive the process of feeding randomized probabilities into the Exponential inverse code. "Exponential. INV (Rand ( ); 1)" for instance, if $\lambda$ = 1. Refer to Figure 5a. It is quite helpful to assign numbers in the first column between 1 and 1 million. The first cell at the top of that column is then used to produce a random variates from the Exponential distribution. The next step is to Double-click on the cell's owner the right corner or move the cell to produce one million variations in a column with two million cells containing random variates fitted to an exponential distribution is the end result (1). The next step after creating a column of random variables need to create EWMA statistics in the subsequent column. The first cell in this column is set to the exact same value (0.576775) as the top cell in the random variates columns, and this value only occurs once. Assuming $\lambda$ to be 0.2, the EWMA equation is then applied to the second cell, as depicted in Figure.3 B, which is represented by cells $x_2$ (the second cell in this column) and $z_1$ (the first cell in the column $z_i$). You can drag and click on this cell twice in the lower right corner. Doing a million EWMA statistical studies on EWMA data is the third and last step. The authors checked and validated the model. The mean and variance of the output variables were computed for each simulation run and compared to theoretical values.

$$Error fraction = \left| \frac{Theoriticalvalue - Simulationvalue}{Theoriticalvalue} \right| \tag{11}$$

In addition, the error fraction was calculated for each simulation run, as shown in equation (11). The error fraction for the variance and mean for all 150 simulation runs were less than 0.01. The error fraction was only 0.02. This validation of the random variable applied for the EWMA functions was excellent. Because of this, the EWMA equation is straightforward and this application has a lower error rate. Figure 4 illustrates how $\lambda$ affects the structure of EWMA statistics with exponential distribution

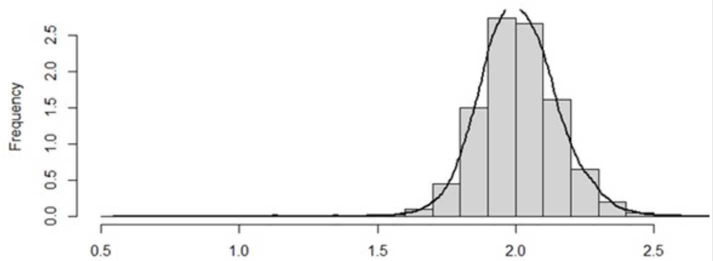
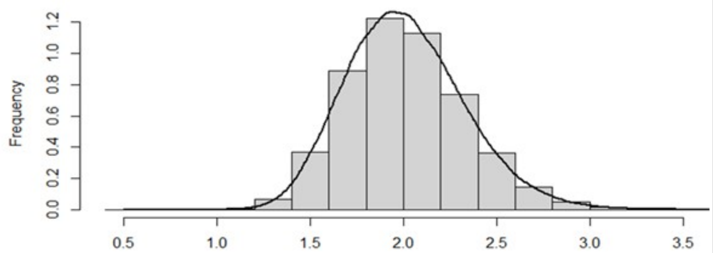
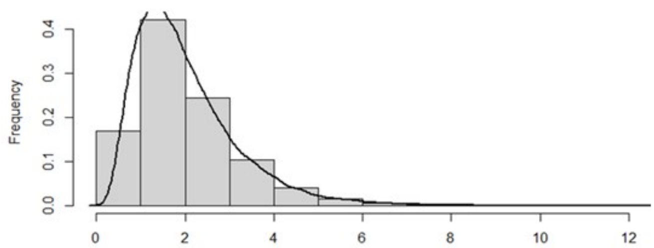
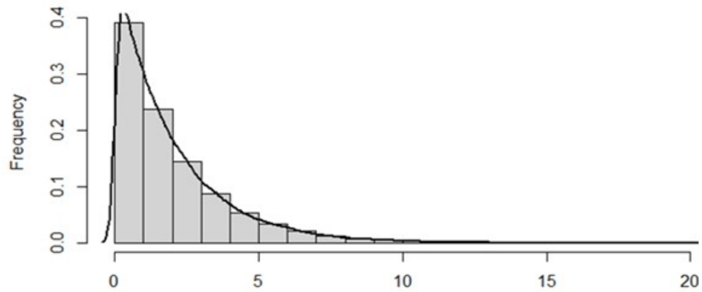
| λ | Shape | Notes |
|---|---|---|
| 0.01 |   | Mean = 0.2814417 Standard deviation = 0.1614081 Which is very close to normal distribution |
| 0.05 |   | Mean = 2.998192 Standard deviation = 0.1615081 |
| 0.5 |   | Mean = 2.997354 Standard deviation = 0.9961142 |
| 0.1 |   | Mean = 2.997308 Standard deviation =1.72673 |

**Figure 4:** *The influence of $\lambda$ in the structure of EWMA statistics with exponential distribution*

## 6. Case research with sensitive analysis

The store chosen for this inquiry is a prominent business around a crowded place metropolis of Maruthamalai, Coimbatore. The ABC store, located in the retail district serves a diverse customer base that includes residents as well as tourists. Recognized for its products and beneficial customer service, the company sees a continuous stream of customers throughout the day. A systematic data collection technique it was executed for this study's client arrival time data. Experienced investigators was discreetly stationed across the store for a few weeks to record customer's exact arrival times. Data collection entailed capturing customer arrival trends. Using real-time arrival data, the purpose is to investigate and understand how temporal dynamics flow is happening.

Understanding customer arrival patterns is critical for managing retail operations and providing better customer service. The case study focuses on the arrival time of customers, the busy capital city. Using empirical observation and data analysis, it was discovered that customer arrival times at this specific firm have an exponential distribution. The retail business at ABC Shop displays market dynamics shaped by city-specific cultural norms and consumer behavior. Customers' arrival timings at a store were investigated and documented throughout a certain time period. That was the case determined to the distribution of arrival timing follows established patterns that correspond to the exponential distribution, which is commonly used to monitor linear arrival times. The exponential distribution is an adaptable framework to describing the variability as well as mutual dependency of customer arrival time intervals, allowing for in-depth analysis and comprehension of the variability and interdependence of customer arrival times, as well as the identification of customer behavior and arrival processes at the retail shop. The first component of arrival times is established during the store's operating hours of 7:00 a.m. to 12:00 p.m. Table 2 displays the period between customer arrivals. The goal of the case study was to understanding the consumer behavior and patterns of arrival in an ABC retail atmosphere. In developing the adherence of client arrival times with the exponential distribution, retail managers and decision-makers with ABC retail shop gained significant insights. These findings have the potential to influence programs for workforce efficiency and queuing management enhancements, and standard customer service enhancement, all of which are based on a better knowledge of the elements that determine arrival times. The efficiency of EWMA and DEWMA may be assessed by measuring conduction sensitivity and varying $\lambda$ values. Please see the Tables. 3 and 4. The case study's customer arrival time data showed significant changes and improvements after using the EWMA and DEWMA functions. The combination of EWMA and DEWMA resulted in smoothed data that provided a clearer picture of underlying patterns and behaviors associated to client arrival, enabling for a more in-depth study of the relevant dynamics involved. Furthermore, by assigning larger weight to recent data sets, the EWMA function allows for real-time study of arrival trends. Customer arrival time trends were quickly discovered, allowing for proactive monitoring and response. Overall, the EWMA and DEWMA functions improved the analysis and interpretation of arriving time data at ABC retail, resulting in better resource utilization, operational scheduling, and outstanding customer management. Table 2 displays the period between customer arrivals.

**Table 2:** *Customers' arrival & inter-arrival times at the ABC shop*

| Arrival no | Arrival time | Inter-arrival time(min) | Arrival no | Arrival time | Inter-arrival time(min) |
|---|---|---|---|---|---|
| **1** | *7.06 a.m.* | *6.2* | *16* | *10.04 a.m.* | *3.6* |
| **2** | *7.31 a.m.* | *25.2* | *17* | *10.17 a.m.* | *13.61* |
| **3** | *7.36 a.m.* | *4.98* | *18* | *10.23 a.m.* | *5.87* |
| **4** | *7.51 a.m.* | *14.55* | *19* | *10.26 a.m.* | *3.01* |
| **5** | *8.05 a.m.* | *14.3* | *20* | *10.39 a.m.* | *12.72* |
| **6** | *8.08 a.m.* | *2.99* | *21* | *10.54 a.m.* | *14.47* |
| **7** | *8.1 a.m.* | *2.39* | *22* | *10.55 a.m.* | *1.11* |
| **8** | *8.35 a.m.* | *24.9* | *23* | *10.56 a.m.* | *0.63* |
| **9** | *9.26 a.m.* | *51.39* | *24* | *10.57 a.m.* | *0.19* |
| **10** | *9.28 a.m.* | *1.74* | *25* | *11.12 a.m.* | *15.15* |
| **11** | *9.3 a.m.* | *2.09* | *26* | *11.16 a.m.* | *4.17* |
| **12** | *9.4 a.m.* | *9.57* | *27* | *11.19 a.m.* | *12.89* |
| **13** | *9.44 a.m.* | *3.98* | *28* | *11.31 a.m.* | *12.09* |
| **14** | *9.54 a.m.* | *10.39* | *29* | *11.55 a.m.* | *24.25* |
| **15** | *9.59 a.m.* | *4.59* | *30* | *11.58 a.m.* | *2.58* |

Table 3 illustrates the Inter-arrival time after using EWMA with varied $\lambda$

**Table 3:** *Inter-arrival time after using EWMA with varied $\lambda$*

| $\lambda$ | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 | 0.05 | 0.01 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Inter-arrival time*** | | | | | | | | | | | |
| ***6.25*** | *6.25* | *6.25* | *6.25* | *6.25* | *6.25* | 6.25 | 6.25 | 6.25 | 6.25 | 6.25 | 6.25 |
| ***25.23*** | *8.148* | *10.046* | *11.945* | *13.843* | *15.741* | 17.639 | 19.538 | 21.436 | 23.334 | 24.283 | 25.042 |
| ***4.99*** | *7.832* | *9.034* | *9.857* | *10.3* | *10.364* | 10.047 | 9.351 | 8.276 | 6.821 | 5.951 | 5.186 |
| ***14.55*** | *8.504* | *10.138* | *11.266* | *12.002* | *12.459* | 12.751 | 12.993 | 13.298 | 13.781 | 14.124 | 14.46 |
| ***14.3*** | *9.084* | *10.971* | *12.176* | *12.921* | *13.379* | 13.681 | 13.908 | 14.1 | 14.248 | 14.291 | 14.302 |
| ***2.99*** | *8.474* | *9.375* | *9.421* | *8.949* | *8.185* | 7.267 | 6.266 | 5.213 | 4.117 | 3.556 | 3.104 |
| ***2.4*** | *7.867* | *7.98* | *7.314* | *6.329* | *5.292* | 4.346 | 3.559 | 2.962 | 2.571 | 2.457 | 2.407 |
| ***24.99*** | *9.58* | *11.383* | *12.619* | *13.795* | *15.144* | 16.736 | 18.564 | 20.588 | 22.753 | 23.868 | 24.769 |
| ***51.39*** | *13.761* | *19.385* | *24.251* | *28.835* | *33.269* | 37.531 | 41.545 | 45.233 | 48.53 | 50.018 | 51.128 |
| ***1.74*** | *12.559* | *15.856* | *17.498* | *17.998* | *17.505* | 16.057 | 13.683 | 10.44 | 6.421 | 4.156 | 2.236 |
| ***2.09*** | *11.513* | *13.104* | *12.877* | *11.636* | *9.799* | 7.679 | 5.57 | 3.762 | 2.526 | 2.196 | 2.094 |
| ***9.57*** | *11.319* | *12.398* | *11.886* | *10.811* | *9.687* | 8.816 | 8.373 | 8.412 | 8.87 | 9.206 | 9.5 |
| ***3.98*** | *10.585* | *10.715* | *9.515* | *8.079* | *6.834* | 5.915 | 5.299 | 4.868 | 4.47 | 4.243 | 4.037 |
| ***10.4*** | *10.566* | *10.651* | *9.78* | *9.007* | *8.616* | 8.605 | 8.869 | 9.292 | 9.806 | 10.091 | 10.335 |
| 4.59 | 9.969 | 9.44 | 8.224 | 7.241 | 6.605 | 6.198 | 5.876 | 5.533 | 5.114 | 4.868 | 4.65 |
| 3.61 | 9.333 | 8.273 | 6.839 | 5.788 | 5.106 | 4.643 | 4.288 | 3.992 | 3.758 | 3.67 | 3.618 |
| 13.62 | 9.761 | 9.342 | 8.872 | 8.92 | 9.362 | 4.643 | 10.818 | 11.692 | 12.631 | 13.12 | 13.517 |
| 5.87 | 9.372 | 8.648 | 7.972 | 7.701 | 7.617 | 4.643 | 7.356 | 7.036 | 6.548 | 6.235 | 5.949 |
| 3.02 | 8.737 | 7.522 | 6.486 | 5.827 | 5.317 | 4.643 | 4.319 | 3.821 | 3.371 | 3.178 | 3.047 |
| 12.73 | 9.136 | 8.564 | 8.359 | 8.588 | 9.023 | 4.643 | 10.206 | 10.948 | 11.794 | 12.252 | 12.633 |
| 14.47 | 9.67 | 9.746 | 10.194 | 10.943 | 11.749 | 4.643 | 13.194 | 13.769 | 14.206 | 14.363 | 14.456 |
| 1.12 | 8.815 | 8.02 | 7.471 | 7.012 | 6.433 | 4.643 | 4.74 | 3.647 | 2.426 | 1.779 | 1.251 |
| 0.63 | 7.996 | 6.542 | 5.419 | 4.46 | 3.532 | 4.643 | 1.863 | 1.234 | 0.81 | 0.688 | 0.636 |
| 0.19 | 7.216 | 5.272 | 3.851 | 2.753 | 1.862 | 4.643 | 0.694 | 0.401 | 0.255 | 0.218 | 0.197 |
| 15.16 | 8.01 | 7.249 | 7.242 | 7.714 | 8.509 | 4.643 | 10.817 | 12.204 | 13.665 | 14.408 | 15.006 |
| 4.17 | 7.626 | 6.634 | 6.322 | 6.298 | 6.342 | 4.643 | 6.167 | 5.781 | 5.124 | 4.686 | 4.283 |
| 2.9 | 7.153 | 5.886 | 5.294 | 4.937 | 4.618 | 4.643 | 3.877 | 3.472 | 3.118 | 2.985 | 2.909 |
| 12.1 | 7.648 | 7.129 | 7.335 | 7.802 | 8.359 | 4.643 | 9.632 | 10.374 | 11.201 | 11.643 | 12.007 |
| 24.26 | 9.309 | 10.555 | 12.412 | 14.384 | 16.308 | 4.643 | 19.87 | 21.481 | 22.952 | 23.627 | 24.135 |
| 2.59 | 8.637 | 8.961 | 9.465 | 9.665 | 9.448 | 4.643 | 7.772 | 6.366 | 4.624 | 3.639 | 2.803 |

Table 4 illustrates the Inter-arrival time after using DEWMA with varied $\lambda$.

**Table 4:** *Inter-arrival time after using DEWMA with varied $\lambda$*

| | *0.9* | *0.8* | *0.7* | *0.6* | *0.5* | 0.4 |
|---|---|---|---|---|---|---|
| ***Inter-arrival time*** | | | | | | |
| ***6.25*** | *6.3* | *6.3* | *6.3* | *6.3* | *6.3* | 6.3 |
| ***25.23*** | *8.3* | *10.8* | *13.7* | *16.9* | *20.5* | 24.5 |
| ***4.99*** | *7.8* | *9* | *9.7* | *10.1* | *10* | 9.6 |
| ***14.55*** | *8.6* | *10.3* | *11.7* | *12.6* | *13.3* | 14.1 |
| ***14.3*** | *9.1* | *11.2* | *12.6* | *13.5* | *14.3* | 15 |
| 2.99 | 8.4 | 9.1 | 8.7 | 7.6 | 6 | 4.2 |
| 2.4 | 7.8 | 7.6 | 6.5 | 4.7 | 2.8 | 0.8 |
| 24.99 | 9.7 | 12 | 14 | 16.1 | 18.8 | 22 |
| 51.39 | 14.2 | 21.1 | 28.1 | 35.8 | 44.2 | 53.2 |
| 1.74 | 12.5 | 15.5 | 16.6 | 16.4 | 15.1 | 12.6 |
| 2.09 | 11.4 | 12.5 | 11.2 | 8.5 | 4.7 | 0.6 |
| 9.57 | 11.3 | 12.1 | 11.1 | 9.2 | 7.1 | 5.2 |
| 3.98 | 10.5 | 10.3 | 8.6 | 6.3 | 4.1 | 2 |
| 10.4 | 10.6 | 10.6 | 9.6 | 8.7 | 8.1 | 7.9 |
| 4.59 | 9.9 | 9.2 | 7.7 | 6.4 | 5.4 | 4.3 |
| 3.61 | 9.3 | 8 | 6.3 | 4.9 | 3.7 | 2.6 |
| 13.62 | 9.8 | 9.5 | 9.3 | 9.8 | 10.8 | 3.4 |
| 5.87 | 9.3 | 8.5 | 7.8 | 7.6 | 7.5 | 3.9 |
| 3.02 | 8.7 | 7.3 | 6 | 5 | 4.1 | 4.2 |
| 12.73 | 9.2 | 8.7 | 8.8 | 9.4 | 10.3 | 4.4 |
| 14.47 | 9.7 | 10 | 10.9 | 12.2 | 13.7 | 4.5 |
| 1.12 | 8.7 | 7.7 | 6.9 | 5.9 | 4.8 | 4.5 |
| 0.63 | 7.9 | 6.2 | 4.6 | 3 | 1.2 | 4.6 |
| 0.19 | 7.1 | 4.9 | 3.1 | 1.5 | 0.1 | 4.6 |
| 15.16 | 8.1 | 7.6 | 8 | 9.2 | 10.8 | 4.6 |
| 4.17 | 7.6 | 6.6 | 6.3 | 6.3 | 6.4 | 4.6 |
| 2.9 | 7.1 | 5.7 | 5 | 4.4 | 3.8 | 4.6 |
| 12.1 | 7.7 | 7.3 | 7.9 | 8.7 | 9.8 | 4.6 |
| 24.26 | 9.5 | 11.3 | 14.1 | 17.4 | 21 | 4.6 |
| 2.59 | 8.6 | 8.8 | 9.1 | 9 | 8.4 | 4.6 |

Figures 5 depict the histograms for inter-arrival time, after applying EWMA and DEWMA, respectively.

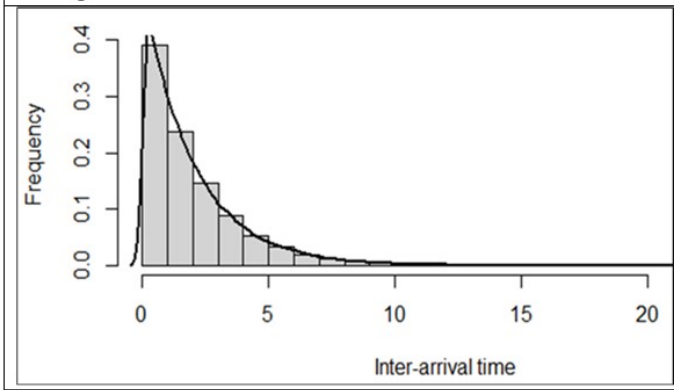
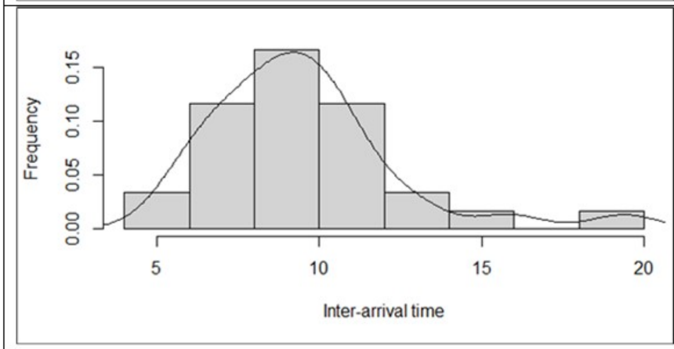
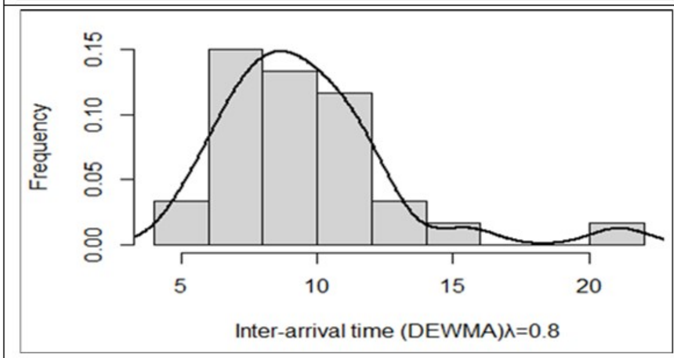
| Histogram | Notes |
|---|---|
|  | (a) Histogram of inter-arrival time<br>Mean = 9.8633<br>Standard deviation = 10.72357<br>N = 30 |
|  | (b) Histogram for inter-arrival time following application EWMA, $\lambda = 0.8$<br>Mean = 9.5023<br>Standard deviation = 2.942628<br>N = 30 |
|  | (C) Histogram for inter-arrival time after using DEWMA, $\lambda = 0.8$<br>Mean = 9.526334<br>Standard deviation = 9.765788<br>N = 30 |

**Figure 5:** *Histogram for inter-arrival time, after applying EWMA and DEWMA*

## 7. Conclusion

Monte Carlo simulation methods were utilized to create exponential distribution data that was evaluated using EWMA control chart functions to determine the value. The overall goal of this study is to assess the impact of distribution parameters in the operation of the EWMA control chart. Furthermore, the investigation is predicated on many key assumptions. The study's findings give an essential new knowledge of how distribution characteristics influence the Effectiveness of the EWMA control charts. Overall, the EWMA and DEWMA functions enhanced the analysis and interpretation of customer's arrival times, leading to superior management and better use of resources. There are a few limitations to consider while interpreting the findings and applying them to real-world situations. More research is required to get overcome these limitations and improve understand the situation. Additional study is needed to solve these limits and obtain an improved better grasp of the situation. Future research may include the creation of new statistical methodologies, namely for improved visualization methods and machine learning anomalies identification. In future research, this approach may be expanded to estimate the variance parameter and used to real data with different distributions.

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