BAYES ESTIMATOR OF PARAMETERS OF BINOMIAL TYPE EXPONENTIAL CLASS SRGM USING GAMMA PRIORS

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Abstract

The Reliability is one of the key characteristics of software that operates flawlessly and in accordance with needs of users. The assessment of Reliability is very important but it is complicated. The one-parameter exponential class failure intensity function is used in this article to quantify the model and assess the software Reliability. The scale parameter and the number of existing total failures are the model's parameters. Using the Bayesian approach, the estimators of parameters are obtained under the assumption that gamma priors are suitable to provide prior information of the parameters. Using risk efficiencies computed under squared error loss, the performance of proposed estimators is studied with their corresponding maximum likelihood estimators. The suggested Bayes estimators are found to outperform over the equivalent maximum likelihood estimators.

Keywords: Binomial process, gamma prior, maximum likelihood estimator (MLE), Exponential class, software reliability growth model (SRGM), confluent hyper-geometric function.

I. Introduction

The modern computers are widely and extensively used worldwide for solving the majority of complex problems pertaining to a variety of fields due to their ability to perform intricate and time-consuming tasks quickly, accurately, and with effective global communication. When completing all of such tasks, very sophisticated computers are used, and they are guided by a series of input instructions, known as a program or Software. Since, Software are necessary components of every computer system, its performance is crucial and has significant role.

Software are developed by human and due to its complexity and size, faults are more likely to occur. As a result, it gets the user's acceptance or rejection. For acceptance of any Software, its reliability is probably most important feature. Reliable software possesses high-quality and meets the needs of users or industries or government organizations. Software must be of an acceptable quality, which is closely connected to reliability qualities, to please the consumers. Since the 1950s, the area of software reliability is being studied by researchers, and several significant results have been produced. A variety of modern Statistical methods may be applied to measure the Software reliability. One of the strategy uses a zero-one method, where a flawed Software has a reliability of zero and a faultless Software has a reliability of one. Another strategy focuses on testing of Software, where software reliability is defined as the proportion of times a software executes an intended function as predicted. Thus, the measurement of Software reliability may be done using this method.

The usage of Software Reliability Growth Models (SRGMs) in the evaluation of software reliability is highly beneficial. The operational profile, accessibility of limited failures and irrational assumptions provide significant difficulties for SRGMs in practice. When estimating the parameters of software reliability model, various fundamental techniques are used which are maximum likelihood method, least squares estimation, Bayesian estimation, the EM method, etc., [5], [6], [7] and [12] have presented the calculation of factors, such as failure rate and total number of failures incorporated in SRGMs.

Software Engineering is the process of developing a software that can balance the dependability, delivery time, and price of the developed software. Software reliability modelling, as described by [8] is another way to represent software reliability using a mathematical function of involve factors, such as fault introduction, fault removal, the operating environment, etc. Software reliability is evaluated mathematically and objectively using the body of Statistics and Probability theory.

The Binomial type models, as categorized by [10] and [11] have been taken into consideration for study in this paper. Here, an attempt is made to derive the Bayes estimators for the parameters of the Binomial type exponential class. The prior distribution for the total number of initial software failures and scale parameters have been taken as the gamma prior distribution assuming that the experimenter is having prior information about both the parameters c.f. [14]- [20].

II. Model Characterization

Based on the following assumptions [12] have modelled the process of software failure using Binomial type models.

- The defect that resulted in a software failure will always be immediately fixed.
- There are θ_0 intrinsic faults in the programme.
- The hazard rates $Z(t)$ for all faults are same.

According to [12, 19], if $f(t)$ is the class of the SRGM and $\lambda(t) = \theta_0 f(t)$ is failure intensity. Considering binomial process and solving the differential equations using boundary conditions $P_0(0) = 1$ and $P_n(0) = 0 \; \forall \; n = 0,1,2, \dots \theta_0$, the following result is obtained.

$$P[M(t) = n] = \binom{\theta_0}{n}\{1 - exp[-\theta_1 t]\}^n \{exp[-\theta_1 t]\}^{\theta_0 - n} \qquad ,n = 0,1,2, \dots, \theta_0.$$

The $P[M(t) = n]$ gives the probability that $M(t) = n$ number of failures encountered at time t has a binomial distribution i.e. Binomial type of the software reliability model [12].

The Binomial Type Exponential Class model has failure intensity

$$\lambda(t) = \theta_0 \theta_1 e^{-\theta_1 t} \qquad , t > 0, \theta_0 > 0 \; and \; \theta_1 > 0 \tag{1}$$

where θ_0, failure rate (θ_1) are the parameters of the model and t be the execution time. This model exhibits failure intensity similar to [4] and [13]. The function of the mean failures is given by

$$\mu(t) = \theta_0[1 - e^{-\theta_1 t}] \qquad , t > 0, \theta_0 > 0 \; and \; \theta_1 > 0 \tag{2}$$

The expected number of failures at time *t* is represented by the Binomial distribution with the mean failure function.

$$\mu(t) = \theta_0\{1 - exp[-\theta_1 t_e]\}$$

and variance of $M(t)$ is

$$var[M(t)] = \theta_0\{1 - exp[-\theta_1 t]\}\{exp[-\theta_1 t]\}.$$

Let m_e be the number of failures that occurred upto execution time t_e , then the likelihood function of Binomial type exponential class model is

$$L(\theta_0,\theta_1|\underline{t}) = e^{-\theta_1 t_e(\theta_0-m_e)}\theta_1^{m_e}e^{-\theta_1 T}\theta_0^{\underline{m_e}} \quad , t_e>0, \theta_1>0,\ \theta_0>0 \text{ and } t_0=0 \tag{3}$$

where

$$T = \sum_{i=1}^{m_e} t_i$$

and

$\theta_0^{\underline{m_e}}$ are falling factorials (cf. [1], [2] and [3]).

The MLEs of θ_0 and θ_1 are obtained by applying standard method of obtaining MLE from equation (3) which comes out to be

$$\sum_{i=1}^{m_e}(\hat{\theta}_{m0}-i+1)^{-1} = \hat{\theta}_{m1}t_e \tag{4}$$

and

$$\hat{\theta}_{m1} = m_e\left[t_e(\hat{\theta}_{m0}-m_e)+T\right]^{-1} \tag{5}$$

The solution for $\hat{\theta}_{m0}$ and $\hat{\theta}_{m1}$ can be obtained by solving (4) and (5) using any standard iterative method.

III. Priors for Model Parameters

If the software professional could somehow predict or guess the information about the total number of failures present in the software and the value of scale parameter θ_1. Let's thus assume that gamma priors are seen to be appropriate for both θ_0 and θ_1 then it would be appropriate to use an informative prior for θ_0 and θ_1. The time-to-first failure distribution for a system with standby exponentially dispersed backups may naturally exhibit the gamma likelihood. Additionally, in practice, the gamma distribution may appear whenever items were tested, and whenever a part of an item or an entire item fails is replaced by an identical one having an exponential failure time distribution with parameter, the total amount of time on test could subsequently follow a gamma probability function. The gamma distribution is based on the fact that the total of i.i.d. random failure times after exponentials with parameters is distributed as gamma [9] and [24]. When an item may fail partially, or when a certain number of partial failures occur before an item fails (such as with redundant systems), the gamma distribution can be employed. Modelling the time to failure or failure rates for products with infant mortality may be done using the continuous Gamma probability.

Many studies have shown that the gamma distribution is feasible for failure rate and sufficiently adaptable for real-world hardware reliability applications in life testing. [7] created a Bayesian SRGM under the gamma prior assumption for the parameter of exponentially distributed periods between model failures. The Bayesian software reliability growth models established by [5] and [6] take into account the gamma prior distribution.

Thus, the gamma prior distributions may be used as informative priors in the current investigation for the parameters θ_0, and θ_1. Therefore,

$$g(\theta_0)\alpha\begin{cases}\theta_0^{\alpha-1}e^{-\beta\theta_0} & ,0<\theta_0<\infty\\ 0 & ,\ Otherwise\end{cases}$$

and

$$g(\theta_1)\alpha\begin{cases}\theta_1^{\eta-1}e^{-\nu\theta_1} & ,0<\theta_1<\infty\\ 0 & ,\ Otherwise\end{cases}$$

where α, β, η, and ν are hyper parameters of considered priors for θ_0 and θ_1 respectively. The hyper parameters η and α are shape parameters and ν and β are scale parameters of the prior distributions. The flexibility in choices of α, β, η, and ν allows the researcher to select the prior model for parameters that best expresses the current state of knowledge about the number of failures and failure rate.

Hence, the joint prior distribution of both parameters θ_0 and θ_1 is given as

$$g(\theta_0,\theta_1)\alpha\begin{cases}\theta_0^{\alpha-1}\theta_1^{\eta-1}e^{-\beta\theta_0}e^{-\nu\theta_1} & ,0<\theta_0,\theta_1<\infty\\ 0 & ,\ Otherwise\end{cases} \tag{6}$$

IV. Joint Posterior and Marginal Posterior Distributions

Assuming the total execution time is t_e, during this time m_e failures are experienced at times $t_i, i = 1,2,\dots,m_e$, θ_0 be the number of failures present in the software and θ_1 be the failure rate then, combining likelihood function (3) with joint prior given by (6), the joint posterior of θ_0 and θ_1 given $t_i, i = 1,2,\dots,m_e (= \underline{t})$ is

$$\pi(\theta_0,\theta_1|\underline{t}) \propto e^{-\theta_1[t_e(\theta_0-m_e)+(T+\nu)]}\theta_1^{m_e+\eta-1}\theta_0^{\alpha-1}e^{-\beta\theta_0}\theta_0^{\underline{m_e}} \qquad ,m_e < \theta_0 < \infty, 0 < \theta_1 < \infty \tag{7}$$

The constant of proportionality (normalizing constant) of above equation is

$$D = \int_{m_e}^{\infty}\int_0^{\infty} e^{-\theta_1[t_e(\theta_0-m_e)+(T+\nu)]}\theta_1^{m_e+\eta-1}\theta_0^{\alpha-1}e^{-\beta\theta_0}\theta_0^{\underline{m_e}}\, d\theta_0\, d\theta_1 \qquad ,m_e < \theta_0 < \infty, 0 < \theta_1 < \infty \tag{8}$$

The above expression of D can be solved using the results given in [1], [2], and [3] as

$$D = C\sum_{m=0}^{m_e} S_{m_e}^{(m)} {m_e}^m \sum_{r=0}^{m+\alpha-1}\binom{m+\alpha-1}{r}\left(\frac{m_e t_e}{(T+\nu)}\right)^{-r} I_1$$

where

$$C = \frac{\Gamma(m_e+\eta){m_e}^{\alpha-1}}{e^{\beta m_e}T_1(T+\nu)^{m_e+\eta}}$$

$$I_1 = \Gamma(r+1)\Psi\left(r+1, r-m_e+\eta+2, \frac{\beta(T+\nu)}{t_e}\right)$$

$\Gamma(.)$ is standard Gamma function and $\Psi(.,.,.)$ is confluent hyper-geometric function defined in [1], [2], and [3].

The marginal posterior of θ_0, say $\pi(\theta_0|\underline{t})$ can be obtained after integrating $\pi(\theta_0,\theta_1|\underline{t})$ over the whole range of θ_1 and it is

$$\pi(\theta_0|\underline{t}) \propto \Gamma(m_e+\eta)\theta_0^{\alpha-1}e^{-\beta\theta_0}\theta_0^{\underline{m_e}}[t_e(\theta_0-m_e)+(T+\nu)]^{-(m_e+\eta)} \tag{9}$$

where

$$\theta_0 \geq m_e, \left(t_e(\theta_0-m_e)+(T+\nu)\right) \geq 0$$

The marginal posterior of θ_1, say $\pi(\theta_1|\underline{t})$ is the solution of $\pi(\theta_1|\underline{t}) = \int_{m_e}^{\infty}\pi(\theta_0,\theta_1|\underline{t})\, d\theta_0$ i.e.

$$\pi(\theta_1|\underline{t}) \propto \theta_1^{m_e+\eta-1}e^{-\theta_1[(T+\nu)-m_e t_e]}\, I_2 \tag{10}$$

where

$$I_2 = \sum_{m=0}^{m_e} S_{m_e}^{(m)}\, e^{-(\beta+\theta_1 t_e)m_e}\sum_{k=0}^{m+\alpha-1}\frac{(m+\alpha-1)!}{k!}{m_e}^k(\beta+\theta_1 t_e)^{-(m+\alpha-k)}$$

V. Bayes Estimates for Model Parameters

The Bayes estimators of θ_0 and θ_1 are posterior mean under the squared error loss function. The Bayes estimator for θ_0 i.e. the posterior mean can be obtained from (9) and is

$$\hat{\theta}_{B0} \propto \frac{\Gamma(m_e+\eta){m_e}^{\alpha}}{e^{\beta m_e}T_1(T+\nu)^{m_e+\eta}}\sum_{m=0}^{m_e} S_{m_e}^{(m)}\, {m_e}^m\sum_{r=0}^{m+\alpha}\binom{m+\alpha}{r}\left[\frac{m_e t_e}{(T+\nu)}\right]^{-r} I_3$$

where

$$I_3 = \Gamma(r+1)\Psi\left(r+1, r-m_e+\eta+2, \frac{\beta(T+\nu)}{t_e}\right)$$

The Bayes estimator for θ_1 is the posterior mean of its marginal posterior distribution (10) is

$$\hat{\theta}_{B1} \propto \frac{\Gamma(m_e+\eta+1){m_e}^{\alpha-1}}{e^{\beta m_e}T_1(T+\nu)^{m_e+\eta+1}}\sum_{m=0}^{m_e} S_{m_e}^{(m)}\, {m_e}^m\sum_{r=0}^{m+\alpha-1}\binom{m+\alpha-1}{r}\left[\frac{m_e t_e}{(T+\nu)}\right]^{-r} I_4$$

Where

$$I_4 = \Gamma(r+1)\Psi\left(r+1, r-m_e+\eta+1, \frac{\beta(T+\nu)}{t_e}\right)$$

VI. Discussion

The proposed Bayes estimators i.e. $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ of total number of failures (θ_0) and failure rate (θ_1) for the parameters of Binomial type exponential class SRGM are obtained by considering gamma priors and are compared with corresponding maximum likelihood estimators $\hat{\theta}_{m0}$ and $\hat{\theta}_{m1}$

respectively. The comparative performance of proposed Bayes estimators against corresponding maximum likelihood estimators has been studied based on the risk efficiencies i.e. $RE_0 = \frac{E\left(\hat{\theta}_{B0}-\theta_0\right)^2}{E\left(\hat{\theta}_{m0}-\theta_0\right)^2}$ and $RE_1 = \frac{E\left(\hat{\theta}_{B1}-\theta_1\right)^2}{E\left(\hat{\theta}_{m1}-\theta_1\right)^2}$. The estimators $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ are based on the prior parameters α, β, η, ν and execution time t_e. The risk efficiencies are calculated by considering different values of these constants and arbitrary values of parameters θ_0 and θ_1 using the Monte Carlo Simulation technique by generating 10[3] samples. An execution time t_e is prefixed and up to this time, sample failures are generated and the risk efficiencies are presented in the following Figure 1 to Figure 14.

Consider the effect of variation in the value of t_e on the risk efficiencies of the proposed Bayes estimator $\hat{\theta}_{B0}$, given in Figure 1 to Figure 4.

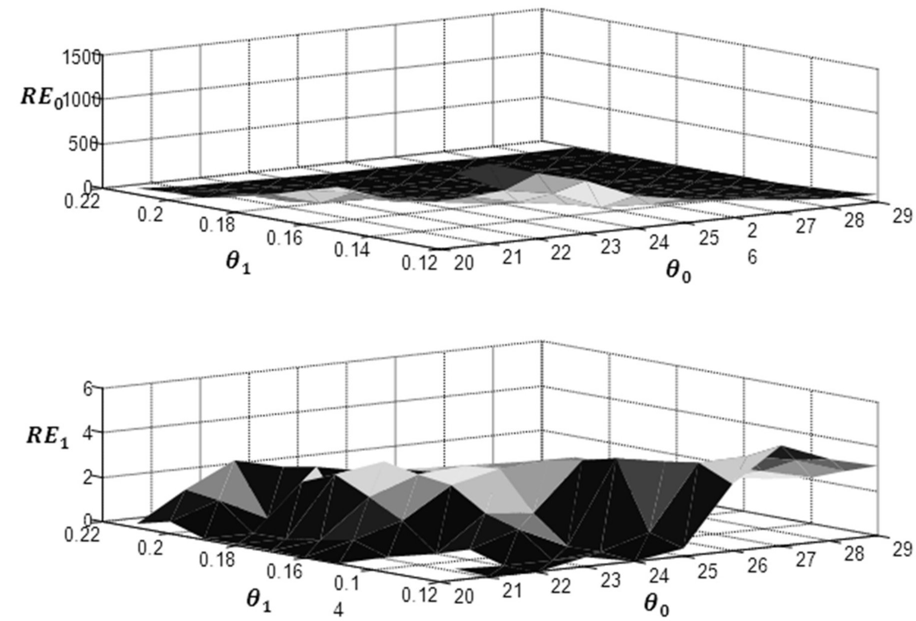

Figure 1: *Risk Efficiencies of $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$, for* $\theta_1(=0.12(0.01)0.21)$, $\theta_0(=20(1)29)$, $\alpha = 30$, $\beta = 10$, $\eta = 10$, $\nu = 10$ *and* $t_e = 3.0$

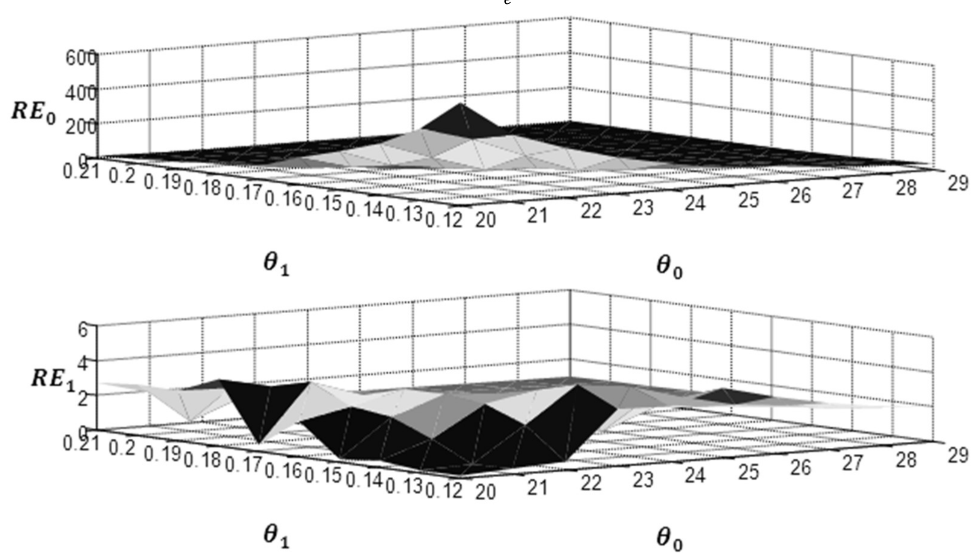

Figure 2: *Risk Efficiencies of $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$, for* $\theta_1(=0.12(0.01)0.21)$, $\theta_0(=20(1)29)$, $\alpha = 30$, $\beta = 10$, $\eta = 10$, $\nu = 10$ *and* $t_e = 3.5$

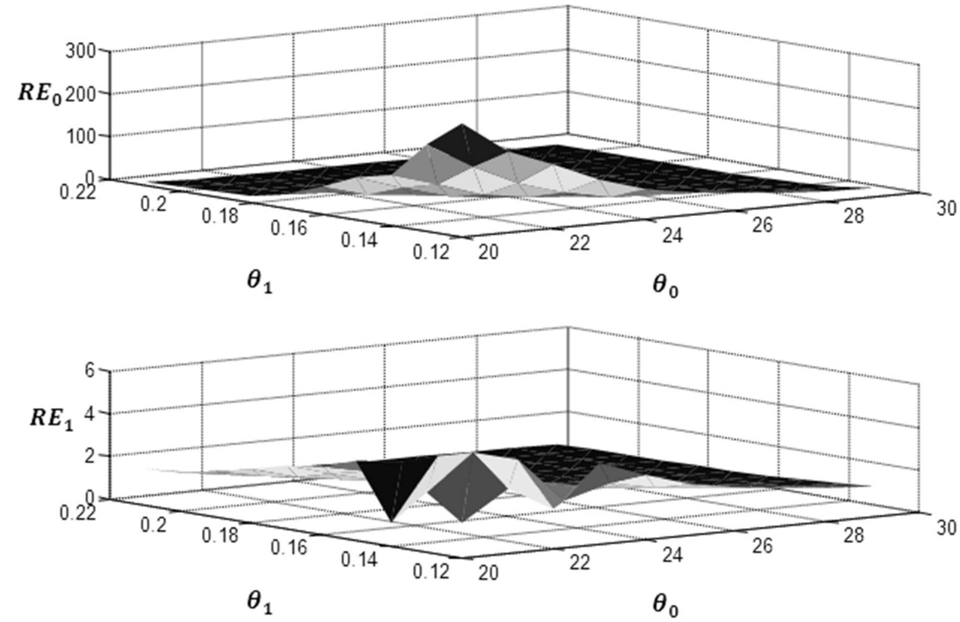

Figure 3: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $\alpha = 30$, $\beta = 10$, $\eta = 10$, $\nu = 10$ *and* $t_e = 4.0$

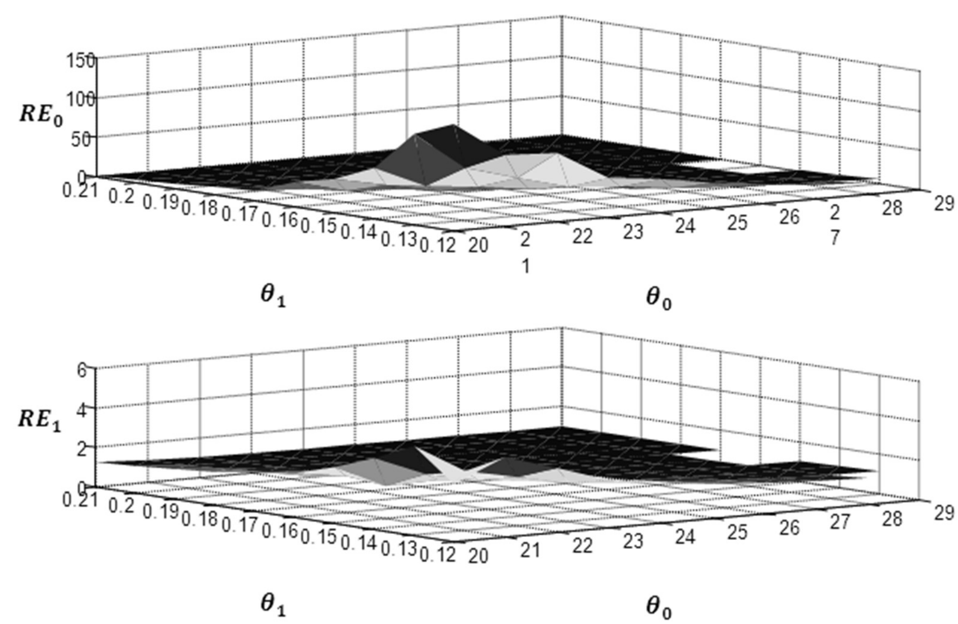

Figure 4: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $\alpha = 30$, $\beta = 10$, $\eta = 10$, $\nu = 10$ *and* $t_e = 4.5$

From these figures, it is seen that for the increase in value of t_e, the risk efficiencies RE_0 of Bayes estimator $\hat{\theta}_{B0}$ decrease as θ_0 and θ_1 increase. The point of maxima varies as the value of t_e changes. Particularly, the risk efficiency RE_0 attains maxima at smaller values of θ_0 and θ_1 for increasing values of t_e.

The variation of shape constant $\alpha(= 30(5)40)$ of proposed prior for the total number of failures, the risk efficiencies of Bayes estimators $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ are presented in Figure 5 and Figure 6.

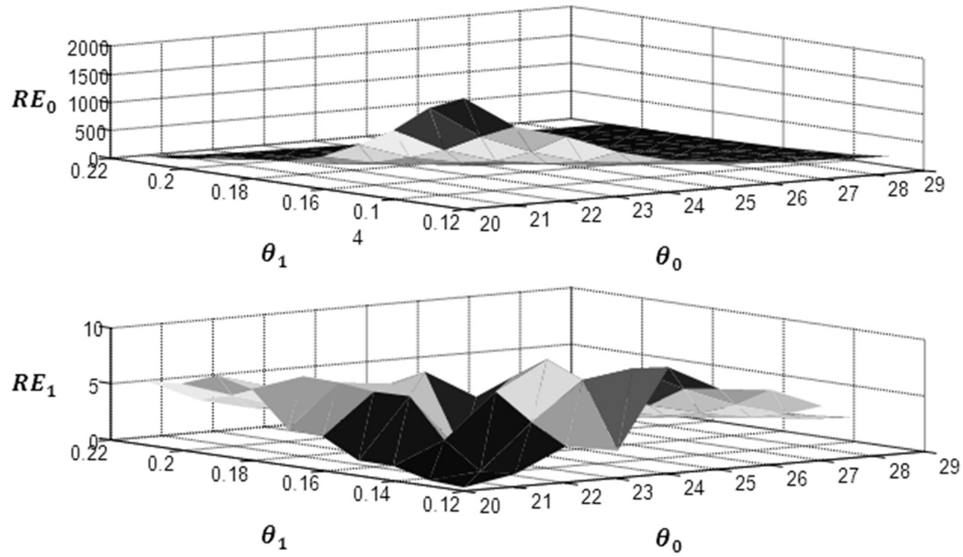

Figure 5: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $t_e = 4.0$, $\beta = 10$, $\eta = 10$, $\nu = 10$ *and* $\alpha = 35$

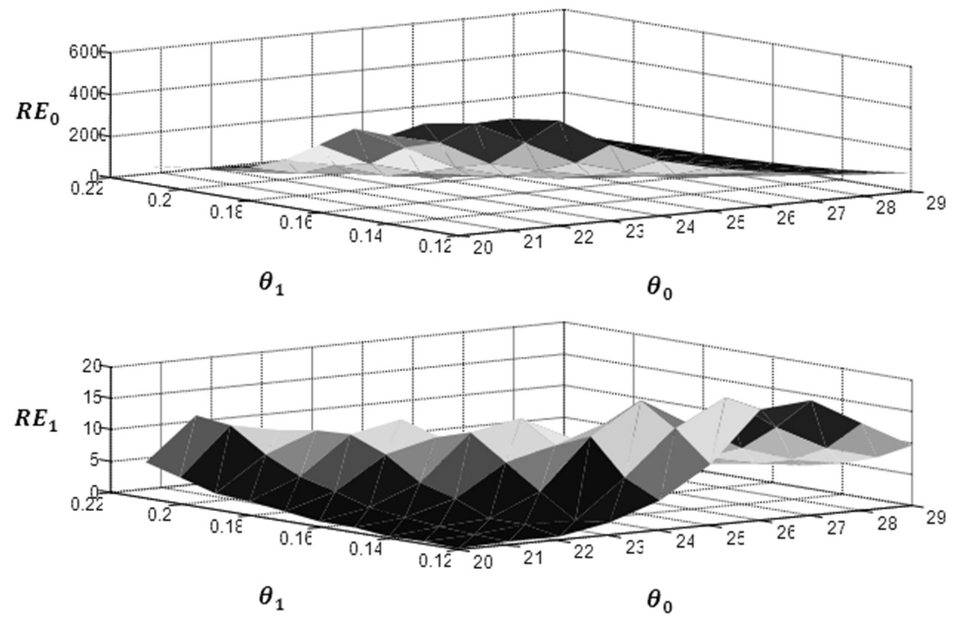

Figure 6: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $t_e = 4.0$, $\beta = 10$, $\eta = 10$, $\nu = 10$ *and* $\alpha = 40$

It is observed that the values of risk efficiencies of both the proposed estimators are increased for increasing the value of α. Here, in these figures the risk efficiencies of both the estimators are increasing for increasing values of α.

The risk efficiencies of Bayes estimators $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ i.e. RE_0 and RE_1 calculated for different values of scale constant $\beta(= 1,10(5)20)$ of prior proposed for θ_0 are summarized in Figure 7 to Figure 8.

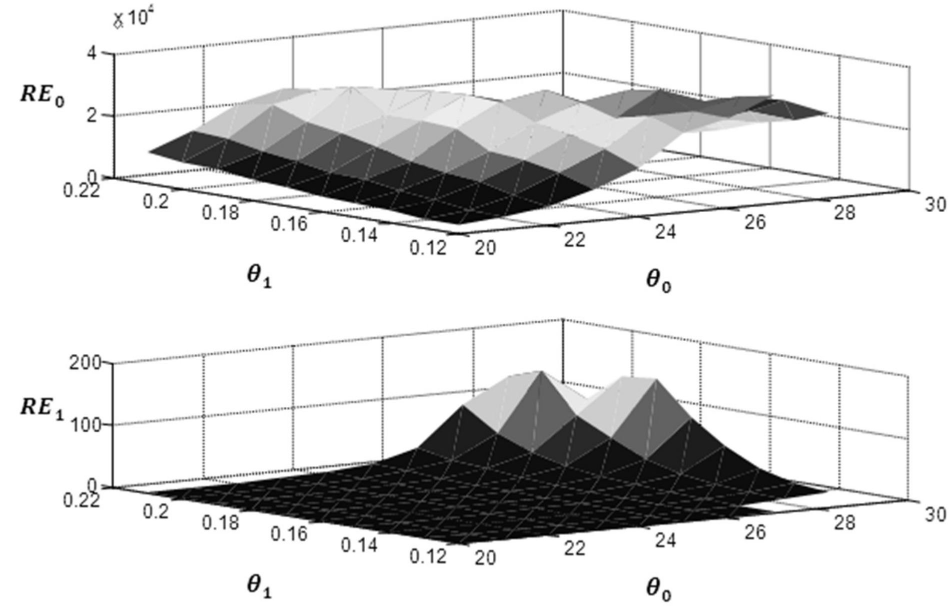

Figure 7: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(=0.12(0.01)0.21)$, $\theta_0(=20(1)29)$, $t_e = 4.0$, $\alpha = 30$, , $\eta = 10$, $\nu = 10$ *and* $\beta = 1$

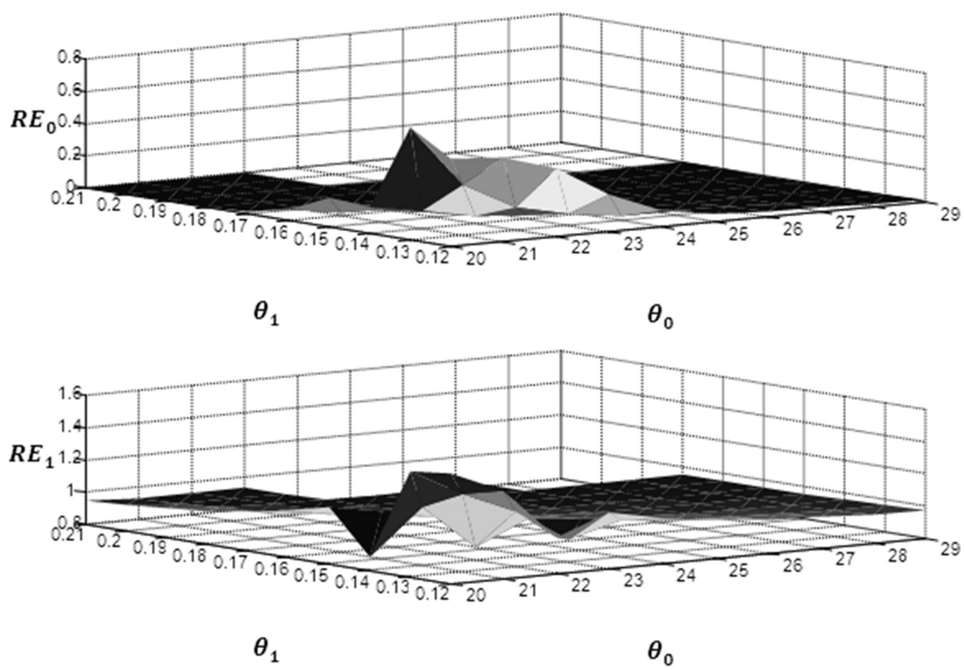

Figure 8: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(=0.12(0.01)0.21)$, $\theta_0(=20(1)29)$, $t_e = 4.0$, $\alpha = 30$, $\eta = 10$, $\nu = 10$ *and* $\beta = 15$

Here, the risk efficiencies of both estimators are decreasing for increasing values of β. It is also seen that, both the proposed Bayes estimators $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ are becoming more inefficient than corresponding maximum likelihood estimators as β increases.

The risk efficiencies of Bayes estimators $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ i.e. RE_0 and RE_1 are also evaluated using various values of shape constant $\eta(=1,10(5)20)$ of prior for θ_1 and are summarized in Figure 9 to Figure 11.

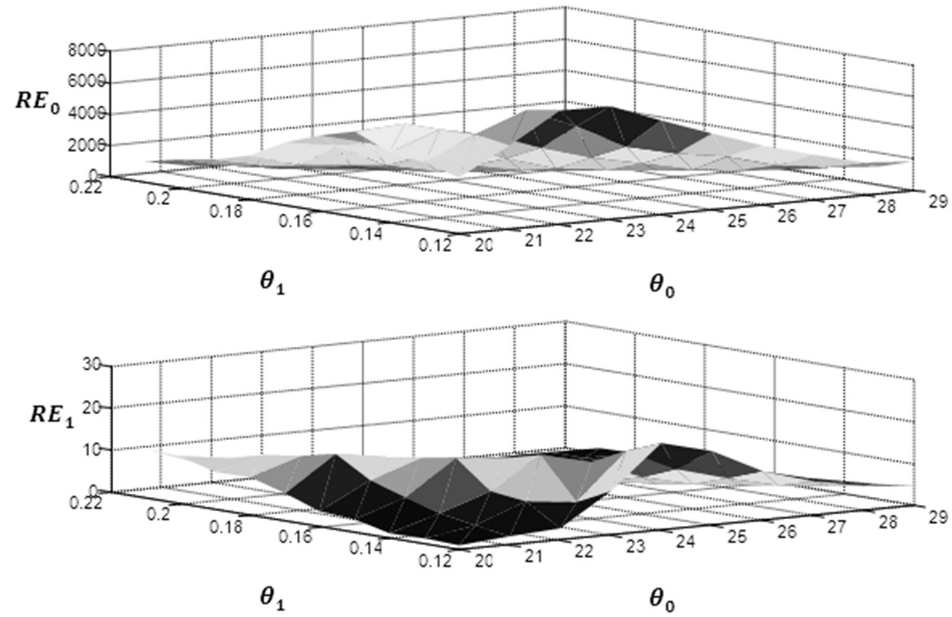

Figure 9: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $t_e = 4.0$, $\alpha = 30$, $\beta = 10$, $\nu = 10$ *and* $\eta = 1$

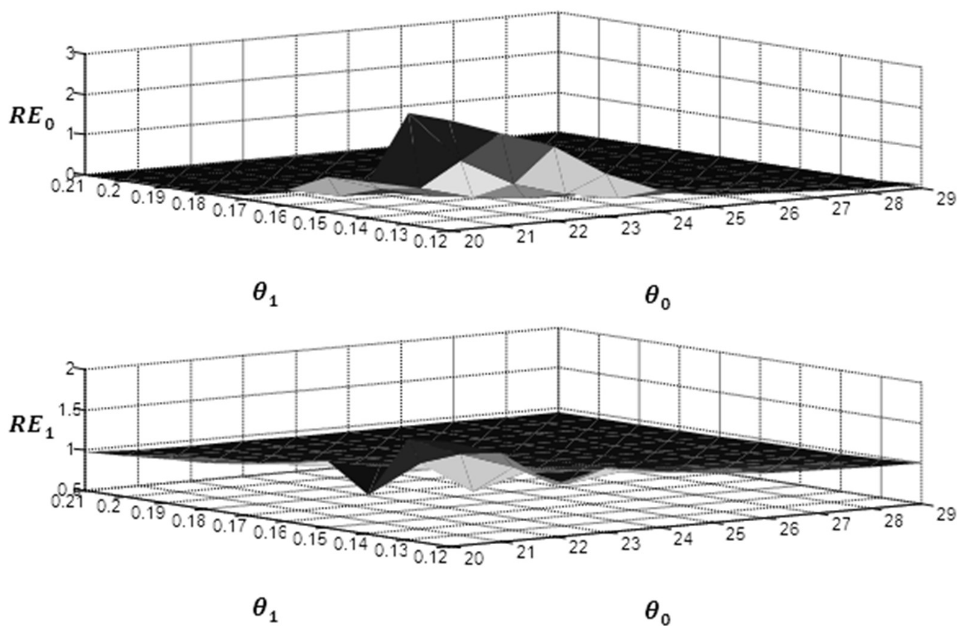

Figure 10: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$*, for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)t_e = 4.0$, $\alpha = 30$, $\beta = 10$, $\nu = 10$ *and* $\eta = 15$

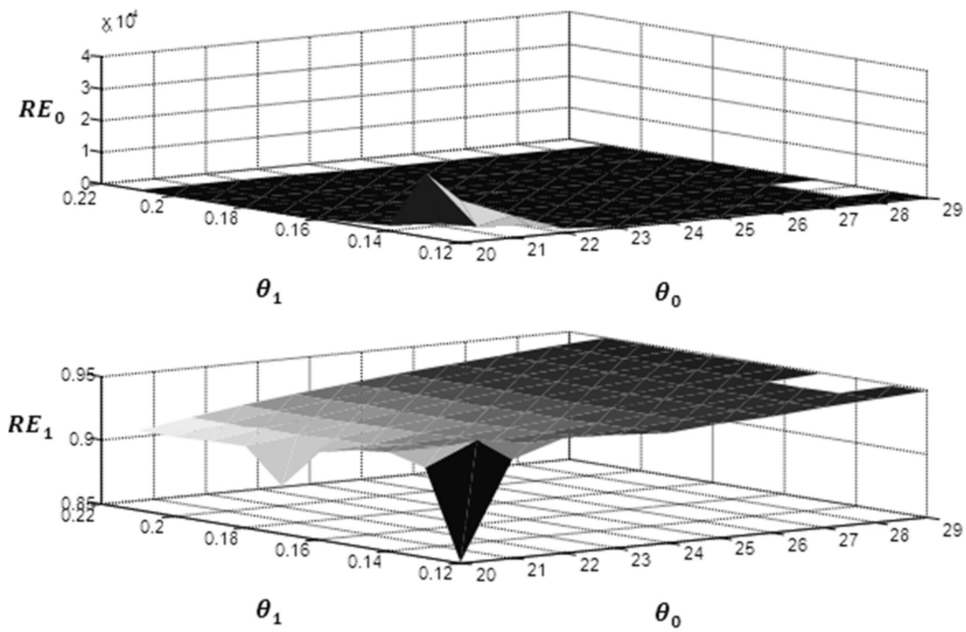

Figure 11: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$, *for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $t_e = 4.0$, $\alpha = 30$, $\beta = 10$, $\nu = 10$ *and* $\eta = 20$

Here, it is observed that the risk efficiencies of both estimators decrease for the increase in the values of η. It is also seen that, both the proposed Bayes estimators $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ are becoming more inefficient than corresponding maximum likelihood estimators as η increasing.

The risk efficiencies of Bayes estimators $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ i.e. RE_0 and RE_1 evaluated using various values of scale constant $\nu(= 1,10(5)20)$ of prior θ_1 and are summarized from Figure 12 to Figure 14.

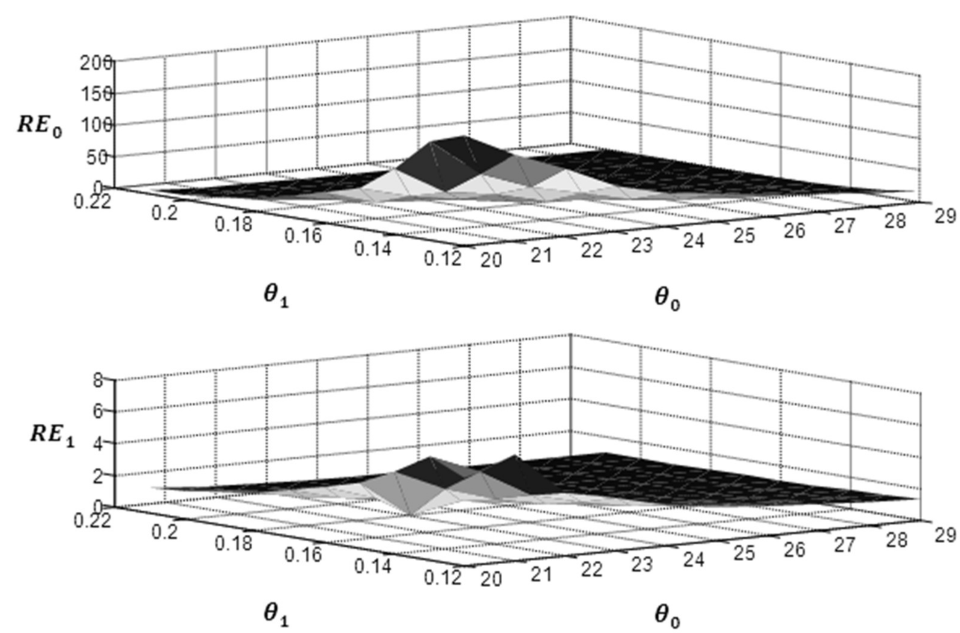

Figure 12: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$, *for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $\alpha = 30$, $\beta = 10$, $\eta = 10$, $t_e = 4.0$ *and* $\nu = 1$

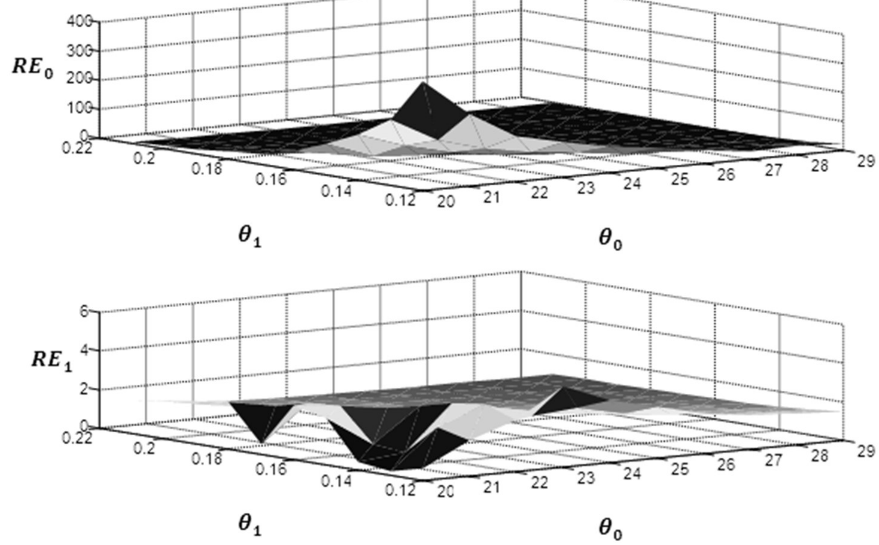

Figure 13: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$, *for* $\theta_1(= 0.12(0.01)0.21)$,$\theta_0(= 20(1)29)$, $t_e = 4.0$, $\alpha = 30$, $\beta = 10$, $\eta = 10$ *and* $\nu = 15$

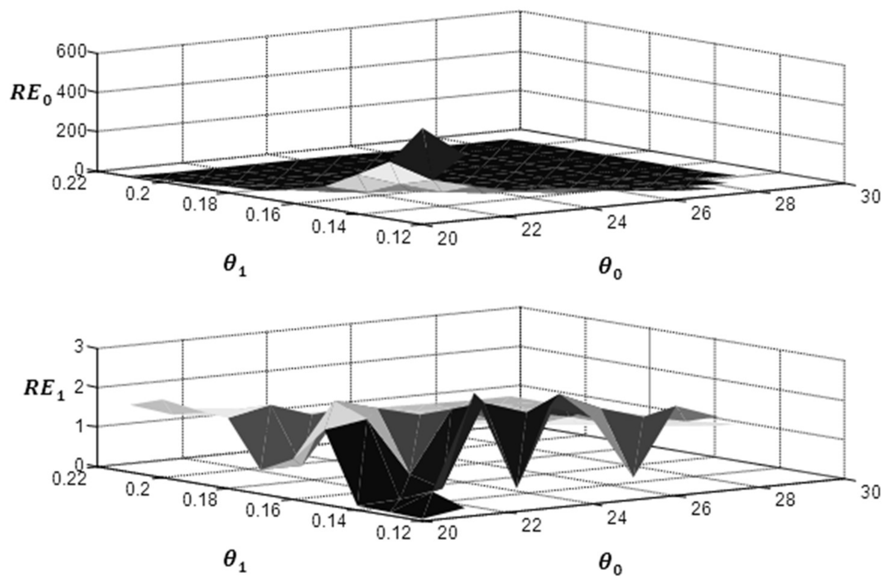

Figure 14: *Risk Efficiencies of* $\hat{\theta}_{B0}$ *and* $\hat{\theta}_{B1}$, *for* $\theta_1(= 0.12(0.01)0.21)$, $\theta_0(= 20(1)29)$, $t_e = 4.0$, $\alpha = 30$, $\beta = 10$, $\eta = 10$, *and* $\nu = 20$

Here, it is seen that the risk efficiencies of $\hat{\theta}_{B0}$ are increasing whereas the risk efficiencies of $\hat{\theta}_{B1}$ are decreasing for increasing the values of ν. It is also seen that the proposed Bayes estimator $\hat{\theta}_{B0}$ is becoming efficient as η increases whereas $\hat{\theta}_{B1}$ becoming more inefficient than the corresponding maximum likelihood estimator.

VII. Conclusions

Both the proposed Bayes estimator of θ_0 and θ_1 i.e. $\hat{\theta}_{B0}$ and $\hat{\theta}_{B1}$ can be preferred over corresponding MLEs if the parameters of gamma priors for model parameters are properly chosen. The value of t_e should be small for moderate values of true parameters and prior constants. The values of shape constant α of prior proposed for θ_0 should be chosen moderately large for smaller values of t_e. The values of scale constant β of prior proposed for the total number of failures i.e. θ_0 should be chosen smaller when values of t_e are small. The values of prior parameters η and ν should be chosen smaller for smaller values of t_e.

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