DETERMINATION OF VITERBI PATH FOR 3 HIDDEN AND 5 OBSERVABLE STATES USING HIDDEN MARKOV MODEL

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Abstract

Hidden markov model (HMM) is a statistical markov model in which the system being modeled and is assumed to be a markov process with unobservable (i.e., Hidden) states. In HMM, the state is not directly visible but the output depend on the state is visible. Each state has a probability distribution over the possible output tokens. The model is referred to as a hidden markov model even if these parameters are known exactly. The viterbi is one of the estimate underlying state path in hidden markov models. In this paper, viterbi path is derived using hidden markov model.

Keywords: Hidden markov model, Viterbi algorithm, Hidden states, Observable states.

I. Introduction

Viterbi algorithm is a dynamic programming algorithm to obtain the maximum posterior probability estimate of the most likely sequence of hidden states and viterbi path results in a sequence of observed events, especially in the context of hidden markov model. Antibiotics are medicines that fight against bacterial infections in people and animals. They work by killing the bacteria or by making it hard for the bacteria to grow and multiply. Antibiotics can be taken in different ways orally (by mouth) this could be, tablets, pills, capsules, or liquids. Another way to take is topically, this might be a cream, ointment, eye drops, or ear drops, through an injection intramuscular or intravenously, this is usually used for more serious infections. Maria Luz Gamiz [7] et al applied hidden markov models in reliability and maintenance. Janani Kalyanam [3] discussed the probabilistic algorithm for list viterbi decoding. Viterbi A.J [10] derived error bounds for convolution codes and asymptotically optimum decoding algorithm. Shinghal R[9] et al described the modification of the viterbi algorithm formally, and a measure of its complexity is derived. The modified algorithm uses heuristic to limit the search through a directed graph or trellis. G Saritha[2] et al discussed the reliability for 4-modular and 5-modular redundancy system by using Markov technique.

II. Mathematical model

Assume there are M possible states to choose from, the state could be any one of {1,2,3,…,m}. The transition probability should be quantified from state *i* to state j asa_{ij}. The transition could happen from any one of the M possible states to another one of the M possible states, there are in total M×M possibilities. They can be arranged in the following matrix representation, known as state transition matrix S.

$$S = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}$$

Here a_{ij},*i*=1,2,3,….m, j=1,2,3,…m are probability values between 0 and 1, each row has to be summed to 1, and a_{ij} can be written as

$$a_{ij} = P\left[X_k = j \mid X_{k-1} = i\right]$$

Therefore,

$S_k^T = S_{k-1}^T A$, i.e., matrix multiplication of previous state with transition,

where $S_k = \begin{bmatrix} P\left[X_k = 1\right] \\ P\left[X_k = 2\right] \\ \text{.....} \\ P\left[X_k = m\right] \end{bmatrix}$

The emission matrix B is the probabilities of a state *i* in an observed value j. The observation has K possible values {0,1, 2,…,k}.

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & b_{22} & \cdots & b_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mk} \end{bmatrix}$$

Here b_{ij} represents the probability of state *i* to emit observable j and can be written as

$$b_{ij} = P\left[Y_k = j \mid X_k = i\right]$$

Therefore,

$O_k^T = S_k^T B$, where $O_k = \begin{bmatrix} P\left[Y_k = 1\right] \\ P\left[Y_k = 2\right] \\ \text{.....} \\ P\left[Y_k = m\right] \end{bmatrix}$

Initial state probability distribution is denoted by $\pi_0$and is given by

$\pi_0 = [P_1 P_2 \ldots P_i \ldots P_m]$

The transition matrix is a regular matrix whose elements are probabilities of one state to another state. The probability between hidden states to observable states is called emission probability, the matrix representation of emission probabilities is called emission matrix. Here the rows represent hidden states and columns represent observable states.

In this paper the hidden state space S= {Nausea, Diarrhea, Stomach pain} and the observable state space B={Amoxicillin + Potassium Clavunate(ap),Cefixime(ce), Amoxicillin(am), Azithromycin (az), Ciprofloxacin(cp)}.

The initial probability $\pi_0 = [0.45,\ 0.3,\ 0.25]$

The transition probability matrix between hidden states is S=

	N	D	S
N	0.5	0.2	0.3
D	0.2	0.6	0.2
S	0.1	0.2	0.7

The emission matrix is B=

	ap	ce	am	az	cp
N	0.33	0.22	0.19	0.14	0.12
D	0.35	0.15	0.15	0.25	0.1
S	0.15	0.35	0.25	0.15	0.1

II. Viterbi Algorithm

For the large number of possibilities forward and backward algorithm cannot be used to get the maximum probability. Viterbi algorithm is used to obtain the maximum posterior probabilities of the most likely sequence of hidden states. The total possibilities are $m = n^t = 3^5 = 243$, where n is the number of hidden states and t is the number of observations. Out of these possibilities, let us consider the viterbi path 2 4 1 3 5. Then

$P(2,N) = P(2/N)P(N) = 0.099$
$P(2,D) = P(2/D)P(D) = 0.045$
$P(2,S) = P(2/S)P(S) = 0.0875$

And the viterbi probabilities are

$V_1(1) = 0.099,\ V_1(2) = 0.045,\ V_1(3) = 0.0875,$

Similarly

$P(4,N) = P(4/N)P(N/N) = 0.07, \quad P(4,D) = 0.05, \ P(4,S) = 0.045$
$P(4,N) = P(4/N)P(N/D) = 0.028, \quad P(4,D) = 0.15, \ P(4,S) = 0.03$
$P(4,N) = P(4/N)P(N/S) = 0.014, \quad P(4,D) = 0.05, P(4,S) = 0.105$

And the viterbi probabilities are

$$V_2(1) = \max\begin{Bmatrix} 0.099 \times 0.07 = 0.00693, \\ 0.045 \times 0.028 = 0.0012, \\ 0.0875 \times 0.014 = 0.001225 \end{Bmatrix} = 0.00693, \quad V_2(2) = 0.00675, \quad V_2(3) = 0.0091875,$$

$$P(1,N) = P(1/N)P(N/N) = 0.165, \quad P(1,D) = 0.07, \quad P(1,S) = 0.045$$
$$P(1,N) = P(1/N)P(N/D) = 0.066, \quad P(1,D) = 0.21, \quad P(1,S) = 0.03$$
$$P(1,N) = P(1/N)P(N/S) = 0.033, \quad P(1,D) = 0.07, \quad P(1,S) = 0.105$$

And the viterbi probabilities are

$$V_3(1) = 0.00114345, \quad V_3(2) = 0.0014175, \quad V_3(3) = 0.0009646875,$$
$$P(3,N) = P(3/N)P(N/N) = 0.095, \quad P(3,D) = 0.03, \quad P(3,S) = 0.075$$
$$P(3,N) = P(3/N)P(N/D) = 0.038, \quad P(3,D) = 0.09, \quad P(3,S) = 0.05$$
$$P(3,N) = P(3/N)P(N/S) = 0.019, \quad P(3,D) = 0.03, \quad P(3,S) = 0.175$$

And the viterbi probabilities are

$$V_4(1) = 0.0001086278, \quad V_4(2) = 0.000127575, \quad V_4(3) = 0.0001688203$$
$$P(5,N) = P(5/N)P(N/N) = 0.06, \quad P(5,D) = 0.02, \quad P(5,S) = 0.03$$
$$P(5,N) = P(5/N)P(N/D) = 0.024, \quad P(5,D) = 0.06, \quad P(5,S) = 0.02$$
$$P(5,N) = P(5/N)P(N/N) = 0.012, \quad P(5,D) = 0.02, \quad P(5,S) = 0.07$$

And the viterbi probabilities are

$$V_5(1) = 0.0000065177, \quad V_5(2) = 0.0000076545, \quad V_5(3) = 0.0000118174,$$

Now,

$\text{Max}\{V_1(1), V_1(2), V_1(3)\}=0.099$
$\text{Max}\{V_2(1), V_2(2), V_2(3)\}=0.0091875$
$\text{Max}\{V_3(1), V_3(2), V_3(3)\}=0.0014175$
$\text{Max}\{V_4(1), V_4(2), V_4(3)\}=0.0001688203$
$\text{Max}\{V_5(1), V_5(2), V_5(3)\}=0.000011817$

The above probabilities are shown in the following diagram and it gives the final path

N ⟶ S ⟶ D ⟶ S ⟶ S

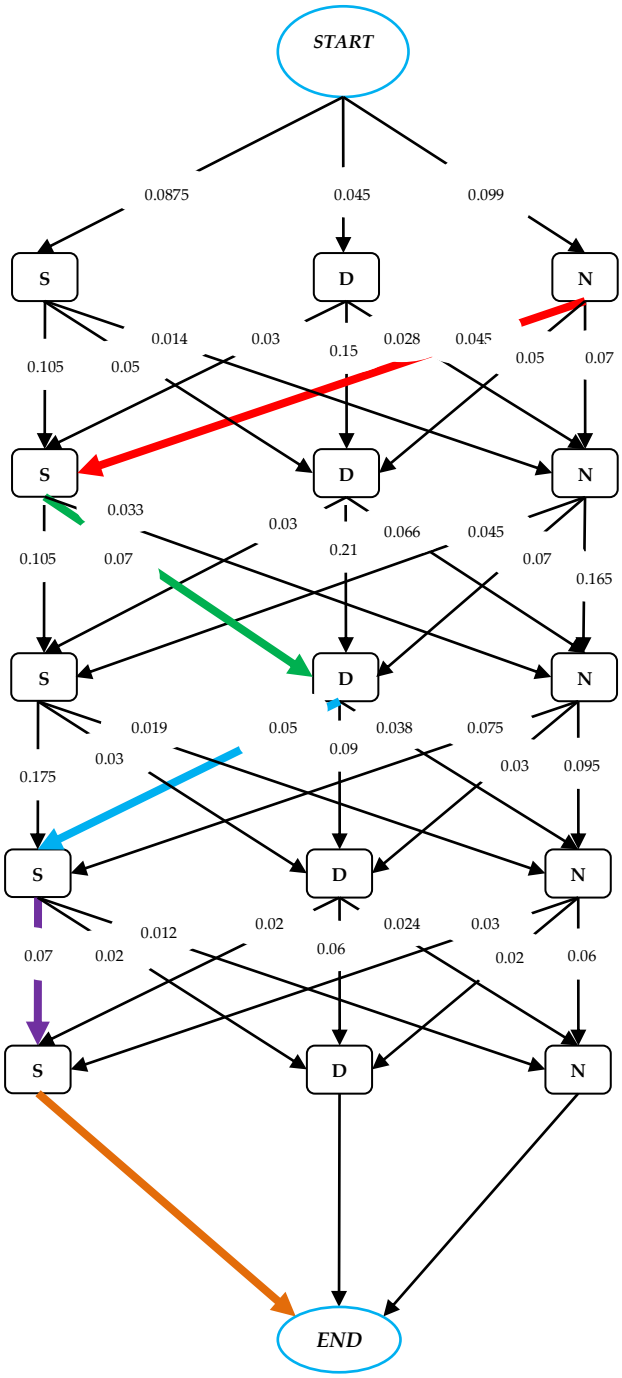

Figure 1: *The different states probabilites*

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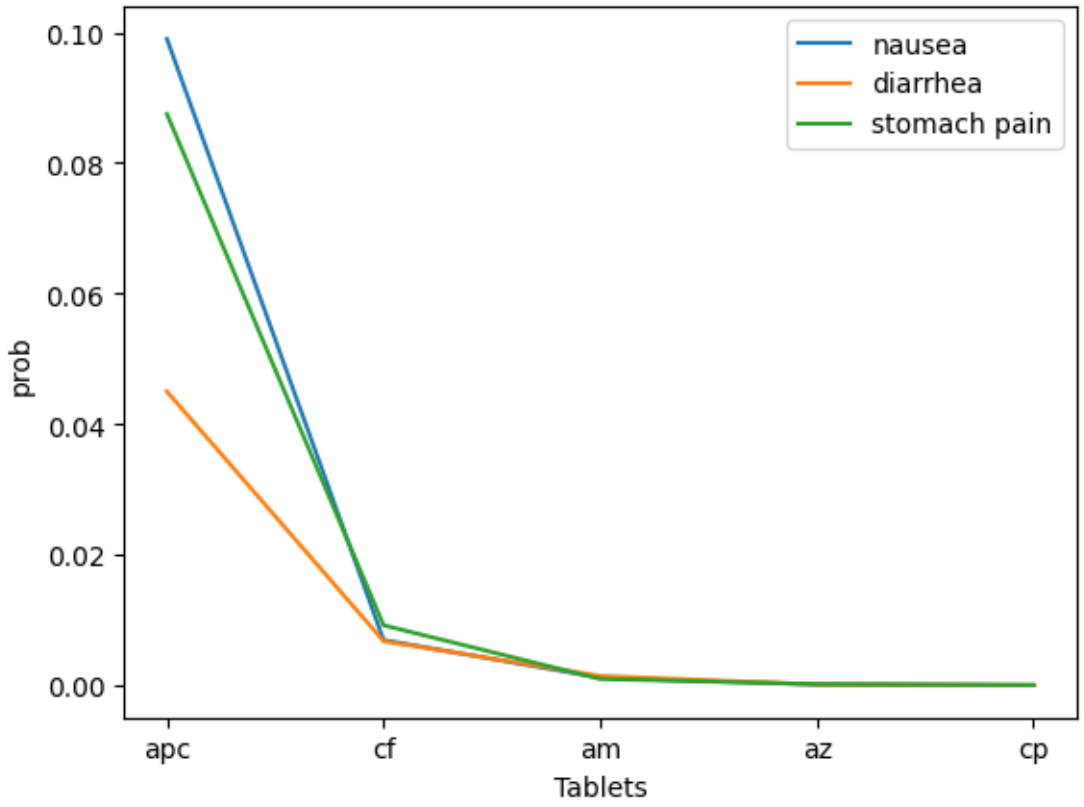

Figure 2: *viterbi probabilities of hidden states*

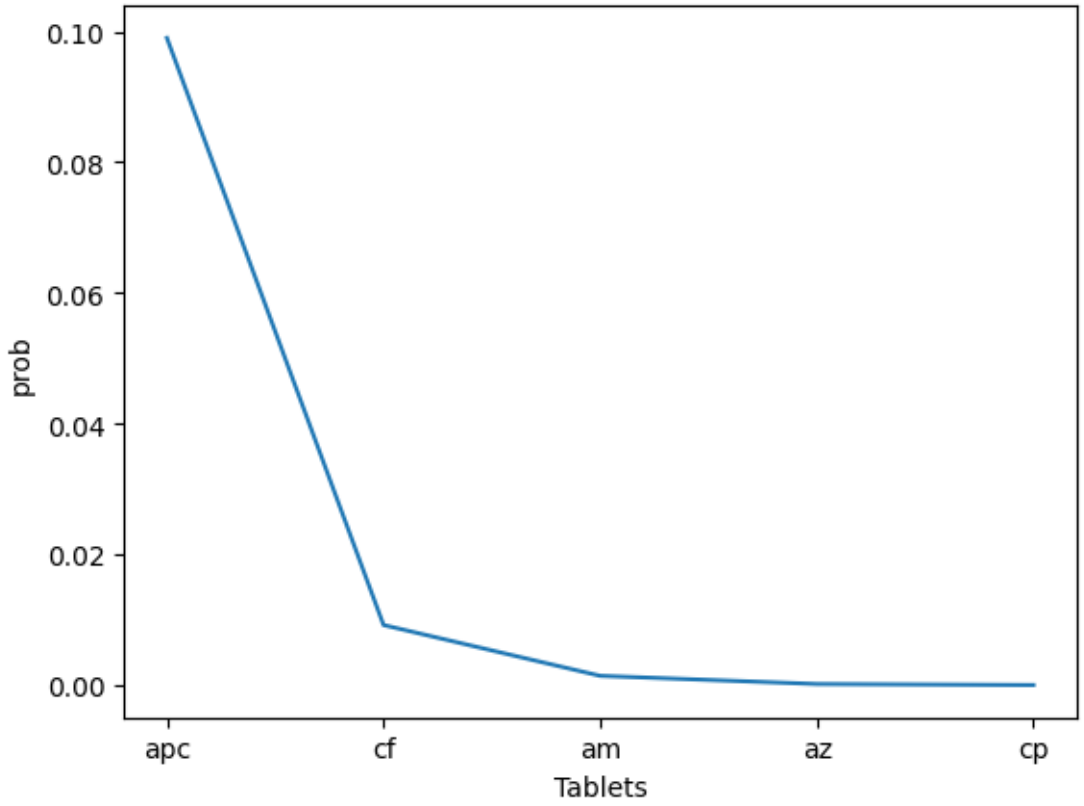

Figure 3: *Maximum probabilities of viterbi path*

From figure 2, we observe that, for tablet Amoxicillin + Potassium Clavunate (apc), the probability for Nausea, Diarrhea, Stomach pain are maximum and the tablet Cefixime(cf), takes the second place and the reaming are almost equal.
From figure 3, we observe that the Maximum probabilities of viterbi path for different antibiotic. Amoxicillin + Potassium Clavunate (ap) gets maximum and the second maximum probability is for Cefixime(ce), and the reaming are almost equal.

IV. Conclusion

In this paper, I used the technique of hidden markov model for 5 observable, 3 hidden states to find the viterbi path. By the path, we observe that the most common side effects of antibiotics is stomach pain. The maximum path for hidden markov model is Nausea, Stomach pain, diarrhea, Stomach pain, Stomach pain with maximum probability 0.0000118174.

I.Acknowledgement

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