A STUDY ON THE IMPACT OF TRANSFORMING PARAMETER IN BOX - COX TRANSFORMATION FOR NON-NORMAL DATA TO ENHANCE PROCESS CAPABILITY

J. Krishnan[1] and R. Vijayaraghavan[2]

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(1). Department of Mathematics, Sri Krishna Adithya College of Arts and Science
Coimbatore – 641042, Tamil Nadu, INDIA
(2). Department of Statistics, Bharathiar University, Coimbatore 641 046,
Tamil Nadu, INDIA
[1]krrishme92@gmail.com, [2]vijaystatbu@gmail.com

Abstract

Process Capability Analysis (PCA) helps to improve and monitor the quality of the manufacturing products in industries. The most commonly and traditionally applied indices are process capability index and process capability ratio. Many statistical tests require the condition that the data to be approximately normally distributed. When it comes to reality the data often do not follow a normal distribution. In such instances, different approaches are employed. Box-Cox Transformation (BCT) is one such methodology that is often used by quality practitioners relying on single transforming parameter lamda to transform the non-normal data into normal data. The widely used approach to decide the transforming parameter lamda is based on the rounded value of lamda instead of an optimal value of lamda. There are two transforming expressions available in BCT method. The choice of the value for lamda in BCT can have a significant impact on the results. This paper concentrates on the impact of data transformation in BCT method through two different expressions based on an optimal as well as a rounded value of lamda. The influence made by the estimates of process capability and process performance indices is also studied in this paper. The result of the analysis clearly indicates that the optimal value of lamda when employed in the first BCT transformation expression to estimate the process capability indices for non-normal data provides improvised results. For data analysis, Ms-Excel and Minitab 21 software has been used in this study.

Keywords: Non-normal, Box-Cox Transformation, MLE, Process Capability, Six sigma

I. Introduction

Process capability analysis (PCA) is a continuous process of monitoring and improving the quality of finished products produced by industries. PCA addresses the issues relating to how well a manufacturing process meets the required specification and it requires most often that the data should obey the assumption of normal distribution. The traditional process capability indices are purely based on normality assumption. When it comes to reality, the data often do not follow a normal distribution. In such instances, different approaches are employed. Data transformation for preserving a somewhat normal distribution has been recommended in [1]. Box - Cox transformation (BCT) is one such methodology that is often used by quality practitioners. The empirical study made in [2] has demonstrated that the findings of transformed data are much superior to the results of the original data (NT methods). Further, NT methods are found to be inadequate in capturing the capability of the process unless the underlying distribution is close to or approximately normal. NT methods are unsatisfactory because the distribution deviates significantly from normal. See, [3]. Process capability indices are calculated using samples of data based on short-term or within group variation, whereas performance indices are calculated using all the data points and long-term or overall variation [4]. The process capability indices are denoted by Cp and Cpk, and process performance indices are denoted by Pp and Ppk. A detailed review on various methods that are chosen for performance comparison in their ability to handle non-normality while computing the process capability indices is presented in [5]. The most commonly and traditionally applied indices by industries are process capability index Cp and process capability ratio Cpk, which are given below in Table 1 along with the respective performance indices, where $\bar{x}$ is the sample mean, USL is the upper specification limit and LSL is the lower specification limit.

Table 1: *Process Capability and Process Performance Indices*

Process capability indices	Process performance indices
$C_p = \frac{USL-LSL}{6\sigma_W}$	$P_p = \frac{USL-LSL}{6\sigma_{overall}}$
$C_{pk} = min\ (C_{PU}, C_{PL})$	$P_{pk} = min\ (C_{PU}, C_{PL})$
$C_{PU} = \frac{USL-\bar{\bar{x}}}{3\sigma_W}, \quad C_{PL} = \frac{\bar{\bar{x}}-LSL}{3\sigma_W}$	$P_{PU} = \frac{USL-\bar{\bar{x}}}{3\sigma_{overall}}, \ P_{PL} = \frac{\bar{\bar{x}}-LSL}{3\sigma_{overall}}$

A detailed review on process capability indices for non-normal data is presented in [3] with an emphasis on Box - Cox transformation (BCT) and on the parameter estimation approach utilizing a search method to estimate the process capabilities. In [6], a method of converting non-normal data into normal data is discussed and the transformed data is analyzed using the process capability indices (PCI). Further, an improved BCT model has been proposed to deal with the non-normal data and to calculate the process capability indices. The choice of the value of the transforming parameter, λ and the conversion formula in BCT might have a significant impact on the results. Hence, in this paper, the estimates of process capability indices are obtained and the analysis is carried out using the optimal as well as the rounded value of λ through BCT to obtain improvised estimates of process capability indices and process performance indices (PPI) within the standard of six sigma level. Thus, the objective of the study in this paper is to investigate the effectiveness of the BCT conversion formula and the optimal as well as the rounded value of λ in data transformation and in process capability analysis for non-normal data. This study would assist in suggesting the most efficient way of utilizing the BCT parameter λ in data transformation to approximate normal data as well as to estimate process capability and PPM values for non-normal data with the least amount of error.

II. Methodology

In data analysis, normally distributed independent observations with constant mean and variance are generally assumed. However, in reality, data frequently do not follow a normal distribution. A family of power transformation, termed as Box - Cox transformation (BCT), for a positive response variable X in such circumstances has been suggested by Box and Cox. See, [7]. The goal of BCT is to stabilize variance and make the data more closely resemble a normal distribution. The following is the conversion formula:

$$x^{\lambda} = \begin{cases} \frac{x^{\lambda}-1}{\lambda}, for\ \lambda \neq 0 \\ \log x, for\ \lambda = 0 \end{cases} \tag{1}$$

It may be noted that since an analysis of variance is unchanged by a linear transformation, the expression given as (1) is equivalent to

$$x^{\lambda} = \begin{cases} x^{\lambda}, for\ \lambda \neq 0 \\ \log x, for\ \lambda = 0 \end{cases} \tag{2}$$

The expression given as (1) is slightly preferable for theoretical analysis since it is continuous at λ = 0 {7}. The major effort in BCT is connected to the transformation of x to xλ, with the parameter λ describing a specific transformation. A single transforming parameter λ is the main source of dependence for this family of transformations and its value is determined using maximum likelihood estimation. One may refer to [7] and [8]. The original non-normal data is used to estimate the value of λ, which is then used to convert the data into approximately or nearer to normal based on the value of λ [6]. Initially, λ is selected within a predetermined range of values. With the selected λ, one would assess the following:

$$\mathrm{L_{max}} = -\frac{1}{2}\ \mathrm{In}\ \widehat{\sigma}^2 + \mathrm{In\ J}(\lambda, \mathrm{X}) \tag{3}$$

$$\mathrm{L_{max}} = -\frac{1}{2}\ \mathrm{In}\ \widehat{\sigma}^2 + (\lambda - 1)\sum_{i=1}^{n} \mathrm{In\ X_i} \tag{4}$$

where

$$(\lambda, \mathrm{X}) = \prod_{i=1}^{n} \frac{\partial \mathrm{W_i}}{\partial \mathrm{X_i}} = \prod_{i=1}^{n} \mathrm{X_i^{\lambda-1}}\ \text{ for all } \lambda \tag{5}$$

From (5), one may have J (λ, X) = $(\lambda - 1)\sum_{i=1}^{n} \mathrm{In\ X_i}$. The estimate of $\hat{\sigma}^2$ for fixed λ is defined by $\hat{\sigma}^2 = S(\lambda)/n$, where $S(\lambda)$ is the residual sum of squares in the analysis of variance of X. Plotting $L_{\max}$ against λ is possible after computing $L_{\max}$ for a number of λ values within the range. The value of λ that maximizes $L_{\max}$ yields the maximum likelihood estimator of λ. Using the expression (1) or (2), the data and specification limits are converted to a normal variate with the optimal value of λ [5]. The transformed observations x^{λ} are assumed to satisfy the normality assumption for an unknown λ. Based on this assumption, the transformed observations are used to estimate the mean and variance [7]. The maximum likelihood estimates of the mean and standard deviation are, respectively, given by

$$\widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \frac{x_i^2 - 1}{\lambda} \tag{6}$$

and

$$\widehat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{X_i^{\lambda}-1}{\lambda} - \frac{1}{n}\sum_{i=1}^{n}\frac{X_i^{\lambda}-1}{\lambda}\right)^2} \tag{7}$$

It has been shown in [6] that, for a common probability distribution, the BCT correctly transforms non-normal data into normal with a success rate of over 97%. Validity of the process capability index based on the transformation method has been verified and concluded that the process capability index using BCT is effective. The choice of the value for λ in a Box - Cox transformation might have a significant impact on the results. In BCT, there are primarily two phases to convert non-normal data into normal data, the first one is to find the transforming parameter λ and then to generate transformed normal data by substituting the non-normal data along with the λ value in the appropriate BCT formula, which may be either expression (1) or its equivalent expression (2). In general, the rounded value of λ is preferred over optimal value of λ because of its ease of transformation, viz., square root transformation when λ = 0.50, cube root transformation when λ = 0.33, fourth root transformation when λ = 0.25, natural log transformation when λ = 0.0, reciprocal square root transformation when λ = - 0.50, reciprocal transformation when λ = - 1.00 and no transformation when λ = 1.00 [3].

III. Numerical Illustrations

Process capability analysis for non-normal data is carried out to assess the capability of the non-normal process in a real-life situation. It is to be noted in [6] that the value of CPU and CPK are 0.73, which are below the benchmark value of 1.33 in industries and hence, the process is not capable. The corresponding PPM values are 14765 ensures that the process is within the standard of three sigma limit only. The rounded value of λ is widely used to transform non-normal data into normal data rather than the optimal value of λ in BCT through maximum likelihood estimation (MLE). Moreover, as pointed out earlier, there are two transforming expressions available in BCT method. One may refer to [7]. It is interesting to note that in [6], the rounded value of λ is taken for data transformation instead of the optimal value of λ. Expression (2) is used to transform the non-normal data into normal data than the expression (1) in BCT. The influence of data transformation utilizing optimal and rounded values of λ via expressions (1) and (2) of BCT method will be examined and the results of estimated process capability analysis will be compared to the results recorded in [6].

A producer, who produces mechanical parts, wants to analyze whether one type of mechanical components comply with the specification. The real data (RD) set pertaining to warping presented in Table 2 is extracted from [6]. In order to assure production quality, the measured warping of mechanical parts should not exceed 9.5 (i.e., USL = 9.5). Before starting with the real data to carry out process capability analysis, it is essential to ensure that the data is normal. According to the result given in [6] and from the summary of the real data set, the p-value is found to be 0.012, which is less than the prescribed 5% level of significance. However, 5% of the data points lie outside the 95% confidence interval and therefore, the hypothesis that the data follow a normal distribution could be rejected, establishing the fact that the data would not follow normal distribution. One may refer to Table 3. Hence, the real data must be transformed in order to ensure normality.

Table 2: *Real Data Set of Warping*

2.000	2.887	0.650	3.612	0.550	6.637	1.525	5.300	1.400	0.350
1.050	3.187	1.262	3.574	3.100	2.400	7.900	4.012	0.975	0.712
3.750	5.899	1.400	2.725	4.887	1.525	4.750	4.350	5.175	0.875
1.612	2.187	3.200	1.312	2.849	0.950	5.274	8.325	6.625	3.550
2.800	2.025	5.287	1.562	1.200	2.987	5.412	3.050	4.737	7.812
3.287	7.037	1.675	2.462	6.225	6.200	0.525	4.387	4.050	4.212
0.425	5.800	3.550	1.050	7.237	2.450	1.500	9.037	6.300	4.037
8.700	4.937	0.950	4.149	3.150	1.687	4.300	1.412	3.825	7.600
4.325	5.475	3.474	5.187	3.850	5.987	3.137	5.337	3.062	2.074
1.762	4.050	0.787	3.212	4.774	2.750	9.112	2.562	5.862	2.650

Table 3: *Summary of Real Data*

Variable	Mean	SD	Min	Median	Max	Skewness	Kurtosis	P-value
RD	3.628	2.178	0.350	3.250	9.112	0.58	-0.31	0.012

BCT makes use of the MLE approach to figure out the single transformation parameter using expressions (3), (4), and (5). As described earlier, there are two methods to transform non-normal data into normal data depending on the value obtained in the Box - Cox plot, namely, the estimated value and the rounded value of λ. It is significant to note from the Box - Cox plot that the rounded value of λ is 0.50, and the estimate (optimal) of λ is 0.45. The corresponding lower and higher confidence limits are 0.24 and 0.70. One may refer to Figure 1 for further details. In [6], the rounded value (RV) of λ is considered instead of optimal value (OV) of λ to transform non-normal data into normal data through BCT using expression (2) and hence, the transforming expression reduces to square root transformation when λ = 0.50. The optimal and rounded values of λ have been taken into consideration while transforming non-normal data into normal data and both the expressions (1) and (2) are utilized to perform process capability analysis.

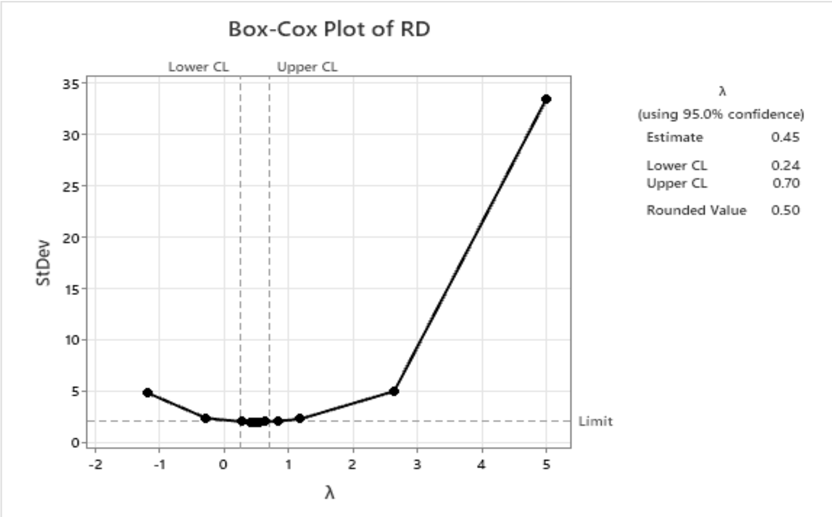

Figure 1: *Estimate of λ through Box - Cox Plot for RD*

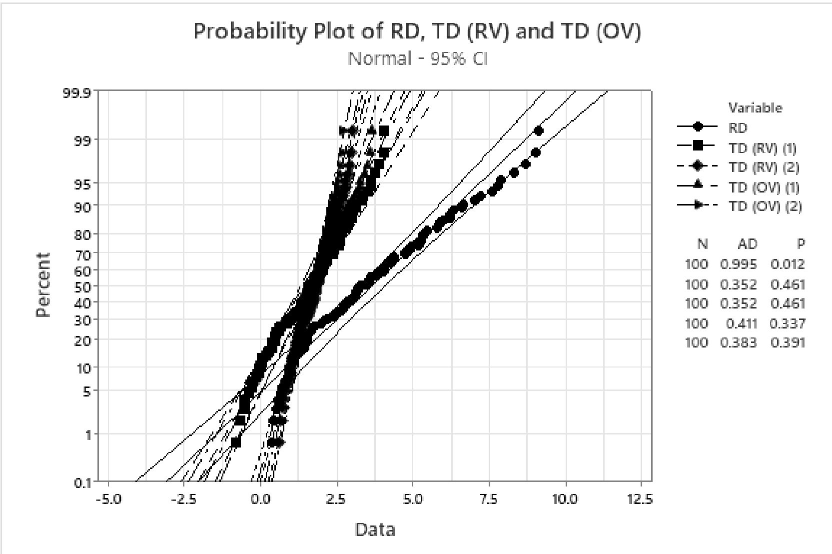

Figure 2: *Probability Plot for RD, TD (RV) and TD (OV) based on expression (1) and (2)*

Here, expressions (1) and (2) are used to convert the non-normal real data into normal data based on optimal and rounded value of λ. Table 4 displays the summary values of transformed data, whereas Figure 2 displays the probability plot of the real data and the transformed data based on RV and OV of λ. The p -values corresponding to the optimal and rounded λ tabulated in Table 4 are greater than the 5% and 1% level of significance, which would indicate that the data would follow a normal distribution. Hence, the transformed data can be used to estimate process capability and carry out process performance analysis.

Table 4: *Summary of Transformed Data using BCT Method*

Variable	λ Value	Mean	SE of Mean	Min	Median	Max	Skewness	Kurtosis	P-value
RD	-	3.628	0.218	0.350	3.250	9.112	0.58	-0.31	0.012
TD (RV) (1)	0.50	1.810	0.059	0.591	1.802	3.018	-0.04	-0.73	0.461
TD (RV) (2)	0.50	1.620	0.119	-0.081	1.605	4.037	-0.04	-0.73	0.462
TD (OV) (1)	0.45	1.697	0.051	0.623	1.699	2.702	-0.10	-0.72	0.337
TD (OV) (2)	0.45	1.509	0.109	-0.084	1.525	3.642	-0.14	-0.70	0.391

* TD (RV) (1) – Transformed data based on rounded value of λ through BCT expression 1 | TD (RV) (2) – Transformed data based on rounded value of λ through BCT expression 2 | TD (OV) (1) – Transformed data based on rounded value of λ through BCT expression 1 | TD (OV) (2) – Transformed data based on rounded value of λ through BCT expression 1

I. Estimate of PCI and PPI Utilizing Rounded Value of λ Through BCT

The estimates of process capability indices CPU and CPK are 1.34 and process performance indices PPU and PPK are 1.32 respectively, when utilizing RV of λ for data transformation through BCT expression (1) and CPU and CPK are 0.73, and PPU and PPK are 0.71 through BCT expression (2). The result obtained from expression (1) based on OV of λ is approximately equal to the guideline value 1.33 and hence, the process is capable to produce the mechanical parts within the given specification limit and the respective PPM values of process capability and process performance indices are 28 and 38 ensuring that the process is better than 5σ limits. On the other hand the result obtained from expression (2) does not meet the standard of guideline value 1.33 in industries and the respective PPM values are 14778 and 16303, which conform that the process is within 3σ limit only and hence, the process may be considered as incapable. Additionally, some data points are beyond specification limits. Hence, when RV of λ is utilized to transform non-normal data into normal data through BCT expression (1), better results would be possible rather than BCT expression (2). One may refer Tables 5 and 6, and Figures 3 and 4 for more information.

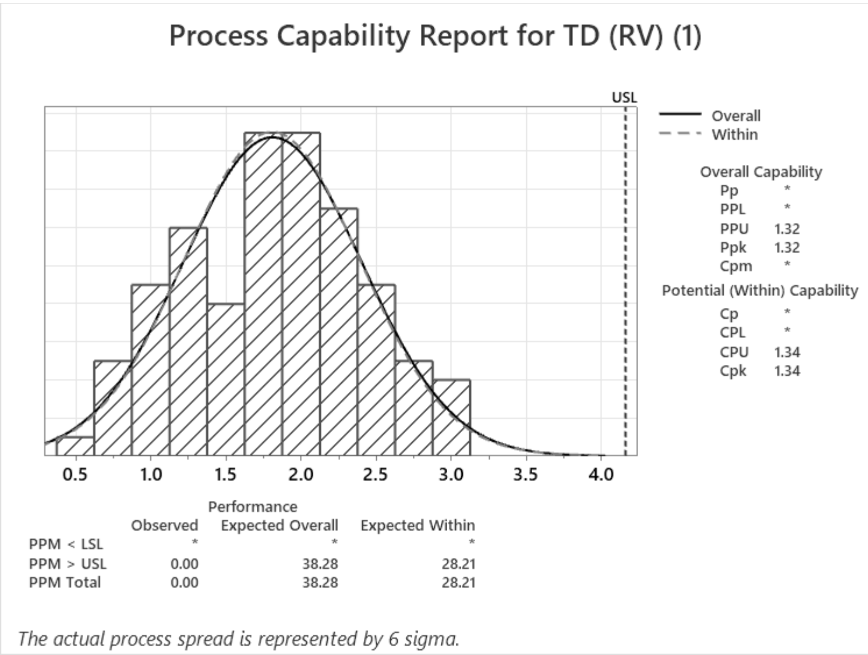

Figure 3: Estimate of Process Capability and Performance Indices for TD (RV) Data Based on Expression (1) in BCT Method

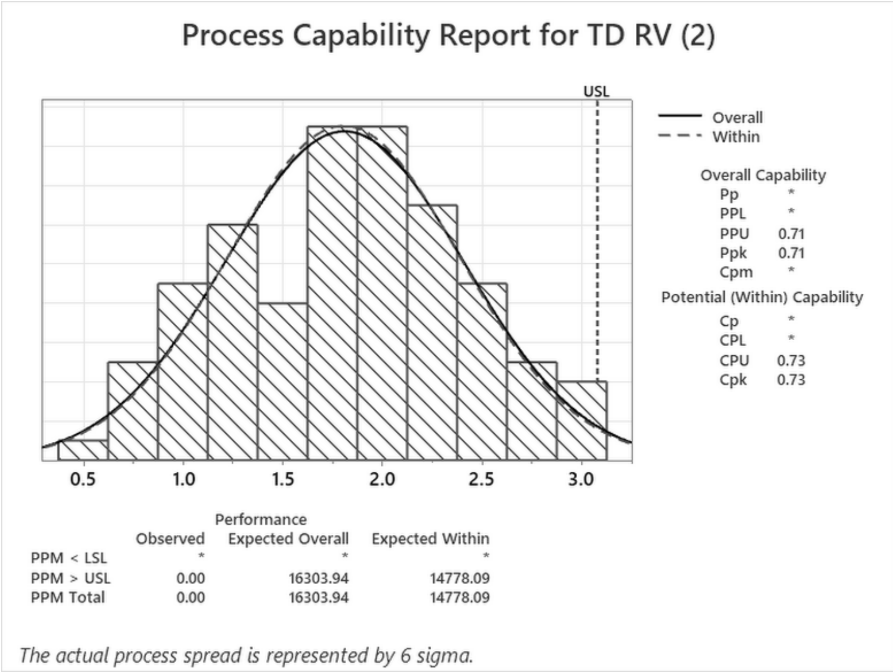

Figure 4: *Estimate of Process Capability and Performance Indices for TD (RV) Data Based on Expression (2) in BCT Method*

Table 5: *Estimate of Process Capability and Performance Indices Based on Transformed Data*

Variable	λ	PCI and PPI of transformation data based on expression (1); $x^{\lambda} = (x^{0.45} - 1)/0.45$ in BCT					PCI and PPI of transformation data based on expression (2); $x^{\lambda} = x^{0.50}$ in BCT				
		USL	C_{pk}	PPM	P_{pk}	PPM	USL	C_{pk}	PPM	P_{pk}	PPM
TD (RV)	0.50	4.164	1.34	28	1.32	38	3.082	073	14778	0.71	16303
TD (OV)	0.45	3.898	1.47	5	1.45	7	2.754	0.71	17001	0.69	18689

Table 6: *Process Fallout in Defective Parts per Million With Respect to Different Sigma Levels*

Sigma Level	Percentage	PPM Values
6	99.9997%	3.4
5	99.98%	233
4	99.4%	6,210
3	93.3%	66,807
2	69.1%	308,537
1	30.9%	691,462

II. Estimate of PCI and PPI Utilizing Optimal Value of λ Through BCT

The estimate of process capability indices CPU and CPK are 1.47 and process performance indices PPU and PPK are 1.45 respectively, when utilizing OV of λ through BCT expression (1) and CPU and CPK are 0.71 and PPU and PPK are 0.69 through BCT expression (2). The result attained using OV of λ through expression (1) is greater than the guideline value 1.33 and the capability of the process is excellent. Hence, the process is capable to produce the mechanical parts within the given specification limit. The respective PPM values of process capability and process performance indices are 5.05 and 7.29, which guarantee that the process is better than 5σ limit and approximately follow 6σ result, whereas the actual PPM value corresponding to 6σ is 3.4 [9].

Conversely, the result observed from expression (2) is below the guideline of 1.33, which would indicate that the process could not be considered as capable and the respective PPM values are 17007 and 18689, ensuring that the process is within the standard of 3σ limit. Hence, making use of OV of λ to transform non-normal data into normal data through BCT expression (1) provides better results than BCT expression (2). One may refer to Table 5 and 6, and Figure 5 and 6 for details.

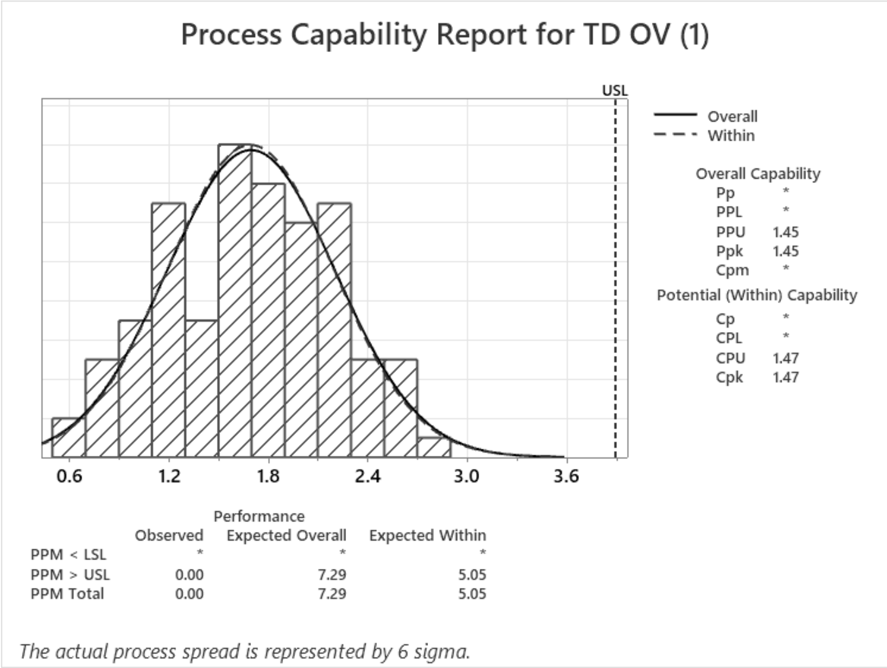

Figure 5: *Estimate of Process Capability and Performance Indices for TD (OV) Data Based on Expression (1) in BCT method*

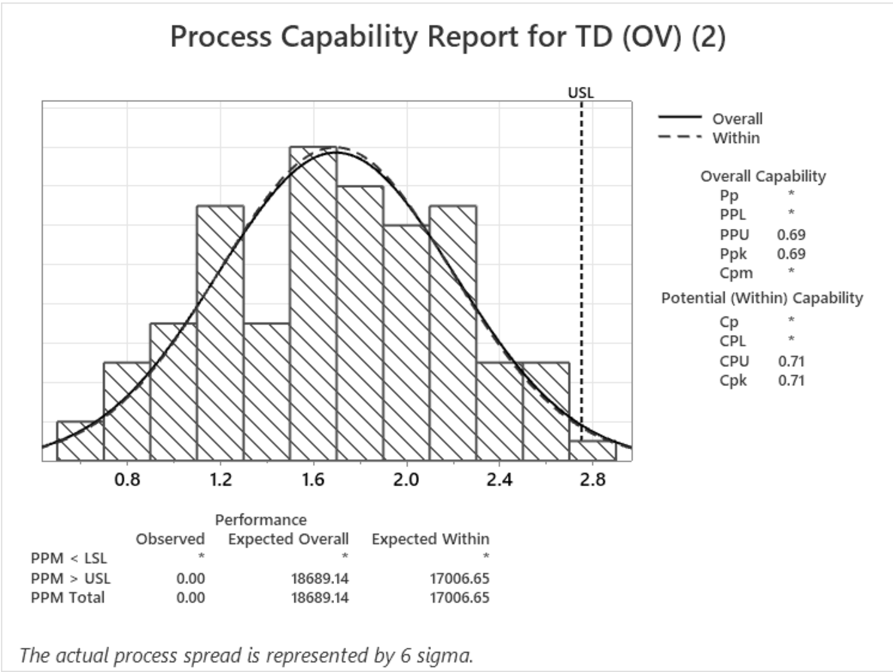

Figure 6: *Estimate of Process Capability and Performance Indices for TD (OV) Data Based on Expression (2) in BCT Method*

A process is categorized as inadequate, if PCI < 1.00; capable, if 1.00 ≤ PCI ≤ 1.33; satisfactory, if 1.33 ≤ PCI ≤ 1.50; excellent, if 1.50 ≤ PCI ≤ 2.00; and super, if ≥ 2.00. Automotive industries use CPK = 1.33 as a benchmark in assessing the capability of the process. If Cp and CPK are greater than or equal to 2 and 1.5, respectively, a process is said to be under six-sigma controls. Similarly, Pp and PPK must be more than 2 and 1.5, respectively, for a process to generate six-sigma results. See, [9]. Table 6 lists the process fallout in PPM in relation to the proportion of good items and PPM values for various sigma levels.

IV. Result and Discussion

The primary goal of this study is to investigate the effect of rounded value (RV) and optimal value (OV) of the transforming parameter λ, on data transformation and estimation of process capability and process performance indices through the BCT method. Furthermore, in order to obtain the transformed normal data and the estimates of process capability indices (PCIs) and process performance indices (PPIs), the identified λ values, such as RV and OV of λ must be substituted in BCT using expressions (1) and (2). The estimates of CPU and CPK in [6] are 0.73 while PPU and PPK are 0.71, respectively. These values are below the industry benchmark value of 1.33, indicating that the process is not capable. Remarkably, the RV of λ is used for estimating PCI and PPI, and also for data transformation rather than the OV of λ. However, as demonstrated by a numerical example in this study, the use of optimal λ value to convert non-normal data into normal data produces results that are as near to normal as possible and have a smaller standard error of mean than the RV of λ. One may refer to Table 4. Hence, the industry benchmark value of 1.33 is met by the estimated CPU and CPK of 1.47 and PPU and PPK of 1.45, respectively, based on OV of λ through BCT expression (1).

Process is, therefore, as demonstrated in Table 5 and 6, thought to be capable of producing manufacturing parts that meet the specification limit. Besides the estimates of process capability and process performance indices, the respective PPM values are essential in assessing the process fallout. It may be noted that only 3.4 defective items out of every million products should have been recorded in a production process that conforms to the standard of 6σ. When using the OV of λ in expression (1) of the BCT method, the process approximately follows 6σ outcome, as indicated by the PPM values of 5.05 and 7.29. One may refer to Table 5, and Figure 5 and 6.

The mean and range charts (X bar – R charts) are drawn for transformed data based on the OV and RV of λ, in order to clearly visualize the statistical control over the process of non-normal real data. All of these data points in Figures 7, 8, 9 and 10 fall within the control limits, indicating that the process is statistically under control. However, compared to the results obtained from a RV of λ, the estimate and PPM values corresponding to transformed data based on OV of λ produced significantly better outcomes.

It is evident from Table 4, and Figures 7, 8, 9 and 10 that when data is transformed using the OV of λ, the standard error of mean obtained is smaller than the one obtained from the RV of λ. Hence, it is necessary to use optimal λ to get better results, as it reflects the exact transforming pattern rather than a rounded value and this ensures all data points as close to normal as possible. Specifically, the utilization of BCT expression (1) to convert non-normal data into normal data produces better results than using BCT expression (2) when an OV of λ is used. As pointed out in [7], the BCT expression (1) is slightly preferable than expression (2) for theoretical analysis because it is continuous at λ = 0. Also, it can be observed from Figures 4 and 6 that some of the data points fall beyond the specification limit conforming that, the process is not capable to produce the mechanical parts within the speciation limit when estimated using RV of λ through expression (2). This result can be compared with [6] to understand that, using OV of λ through BCT expression (1) produces improvised results that are nearer to the standard of 6σ than using Rounded RV of λ.

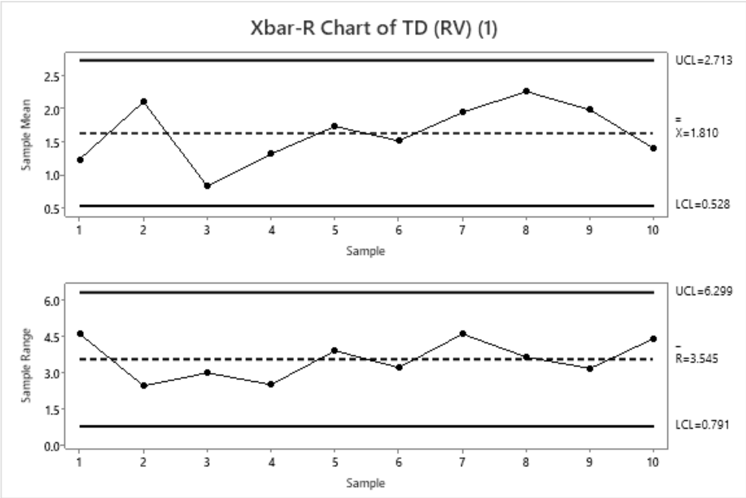

Figure 7: *Xbar and R Chart for Transformed Data using Rounded Value of λ through expression (1)*

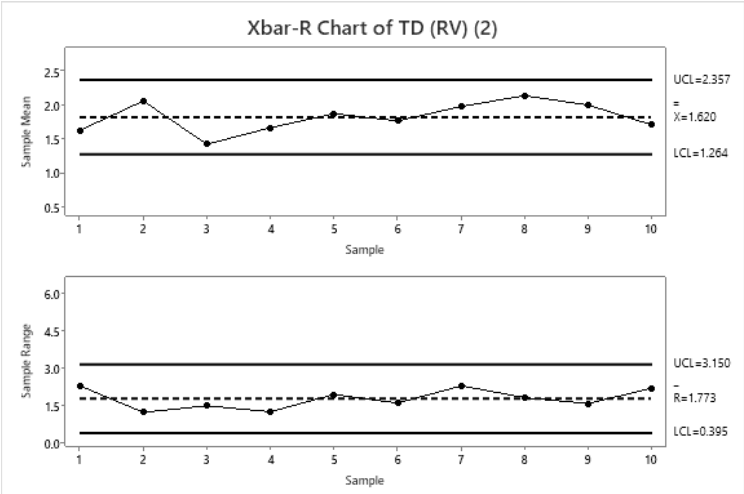

Figure 8: *Xbar and R Chart for Transformed Data using Rounded Value of λ through expression (2)*

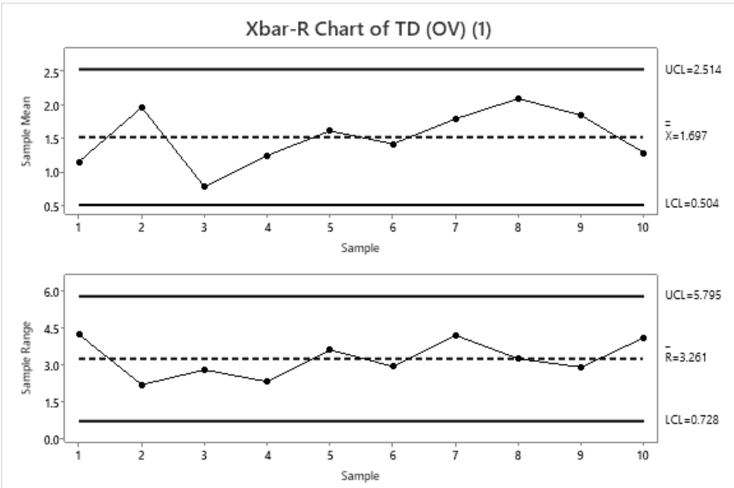

Figure 9: *Xbar and R Charts for Transformed Data using Optimal Value of λ through expression (1)*

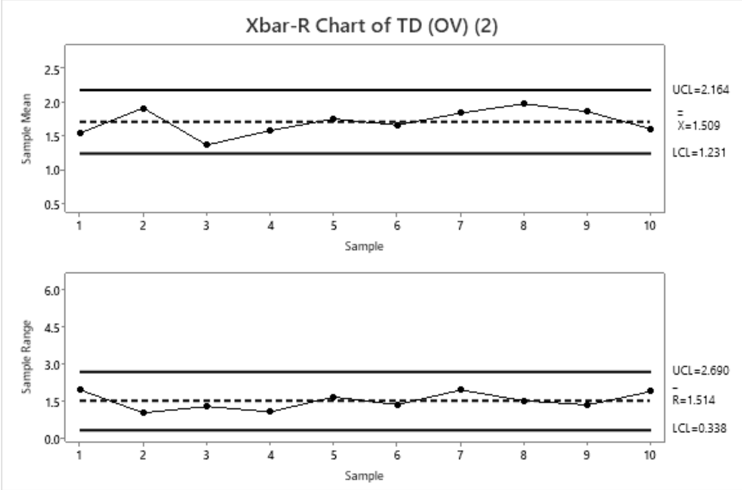

Figure 10: *Xbar and R Charts for Transformed Data using Optimal Value of λ through expression (2)*

V. Conclusion

The methodology of continuously assessing and enhancing the quality of manufacturing products in industries is known as process capability analysis. In order to handle the problems pertaining to how well a manufacturing process satisfies the necessary specifications, PCA needs that the data should follow the normal distribution. If the normality assumptions are violated or failed, some adjustments must be made to the traditional process capability indices, which would solely depend on normality assumptions. A significant methodology for handling non-normal data is data transformation. One such methodology is the Box-Cox Transformation. Using a single transforming parameter λ, which can be either a rounded value from the MLE approach or an estimated (optimal) value, the non-normal data can be converted into normal data. By taking into account of the objective of this paper, both optimal and rounded value of λ are considered for data transformation through two BCT expressions. Based on the data analysis, improvised estimates of PCIs and PPIs are obtained, when utilizing an optimal value of λ through BCT expression (1) rather than the rounded value of λ through BCT expression (2). The corresponding PPM values of the PCIs and PPIs are very smaller and approximately follow the case of standard of 6σ. Furthermore, it is evident that the transformed data is nearer to normal with a smaller standard error of mean when utilizing optimal value of λ. Thus, one can conclude that the improvised estimates would generally be obtained by utilizing optimal value of λ than the rounded value of λ when using Box-Cox transformation. In particular, the most effective results for PCI's and PPI's are quite possible only when utilizing optimal value of λ through BCT expression (1) rather than BCT expression (2).

References

[1] Kane, V. E. (1986), Process Capability Indices, Journal of Quality Technology, 18, 41 – 52.

[2] Gunter, B. H. (1989). The Use and Abuse of Cpk, Quality Progress, 22, 108 – 109.

[3] Swamy, D. R., Nagesh, P., and Wooluru, Y. (2016). Process Capability Indices for Non-normal Distribution – A Review, Proceedings of the International Conference on Operations Research and Management, January 21 – 22, 2016, Mysuru, India.

[4] Sennaroglu, B., and Senvar, O. (2015). Performance Comparison of Box-Cox Transformation and Weighted Variance Methods with Weibull Distribution, Journal of Aeronautics and Space Technologies, 8, 49 – 55.

[5] Tang, L. C., and Than, S. E. (1999). Computing Process Capability Indices for Non-normal Data: A Review and Comparative Study, Quality and Reliability Engineering International, 15, 339 – 353.

[6] Yang Y and Zhu H (2018). A Study on Non-normal Process Capability Analysis based on Box-Cox Transformation, Proceedings of the 3rd International Conference on Computational Intelligence and Applications (ICCIA), Hong Kong, China, IEEE, 240 – 243.

[7] Box, G. E. P., and Cox, D. R. (1964). An Analysis of Transformations. Journal of the Royal Statistical Society: Series B (Methodological), 26, 211 - 243.

[8] Asar, O., Ilk, O., and Dag, O. (2017). Estimating Box-Cox Power Transformation Parameter via Goodness-of-Fit Tests, Communications in Statistics - Simulation and Computation, 46, 91 – 105.

[9] Pearn, W. L., and Chen, K. -S. (2002). One sided Capability Indices CPU and CPL: Decision Making with Sample Information, International Journal of Quality & Reliability Management, 19, 221 – 245.