CRITICAL ANALYSIS OF FAILURE AND REPAIR RATES OF POLY-TUBE MANUFACTURING PLANT USING PSO

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Abstract

A proper maintenance strategy is essential for the optimal performance of poly tube manufacturing to ensure high reliability. It involves a complex structure consisting of many components interconnected in series or parallel configurations. This project's contribution is the development of a method for evaluating the performance of an industrial system using previously unknown data. The RAM index, influenced by failure and repair rates, has been devised to identify the system's most critical component that impacts reliability, availability, and maintainability, collectively known as RAM. For performance analysis, a Markov-based simulation system model has been formulated and resolved to refine the results through particle swarm optimization (PSO). The transition diagram facilitates the construction of ordinary differential equations (ODEs), which represent various operational states such as full capacity, reduced capacity, and failure. These ODEs are then solved using initial and boundary condition.

Keywords: Availability analysis, Supplementary variable Technique, Particle swarm optimization (PSO),Critical Analysis.

I. Introduction

In contemporary settings, the advancement of technology has diminished the need for physical labor. The creation of various machines and equipment results in less manual work and more precise outcomes. Therefore, for optimal performance, the efficiency of this equipment is crucial, which relies entirely on its operation and maintenance, as well as that of its components. Reliability can be defined as the likelihood of success at a given time t, meaning the probability that a machine designed to fulfill its function within a set timeframe under certain external conditions will do so successfully. A system or device is considered highly reliable or dependable when it executes the intended task flawlessly without encountering any issues. However, consistent usage of a system inevitably leads to wear and tear of parts, meaning no system can maintain maximum efficiency indefinitely. Consequently, a system's reliability and efficiency are compromised when specific components deteriorate and fail. The motive of the study can help the manufacturing plant to get maximum production by ensuring that the system is as fault-free as possible through effective management, control and maintenance. This paper deals with the availability of the system having different numbers for the rates of failing and repairing. For the first time, an evolutionary optimization approach, namely PSO, is used to anticipate accessibility in the process sector. The existing methods including Markov and Genetic Algorithm confirmed or validated the outcomes generated for optimum allocation/ availability. This is customary that these outcomes are valuable for the management of the industry for adopting a suitable maintenance program and strategies.

Cox [1] employed a supplemental variable approach to determine the system's dependability and availability. After that, many researchers reported the system's reliability via the supplementary variable technique. Ying-Shen et al. [2] developed a series-parallel system to find the availability of GA. Sagayaraj et al. [4] documented the system's reliability via a mixed series-parallel combination. The highest degree of dependability and availability is important not just to minimize total production costs but also to limit the danger of risks (Yang et al., [5]). Chaudhary et al. [6] evaluated the "reliability, availability, and maintainability" (RAM) in a cement factory with various failure and repair rates. The study's goal is to identify essential subsystems of a cement plant so that effective solutions to enhance their RAM characteristics can be offered, resulting in an improvement in cement plant capacity utilization. TPP availability, dependability, and planning have transformed into important needs in recent years as society's demand for energy has increased (Kuo and Ke, [7]). Sunita et al. [8] discussed the sensitive analysis of the thresher plant under study. Sunita et al. [9] discussed RAP via a constraint optimization genetic algorithm. Sunita et al. [10] find the solution to constrained problems using particle swarm optimization. RAM (reliability, availability, and maintainability) of threshing machines in agriculture was observed by Anchal et al. [11]. Vanita et al. [12] highlighted the effect on profit and availability of a briquette machine under the minor and major faults handled by two repairmen. An effort is also being made to assess the plant's dependability and reliability by using continual failure and repair rates. A deep learning process was examined by Singla et al. [13] in order to optimize the reliability parameters and boost industry revenues and manufacturing of a 2:3 good system. Singla et al. [14] investigate a failing system by applying a genetic algorithm to ascertain the reliability metrics influenced by the rate of degradation and the rate of preventive maintenance.

So far, the polytube sector has received little attention, despite the fact that it plays a significant part in our everyday lives. In any sector, this normality assumption for the rates of failing and repairing are not feasible. Keeping this in mind, we analyzed a four-unit Polytube sector subjected to fluctuating subsystems' failing and repairing rates in the current study, and we used supplementary variable technique to explore the reliable model of the Polytube sector. An effort is also being made to assess the plant's dependability/reliability by using continual failing and repairing rates.

The goal of the presented work is to maximize system's availability with respect to each unit while maintaining constant operation over time and at varying rates. The goal of this work is to concentrate on how sensitivity of units of the system is reliable to the availability. The paper is structured as follows: Section 2 provides an explanation of the model's specifics, including the state overview, assumptions, notations, and model frame. The mathematical representations is covered in Section 3. The methodology used to know the effect different rates is presented in section 4. The system's critical analysis is discussed in Section 5. Section 6 has concluded with the results discussion. Presented in Section 7 is the conclusion.

II. Model descriptions and Symbols

2.1 System description

Due to the fact that iron pipe corrodes rapidly in damp and humid environments, shortening its lifespan and making that fragile and more leakage susceptible. Polytube industries, making plastic pipes, is crucial in our daily lives since these are used to transport portable water, fluids (liquids other than water and gases) from one location to another. This study aimed at the "Polytube industry", which is made up of four subsystems: "Mixture", "Extruder", "Die" and "Cutter" as shown in figure

1. The following is a detailed system's description along with the notations, as necessary for the formulation of mathematical structure:

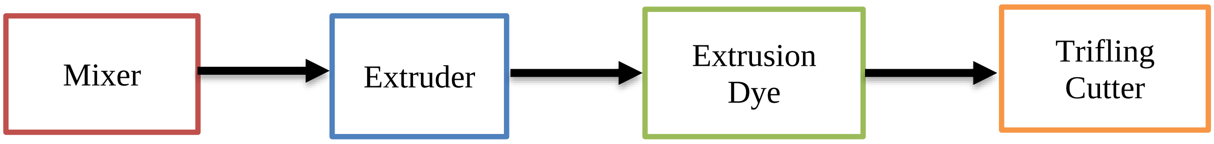

Figure 1 : *Structure of Polytube Manufacturing Plant*

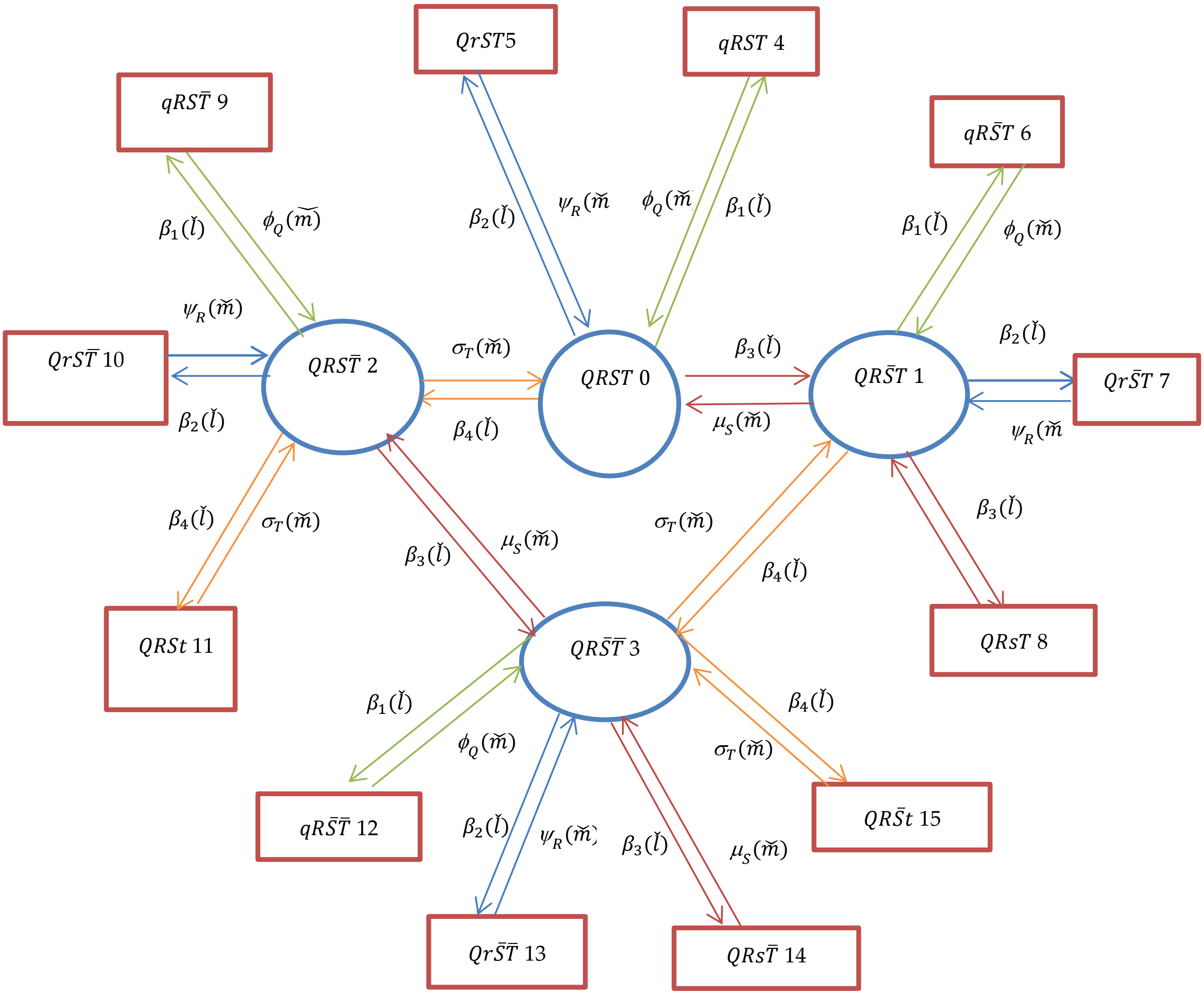

Figure 2 : *Transitions States of Polytube Manufacturing Plant.*

- Sub-system Q (Mixture) Its failure results in the system's total failure.
- Sub-system R (Extruder) Its failure results in the system's total failure.
- Sub-system S (Dye) This is utilized for making various diameters of pipe

- Sub-system T (Cutter) This subsystem consists of 2 series - connected elements. The first item is the blade, that shreds a pipe, while the 2nd second element is the engine (motor), that slice a pipe in various sizes.

2.2 Symbols

Table 1: *Various notations regarding model*

Q, R, S, T	Signify that the subsystem is fully operational.
$\bar{S}, \bar{T}$	Designate the minimized configuration of the subsystems "S" and "T"
q, r, s, t	Designate the subsystem's failure condition.
$\beta_i(\check{l})(\mathrm{i} = 1, \dots, 4)$	Represents subsystem's Q, R, S, and T rate of failing respectively.
$\phi_Q(\breve{m}), \psi_R(\breve{m}), \mu_S(\breve{m}), and\ \sigma_T(\breve{m})$	Repair rates of Q, R, S and T, respectively.
$P_o(t)$	Specify fully operational system without any failure
$P_i(\check{l}, \breve{m}, t)(i = 1, \dots, 16)$	Represents the probability of the industry in state I, at time t, with an expired failure time $\check{l}$ and an elapsed time of repair $\check{l}$.

2.3 Assumptions

The current study is based on the following assumptions

- Repairing ((R_r) and failing rates(F_r) are not dependent on each other ,independent to each other .
- A repairing subsystem is as good as original.
- Repair services are available.

III. Mathematical presentation of the system

3.1.Rates of Failing ($\boldsymbol{F_r}$)and repairing ($\boldsymbol{R_r}$) are taken as variable

If there are variable rates of failing (F_r) and repairing (R_r) for the transitory state, the differential difference equation using Chapman Kolmogrov's rule connected with the state transition diagram (fig. 1) are as follow:

$$P_0(t+\Delta t) = [1 - \beta_1(\check{l})\Delta t - \beta_2(\check{l})\Delta t - \beta_3(\check{l})\Delta t - \beta_4(\check{l})\Delta t]P_0(t) + \int \mu_S(\breve{m})P_1(\check{l}, \breve{m}, t)d\check{l}\,\Delta t$$
$$+ \int \sigma_T(\breve{m})P_2(\check{l}, \breve{m}, t)d\check{l}\Delta t + \int \phi_Q(\breve{m})P_4(\check{l}, \breve{m}, t)d\check{l}\,\Delta t + \int \psi_R(\breve{m})P_5(\check{l}, \breve{m}, t)d\check{l}\,\Delta t$$

$$P_0(t+\Delta t) - P_0(t)$$
$$= -[\beta_1(\check{l})\Delta t + \beta_2(\check{l})\Delta t + \beta_3(\check{l})\Delta t + \beta_4(\check{l})\Delta t]P_0(t) + \int \mu_S(\breve{m})P_1(\check{l}, \breve{m}, t)dl\,\Delta t$$
$$+ \int \sigma_T(\breve{m})P_2(\check{l}, \breve{m}, t)d\check{l}\Delta t + \int \phi_Q(\breve{m})P_4(\check{l}, \breve{m}, t)d\check{l}\,\Delta t + \int \psi_R(\breve{m})P_5(\check{l}, \breve{m}, t)d\check{l}\,\Delta t$$

Dividing both sides byΔt,we get

$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t}$$
$$= -[\beta_1(\check{l})+\beta_2(\check{l})+\beta_3(\check{l})+\beta_4(\check{l})]P_0(t)+\int \mu_S(\breve{m})P_1(\check{l},\breve{m},t)d\check{l}$$
$$+\int \sigma_T(\breve{m})P_2(\check{l},\breve{m},t)d\check{l}+\int \phi_Q(\breve{m})P_4(\check{l},\breve{m},t)d\check{l}+\int \psi_R(\breve{m})P_5(\check{l},\breve{m},t)d\check{l}$$

$$\left[\frac{d}{dt}+Z_0\right]P_0(t)=E_0 \quad (1)$$

Likewise, we may construct the differential equation for the other states as:

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \check{l}}+\frac{\partial}{\partial \breve{m}}+Z_i(\check{l},\breve{m})\right]P_i(\check{l},\breve{m},t)=E_i(\check{l},\breve{m},t) \quad \text{for i=1,2 and 3} \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \check{l}}+\frac{\partial}{\partial \breve{m}}+\phi_Q(\breve{m})\right]P_j(\check{l},\breve{m},t)=\beta_1(\check{l})P_k(t) \quad \text{for j=4,6,9 and 12 and k=0 to 3 respectively} \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \check{l}}+\frac{\partial}{\partial \breve{m}}+\psi_R(\breve{m})\right]P_p(\check{l},\breve{m},t)=\beta_2(\check{l})P_k(t) \quad \text{for p=5,7,10 and 13 and k=0 to 3 respectively} \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \check{l}}+\frac{\partial}{\partial \breve{m}}+\mu_S(\breve{m})\right]P_s(\check{l},\breve{m},t)=\beta_3(\check{l})P_r(\check{l},\breve{m},t) \quad \text{for s=8 and 14 and r=1 and 3 respectively} \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \check{l}}+\frac{\partial}{\partial \breve{m}}+\sigma_T(\breve{m})\right]P_u(\check{l},\breve{m},t)=\beta_4(\check{l})P_v(\check{l},\breve{m},t) \quad \text{for u=11 and 15 and v= 2 and 3 respectively} \quad (6)$$

Where,

$$Z_0=\beta_1(\check{l})+\beta_2(\check{l})+\beta_3(\check{l})+\beta_4(\check{l})$$
$$E_0=\int \mu_S(\breve{m})P_1(\check{l},\breve{m},t)dl+\int \sigma_T(\breve{m})P_2(\check{l},\breve{m},t)dl+\int \phi_Q(\breve{m})P_4(\check{l},\breve{m},t)dl+\int \psi_R(\breve{m})P_5(\check{l},\breve{m},t)dl$$
$$Z_1(\check{l},\breve{m})=\mu_S(\breve{m})+\beta_1(\check{l})+\beta_2(\check{l})+\beta_3(\check{l})+\beta_4(\check{l})$$
$$E_1(\check{l},\breve{m},t)=\beta_3(\check{l})P_0(t)+\phi_Q(\breve{m})P_6(\check{l},\breve{m},t)+\psi_R(\breve{m})P_7(\check{l},\breve{m},t)+\mu_S(\breve{m})P_8(\check{l},\breve{m},t)+\sigma_T(\breve{m})$$
$$Z_2(\check{l},\breve{m})=\sigma_T(\breve{m})+\beta_1(\check{l})+\beta_2(\check{l})+\beta_3(\check{l})+\beta_4(\check{l})$$
$$E_2(\check{l},\breve{m},t)=\beta_4(\check{l})P_0(t)+\phi_Q(\breve{m})P_9(\check{l},\breve{m},t)+\psi_R(\breve{m})P_{10}(\check{l},\breve{m},t)+\mu_S(\breve{m})P_3(\check{l},\breve{m},t)+\sigma_T(m)P_{11}(\check{l},\breve{m},t)$$
$$Z_3(\check{l},\breve{m})=\sigma_T(\breve{m})+\mu_S(\breve{m})+\beta_1(\check{l})+\beta_2(\check{l})+\beta_3(\check{l})+\beta_4(\check{l})$$
$$E_3(\check{l},\breve{m},t)=\beta_4(\check{l})P_1(\check{l},\breve{m},t)+\beta_3(l)P_2(\check{l},\breve{m},t)+\phi_Q(\breve{m})P_{12}(\check{l},\breve{m},t)+\psi_R(\breve{m})P_{13}(\check{l},\breve{m},t)$$
$$+\mu_S(\breve{m})P_{14}(\check{l},\breve{m},t)+\sigma_T(\breve{m})P_{15}(\check{l},\breve{m},t)$$

Boundary Conditions:

$$P_a(0,\breve{m},t)=\beta_b(\check{l})P_0(t) \quad \text{for a=1,2,4 and 5 and b=3,4,1and 2 respectively} \quad (7)$$
$$P_3(0,\breve{m},t)=\int \beta_4(\check{l})P_1(\check{l},\breve{m},t)\,d\check{l}+\int \beta_3(\check{l})P_2(\check{l},\breve{m},t)d\check{l} \quad (8)$$
$$P_c(0,\breve{m},t)=\int \beta_d(\check{l})P_1(\check{l},\breve{m},t)\,d\check{l} \quad \text{for c=6 to 8 and d= 1 to 3 respectively} \quad (9)$$
$$P_o(0,\breve{m},t)=\int \beta_p(\check{l})P_2(\check{l},\breve{m},t)\,d\check{l} \quad \text{for o=9 to 11 and p=1,2 and 4 respectively} \quad (10)$$
$$P_{a_1}(0,\breve{m},t)=\int \beta_{a_2}(\check{l})P_3(\check{l},\breve{m},t)\,d\check{l} \quad \text{for } a_1\text{=12 to 15 and } a_2\text{=1 to 4 respectively} \quad (11)$$

Initial Condition:

$$P_i(\check{l},\breve{m},0)=0; \quad (i=1\dots 15) \quad (12)$$
$$P_0(0)=1 \quad (13)$$

For finding the systems' reliability R_{re} (t)), the solutions of differential equations (DE) (1-6) have been obtained. Shakuntla et.al (2011) used Lagrange's method to solve Chapman-Kolmogorov differential equation (DE) with constant rates of failing and repairing. To obtain the probability $P_i(t)(i=1\ldots 15)$, every state equation (2-6) and the initial conditions (7-11) were solved:

$$P_{15}(\check{l},\breve{m},t)=e^{-\int \sigma_T(\breve{m})d\breve{m}}\begin{bmatrix}\int \beta_4(\breve{m}-\check{l})P_3(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l}\\+\int \beta_4(\check{l})P_3(\check{l},\breve{m},t)e^{\int \sigma_T(\breve{m})d\breve{m}}d\check{l}\end{bmatrix} \quad (14)$$

$$P_{14}(\check{l},\breve{m},t)=e^{-\int \mu_S(\breve{m})d\breve{m}}\begin{bmatrix}\int \beta_3(\breve{m}-\check{l})P_3(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l}\\+\int \beta_3(\check{l})P_3(\check{l},\breve{m},t)e^{\int \mu_S(\breve{m})d\breve{m}}d\check{l}\end{bmatrix} \quad (15)$$

$$P_{13}(\check{l},\breve{m},t)=e^{-\int \psi_R(\breve{m})d\breve{m}}\begin{bmatrix}\int \beta_2(\breve{m}-\check{l})P_3(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l}\\+\int \beta_2(\check{l})P_3(\check{l},\breve{m},t)e^{\int \psi_R(\breve{m})d\breve{m}}d\check{l}\end{bmatrix} \quad (16)$$

$$P_{12}(\check{l},\breve{m},t)=e^{-\int \phi_Q(\breve{m})d\breve{m}}\begin{bmatrix}\int \beta_1(\breve{m}-\check{l})P_3(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l}\\+\int \beta_1(\check{l})P_3(\check{l},\breve{m},t)e^{\int \phi_Q(\breve{m})d\breve{m}}d\check{l}\end{bmatrix} \quad (17)$$

$$P_{11}(\check{l},\breve{m},t) = e^{-\int \sigma_T(\breve{m})d\breve{m}} \begin{bmatrix} \int \beta_4(\breve{m}-\check{l})P_2(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l} \\ + \int \beta_4(\check{l})P_2(\check{l},\breve{m},t)e^{\int \sigma_T(\breve{m})d\breve{m}}d\check{l} \end{bmatrix} \quad (18)$$

$$P_{10}(\check{l},\breve{m},t) = e^{-\int \psi_R(\breve{m})d\breve{m}} \begin{bmatrix} \int \beta_2(\breve{m}-\check{l})P_2(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l} \\ + \int \beta_2(\check{l})P_2(\check{l},\breve{m},t)e^{\int \psi_R(\breve{m})d\breve{m}}d\check{l} \end{bmatrix} \quad (19)$$

$$P_9(\check{l},\breve{m},t) = e^{-\int \phi_Q(\breve{m})d\breve{m}} \begin{bmatrix} \int \beta_1(m-l)P_2(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l} \\ + \int \beta_1(\check{l})P_2(l,m,t)e^{\int \phi_Q(\breve{m})d\breve{m}}d\check{l} \end{bmatrix} \quad (20)$$

$$P_8(\check{l},\breve{m},t) = e^{-\int \mu_S\mu(m)dm} \begin{bmatrix} \int \beta_3(m-l)P_1\check{l},\breve{m}-\check{l},t-\check{l}\,d\check{l} \\ + \int \beta_3(\check{l})P_1(l,m,t)e^{\int \mu_S(\breve{m})d\breve{m}}d\check{l} \end{bmatrix} \quad (21)$$

$$P_7(\check{l},\breve{m},t) = e^{-\int \psi_R(m)dm} \begin{bmatrix} \int \beta_2(\breve{m}-\check{l})P_1(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l} \\ + \int \beta_2(\check{l})P_1(\check{l},\breve{m},t)e^{\int \psi_R(m)dm}d\check{l} \end{bmatrix} \quad (22)$$

$$P_6(\check{l},\breve{m},t) = e^{-\int \phi_Q(m)dm} \begin{bmatrix} \int \beta_1(\breve{m}-\check{l})P_1(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l} \\ + \int \beta_1(\breve{m})P_1(\check{l},\breve{m},t)e^{\int \phi_Q(\breve{m})d\breve{m}}d\check{l} \end{bmatrix} \quad (23)$$

$$P_5(\check{l},\breve{m},t) = e^{-\int \psi_R(m)dm} \begin{bmatrix} \beta_2(\breve{m}-\check{l})P_0(t-\check{l}) \\ + \int \beta_2(\check{l})P_0(t)e^{\int \psi_R(\breve{m})d\breve{m}}d\check{l} \end{bmatrix} \quad (24)$$

$$P_4(\check{l},\breve{m},t) = e^{-\int \phi_Q(m)dm} \begin{bmatrix} \beta_1(\breve{m}-\check{l})P_0(t-\check{l}) \\ + \int \beta_1(\check{l})P_0(t)e^{\int \phi_Q(\breve{m})d\breve{m}}d\check{l} \end{bmatrix} \quad (25)$$

$$P_3(\check{l},\breve{m},t) = e^{-\int Z_3(\check{l},\breve{m})d\check{l}} \begin{bmatrix} \int E_3(\check{l},\breve{m},t)e^{\int Z_3(\check{l},\breve{m})dl}d\check{l} \\ + \int \beta_4(\breve{m}-\check{l})P_1(\check{l},\breve{m}-\check{l},t-\check{l})\,d\check{l} \\ + \int \beta_3(b-a)P_2(\check{l},\breve{m}-\check{l},t-\check{l}) \end{bmatrix} \quad (26)$$

$$P_2(\check{l},\breve{m},t) = e^{-\int Z_2(\check{l},\breve{m})d\check{l}} \begin{bmatrix} \int E_2(\check{l},\breve{m},t)e^{\int Z_2(\check{l},\breve{m})dm}d\check{l} \\ +\beta_4(\breve{m}-\check{l})P_0(t-\check{l}) \end{bmatrix} \quad (27)$$

$$P_1(\check{l},\breve{m},t) = e^{-\int Z_1(\check{l},\breve{m})d\check{l}} \begin{bmatrix} \int E_1(\check{l},\breve{m},t)e^{\int Z_1(\check{l},\breve{m})dl}dl \\ +\beta_3(\breve{m}-\check{l})P_0(t-\check{l}) \end{bmatrix} \quad (28)$$

$$P_0(t) = e^{-Z_0t}[1 + \int E_0(t)e^{Z_0t}dt] \quad (29)$$

If the manufacturing plant supply the rates of failing and repairing, we may find the reliability $R_{re}(t)$ in concern to probability$P_0(t)$ & via equation (1). Hence, Reliability $R_{re}(t)$ of manufacturing plant is given by

$$R_{re}(t) = P_0(t) + \int \sum_{i=1}^{3} P_i(\check{l},\breve{m},t)\,d\check{l}d\breve{m} \quad (30)$$

3.2.Failure$\mathbf{F_r}$ and $\mathbf{R_r}$ Repair rates are constant

When both the rates of failing and repairing are consistent, the system of equations (1-6) collapses to simple differential equations (DE) form, as shown below:

$$\left[\frac{d}{dt} + C\right] P_0(t) = \mu_S P_1(t) + \sigma_T P_2(t) + \phi_Q P_4(t) + \psi_R P_5(t) \quad (31)$$

$$\left[\frac{d}{dt} + \mu_S + C\right] P_1(t) = \beta_3 P_0(t) + \phi_Q P_6(t) + \psi_R P_7(t) + \mu_S P_8(t) + \sigma_T P_3(t) \quad (32)$$

$$\left[\frac{d}{dt} + \sigma_T + C\right] P_2(t) = \beta_4 P_0(t) + \phi_Q P_9(t) + \psi_R P_{10}(t) + \mu_S P_3(t) + \sigma_T P_{11} \quad (33)$$

$$\left[\frac{d}{dt} + \sigma_T + \mu_S + C\right] P_3(t) = \beta_4 P_1(t) + \beta_3 P_2(t) + \phi_Q P_{12}(t) + \psi_R P_{13} + \mu_S P_{14}(t) + \sigma_T P_{15}(t) \quad (34)$$

$\left[\frac{d}{dt} + \phi_Q\right] P_j(t) = \beta_1 P_k(t)$ for j=4,6,9 and 12 and k=0 to 3 respectively (35)

$\left[\frac{d}{dt} + \psi_R\right] P_p(t) = \beta_2 P_k(t)$ for p=5,7,10 and 13 and k=0 to 3 respectively (36)

$\left[\frac{d}{dt} + \mu_S\right] P_s(t) = \beta_3 P_r(t)$ for s=8 and 14 and r=1 and 3 respectively (37)

$\left[\frac{d}{dt} + \sigma_T\right] P_u(t) = \beta_4 P_v(t)$ for u=11 and 15 and v= 2 and 3 respectively (38)

Where C=$\beta_1 + \beta_2 + \beta_3 + \beta_4$

Initial conditions: The initial conditions of the subsystems are as under:

$$P_i(0) = \begin{cases} 1, & i = 0 \\ 0, & \text{otherwse} \end{cases} \quad (39)$$

One can obtain the state probabilities $P_i(i = 1, \ldots, 15)$ by solving differential equations (14-29) along with initial boundations (30).

3.3. Steady State

Manufacturing firms are continuously looking for long-term availability in order to meet their goals. This may be calculated numerically concerning $\frac{d}{dt} \to 0$ as $t \to \infty$ into the system of equations (31-38) therefore; the system of equations (31-38) reduces to the following system of linear equations:

$$[C]P_0 - \mu_S P_1 - \sigma_T P_2 - \phi_Q P_4 - \psi_R P_5 = 0 \quad (40)$$

$$[\mu_S + C]P_1 - \beta_3 P_0 - \phi_Q P_6 - \psi_R P_7 - \mu_S P_8 - \sigma_T P_3 = 0 \quad (41)$$

$$[\sigma_T + C]P_2 - \beta_4 P_0 - \phi_Q P_9 - \psi_R P_{10} - \mu_S P_3 - \sigma_T P_{11} = 0 \quad (42)$$

$$[\sigma_T + \mu_S + C]P_3 - \beta_4 P_1 - \beta_3 P_2 - \phi_Q P_{12} - \psi_R P_{13} - \mu_S P_{14} - \sigma_T P_{15} = 0 \quad (43)$$

$\phi_Q P_j - \beta_1 P_k = 0$ for j=4,6,9 and 12 and k=0 to 3 respectively (44)

$\psi_R P_p - \beta_2 P_k = 0$ for p=5,7,10 and 13 and k=0 to 3 respectively (45)

$\mu_S P_s - \beta_3 P_r = 0$ for s=8 and 14 and r=1 and 3 respectively (46)

$\sigma_T P_u - \beta_4 P_v = 0$ for u=11 and 15 and v=2 and 3 respectively (47)

The availability $A_{av}(t)$ of the system can be computed as,

$$A_{av}(t) = \sum_{i=0}^{3} P_i(t) \quad (48)$$

The system's availability $A_{av}(t)$, specified in equation (48) is evaluated at different values for various rates of failing and repairing. This might noticed here that we have only considered the most important subsystems(Q, R, S, T)

At the final point, the steady state availability has been solved by using the system of linear equations (31-38) recursively be expressing all the probabilities in terms of P_0. These are obtaining as below:

$P_i = N_i P_0$ for i= 1 to 3 (49)

$P_j = \frac{\beta_1}{\phi_Q} P_k$ for j=4,6,9 and 12 and k=0 to 3 respectively (50)

$P_p = \frac{\beta_2}{\psi_R} P_k$ for p=5,7,10 and 13 and k=0 to 3 respectively (51)

$P_s = \frac{\beta_3}{\mu_S} P_r$ for s=8 and 14 and r=1 and 3 respectively (52)

$P_u = \frac{\beta_4}{\mu_S} P_v$ for u=11 and 15 and v=2 and 3 respectively (53)

where

$$N_1 = \frac{\beta_3}{U_3} + \frac{\beta_3 \sigma_T \beta_4}{U_1 U_2 U_3}, \quad N_2 = \frac{\beta_4}{U_2} + \frac{\mu_S \beta_4 N_1}{U_1 U_2}, N_3 = \frac{\beta_4}{U_1} N_1 + \frac{\beta_3}{U_1} N_2$$

$$U_1 = \sigma_T + \mu_S, U_2 = \sigma_T + \beta_3 - \frac{\mu_S \beta_3}{U_1}, \quad U_3 = \mu_S + \beta_4 - \frac{\sigma_T \beta_4}{U_1} - \frac{\mu_S \beta_3 \sigma_T \beta_4}{U_1 U_2 U_1}$$

Now using the normalizing conditions$\sum_{i=0}^{15} P_i = 1$, we get

$$P_0 = \left[1 + \frac{\beta_1}{\phi_Q} + \frac{\beta_2}{\psi_R} + \left(1 + \frac{\beta_1}{\phi_Q} + \frac{\beta_2}{\psi_R} + \frac{\beta_3}{\mu_S}\right) N_1 + \left(1 + \frac{\beta_1}{\phi_Q} + \frac{\beta_2}{\psi_R} + \frac{\beta_4}{\sigma_T}\right) N_2 + (1 + \frac{\beta_1}{\phi_Q} + \frac{\beta_2}{\psi_R} + \frac{\beta_3}{\mu_S} + \frac{\beta_4}{\sigma_T}) N_3\right]^{-1} \quad (54)$$

As a result, the manufacturing plant's steady-state availability is attained as:

$$A_{av}(\infty) = \sum_{i=0}^{3} P_i = [1 + N_1 + N_2 + N_3]P_0 \quad (55)$$

IV. Methodology used for availability analysis of poly tube plant using PSO

4.1. Introduction

Inspired by swarming behaviors found in nature, such as flocks of fish and birds, Particle Swarm Optimization (PSO) is a potent meta-heuristic optimization algorithm, also called a stochastic search algorithm, based on population dynamics. It is a computational method used to optimize the problem. It performs its task of optimization by improving particle solutions. This algorithm works with some parameters, like particle size, population, position, velocity, search space, etc. In PSO, the population, like the bird group, represents a swarm, and each member of the swarm represents a particle. Every particle's movement is dictated by its local position. Each particle has velocities that direct the flight of particles. The search space refers to the spectrum in which the technique calculates the most effective regulatory variables. The value will be reset if the searching space is exceeded by any particle's optimal control value.

4.2. Working procedure of PSO with an example

To better understand how PSO operates, let's look at an example. A flock of birds flying aimlessly in an area, trying to find a single piece of food. Not a single bird is aware of the location of the food, i.e., they are aware of their progress in each iteration, even though they are unsure of the ideal eating position. They launch themselves in different directions and adhere to the PSO's search plan, i.e., swiftly follow that bird that is close to food. Each particle or bird, starting from a randomly selected population, moves through the searching space in randomly chosen directions while recalling its best historical positions and those of its neighbors, i.e., the highest ranking globally. Follow that bird to the global best position to obtain the optimal value, i.e., food.

4.3. PSO Algorithm Fundamentals

Step I	Start
Step II	Initializes particles with velocity vectors (μ)and random positions
Step III	Use of Fitness equation: Find out the fitness of the particles
Step IV	Evaluate and Update p best and g best
Step V	Numerically solved and updating of the position of the velocity vectors
Step VI	Numerically solved and updating of the position of the velocity vectors.
Step VII	Numerically solved and updating of the position of the particles.
Step VIII	Termination Satisfies?
Step IX	Stop.

Table 2 : *Different notations used during Algorithm*

μ_{ij}^{n}	Represents particle's velocity vector i at time t in dimension j.
r_{ij}^{t}	Represents particle's position vector i at time t in dimension j.
$P_{Best,t}^{i}$	Represents particle's best position i through initializing time t in dimension j.
$L_{Best,t}^{i}$	The best position that any particle has had in the neighborhood of particle i in dimension j initialization through time t.
$a_{1,} and\ a_2$	Designates the constants of positive acceleration that are employed to balance the social and cognitive aspects, respectively.
$r_{1j}^{t}\ and\ r_{2j}^{t}$	Time-dependent random integers drawn from a uniform distribution.

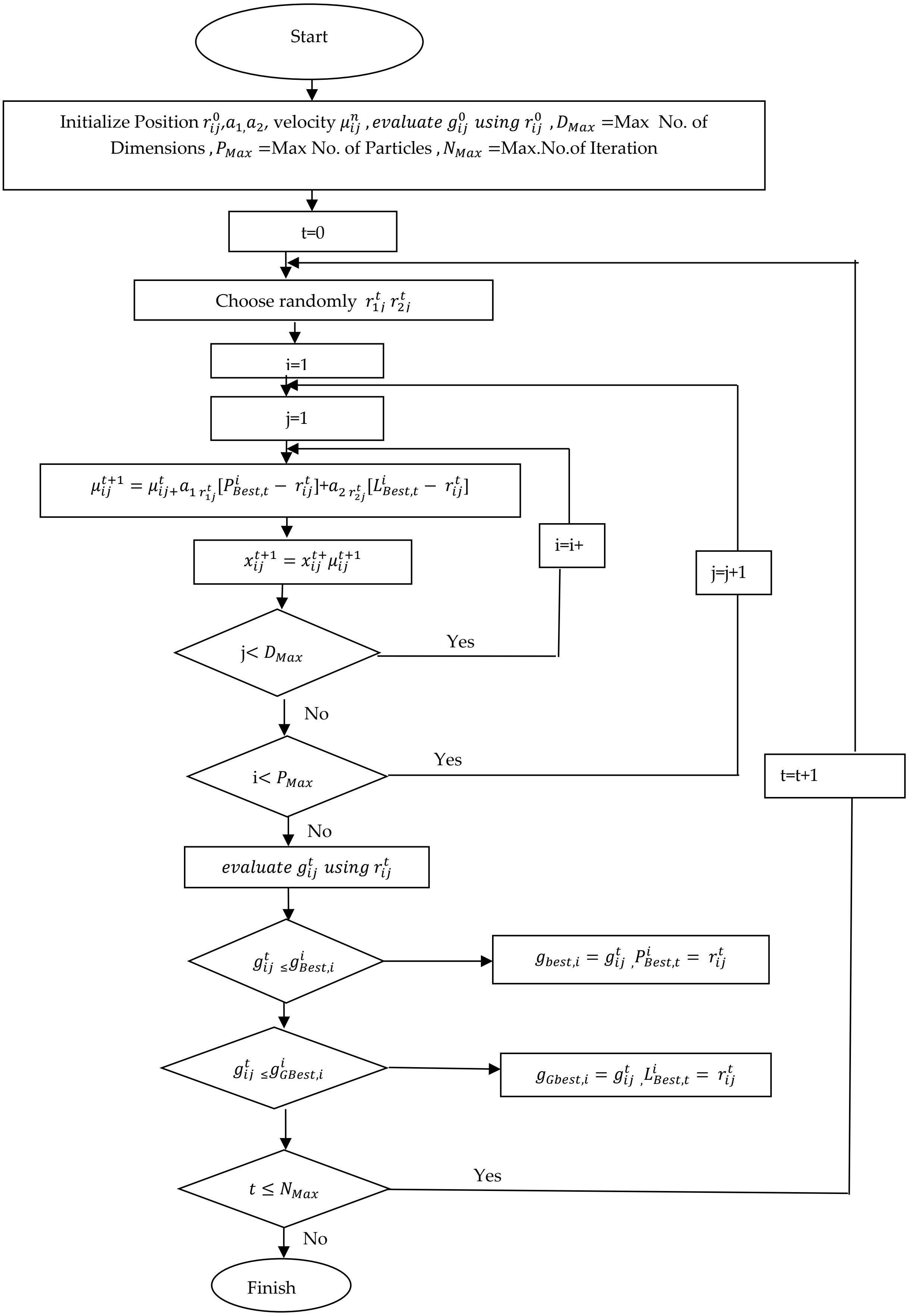

Figure 3: *Flow diagram of PSO*

4.4. Availability analysis of the industry with various failure($\mathbf{F_r}$) **and** repair($\mathbf{R_r}$) rate.

After applying the methodology of PSO over the fitness function i.e. an equation(55) for availability over different rate of failure and repair for each subsystem Q,R, S and T. We get the following variation presented from table 3 to 6 and corresponds figure 4 to 7.

Table 3. *Effect on Availability(A_{av}) of Mixture with various combination of Failure (F_r)and Repair Rate (R_r).*

β_1	A_{av}	σ_T	A_{av}
0.003122	0.31321	0.011276	0.50199
0.005446	0.30147	0.018296	0.66397
0.007046	0.25008	0.026777	0.70571
0.009462	0.14193	0.033979	0.81269
0.011776	0.13373	0.040393	0.85321
0.013864	0.13325	0.047671	0.93818
0.015398	0.08774	0.054925	0.95747
0.017696	0.07875	0.061103	0.99269

Table 4. *Effect on Availability(A_{av}) of Extruder with various combination of Failure (F_r)and Repair Rate (R_r)*

β_2	A_{av}	ϕ_Q	A_{av}
0.003517	0.31321	0.004498	0.40221
0.004683	0.27397	0.011015	0.52269
0.005914	0.15675	0.018255	0.59321
0.00665	0.13373	0.025938	0.64108
0.007282	0.102	0.032681	0.69269
0.008272	0.04782	0.039192	0.69269
0.009272	0.04782	0.046988	0.84675
0.010744	0.03037	0.053954	0.96397

Table 5. *Effect on Availability(A_{av}) of Dye with various combination of Failure (F_r)and Repair Rate (R_r)*

β_3	A_{av}	ψ_R	A_{av}
0.006091	0.37162	0.002535	0.57981
0.008081	0.31108	0.00958	0.62774
0.010847	0.24269	0.01655	0.67675
0.012656	0.19654	0.023479	0.74269
0.014349	0.08774	0.030528	0.8029
0.016375	0.06323	0.037996	0.86818
0.018533	0.05037	0.046239	0.89321
0.020701	0.11634	0.053487	0.90571

Table 6. *Effect on Availability(A_{av}) of Die with various combination of Failure (F_r)and Repair Rate (R_r)*

β_4	A_{av}	μ_S	A_{av}
0.007677	0.31321	0.003084	0.58774
0.008446	0.16134	0.010663	0.61221
0.009871	0.15675	0.017025	0.69682
0.010192	0.08774	0.024024	0.78571
0.011578	0.07981	0.031726	0.82397
0.012815	0.0629	0.037879	0.87321
0.013071	0.04774	0.044432	0.89269
0.014671	0.02037	0.051264	0.89408

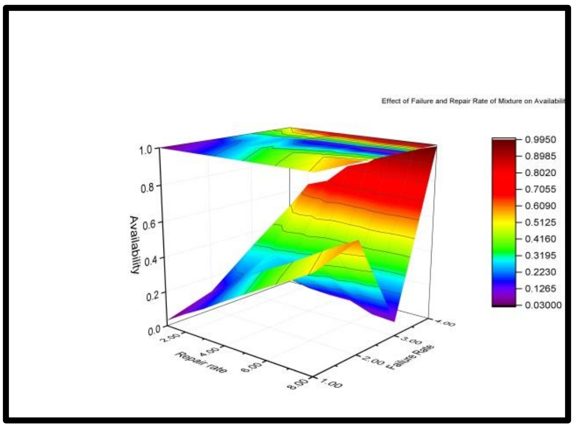

Figure 4 : *Effect on Availability(A_{av}) of Mixture with various combination of Failure (β_1)and Repair Rate (σ_T).*

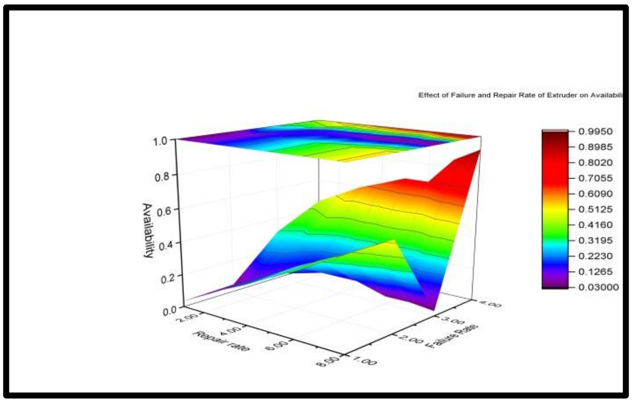

Figure 5 : *Effect on Availability(A_{av}) of Extruder with various combination of Failure (β_2) and Repair Rate (ϕ_Q).*

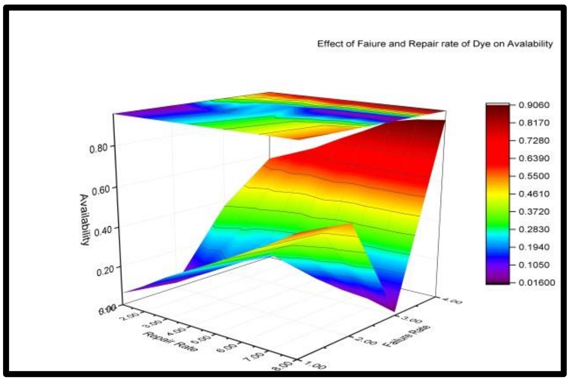

Figure 6 : *Effect on Availability(A_{av}) of Die with various combination of failure (β_3) and repair rate(ψ_R).*

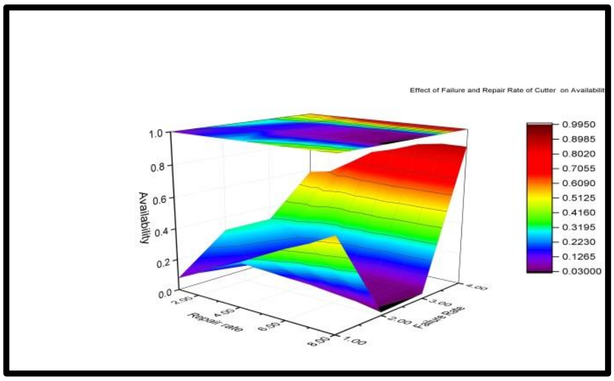

Figure 7 : *Effect on Availability(A_{av}) of Cutter with various combination of Failure (β_4)and Repair Rate (μ_S).*

V. Critical Analysis

The Critical Analysis has been done to demonstrate the impact of the subsystem on the overall system's availability than any of the other subsystems. Over long running of system ,How the availability percentage increased or decreased of a unit with respect to failure or repair rate with passage of time with comparison of other? To know the preference of subsystem to give more attention to met the requirements up to more efficient level, a critical analysis has been applied on the results obtain.

Table 7. *Critical Analysis of system component for their available performance time*

Systems	Failure Rate(F_r)	Decreases in Availability (A_{av})	Repair Rate(R_r)	Increase in availability (A_{av})	Repair Ranking (R_r)
Mixture	0.003122-0.017696-	23%	0.003122-0.061103	49%	II
Extruder	0.003517-0.010744	28%	0.004498-0.053954	56%	I
Dye	0.006091-	26%	0.002535-	32%	III

	0.020701		0.053487		
Cutter	0.007677-0.014671	19%	0.003084-0.051264	30%	IV

VI. Result and Observations

The following results are obtained after applying the methodology.

- Table 3 and figure 4 shown the variation in availability of component mixture with respect to both failure and repair rate which are depicted that with increase failure rate the availability decreases and with increasing repair rate, availability increases.
- Table 4 and figure 5 resulted about the behaviour of component extruder’s availability with respect to both failure and repair rate which are depicted that with increase failure rate the availability decreases and with increasing repair rate, availability increases.
- Table 5 and figure 6 shown the variation in availability of component dye with respect to both failure and repair rate which are depicted that with increase failure rate the availability decreases and with increasing repair rate, availability increases.
- Table 6 and figure 7 shown the variation in availability of component cutter with respect to both failure and repair rate which are depicted that with increasing failure rate the availability decreases and with increasing repair rate, availability increases.
- Table 7 analyzed the component performance based on availability with failure and repair rate.

VII. Concluding observations

Availability of polytube is optimized by using particle swam optimization technique. During PSO maximum allowable velocity and weight parameters play a vital rule for analysis. For various combination of failure and repair rate many runs were performed. The optimal availability is achieved 99% at Failure rate increase from (0.003517-0.010744) and repair rate (0.004498-0.053954). The fundamental modeling of the manufacturing plant represents the flow of row material from one subsystem to another subsystem. With the help of transition diagram all the possible stages (Failure, Repair and reduced) are helpful to find the probabilities of every stage. To calculate (steady state and transit state) the sensitive analysis of failure and repair rate of the subsystem with maintenance strategy play an important rule for the management to get maximum availability of the system without failure. The table's (3-6) reveals the numerical analysis of the optimized availability of polytube manufacturing plant. Further the results are explained graphically from the (Figure3-6). The comparative analysis demonstrates that the subsystem Extruder has the greatest impact on the overall system's availability than any of the other subsystems. Other subsystems have a minor impact on the availability of the polytube manufacturing facility. For finding the optimal solution PSO techniques is very helpful . With the help of this techniques one can use different combination of failure and repair rate to get the optimal solution.

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