ANALYSIS OF THE MULTIPLE WORKING VACATIONS, BATCH SERVICE AND RENEGING QUEUING SYSTEM UNDER SINGLE SERVER POLICY

Lidiya P*, K Julia Rose Mary

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[1,2]Department Of Mathematics, Nirmala College For Women, Coimbatore
*lidiyap481@gmail.com
juliakulandaisamy@gmail.com

Abstract

In this paper, we analysed the multiple working vacation queuing model with reneging under a single server policy. Reneging describes the situation where a customer or entity decides to leave the queue before being served. The presence of reneging behaviour affects queue and service efficiency, as customers leaving the queue prematurely can impact overall system performance and customer satisfaction. In this model, customers arrive at a service facility and form a queue to be served by a single server. The arrival follows the Poisson distribution, and the service follows the exponential process. Batches of customers are served under the General Bulk Service Rule. In GBSR, rather than individual customer arriving at a queue one by one, customers arrive in groups or batches. Thus, each batch of service contains a minimum of $'a'$ units and a maximum of $'b'$ units of customers. The steady-state equation, the various performance measures for the system, and particular cases of the described model are derived.

Keywords: Reneging, Multiple Working vacations (MWV), Queue length, Bulk service, System size

1. Introduction

Erlang developed queuing theory while working for the Telephone Company to analyse the behaviour of telephone traffic and optimise the capacity of telephone exchanges. His work laid the foundation for the study of waiting lines and has since been widely applied in various fields to improve system performance and efficiency. The main objective of queuing theory is to understand and optimise the performance of systems that involve waiting lines. By studying factors such as arrival rates, service rates, queue lengths, and waiting times, queuing theory provides insights into how to improve efficiency, and reduce waiting times.

In queuing theory, a vacation queuing model is a type of queuing system where the server may take breaks or go on vacation, leading to periods of time when service is not available. This type of model is often used in scenarios where service providers have scheduled breaks, such as in customer service centres, healthcare facilities, or manufacturing processes. Analysing and optimising vacation queuing models involves considering factors of the duration and frequency of vacations, the impact on service during vacation periods, and strategies to minimise the effects of downtime on customer satisfaction. The concept of "multiple working vacations" refers to a scenario where the server in a queuing system takes several breaks or vacations during their work.

The concept of the GBS rule was indeed introduced by Neuts. The GBS rule, which he introduced, is used to analyse queuing systems where customers arrive in batches and are served as a single entity with a fixed service time for the entire batch. In a batch arrival process, rather than individual customers arriving at a queue one by one, customers arrive in groups or batches.

Reneging is a term used in queuing theory to describe the situation where a customer or entity decides to leave the queue before being served. This reneging occurs when customers experience long waiting times or delays in service. Customers who renege may seek alternative service providers or simply give up. This can result in lost business opportunities and decreased customer satisfaction. Various strategies can be used to address reneging, such as optimising service processes to reduce waiting times, providing clear communication about the expected waiting times, management techniques that minimise reneging rates.

This paper analyses the queuing system that combines multiple working vacations, batch service, a single server, and reneging behaviour. For this model, we obtained steady state equations, measures of performances, and analysed the particular cases.

2. Review of literature

Research on vacation and reneging queuing models has gained significant attention from researchers in the field of queuing theory. The $MAP/PH/1$ queuing system, including setup, shutdown, multiple vacations, standby server, malfunction, maintenance, and reneging, was examined by [4]. In their paper, the matrix-analytic technique has been used to investigate the total number of consumers existing in the system within a steady-state probability matrix. The non-Markovian approach, which includes longer vacations, renewal processes, and service interruptions followed by repair stages, was studied by [25]. Fixing the M/M/2 machine issue with rushing clients. Under multiple working vacations and strategy (0, Q, N, M) plans, machines are considered to be fixed proposed by [15]. A single server with limitless capacity the Markovian queue structure, is analysed in [12] thesis, has multiple working vacations, an adjustment period, and reneging, by considering both limited capacities and continuous-time queuing systems. An M/M/C queuing model involving changing working vacations was examined by [26]. Additionally, the model's cost function is developed, and the quadratic fit search method is used to look into its optimisation.

Single server's batch of arrival queuing concept for the system, which provides customers who renege during server vacation and system failure times with three stages of heterogeneous services introduced by [7]. The supplemental parameter method has been used to create steady-state probability distribution functions for the queue size. A boundless capacity of one-server Markovian queue mechanism with one working vacation, reneging, and retention of reneged clients evaluated in the thesis of [11]v. A different working vacation queue method involving a second optional service, an unstable server, and the retention of reneged clients was investigated by [21]. They also covered an optimisation problem under a certain cost model. With a practical retention plan for reneged clients and Bernoulli's planned altered vacation regulations, [23] developed a multi-server finite capacity queuing method. The steady-state probability is obtained using the matrix analytical approach. Measures of performance that are produced with an application are also developed and dealt with using the particle swarming optimisation (PSO) meta-heuristic. State-dependent reneging, maintenance of reneged clients, and infinite-capacity single-server Markovian line systems with one working vacation evaluated by [1].

Using Bernoulli responses and client impatience in the context of several vacations, [3] derived a multi-server queuing model. Host vacations, malfunctions, and join or balk tactical behaviours. Additionally, demonstrated how important the reneging option is when the initial system involving non-strategic clients is unreliable researched by [6]. The Markovian queuing structure, which includes working vacation, Bernoulli scheduling disruption, initialization time according to suggestions, reneging of impatient clients, and retention of reneged clients evaluated by [9]. The M/M/1 feedback line with backward balking, backward reneging, and multiple operating vacations was investigated by [14]. The matrix methodology and the ant colony optimisation (ACO) strategy are used to generate the steady-state system length estimates for the model. Reneging, multiple vacations, and set-up period queuing methods were proposed by [2]. The server provides guidance in three stages; the first two are required, while the third is optional. The server's support duties will be completed if it needs to take a required vacation during that period.

Buffer-modified reverse balking in a single-server finite capacity feedback queuing system, as well as the retention of impatient clients investigated by [24]. The infinite buffering M/M/1 queue with variation working vacations subject to Bernoulli scheduling vacation interruption, in which consumers balk with a probability, was studied by [19]. For various server states, determine a closed-form formulation of the system's capacity and the steady-state probabilities. Evaluate the cost optimisation problem using the quadratic fit-finding technique. M/M/1/N feedback on the operating vacation queuing system, including reneging, was presented by [8]. They implement the Markov process approach to generate the steady-state probability equations. The matrix approach was applied to solve the steady-state probabilities, and a cost model was also developed. Hospitalisation and queuing management processes, including decisions to discharge patients too soon and noncompliance examined by [27]. In order to minimise the hospital's total projected costs, they formulate their solution as an infinite-horizon total discounted cost Markov decision process that balances bed utilisation and patient experiences while they wait for admission. To explain how certain important system factors affect the optimal adaptive policy. An adaptable queuing system, including the retention of reneging consumers, was proposed by [22]. Furthermore, the costs and performance analysis, along with the steady-state and transient-state measures of performance, provide numerical illustrations for demonstrating the model's usefulness in assessing wait times in the service field. The M/M/1 drive-thru lines in order to gain insight into the possibility of reneging occurrences and determine how customers' sensitivity to entry times affects their choice of exiting the line early. By adding reneging behaviour to the queuing theory with fundamental equations, they suggest improving the theory's applicability and improving its reflection and interpretation of queuing issues in the real world discussed by [5]. Three distinct categories of client behavior, such as balking, interruptions, and reneging, were examined by [28]. They researched multi-server preference queues, including client balking, interruptions, and reneging, from an analytical and practical perspective. The idea of the client-reneging effect was introduced by [18]. They analysed reneging versus no-reneging, where customers are balanced and strategic. Client tactics and they take into consideration the fluid on-off concept of the standard queue, including vacations and failures.

Also, how client reneging impacted the formulation of a queue system with two separate shifts. It shows the manner in which lost consumers and costs are analysed in an engaged service policy. Utility operations are also used as decision-making tools analysed by [10]. The F-policy, server keeping up, reneging, and balking queue procedures using only one server and limited capacity were determined by [13]. The recursive approach is used to attain the steady state. Multiple working vacations under breakdown, types of breakdown for heterogeneous arrival queuing model analysed by [16,17]. Multiple working vacations under heterogeneous with encouraged arrival evaluated by [20]. With the help of the appropriate literature, we are able to evaluate the queuing model with reneging under the MWV single server policy.

3. Methodology

In this paper analysed the $M/M(a,b)/1/MWV$ queuing system with reneging. Instead of the server being fully idle during the vacation period, the server serves at a different rate during multiple working vacations. The service rate varies depending on the arrival state. Server provides service during the regular busy period with parameter μ_{rb} and under multiple working vacations, the server provides service with parameter μ_{wv} with exponential distribution. Customer arrive at the system with the parameter λ_v it follows Poisson distribution. In this model, batches of customers are served under the General Bulk Service Rule. Thus, each batch of service contains a minimum of $'a'$ units and a maximum of $'b'$ units of customers. Suppose the number of customers waiting in the queue is less than $'a'$ the server begins a vacation random variable V with parameter ξ. Let

$$\begin{aligned} R_n^I(t) &= Pr\{N_c(t) = n, L(t) = 0\} \qquad 0 \le n \le a-1 \\ Q_n^V(t) &= Pr\{N_c(t) = n, L(t) = 1\} \qquad n \ge 0 \\ P_n^B(t) &= Pr\{N_c(t) = n, L(t) = 2\} \qquad n \ge 0 \end{aligned}$$

$L(t) = 0$, the size of the queue and system are same.
$L(t) = 1$ *or* 2 , the total number of customers in the system is the sum of the number of customers in queue and the size of the service batches that contains particular $a \le x \le b$ customers.
Hence the probabilities of the steady state are ,

$$Q_n^V = \lim_{t\to\infty} Q_n^V(t); \qquad R_n^I = \lim_{t\to\infty} R_n^I(t); \qquad P_n^B = \lim_{t\to\infty} P_n^B(t);$$

exist and the Chapman Kolmogrove equations satisfied by them in the steady state are given by,

$$\lambda_v R_0^I = \mu_{rb} P_0^B + \mu_{wv} Q_0^V \tag{1}$$

$$\lambda_v R_n^I = \lambda_v R_{n-1}^I + (\mu_{rb} + (n-1)\alpha) P_n^B + (\mu_{wv} + (n-1)\alpha) Q_n^V; 1 \le n \le a-1 \tag{2}$$

$$(\lambda_v + \xi + \mu_{wv}) Q_0^V = \lambda_v R_{a-1}^I + \sum_{n=a}^{b} (\mu_{wv} + (n-1)\alpha) Q_n^V \tag{3}$$

$$(\lambda_v + \xi + (\mu_{wv} + (n-1)\alpha)) Q_n^V = \lambda_v Q_{n-1}^V + (\mu_{wv} + (b+n-1)\alpha) Q_{n+b}^V; \qquad n \ge 1 \tag{4}$$

$$(\lambda_v + \mu_{rb}) P_0^B = \sum_{n=a}^{b} (\mu_{rb} + (n-1)\alpha) P_n^B + \xi Q_0^V \tag{5}$$

$$(\lambda_v + (\mu_{rb} + (n-1)\alpha)) P_n^B = \lambda_v P_{n-1}^B + (\mu_{rb} + (b+n-1)\alpha) P_{n+b}^B + \xi Q_n^V;\ n \ge 1 \tag{6}$$

4. Steady state solution

To solve the steady state equation, the forward shifting operator E on P_n^B and Q_n^V are introduced then,

$$E(P_n^B) = P_{n+1}^B; \qquad E(Q_n^V) = Q_{n+1}^V \qquad for\ n \ge 0$$

Thus the (4) gives homogeneous difference equation as,

$$[\lambda_v + (\mu_{wv} + (b+n-1)\alpha) E^{b+1} - (\lambda_v + \xi + (\mu_{wv} + (n-1)\alpha) E] Q_n^V = 0 \tag{7}$$

The characteristics equation of (7) is obtained as,

$$z(u) = \lambda_v + (\mu_{wv} + (b+n-1)\alpha) u^{b+1} - (\lambda_v + \xi + (\mu_{wv} + (n-1)\alpha)) u = 0 \tag{8}$$

by taking $x(u) = (\lambda_v + (\mu_{wv} + (b+n-1)\alpha))$ and $y(u) = (\lambda_v + \xi + (\mu_{wv} + (n-1)\alpha)$, it is found that $|y(u)| < |x(u)|$ on $|u| = 1$. By Rouche's theorem $z(u)$ has unique root r_v inside the contour $|u| = 1$. (7) has a homogeneous solution as,

$$Q_n^V = r_v^n Q_0^V \tag{9}$$

From (6) we get,

$$[\lambda_v + (\mu_{rb} + (b+n-1)\alpha) E^{b+1} - (\lambda_v + (\mu_{rb} + (n-1)\alpha) E] P_n^B = -\xi r_v^{n+1} Q_0^V \tag{10}$$

By applying Rouche's theorem to (10) as,

$$[\lambda_v + (\mu_{rb} + (b+n-1)\alpha) E^{b+1} - (\lambda_v + (\mu_{rb} + (n-1)\alpha) E] P_n^B = 0$$

The above equation has unique root r with $|r| < 1$. Also (10) gives a non-homogeneous solution as,

$$P_n^B = \left[Zr^n - \frac{\xi r_v^{n+1}}{[\lambda_v + (\mu_{rb} + (b+n-1)\alpha) r_v^{b+1} - (\lambda_v + (\mu_{rb} + (n-1)\alpha) r_v]} \right] Q_0^V \tag{11}$$

$$P_n^B = (Zr^n + Z^* r_v^n) Q_0^V \tag{12}$$

Where

$$Z^* = \frac{\xi r_v}{[\lambda_v (r_v - 1) + \mu_{rb} r_v (1 - r_v^b) + \alpha r_v ((n-1) - (b+n-1) r_v^b)]} \tag{13}$$

The expression for R_n^I is obtained by adding (1) & (2) and substitute P_n^B and Q_n^V values,

$$R_n^I = \left\{ Z \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{(1 - r^{n+1})}{(1-r)} \right) + \frac{\alpha r^2}{\lambda_v} \left(\frac{1 - r_v^{n-1}}{1 - r_v} \right) \right] + Z^* \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{(1 - r^{n+1})}{(1-r)} \right) + \frac{\alpha r_v^2}{\lambda_v} \left(\frac{1 - r_v^{n-1}}{1 - r_v} \right) \right] + \frac{\mu_{wv}}{\lambda_v} \left(\frac{1 - r_v^{n+1}}{1 - r_v} \right) + \alpha r_v^2 \left(\frac{1 - r^{n-1}}{1 - r_v} \right) \right\} Q_0^V$$

Now to calculate Z, considering (5) and substitute P_n^B and Q_n^V values we find,

$$Z \left[(\lambda_v + \mu_{rb}) - \frac{\mu_{rb}(r^a - r^{b+1})}{(1-r)} - fracr^a \alpha((a-1) - (b-1) r^{b-a+1} 1 - r \right] = \xi - Z^* \left((\lambda_v + \mu_{rb}) - \frac{\mu_{rb}(r_v^a - r_v^{b+1})}{(1 - r_v)} - \frac{r_v^a \alpha((a-1) - (b-1) r_v^{b-a+1}}{(1 - r_v)} \right)$$

the above expression can be simplified as,

$$\frac{Z \mu_{rb}(1 - r^a)}{(1-r)} = \frac{\xi}{(1 - r_v)} - \frac{Z^* \mu_{rb}(1 - r_v^a)}{(1 - r_v)} \tag{14}$$

Hence the probability of queue size of the steady-state equation in terms of Q_0^V are obtained,

$$Q_n^V = (r_v^n) Q_0^V \qquad n \geq 0 \tag{15}$$

$$P_n^B = (Zr^n + Z^* r_v^n) Q_0^V \qquad n \geq 0 \tag{16}$$

where

$$Z = \frac{(1-r)}{\mu_{rb}(1 - r^a)} \left[\frac{\xi}{(1 - r_v)} - \frac{Z^* \mu_{rb}(1 - r_v^a)}{(1 - r_v)} \right] \tag{17}$$

$$Z^* = \frac{\xi r_v}{[\lambda_v (r_v - 1) + \mu_{rb} r_v (1 - r_v^b) + \alpha r_v ((n-1) - (b+n-1) r_v^b)]} \tag{18}$$

and

$$R_n^I = \left\{ Z \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{(1 - r^{n+1})}{(1-r)} \right) + \frac{\alpha r^2}{\lambda_v} \left(\frac{1 - r_v^{n-1}}{1 - r_v} \right) \right] + Z^* \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{(1 - r^{n+1})}{(1-r)} \right) + \frac{\alpha r_v^2}{\lambda_v} \left(\frac{1 - r_v^{n-1}}{1 - r_v} \right) \right] + \frac{\mu_{wv}}{\lambda_v} \left(\frac{1 - r_v^{n+1}}{1 - r_v} \right) + \alpha r_v^2 \left(\frac{1 - r^{n-1}}{1 - r_v} \right) \right\} Q_0^V \tag{19}$$

by using normalizing condition and calculated the value of Q_0^V

$$\sum_{n=0}^{\infty} Q_n^V + \sum_{n=0}^{\infty} P_n^B + \sum_{n=0}^{a-1} R_n^I = 1$$

By substituting P_n^B, Q_n^V and R_n^I we observe that,

$$\sum_{n=0}^{\infty} r_v^n Q_0^V + \sum_{n=0}^{\infty}(Zr^n + Z^* r_v^n)Q_0^V + \sum_{n=0}^{a-1}\Bigg[R_n^I = \Bigg\{Z\left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{(1-r^{n+1})}{(1-r)}\right) + \frac{\alpha r^2}{\lambda_v}\left(\frac{1-r_v^{n-1}}{1-r_v}\right)\right] + Z^*\left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{(1-r^{n+1})}{(1-r)}\right) + \frac{\alpha r_v^2}{\lambda_v}\left(\frac{1-r_v^{n-1}}{1-r_v}\right)\right] + \frac{\mu_{wv}}{\lambda_v}\left(\frac{1-r_v^{n+1}}{1-r_v}\right) + \alpha r_v^2\left(\frac{1-r^{n-1}}{1-r_v}\right)\Bigg\}Q_0^V = 0$$

Then,

$$(Q_0^V)^{-1} = \omega(r_v, \mu_{wv}) + Z\omega(r, \mu_{rb}) + Z^*\omega(r_v, \mu_{rb}) + \Gamma(r_v) + Z\Gamma(r) + Z^*\Gamma(r_v) \tag{20}$$

where

$$\omega(x,y) = \frac{1}{(1-x)}\left(1 + \frac{y}{\lambda_v}(c - \frac{x(1-x^a)}{(1-x)})\right)$$

$$\Gamma(x) = \left[\frac{\alpha x}{\lambda_v(1-x)}(cx - \frac{(1-x^a)}{(1-x)}\right]$$

5. Performance Measures

In this section, the performance measures of expected queue length, expected waiting time of the queue, expected system length and expected waiting time of the queue for multiple working vacations model with reverse balking under types of breakdowns model are derived.

5.1. Mean queue length

The expected queue length is given by,

$$L_q = \sum_{n=1}^{\infty} n(Q_n^V + P_n^B) + \sum_{n=1}^{a-1} nR_n^I$$

By substituting P_n^B, Q_n^V and R_n^I we observe that,

$$L_q = \sum_{n=1}^{\infty} n(r_v^n Q_0^V) + \sum_{n=1}^{\infty} n(Zr^n + Z^* r_v^n)Q_0^V + \sum_{n=1}^{a-1} n\Bigg\{Z\left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{(1-r^{n+1})}{(1-r)}\right) + \frac{\alpha r^2}{\lambda_v}\left(\frac{1-r_v^{n-1}}{1-r_v}\right)\right] + Z^*\left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{(1-r^{n+1})}{(1-r)}\right) + \frac{\alpha r_v^2}{\lambda_v}\left(\frac{1-r_v^{n-1}}{1-r_v}\right)\right] + \frac{\mu_{wv}}{\lambda_v}\left(\frac{1-r_v^{n+1}}{1-r_v}\right) + \alpha r_v^2\left(\frac{1-r^{n-1}}{1-r_v}\right)\Bigg\}Q_0^V$$

$$L_q = Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv}) + Z\Gamma^*(r) + Z^*\Gamma^*(r_v) + \Gamma^*(r_v) \tag{21}$$

where

$$\omega^*(x,y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda_v(1-x)}\left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2}\right] \tag{22}$$

$$\Gamma^*(x) = \frac{\alpha x^2}{\lambda_v(1-x)}\left\{\frac{a(a-1)}{2} + \frac{ax^{a-1}(1-x) + x^a - 1}{(1-x)^2}\right\} \tag{23}$$

and Z & Z^* are given by (17) & (18).

5.2. Mean System Length

The expected system length is given by,

$$L_s = L_q + \rho$$

$$L_s = \{Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv}) + Z\Gamma^*(r) + Z^*\Gamma^*(r_v) + \Gamma^*(r_v)\} + \rho$$

where $\omega^*(x, y)$ and $\Gamma^*(x)$ are given by (22) & (23).

5.3. Mean Waiting Time of the Queue

The expected waiting time of the queue is given by,

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv}) + Z\Gamma^*(r) + Z^*\Gamma^*(r_v) + \Gamma^*(r_v)}{\lambda_v}$$

where $\omega^*(x, y)$ and $\Gamma^*(x)$ are given by (22) & (23).

5.4. Mean Waiting Time of the System

The expected waiting time of the system is given by,

$$W_s = \frac{L_s}{\lambda}$$

$$W_s = \frac{\{Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv}) + Z\Gamma^*(r) + Z^*\Gamma^*(r_v) + \Gamma^*(r_v)\} + \rho}{\lambda_v}$$

where $\omega^*(x, y)$ and $\Gamma^*(x)$ are given by (22) & (23).
If $Pr_{(wv)}$, $Pr_{(busy)}$ *and* $Pr_{(idle)}$ denote the probability that the server in idle, regular busy and busy vacation period then

$$Pr_{(idle)} = \sum_{n=0}^{a-1} R_n^I \tag{24}$$

where the R_n^I is given by (19).

$$Pr_{(busy)} = \sum_{n=0}^{\infty} P_n^B = \left(\frac{Z}{(1-r)} + \frac{Z^*}{(1-r_v)}\right) Q_0^V \tag{25}$$

$$Pr_{(wv)} = \sum_{n=0}^{\infty} Q_n^V = \frac{Q_0^V}{(1-r_v)} \tag{26}$$

6. Particular cases

6.1. Classical $M/M(a,b)/1/MWV$ model

By letting $\alpha = 0$ (21) and we obtain,

$$Q_n^V = (r_v^n) Q_0^V \qquad n \geq 0$$

$$P_n^B = (Zr^n + Z^* r_v^n) Q_0^V \qquad n \geq 0$$

$$R_n^I = \left[\frac{\mu_{rb}}{\lambda_v}(Zg_n(r) + Z^* g_n(r_v) + g_n(r_v)\right] Q_0^V \qquad 0 \leq n \leq a-1$$

where

$$Z = \frac{(1-r)}{\mu_{rb}(1-r^a)}\left[\frac{\xi}{(1-r_v)} - \frac{Z^* \mu_{rb}(1-r_v^a)}{(1-r_v)}\right]$$

$$Z^* = \frac{\xi r_v}{\lambda_v(r_v - 1) + \mu_{rb} r_v (1 - r_v^b)}$$

Further

$$L_q = Z\omega^*(r, \mu_{rb}) + Z^* \omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv})$$

where

$$\omega^*(x,y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda_v(1-x)}\left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2}\right]$$

Thus, observed that our specified model coincides with the $M/M(a,b)/1/MWV$ queuing model analysed by J.R.Mary and A.Begum (2011).

Conclusion

In this study, the $M/M(a,b)/1/MWV$ queuing model with reneging is analyzed. In this model, GBSR is followed. The steady-state solution, the various performance measures for the system, and particular cases are calculated.

Reneging factors into queuing systems, making it difficult for service providers to predict and plan for service demand. Reducing reneging in a queuing system is important for improving customer satisfaction, optimising service efficiency, and maximising revenue opportunities. The impact of reneging on queuing systems can affect both customers and service providers. By implementing a system with multiple working vacations effective queue management techniques, optimising service processes, and improving the overall customer experience, it is possible to minimise reneging behaviour and enhance the performance of the queuing system. Further in the future, the model may be extended to the arrival of multiple working vacations queue with the concept of balking.

Conflict of Interest

The authors declare that there is no conflict of interest.

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