PROFIT AND AVAILABILITY ANALYSIS OF UTENSIL INDUSTRY SUBJECT TO REPAIR FACILITY

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Abstract

The main objective of the paper is to optimize the availability and profit values of the utensil industry. There are three distinct units in the utensil industry and two of them work in reduced state. It is assumed that unit A is failed in complete failure mode while units B and C completely failed through partial failure mode. The system is in a failing condition when one of the units completely fails. There is a qualified technician to fix the fault in the system. Timelines for failure and repair are unrelated to one another. The repair time is exponential while the failure time distribution is general. Many factors, including mean time to system failure, availability, busy period, estimated number of server visits and profit values are calculated from tables.

Keywords: Reliability, base state, mean sojourn time, availability and profit.

I. Introduction

Manufacturers and industrialists continuously produce goods to meet the rising demand for goods, which they can do by optimizing their manufacturing procedures. For product development, reliability engineering presents an integrated approach that helps in the design and maintenance of the products. To calculate the profit values, it is important to analyze the availability of the system. This study examines the MTSF, availability and profit values of the housewares sector, emphasizing the need of utilizing the regenerative point graphical technique for priority in repair under specific circumstances. A large amount of research work has been done on repairable systems such that Kapur and Kapoor [5] analyzed the behaviour of a two unit system subject to repair facility and one spare unit is kept in cold standby mode. Gnedenko and Igor [3] explored reliability and probability measures to solve the complexity of repairable system. Jack and Murthy [4] discovered the role of limited warranty and extended warranty for the product. Wang and Zhang [10] examined the repairable system of two non identical components under repair facility using geometric distributions. Kumar and Goel [7] threw light on the preventive maintenance in two unit cold standby repairable system under general distributions. Chaudhary and Tomar [2] threw light on the stochastic behavior of a two unit cold standby system under inspection. Kumar et al. [6] described the effects of washing unit in the paper industry by using the regenerative point graphical technique. Levitin et al. [8] explored the results of optimal preventive replacement of failed units in a cold standby system by using the poisson process. Agarwal et al. [1] analyzed the performance and reliability of water treatment plant under repair facility. Sengar and Mangey [9] examined the availability and profit values of complicated repairable system with inspection using copula methodology.

II. System Assumptions

There are following system assumptions:

- The utensil industry has three distinct units such that cutting system, pressing system, spinning and buffing system.
- Sheets are cut into circular sheets with the help of cutting system (A).
- Sheets are converted into the shape of utensils by using pressing technology (B).
- Spinning and buffing machinery give the final shape and polish to the utensil.
- It is considered that units B and C may be in a complete failed state through partial failure but unit A is in only complete failed state.
- Failure time follows general distribution whereas repair time follows the exponential distribution.
- A technician is always available to repair the failed unit.
- The failed unit works like a new unit after repair.

III. System Notations

There are following system notations:

$i \xrightarrow{Sr} j$	r^{th} directed simple path from state 'i' to state 'j' where 'r' takes the positive integral values for different directions from state 'i' to state 'j'.
$\xi \xrightarrow{sff} i$	A directed simple failure free path from state ξ to state 'i'.
$m-cycle$	A circuit (may be formed through regenerative or non regenerative / failed state) whose terminals are at the regenerative state 'm'.
$m-\overline{cycle}$	A circuit (may be formed through the unfailed regenerative or non regenerative state) whose terminals are at the regenerative 'm' state.
$U_{k,k}$	Probability factor of the state 'k' reachable from the terminal state 'k' of 'k' cycle.
$U_{\overline{k,k}}$	The probability factor of state 'k' reachable from the terminal state 'k' of $k\ \overline{cycle}$.
μ_i	Mean sojourn time spent in the state 'i' before visiting any other states.
μ_i'	Total unconditional time spent before transiting to any other regenerative state while the system entered regenerative state 'i' at t=0.
η_i	Expected waiting time spent while doing a job given that the system entered to the regenerative state 'i' at t=0.
A/a	System first unit is in the operative state/failed state.
$B/\overline{B}/b$	System second unit is in the operative state/reduced state/failed state.
$C/\overline{C}/c$	System third unit is in the operative state/reduced state/failed state.
$\alpha_1\ /\ \alpha_3$	The constant partial failure rate of the unit B/C respectively.
$\alpha_2\ /\alpha_4$	The constant complete failure rate of the unit B/C respectively.
α_5	The constant complete failure rate of unit A.
$f_1(t)/F_1(t)$	PDF/CDF of repair time of unit B from partial failed state.
$f_2(t)/F_2(t)$	PDF/CDF of repair time of unit B from complete failed state.
$f_3(t)/F_3(t)$	PDF/CDF of repair time of unit C from partial failed state.
$f_4(t)/F_4(t)$	PDF/CDF of repair time of unit C from complete failed state.
$f_5(t)/F_5(t)$	PDF/CDF of repair time of unit A from complete failed state.

IV. Transition Diagram and Their Descriptions

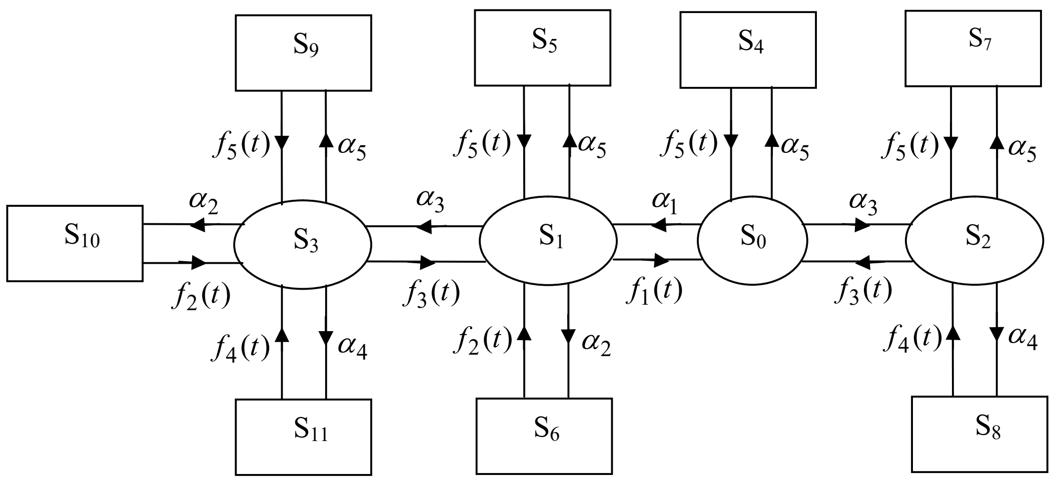

Figure 1: *State Transition Diagram*

In the system transition diagram, there are following states

where, $S_0 = ABC$, $S_1 = A\bar{B}C$, $S_2 = AB\bar{C}$, $S_3 = A\overline{BC}$, $S_4 = aBC$, $S_5 = a\bar{B}C$
$S_6 = AbC$, $S_7 = aB\bar{C}$, $S_8 = ABc$, $S_9 = a\overline{BC}$, $S_{10} = Ab\bar{C}$, $S_{11} = A\bar{B}c$

V. Transition Probabilities

The transition probabilities are following

$$p_{0,1} = \alpha_1 /(\alpha_1 + \alpha_3 + \alpha_5),\ p_{0,2} = \alpha_2 /(\alpha_1 + \alpha_3 + \alpha_5)$$
$$p_{0,4} = \alpha_5 /(\alpha_1 + \alpha_3 + \alpha_5),\ p_{1,0} = \beta_1 /(\beta_1 + \alpha_2 + \alpha_3 + \alpha_5)$$
$$p_{1,3} = \alpha_3 /(\beta_1 + \alpha_2 + \alpha_3 + \alpha_5),\ p_{1,5} = \alpha_5 /(\beta_1 + \alpha_2 + \alpha_3 + \alpha_5)$$
$$p_{1,6} = \alpha_2 /(\beta_1 + \alpha_2 + \alpha_3 + \alpha_5),\ p_{2,0} = \beta_3 /(\beta_3 + \alpha_4 + \alpha_5)$$
$$p_{2,7} = \alpha_5 /(\beta_3 + \alpha_4 + \alpha_5),\ p_{2,8} = \alpha_4 /(\beta_3 + \alpha_4 + \alpha_5)$$
$$p_{3,1} = \beta_3 /(\beta_3 + \alpha_2 + \alpha_4 + \alpha_5),\ p_{3,9} = \alpha_5 /(\beta_3 + \alpha_2 + \alpha_4 + \alpha_5)$$
$$p_{3,10} = \alpha_2 /(\beta_3 + \alpha_2 + \alpha_4 + \alpha_5),\ p_{3,11} = \alpha_4 /(\beta_3 + \alpha_2 + \alpha_4 + \alpha_5)$$
$$p_{4,0} = p_{5,1} = p_{6,1} = p_{7,2} = p_{8,2} = p_{9,3} = p_{10,3} = p_{11,3} = 1 \tag{1}$$

It has been concluded that

$$p_{0,1} + p_{0,2} + p_{0,4} = 1,\ p_{1,0} + p_{1,3} + p_{1,5} + p_{1,6} = 1$$
$$p_{2,0} + p_{2,7} + p_{2,8} = 1,\ p_{3,1} + p_{3,9} + p_{3,10} + p_{3,11} = 1 \tag{2}$$

VI. Mean Sojourn Time

Let μ_i represents the mean sojourn time. Mathematically, the time taken by a system in a particular state becomes

$$\mu_i = \sum_j m_{i,j} = \int_0^\infty P(T > t)\, dt\ .$$

and $\mu_0 = 1/(\alpha_1 + \alpha_3 + \alpha_5)$, $\mu_1 = 1/(\beta_1 + \alpha_2 + \alpha_3 + \alpha_5)$, $\mu_2 = 1/(\beta_2 + \alpha_4 + \alpha_5)$
$\mu_3 = 1/(\beta_3 + \alpha_2 + \alpha_4 + \alpha_5)$, $\mu_4(t) = \mu_5(t) = 1/(\alpha_5)$, $\mu_6 = \mu_{10} = 1/(\alpha_2)$
$\mu_7 = \mu_9 = 1/(\alpha_5)$, $\mu_8 = \mu_{11} = 1/(\alpha_4)$ (3)

VII. Evaluation of Parameters

Using the regenerative point graphical technique, all reliability parameters (including mean time to system failure, availability, busy period of technician and expected number of technician visits) are calculated. It is considered that

$f_1(t) = \beta_1 e^{-\beta_1 t}$, $f_2(t) = \beta_2 e^{-\beta_2 t}$, $f_3(t) = \beta_3 e^{-\beta_3 t}$, $f_4(t) = \beta_4 e^{-\beta_4 t}$, $f_5(t) = \beta_5 e^{-\beta_5 t}$
and, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha$, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta$.

I. Mean Time to System Failure

The system can transition to regenerative un-failed states (i=0, 1, 2, 3) using initial state 0 before reaching any failed state (using base state ξ=0). At that point, MTSF becomes

$$T_0 = \left[\sum_{i=0}^{3} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr(sff)} i\right) \right\} . \mu_i}{\prod_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[1 - \sum Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr(sff)} 0\right) \right\}}{\prod_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$= [\mu_0 + p_{0,1}\mu_1 + p_{0,2}\mu_2 + p_{0,1}p_{1,3}\mu_3]/[1 - p_{0,1}p_{1,0} - p_{0,2}p_{2,0}]$$

$$= \frac{[(\beta + 3\alpha)(\beta + 4\alpha) + \alpha^2](\beta + 2\alpha)}{(\beta + 3\alpha)\left[3\alpha(\beta + 3\alpha) + (\beta + 2\alpha) - \alpha\beta(\beta + 2\alpha) - \alpha\beta(\beta + 3\alpha)\right]} \quad (4)$$

II. Availability of the System

The system is available for use at regenerative states j=0, 1, 2, 3 with ξ=0 then the availability of system is defined as

$$A_0 = \left[\sum_{j=0}^{3} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} j\right) \right\} . f_j . \mu_j}{\prod_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{11} Sr \left\{ \frac{\left\{ pr\left(0 \xrightarrow{Sr} i\right) \right\} . \mu_i'}{\prod_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$= \frac{[U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3]}{\left[\begin{array}{l} U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4 + U_{0,5}\mu_5 \\ + U_{0,6}\mu_6 + U_{0,7}\mu_7 + U_{0,8}\mu_8 + U_{0,9}\mu_9 + U_{0,10}\mu_{10} + U_{0,11}\mu_{11} \end{array} \right]}$$

$$= \frac{\beta^3(\beta+2\alpha)[(\beta+3\alpha)^3+\alpha(\beta+2\alpha)(\beta+3\alpha)]}{\begin{bmatrix} \alpha^3(\beta+2\alpha)[(\beta+3\alpha)^3+\alpha(\beta+2\alpha)(\beta+3\alpha)] \\ +\alpha\beta^2(\beta+2\alpha)(\beta+3\alpha)^3(\beta+2\alpha) \\ +[\alpha^2\beta(2\beta+5\alpha)]\beta(\beta+2\alpha)(\beta+3\alpha)+3\alpha^3\beta^2(\beta+2\alpha)^2 \end{bmatrix}} \tag{5}$$

III. Busy Period of the Technician

The technician is busy due to repair of the failed unit at regenerative states *j*= 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 with base state ξ = 0 then the fraction of time for which the technician remains busy is defined as

$$B_0 = \left[\sum_{j=1}^{11} Sr\left\{\frac{\left\{pr(0\xrightarrow{Sr}j)\right\}.\eta_j}{\Pi_{k_1\neq 0}\left\{1-V_{\overline{k_1k_1}}\right\}}\right\}\right] \div \left[\sum_{i=0}^{11} Sr\left\{\frac{\left\{pr(0\xrightarrow{Sr}i)\right\}.\mu_i'}{\Pi_{k_2\neq 0}\left\{1-V_{\overline{k_2k_2}}\right\}}\right\}\right]$$

$$= \frac{\begin{bmatrix} U_{0,1}\mu_1+U_{0,2}\mu_2+U_{0,3}\mu_3+U_{0,4}\mu_4+U_{0,5}\mu_5+U_{0,6}\mu_6 \\ +U_{0,7}\mu_7+U_{0,8}\mu_8+U_{0,9}\mu_9+U_{0,10}\mu_{10}+U_{0,11}\mu_{11} \end{bmatrix}}{\begin{bmatrix} U_{0,0}\mu_0+U_{0,1}\mu_1+U_{0,2}\mu_2+U_{0,3}\mu_3+U_{0,4}\mu_4+U_{0,5}\mu_5 \\ +U_{0,6}\mu_6+U_{0,7}\mu_7+U_{0,8}\mu_8+U_{0,9}\mu_9+U_{0,10}\mu_{10}+U_{0,11}\mu_{11} \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} \beta^3(\beta+2\alpha)[(\beta+3\alpha)^2\alpha+\alpha(\beta+2\alpha)(\beta+3\alpha)] \\ +\alpha\beta^2(\beta+2\alpha)(\beta+3\alpha)^3(\beta+2\alpha) \\ +[\alpha^2\beta(2\beta+5\alpha)]\beta(\beta+2\alpha)(\beta+3\alpha)+3\alpha^3\beta^2(\beta+2\alpha)^2 \end{bmatrix}}{\begin{bmatrix} \alpha^3(\beta+2\alpha)[(\beta+3\alpha)^3+\alpha(\beta+2\alpha)(\beta+3\alpha)] \\ +\alpha\beta^2(\beta+2\alpha)(\beta+3\alpha)^3(\beta+2\alpha) \\ +[\alpha^2\beta(2\beta+5\alpha)]\beta(\beta+2\alpha)(\beta+3\alpha)+3\alpha^3\beta^2(\beta+2\alpha)^2 \end{bmatrix}} \tag{6}$$

IV. Estimated Number of Visits Made by the Technician

The technician visits at regenerative states *j*= 1, 2, 3 with ξ=0 then the number of visits by the repairman is defined as

$$V_0 = \left[\sum_{j=1}^{3} Sr\left\{\frac{\left\{pr(0\xrightarrow{Sr}j)\right\}}{\Pi_{k_1\neq 0}\left\{1-V_{\overline{k_1k_1}}\right\}}\right\}\right] \div \left[\sum_{i=0}^{11} Sr\left\{\frac{\left\{pr(0\xrightarrow{Sr}i)\right\}.\mu_i'}{\Pi_{k_2\neq 0}\left\{1-V_{\overline{k_2k_2}}\right\}}\right\}\right]$$

$$= \frac{[U_{0,1}\mu_1+U_{0,2}\mu_2+U_{0,3}\mu_3]}{\begin{bmatrix} U_{0,0}\mu_0+U_{0,1}\mu_1+U_{0,2}\mu_2+U_{0,3}\mu_3+U_{0,4}\mu_4+U_{0,5}\mu_5 \\ +U_{0,6}\mu_6+U_{0,7}\mu_7+U_{0,8}\mu_8+U_{0,9}\mu_9+U_{0,10}\mu_{10}+U_{0,11}\mu_{11} \end{bmatrix}}$$

$$= \frac{\beta^3(\beta+2\alpha)[(\beta+3\alpha)^2\alpha+\alpha(\beta+2\alpha)(\beta+3\alpha)]}{\begin{bmatrix}\alpha^3(\beta+2\alpha)[(\beta+3\alpha)^3+\alpha(\beta+2\alpha)(\beta+3\alpha)] \\ +\alpha\beta^2(\beta+2\alpha)(\beta+3\alpha)^3(\beta+2\alpha) \\ +[\alpha^2\beta(2\beta+5\alpha)]\beta(\beta+2\alpha)(\beta+3\alpha)+3\alpha^3\beta^2(\beta+2\alpha)^2\end{bmatrix}} \tag{7}$$

V. Profit Analysis

If a system produces revenue for its developer, it is considered valuable. The availability of the system, busy period of the technician and expected number of visits by the technician are taken into consideration to calculate the profit values of the system. The profit function may be used to do the profit analysis of the system and it is given by

$$P = E_0A_0 - E_1B_0 - E_2V_0 \tag{8}$$

where, E_0 =8000 (Pay per unit uptime of the system)

E_1 =500 (Charge per unit time for which technician is busy)

E_2 =200 (Charge per visit of the technician)

VIII. Discussion

Table 1 describes the nature of the mean time to system failure of the utensil industry. It has an

Table 1: *MTSF vs. Repair Rate*

β ↓	α=0.025	α=0.04	α=0.06
0.01	4.357262	4.200299	3.696809
0.02	4.600326	4.405797	3.903394
0.03	4.832536	4.599156	4.102564
0.04	5.054602	4.781421	4.29471
0.05	5.267176	4.953519	4.480198
0.06	5.470852	5.116279	4.659367
0.07	5.666179	5.27044	4.832536
0.08	5.853659	5.416667	5.25355
0.09	6.033755	5.555556	5.162037
0.10	6.206897	5.687646	5.318907

increasing trend corresponding to increment in repair rate (β) and has decreasing trend corresponding to an increment in failure rate (α). In the above table, the values of parameters are α= 0.025, 0.04, 0.06 and β=0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 respectively. When the value of repair rate enhances then MTSF values are also enhanced. When α=0.025 changes into α=0.04, 0.06 then MTSF values are declined.

Table 2 explores the increasing trends of availability with respect to increments in repair rate (β) and has decreasing trends corresponding to increments in failure rate (α). When the value of the repair rate is enhanced then the availability values are also enhanced. Also, when the failure rate of unit changes α=0.025 to 0.04, 0.06 then the availability of system declines.

Table 2: *Availability vs. Repair Rate*

β ↓	α=0.025	α=0.04	α=0.06
0.01	0.623324	0.604782	0.542904
0.02	0.628307	0.609813	0.547959
0.03	0.633159	0.614717	0.552904
0.04	0.637887	0.619499	0.557741
0.05	0.642494	0.624164	0.562476
0.06	0.646985	0.628716	0.56711
0.07	0.651365	0.633159	0.571646
0.08	0.655637	0.637497	0.576089
0.09	0.659806	0.641734	0.58044
0.10	0.663876	0.645873	0.584703

Table 3 explores the trend of profit values with respect to repair rate (β) and its value increase corresponding to increments in repair rate (β) and decrease corresponding to increments in failure rate (α). It is concluded that when the value of the repair rate enhances then profit values are also enhanced but when the failure rate of the unit changes α=0.025 to 0.04, 0.06 then the profit of the system declines.

Table 3: *Profit vs. Repair Rate*

β ↓	α=0.025	α=0.04	α=0.06
0.01	2438.338	2386.076	2019.52
0.02	2467.262	2415.876	2049.467
0.03	2495.431	2444.927	2078.759
0.04	2522.874	2473.257	2107.417
0.05	2549.618	2500.892	2135.461
0.06	2575.691	2527.857	2162.912
0.07	2601.117	2554.178	2189.787
0.08	2625.919	2579.875	2216.104
0.09	2650.121	2604.972	2241.881
0.10	2673.744	2629.49	2267.135

IX. Conclusion

The regenerative point graphical technique is used to calculate the performance of the utensil industry. According to the given tables, it is clear that MTSF, availability and profit values increased with increment in repair rate but these reliability measures decreased with increment in failure rate. It is observed that if the system is more available then it gives more profit to the developer. It is evident that industries can examine the behavior of products and system components with the help of the regenerative point graphic technique.

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