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BULK ARRIVING RETRIAL QUEUE WITH G-QUEUE AND RENEGING CLIENTS

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Abstract

We consider a server queue with negative clients (G-Queue) in this effort, where clients are serviced one after the other in batches in a system of variable size. Additionally, we presumptively have a general distribution for the service times, delay times, and repair times. For various states, we concrete the probability-generating functions for the number of customers in the orbit. We scrutinize a single server queue with batches of reneging or balking clients in a system of variable size in this work. Different performance measures and unique situations are examined. The outcomes of this work have applications in satellite communication, software-design for various computer-communication systems and mailing systems among other things.

Keywords: G-Queue, Retrial Queue, Bulk, Reneging Clients, Sudden Breakdown

1. Introduction

The concept of positive and negative consumers coming in a queueing system received further interest and was researched due to its usage in organizations, industry, manufacturing, computer field, and network systems. This study [7] proposed such queues (G-Queues) for the first time to simulate neural networks. In [10] tremendous improvements have been made to the wait times for retrials and vacations. Adapted from [2], in [15] discussed the $M/G/1$ retrial queueing system, which has two service phases and immediate feedback. In this system, the regular busy server is impacted by the arrival of negative customers. By incorporating the idea of G-queues with immediate feedback used by [17]. A finite-source retrial queueing system is considered in [14] along with impatient clients and catastrophic failures. In [13] considered a modified Bernoulli vacation schedule with negative arrivals, reneging and starting failure.

Many authors have examined the queueing issues caused by different combinations of server vacations. A literature review on queues with server vacations can always be found in [6]. Consider this reliability modelling with G-queues in [8], when server failures are described by the arrival of negative clients that cause certain clients to lose service. It was taken into account by [12] to evaluate the queue containing feedback and server vacations (optional) utilizing an SVP (single vacation policy). In [9] examined a bulk- arriving with a server(starting) and more J service options. A batch arrival queue with an additional service channel was researched by [5] under -policy. Retrial queueing technique and balking clients are delved by [3, 4].

Both [1, 11] provided retry queues that take into consideration server faults and repair. The queueing indices and reliability features of an RRQM (repairable retrial queueing model) were investigated by [16] in terms of reliability. On the M/G/1 retrial queue model with service, we have reviewed a variety of academic publications. This work is motivated by the Retrial Queue model (RQM), which includes service and repair.

The remainder of this article is categorized as follows. We provide a brief mathematical overview and its application of the model is specific in Section 2. The notations and the number of consumers in the orbit/system at a steady state are shown in Section 3 and Section 4. The system performance metrics and numerical outcomes are presented in Section 5 and Section 6. The work's conclusion is stated in section 7.

2. The Mathematical Model's descriptions

Consider an *SSRQM* (single server retry queueing system) with negative and positive independent arrivals. Assume that both categories of customers enter the system using separate Poisson processes with rates of λ and δ respectively. The bulk size Y is a RV (random variable) with *df* (distribution function) $P(\tilde{\tilde{Y}} = k) = \tilde{\tilde{T}}_k, k = 1, 2, \ldots .,$.

If a huge proportion of positive consumers discover the server free upon arrival, any newly incoming customer begins his service, and others join the orbit. When positive customer enter the service with *prob.,* (probability) 1-$\tilde{\tilde{b}}$ and exit with probability $\tilde{\tilde{b}}$, balking (or reneging) may occur.

One of the arrivals starts his service, and others join the orbital, if a batch of affirmative clients finds the server unoccupied upon arrival. The generic distribution for the retrial queue is *DF*(distribution function) $A_l(\tilde{x}_1)$ with associated it LST(Laplace Stieltjes transform) $A_l^*(s)$ and *HR*(Hazard rate) $Y(\tilde{x}_1)d\tilde{x} = \frac{dA_l(\tilde{x}_1)}{1-A_l(\tilde{x}_1)}$.

This service-time also follows a generic distribution with *DF* $B_l(\tilde{x}_1)$, *LST* $B_l^*(s)$, n^{th} factorial moments $\tilde{\tilde{i}}_n$ and its *HR*, $\mu(\tilde{x}_1)d\tilde{x} = \frac{dB_l(\tilde{x}_1)}{1-B_l(\tilde{x}_1)}$. By the Poisson process, negative consumers come individually at a rate of δ. A server breakdown occurs when a negative client gets into the system, removing the server's functioning positive client. The server stops service and waits for repairs to begin whenever it fails.

This waiting time of the server is known as delay time. The Delay time follows a general distribution with *DF* $E_l(\tilde{x}_1)$, *LST* $E_l^*(s)$, n^{th} factorial moments $\tilde{\tilde{k}}_n$ and its *HR*, $\chi(\tilde{x_1})d\tilde{x_1} = \frac{dE_l(\tilde{x_1})}{1-E_l(\tilde{x}_1)}$. When a negative customer comes up, the system no longer has the positive customer in service, which forces the server to breakdown. When a server breaks down, it stops service and waits for repair to begin. The server's waiting period of time is known as the delay time. Furthermore, the repair time has a general distribution with *DF* $F_l(\tilde{x}_1)$, *LST*$F_l^*(s)$, n^{th} factorial moments $\tilde{\tilde{l}}_n$ and its *HR*, $\xi(\tilde{x_1}) = \frac{dF_l(\tilde{x}_1)}{1-F_l(\tilde{x}_1)}$.

2.1. Application of the Model in Real Life

The size of the message buffer (orbit) of a CPS (computer processing system), where messages (customers) are received at a time. The work of processing communications falls on the processor (server). A virus infection (a negative customer) might affect the active mail server, and electronic failures (breakdowns) could occur at any time throughout the service term and require urgent repair. At that time, if the processor is not available, FCFS temporarily stores the messages in a buffer to be served later (retrial time). When all messages have been treated (processed) and there are no pending new messages, the processor will carry out several maintenance procedures, such as virus scanning, to improve the computer's performance. The processor checks the messages after each maintenance process is done before deciding whether to restore the rate of the standard services. If the system is currently empty of messages, the processor may decide to do another maintenance task.

3. Probability Notations

The system's stochastic processes are all considered to be independent from one another. We now introduce some more notations that will be utilized in this model's mathematical formulation. Let $C(\tilde{t})$ be the server state, where $C(\tilde{t})$

$$C(\tilde{t}) = \begin{cases} 0 & \longrightarrow \text{ idle (server)} \\ 1 & \longrightarrow \text{ busy (server)} \\ 2 & \rightarrowtail \text{ server is repair(waiting process)} \\ 3 & \longrightarrow \text{ server is repair(under process)} \end{cases}$$

Then the process $\{C(\tilde{t}), \tilde{\tilde{N}}(\tilde{t}); \tilde{t} \geq 0\}$ is a Markov Process.
Define the following probabilities are, for $\tilde{t} \geq 0$

$\tilde{\tilde{I}}_0(\tilde{t}) = P\{\{C(\tilde{t}) = 0, \tilde{\tilde{N}}(\tilde{t}) = 0\}$

$\tilde{\tilde{I}}_{\tilde{n}}(\tilde{x}_1, \tilde{t})d\tilde{x}_1 = P\{\{C(\tilde{t}) = 0, \tilde{\tilde{N}}(\tilde{t}) = \tilde{n}\}, \tilde{x}_1 < A_l^0(\tilde{t}) \leq \tilde{x}_1 + d\tilde{x}_1, \ \ \tilde{n} \geq 1,$

$\tilde{\tilde{M}}_{\tilde{n}}(\tilde{x}_1, \tilde{t})d\tilde{x}_1 = P\{\{C(\tilde{t}) = 1, \tilde{\tilde{N}}(\tilde{t}) = \tilde{n}\}, \tilde{x}_1 < B_l^0(\tilde{t}) \leq \tilde{x}_1 + d\tilde{x}_1, \ \ \tilde{n} \geq 0$

$\tilde{\tilde{Q}}_{\tilde{n}}(\tilde{x}_1, \tilde{t})d\tilde{x}_1 = P\{\{C(\tilde{t}) = 2, \tilde{\tilde{N}}(\tilde{t}) = \tilde{n}\}, \tilde{x}_1 < E_l^0 \leq \tilde{x}_1 + d\tilde{x}_1$

$\tilde{\tilde{R}}_{\tilde{n}}(\tilde{x}_1, \tilde{t})d\tilde{x}_1 = P\{\{C(\tilde{t}) = 3, \tilde{\tilde{N}}(\tilde{t}) = \tilde{n}\}, \tilde{x}_1 < F_l^0 \leq \tilde{x}_1 + d\tilde{x}_1$

4. Steady State Equations

The collection of equations governing the dynamics of the system behaviour in steady state is obtained using the *SVM*(supplementary variable method) as follows:

$$\tilde{\tilde{b}}\lambda\tilde{\tilde{I}}_0 = \int_0^\infty \tilde{\tilde{I}}_0(\tilde{x}_1)\Theta(\tilde{x}_1)d\tilde{x}_1 + \int_0^\infty \tilde{\tilde{R}}_0\xi(\tilde{x}_1)d\tilde{x}_1 \tag{1}$$

$$\frac{d\tilde{\tilde{I}}_{\tilde{n}}(\tilde{x}_1)}{d\tilde{x}_1} + (\lambda + \Upsilon(\tilde{x}_1))\tilde{\tilde{I}}_{\tilde{n}}(\tilde{x}_1) = 0, \ \tilde{n} \geq 1 \tag{2}$$

$$\frac{d\tilde{\tilde{M}}_{\tilde{n}}(\tilde{x}_1)}{d\tilde{x}_1} + (\tilde{\tilde{b}}\lambda + \delta + \mu(\tilde{x_1}))\tilde{\tilde{M}}_{\tilde{n}}(\tilde{x}_1) = \tilde{\tilde{b}}\lambda\sum_{k=0}^{\infty}\tilde{\tilde{T}}_k\tilde{\tilde{M}}_{\tilde{n}-k}(\tilde{x}_1) \tag{3}$$

$$\frac{d\tilde{\tilde{Q}}_{\tilde{n}}(\tilde{x}_1)}{d\tilde{x}_1} + (\tilde{\tilde{b}}\lambda + \chi(\tilde{x}_1))\tilde{\tilde{Q}}_{\tilde{n}}(\tilde{x}_1) = \tilde{\tilde{b}}\lambda\sum_{k=0}^{\infty}\tilde{\tilde{T}}_k\tilde{\tilde{Q}}_{\tilde{\tilde{n}}-k}(\tilde{x}_1) \tag{4}$$

$$\frac{d\tilde{\tilde{R}}_{\tilde{n}}(\tilde{x}_1)}{d\tilde{x}_1} + (\tilde{\tilde{b}}\lambda + \xi(\tilde{x}_1))\tilde{\tilde{R}}_{\tilde{n}}(\tilde{x}_1) = \tilde{\tilde{b}}\lambda\sum_{k=0}^{\infty}\tilde{\tilde{T}}_k\tilde{\tilde{R}}_{\tilde{n}-k}(\tilde{x}_1) \tag{5}$$

The *B.c* (boundary conditions) are

$$\tilde{\tilde{I}}_{\tilde{n}}(0) = \int_0^\infty \tilde{\tilde{I}}_0(\tilde{x}_1)\Theta(\tilde{x}_1)d\tilde{x_1} + \int_0^\infty \tilde{\tilde{R}}_0\xi(\tilde{x}_1)d\tilde{x_1} - \tilde{\tilde{b}}\lambda\tilde{\tilde{I}}_0 \tag{6}$$

$$\tilde{\tilde{M}}_0(0) = \lambda\tilde{\tilde{T}}_1\tilde{\tilde{I}}_0 + \int_0^\infty \tilde{\tilde{I}}_1(\tilde{x}_1)\Upsilon(\tilde{x}_1)d(\tilde{x}_1) \tag{7}$$

$$\tilde{\tilde{M}}_{\tilde{n}}(0) = \lambda\tilde{\tilde{T}}_{\tilde{n}+1}\tilde{\tilde{I}}_0 + \int_0^\infty \tilde{\tilde{I}}_{\tilde{n}+1}(\tilde{x}_1)\Upsilon(\tilde{x}_1)d(\tilde{x}_1) + \lambda\sum_{k=0}^{\infty}\tilde{\tilde{T}}_k\int_0^\infty \tilde{\tilde{M}}_{\tilde{n}-k+1}(\tilde{x}_1)d(\tilde{x}_1) \tag{8}$$

$$\tilde{\tilde{Q}}_{\tilde{n}}(0) = \delta \int_0^\infty \tilde{\tilde{M}}_{\tilde{n}}(\tilde{x}_1) d(\tilde{x}_1) \tag{9}$$

$$\tilde{\tilde{R}}_{\tilde{n}}(0) = \int_0^\infty \tilde{\tilde{Q}}_{\tilde{n}}(\tilde{x}_1) \chi(\tilde{x}_1) d(\tilde{x}_1) \tag{10}$$

Normalization Condition is

$$\tilde{\tilde{I}}_0 + \sum_{\tilde{n}=1}^{\infty} \int_0^\infty \tilde{\tilde{I}}_{\tilde{n}}(\tilde{x}_1) d(\tilde{x}_1) + \sum_{\tilde{n}=1}^{\infty} \int_0^\infty \tilde{\tilde{M}}_{\tilde{n}}(\tilde{x}_1) d(\tilde{x}_1) + \sum_{\tilde{n}=0}^{\infty} \int_0^\infty \tilde{\tilde{Q}}_{\tilde{n}}(\tilde{x}_1) d(\tilde{x}_1) + \sum_{\tilde{n}=0}^{\infty} \int_0^\infty \tilde{\tilde{R}}_{\tilde{n}}(\tilde{x}_1) d(\tilde{x}_1) \tag{11}$$

The following findings are obtained by multiply equ (2) - (10) by $\tilde{z_1}^{\tilde{n}}$ and adding all values(possible) of $\tilde{n}$:

$$\frac{d\tilde{\tilde{I}}(\tilde{x_1}, \tilde{z}_1)}{d\tilde{x}_1} + (\lambda + \Upsilon(\tilde{x}_1))\tilde{\tilde{I}}(\tilde{x}_1, \tilde{z}_1) = 0 \tag{12}$$

$$\frac{d\tilde{\tilde{M}}(\tilde{x}_1, \tilde{z}_1)}{d\tilde{x}_1} + (\tilde{\tilde{b}}\lambda(1 - \tilde{\tilde{T}}(\tilde{z_1})) + \delta + \mu(\tilde{x}_1)\tilde{\tilde{M}}(\tilde{x_1}, \tilde{z}_1) = 0 \tag{13}$$

$$\frac{d\tilde{\tilde{Q}}(\tilde{x}_1, \tilde{z}_1)}{d\tilde{x}_1} + (\tilde{\tilde{b}}\lambda(1 - \tilde{\tilde{T}}(\tilde{z}_1)) + \chi(\tilde{x}_1))\tilde{\tilde{Q}}(\tilde{x}_1, \tilde{z}_1) = 0 \tag{14}$$

$$\frac{d\tilde{\tilde{R}}(\tilde{x}_1, \tilde{z}_1)}{d\tilde{x}_1} + (\tilde{\tilde{b}}\lambda(1 - \tilde{\tilde{T}}(\tilde{z}_1)) + \xi(\tilde{x}_1))\tilde{\tilde{R}}(\tilde{x}_1, \tilde{z}_1) = 0 \tag{15}$$

Equations (12) to (15), using to solve partial differential

$$\tilde{\tilde{I}}(\tilde{x}_1, \tilde{z}_1) = \tilde{\tilde{I}}(0, \tilde{z}_1)[1 - A_l(\tilde{x}_1)]e^{-\lambda \tilde{x}_1} \tag{16}$$

$$\tilde{\tilde{M}}(\tilde{x}_1, \tilde{z}_1) = \tilde{\tilde{M}}(0, \tilde{z}_1)[1 - B_l(\tilde{x}_1)]e^{-\tilde{N}(\tilde{z}_1)\tilde{x}_1} \tag{17}$$

$$\tilde{\tilde{Q}}(\tilde{x}_1, \tilde{z}_1) = \tilde{\tilde{Q}}(0, \tilde{z}_1)[1 - E_l(\tilde{x}_1)]e^{-O(\tilde{z}_1)\tilde{x}_1} \tag{18}$$

$$\tilde{\tilde{R}}(\tilde{x}_1, \tilde{z}_1) = \tilde{\tilde{R}}(0, \tilde{z}_1)[1 - F_l(\tilde{x}_1)]e^{O(\tilde{z}_1)\tilde{x}_1} \tag{19}$$

where $N(\tilde{z_1}) = O(\tilde{z_1}) + \delta$, and $O(\tilde{z_1}) = \tilde{\tilde{b}}\lambda(1 - \tilde{\tilde{T}}(\tilde{z}_1))$

$$\tilde{\tilde{I}}(0, \tilde{z}_1) = \int_0^\infty \tilde{\tilde{M}}(\tilde{x}_1, \tilde{z}_1)\Theta(\tilde{x}_1) d\tilde{x}_1 + \int_0^\infty \tilde{\tilde{R}}(\tilde{x}_1, \tilde{z}_1)\xi(\tilde{x}_1) d\tilde{x}_1 - \tilde{\tilde{b}}\lambda\tilde{\tilde{I}}_0 \tag{20}$$

$$\tilde{\tilde{M}}(0, \tilde{z}_1) = \frac{1}{\tilde{z}_1} \int_0^\infty \tilde{\tilde{M}}(\tilde{x}_1, \tilde{z}_1)\Upsilon(\tilde{x}_1) d\tilde{x_1} + \frac{\lambda\tilde{\tilde{T}}(\tilde{z}_1)}{\tilde{z}_1}\left[\int_0^\infty \tilde{\tilde{I}}(\tilde{x}_1, \tilde{z}_1) d\tilde{x_1} + \tilde{b}\lambda\tilde{\tilde{I}}_0\right] \tag{21}$$

$$\tilde{\tilde{Q}}(0, \tilde{z}_1) = \delta \int_0^\infty \tilde{\tilde{M}}(\tilde{x_1}, \tilde{z}_1) d\tilde{x_1} \tag{22}$$

$$\tilde{\tilde{R}}(0, \tilde{z}_1) = \int_0^\infty \tilde{\tilde{D}}(\tilde{x_1}, \tilde{z}_1)\xi(\tilde{x}_1) d\tilde{x_1} \tag{23}$$

The orbital size partial *PGF* (probability generating function) while the server is inactive, active, waiting for repair, under repair

$$\tilde{\tilde{I}}(\tilde{z}_1) = \frac{\begin{bmatrix} \tilde{\tilde{I}}_0(1 - A_l^*(\lambda))\tilde{\tilde{b}}\{N(\tilde{z}_1)B_l^*(N(\tilde{Z}_1)) \\ +\delta\tilde{\tilde{T}}(z_1)(1 - B_l^*(N(\tilde{z}_1)))E^*O(\tilde{z}_1)F^*(O(\tilde{z_1}))\} \end{bmatrix}}{\begin{bmatrix} \tilde{z_1}N(\tilde{z}_1) - \{[A_l^*(\lambda) + \tilde{\tilde{T}}_{\tilde{z}_1}(1 - A_l^*(\lambda))]N(\tilde{z}_1)B_l^*N(\tilde{z}_1) \\ +\delta(1 - B_l^*(N(\tilde{z}_1)))E_l^*(O(\tilde{z}_1))F_l^*(O(\tilde{z_1}))\} \end{bmatrix}} \tag{24}$$

$$\tilde{\tilde{M}}(\tilde{z}_1) = \frac{\tilde{\tilde{I}}_0 A_l^*(\lambda)(B_l^*(N(\tilde{z}_1)) - 1)}{\begin{bmatrix} \tilde{z_1} N(\tilde{z}_1) - \{[A_l^*(\lambda) + \tilde{\tilde{T}}_{\tilde{z}_1}(1 - A_l^*(\lambda))] N(\tilde{z}_1) B_l^* N(\tilde{z}_1) \\ +\delta(1 - B_l^*(N(\tilde{z}_1))) E_l^*(O(\tilde{z}_1)) F_l^*(O(\tilde{z_1}))\} \end{bmatrix}} \tag{25}$$

$$\tilde{\tilde{Q}}(\tilde{z}_1) = \frac{\tilde{\tilde{I}}_0 A_l^*(\lambda)\delta(E_l^*(N(\tilde{z}_1)) - 1)(1 - B_l^*(N(\tilde{z}_1)))}{\begin{bmatrix} \tilde{z_1} N(\tilde{z}_1) - \{[A_l^*(\lambda) + \tilde{\tilde{T}}_{\tilde{z}_1}(1 - A_l^*(\lambda))] N(\tilde{z}_1) B_l^* N(\tilde{z}_1) \\ +\delta(1 - B_l^*(N(\tilde{z}_1))) E_l^*(O(\tilde{z}_1)) F_l^*(O(\tilde{z_1}))\} \end{bmatrix}} \tag{26}$$

$$\tilde{\tilde{R}}(\tilde{z}_1) = \frac{\tilde{\tilde{I}}_0 A_l^*(\lambda)\delta(F_l^*(N(\tilde{z}_1)) - 1)(1 - B_l^*(N(\tilde{z})))E_l^*(N(\tilde{z}_1))}{\begin{bmatrix} \tilde{z_1} N(\tilde{z}_1) - \{[A_l^*(\lambda) + \tilde{\tilde{T}}_{\tilde{z}_1}(1 - A_l^*(\lambda))] N(\tilde{z}_1) B_l^* N(\tilde{z}_1) \\ +\delta(1 - B_l^*(N(\tilde{z}_1))) E_l^*(O(\tilde{z}_1)) F_l^*(O(\tilde{z_1}))\} \end{bmatrix}} \tag{27}$$

Since $\tilde{\tilde{I}}_0$ can be calculated using the normalization condition and represents the probability that the server would be idle while there are no customers in the orbit,

$$\tilde{\tilde{I}}_0 = \left\{ \frac{\begin{matrix} \delta(1 - \tilde{\tilde{j}}_1 + \tilde{\tilde{j}}_1 A_l{}^*(\lambda)) - \lambda \tilde{\tilde{j}}_1 (1 - B_l^*(\delta)) \\ (1 + \delta\tilde{\tilde{k}}_1 + \delta\tilde{\tilde{l}}_1) \end{matrix}}{\begin{matrix} \delta(1 - \tilde{\tilde{b}})(1 - \tilde{\tilde{j}}_1 + \tilde{\tilde{j}}_1 A_l{}^*(\lambda)) + \tilde{\tilde{b}}\delta A_l{}^*(\lambda) \\ -(1 - \tilde{\tilde{b}})(A_l{}^*(\lambda))\lambda\tilde{\tilde{j}}_1(1 - B_l^*(\delta))(1 + \delta\tilde{\tilde{k}}_1 + \delta\tilde{\tilde{l}}_1) \end{matrix}} \right\}$$

We establish the following definitions for the *PGF* for the system's customers:

$$\tilde{\tilde{K}}_l(\tilde{z}_1) = \tilde{\tilde{I}}_0 + \tilde{\tilde{I}}(\tilde{z}_1) + \tilde{\tilde{M}}(\tilde{z}_1) + \tilde{\tilde{Q}}(\tilde{z}_1) + \tilde{\tilde{R}}(\tilde{z}_1)$$
$$\tilde{\tilde{K}}_l(\tilde{z}_1) = \tilde{\tilde{I}}_0\{\tfrac{N1}{D1}\}$$

$$N1 = \left\{ \begin{matrix} \tilde{z_1} N(\tilde{z_1})[1 - \tilde{\tilde{b}}(1 - A_l{}^*(\lambda))] + (\tilde{\tilde{b}} - 1)(1 - A_l{}^*(\lambda))\tilde{\tilde{T}}(\tilde{z})[N(\tilde{z})B^*(N(\tilde{z_{11}})) \\ +\delta(1 - B_l^*(N(\tilde{z}_1)))\tilde{E^*}(O(\tilde{z}_1))F_l^*(O(\tilde{z}_1))] - A^*(\lambda)[B_l^*(N(\tilde{z}_1)) \\ +(1 - B_l^*(N(\tilde{z}_1)))(\tilde{z_1}(O(\tilde{z}_1)) + \delta)] \end{matrix} \right\}$$

$$D1 = \left\{ \begin{matrix} \tilde{z_1} N(\tilde{z}_1) - [A_l^*(\lambda) + \tilde{\tilde{T}}_{\tilde{z_1}}(1 - A_l^*(\lambda))]\{N(\tilde{z_1})B_l^*(N(\tilde{z}_1)) + \delta(1 - B_l^*(N(\tilde{z_1}))) \\ E_l^*(O(\tilde{z_1}))F_l^*(O(\tilde{z}_1))\} \end{matrix} \right\}$$

We define the probability generating functions of the number of customers in the orbit, where $\tilde{\tilde{H}}_l(\tilde{z}_1) = \tilde{\tilde{I}}_0 + \tilde{\tilde{I}}(\tilde{z_1}) + \tilde{z_1}\tilde{\tilde{M}}(\tilde{z}_1) + \tilde{\tilde{Q}}(\tilde{z_1}) + \tilde{\tilde{R}}(\tilde{z}_1)$ and $\tilde{\tilde{H}}_l(\tilde{z}_1) = \tilde{\tilde{I}}_0 \frac{N2}{D1}$

$$N2 = \left\{ \begin{matrix} \tilde{z_1} N(\tilde{z_1})[1 - \tilde{\tilde{b}}(1 - A_l{}^*(\lambda))] - A_l{}^*(\lambda)[B_l^*(N(\tilde{z_1})) \\ +(1 - B_l^*(N(\tilde{z_1})))(\tilde{z_1}(O(\tilde{z}_1)) + \delta)] \\ +(\tilde{\tilde{b}} - 1)(1 - A_l{}^*(\lambda))\{\tilde{\tilde{T}}(\tilde{z}_1)[N(\tilde{z}_1)B_l^*(N(\tilde{z}_1)) \\ +\delta(1 - B_l^*(N(\tilde{z}_1)))E_l^*(O(\tilde{z}_1))F_l^*(O(\tilde{z}_1))]\} \end{matrix} \right\}$$

5. Performance Measures

We derive the system performance of our model.
By differentiating $\tilde{\tilde{K}}_l(\tilde{z}_1)$ with respect to $\tilde{z_1}$ and evaluating at $\tilde{z_1} = 1$, the average number of consumers in the system $\tilde{\tilde{L}}_s$ in steady-statestate conditions may be determined.

$$\tilde{\tilde{L}}_s = \lim_{\tilde{z} \to 1} \tilde{\tilde{K}}'(\tilde{z})$$

$$\tilde{\tilde{L}}_s = \tilde{\tilde{I}}_0 \frac{(Dr' Nr_1'' - Nr_1' Dr'')}{2(Dr')^2}$$

$$Nr_1' = \begin{Bmatrix} (-\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1 + \delta)[1 - \tilde{\tilde{b}}(1 - A_l^*(\lambda))] + A_l^*(\lambda)\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1 + (\tilde{\tilde{b}} - 1)(1 - A_l^*(\lambda)) \\ \{\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1[-B_l^*(\delta) + \delta(1 - B_l^*(\delta))(\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1)] + \delta \tilde{\tilde{j}}_1\} \end{Bmatrix}$$

$$Nr_1'' = -\begin{Bmatrix} [\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2 + 2\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1][1 - \tilde{\tilde{b}}(1 - A_l^*(\lambda))] + A_l^*(\lambda)B_l^*(\delta)\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2 \\ +(\tilde{\tilde{b}} - 1)(1 - A_l^*(\lambda))\{2\tilde{\tilde{j}}_1(-\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1 B_l^*(\delta) \\ +\delta(1 - B_l^*(\delta))\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1(\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1)) + \delta \tilde{\tilde{j}}_2 - (\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2 B_l^*(\delta) \\ +2B_l^*(\delta)\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_i \tilde{\tilde{i}}_1 + 2\delta(B_l^*\lambda \tilde{\tilde{j}}_1)^2 \tilde{\tilde{i}}_1(\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1)) \\ +\delta(1 - B_l^*(\delta))[(\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1)^2(\tilde{\tilde{l}}_2 + \tilde{\tilde{k}}_2) \\ +\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2(\tilde{\tilde{l}}_1 + \tilde{\tilde{k}}_1) + 2(\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1)^2 \tilde{\tilde{k}}_1 \tilde{\tilde{l}}_1]\} \end{Bmatrix}$$

$$Dr' = \delta(1 - \tilde{\tilde{j}}_1 + \tilde{\tilde{j}}_1 A_l^*(\lambda)) - \lambda \tilde{\tilde{j}}_1(1 - B_l^*(\delta))[1 + \delta(\tilde{\tilde{l}}_1 + \tilde{\tilde{k}}_1)]$$

$$Dr'' = \begin{Bmatrix} -\lambda \tilde{\tilde{j}}_2(1 - B_l^*(\delta)) - 2\lambda \tilde{\tilde{j}}_2 - \delta \tilde{\tilde{j}}_2(1 - A_l^*(\lambda)) \\ -2\tilde{\tilde{j}}_1(1 - A_l^*(\lambda))(-\lambda \tilde{\tilde{j}}_1 B_l^*(\delta) + \delta\lambda \tilde{\tilde{j}}_1(1 - B_l^*(\delta))(\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1)) \\ +2(\lambda \tilde{\tilde{j}}_1)^2 \tilde{\tilde{i}}_1(\delta + (\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1)) - 2\delta(\lambda \tilde{\tilde{j}}_1)^2(1 - B_l^*(\delta))\tilde{\tilde{k}}_1 \tilde{\tilde{l}}_1 \\ -\delta\lambda \tilde{\tilde{l}}_2(\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1)(1 - B_l^*(\delta)) - (\lambda \tilde{\tilde{j}}_1)^2(1 - B_l^*(\delta))(\tilde{\tilde{k}}_2 + \tilde{\tilde{l}}_2) \end{Bmatrix}$$

By differentiating $\tilde{\tilde{H}}_l(\tilde{z}_1)$ with respect to $\tilde{z}_1$ and evaluating at $\tilde{z}_1 = 1$, the average number of consumers in the orbit $\tilde{\tilde{L}}_q$ in steady-statestate conditions may be determined

$$\tilde{\tilde{L}}_q = \lim_{\tilde{z} \to 1} \tilde{\tilde{H}}_l'(\tilde{z})$$

$$\tilde{\tilde{L}}_q = \tilde{\tilde{I}}_0 \frac{(Dr' Nr_2'' - Nr_2' Dr'')}{2(Dr')^2}$$

$$Nr_2' = \begin{Bmatrix} [-\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1 + \delta][1 - \tilde{\tilde{b}}(1 - A_l^*(\lambda))] + A_l^*(\lambda)\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1 A_l^*(\lambda) \\ +(\tilde{\tilde{b}} - 1)(1 - A_l^*(\lambda))\{ -\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1 B_l^*(\delta) \\ +\delta[1 - B_l^*(\delta)]\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1(\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1) + \delta \tilde{\tilde{j}}_1\} \end{Bmatrix}$$

$$Nr_2'' = \begin{Bmatrix} -[\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2 + 2\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1][1 - \tilde{\tilde{b}}(1 - A_l^*(\lambda))] + A_l^*(\lambda)[\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2 B_l^*(\delta) + \delta\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1 \tilde{\tilde{i}}_1 \\ +\delta(\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1)^2 \tilde{\tilde{i}}_2 + \delta\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2 \tilde{\tilde{i}}_1 - \tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2(1 - B^*(\delta))] \\ +(\tilde{\tilde{b}} - 1)(1 - A_l^*(\lambda))2[\tilde{\tilde{j}}_1(-\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1)B_l^*(\delta) + \delta(1 - B_l^*(\delta))\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1(\tilde{\tilde{k}}_1 \\ +\tilde{\tilde{l}}_1) - ((\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2)B_l^*(\delta)) + 2(\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1)^2[\tilde{\tilde{i}}_1 + \delta \tilde{\tilde{i}}_1(\tilde{\tilde{k}}_1 + \tilde{\tilde{l}}_1)] \\ +[1 - B_l^*(\delta)]((\tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_1)^2[\tilde{\tilde{k}}_2 + \tilde{\tilde{l}}_2 + 2\tilde{\tilde{k}}_1 \tilde{\tilde{l}}_1] + \tilde{\tilde{b}}\lambda \tilde{\tilde{j}}_2(\tilde{\tilde{l}}_1 + \tilde{\tilde{k}}_1))] \end{Bmatrix}$$

6. Numerical Results

We demonstrate the various settings on system behavior measurements in this section using MATLAB. We examine retrial times, service times, service times with a reduced speed, vacation times, delayed repair times, and exponentially distributed repair times. To meet the stability condition, the numerical measurements are chosen at random. Regarded predicted values of our model's varying metrics, such as the typical queue size and the probability that the server isn't active while retries, and the likelihood that the server is idle overall. and the likelihood that the server is idle overall. $\lambda = 1, \delta = 1, \tilde{\tilde{j}}_1 = 1, \tilde{\tilde{j}}_1 = 0, \mu = 15, \chi = 0.9, \xi = 0.8, \tilde{\tilde{b}} = 0.6$.

In Table 1 represents the effect of retrial rate (Υ) on $\tilde{\tilde{I}}_0$, $\tilde{\tilde{I}}(1)$, $\tilde{\tilde{M}}(1)$, $\tilde{\tilde{Q}}(1)$ and L_q

Table 1: *The effect of retrial rate (Υ) on $\tilde{\tilde{I}}_0$, $\tilde{\tilde{I}}(1)$, $\tilde{\tilde{M}}(1)$, $\tilde{\tilde{Q}}(1)$ and L_q*

Υ	$\tilde{\tilde{I}}_0$	$\tilde{\tilde{I}}(1)$	$\tilde{\tilde{M}}(1)$	$\tilde{\tilde{Q}}(1)$	$\tilde{\tilde{R}}(1)$	L_q
6	0.9169	0.0079	0.0628	0.0055	0.0105	1.2886
7	0.9171	0.0067	0.0627	0.0054	0.0104	1.3339
8	0.9176	0.0059	0.0626	0.0053	0.0103	1.3692
9	0.9180	0.0052	0.0625	0.0052	0.0102	1.3976
10	0.9185	0.0047	0.0624	0.0051	0.0101	1.4208

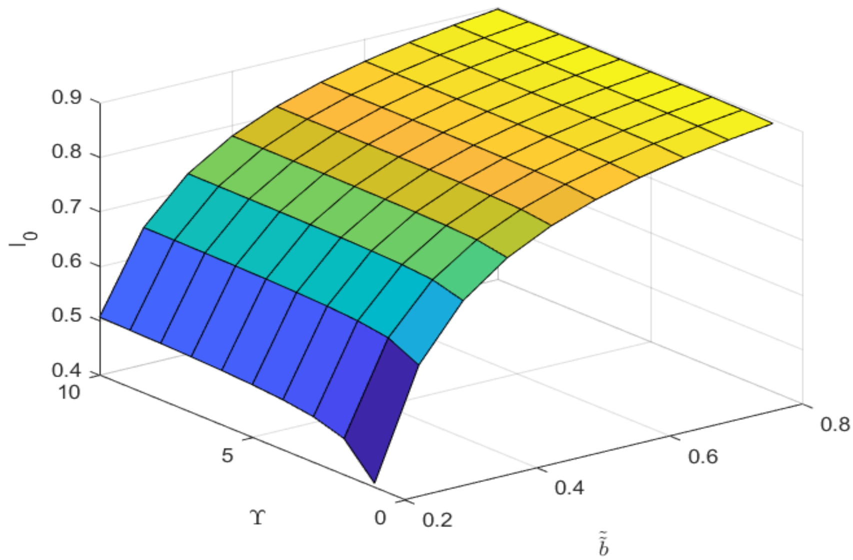

Figure 1: *I_0 versus $\tilde{\tilde{b}}$ and Υ*

Figures provide illustrations of three-dimensional graphs (1–5).
Figure 1 demonstrates $\tilde{\tilde{b}}$ and Υ increases I_0 also increases, Figure 2 demonstrates $\tilde{\tilde{b}}$ and μ increases I_0 also increases , Figure 3 demonstrates $\tilde{\tilde{b}}$ and Υ increases I_0 also increases , Figure 4 demonstrates shows μ increases $\tilde{\tilde{L}}_q$and $\tilde{\tilde{W}}_q$ also increases , Figure 5 demonstrates Υ increases $\tilde{\tilde{L}}_q$and $\tilde{\tilde{W}}_q$ also increases. Through the aforementioned numerical examples, we got able to see how parameters influenced the system's performance metrics and determine that the findings were accurate for real-world applications.

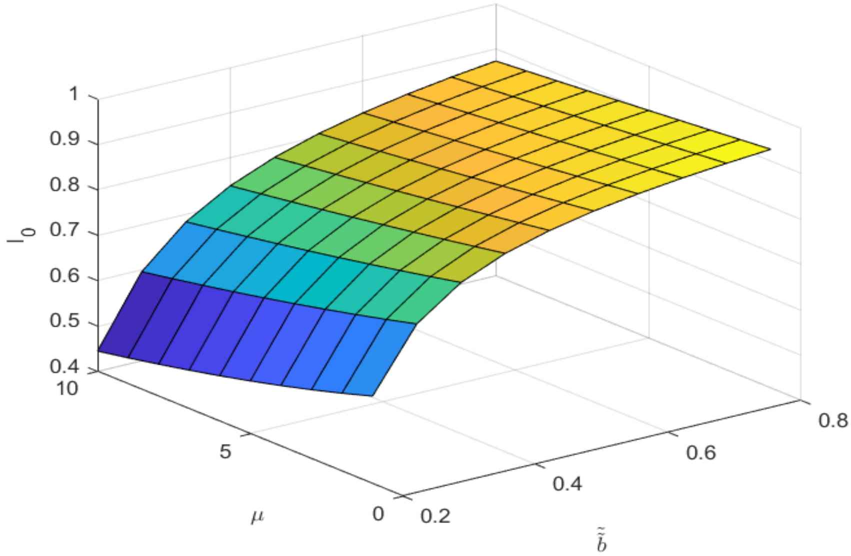

Figure 2: *I_0 versus $\tilde{\tilde{b}}$ and μ*

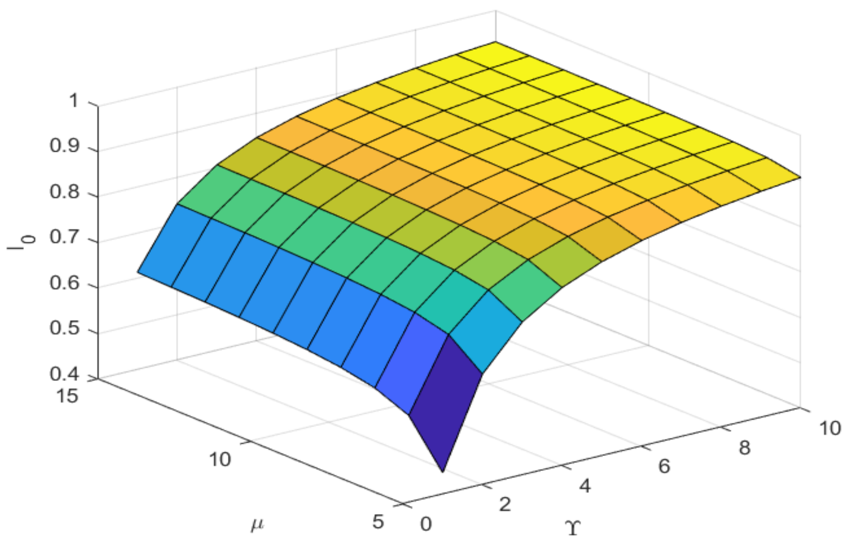

Figure 3: *I_0 versus μ and Υ*

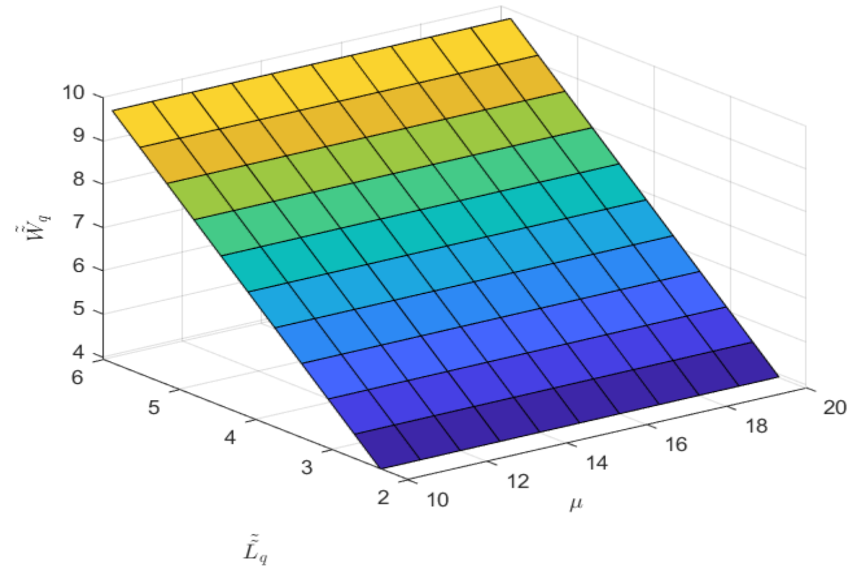

Figure 4: *μ versus $\tilde{\tilde{L}}_q$ and $\tilde{\tilde{W}}_q$*

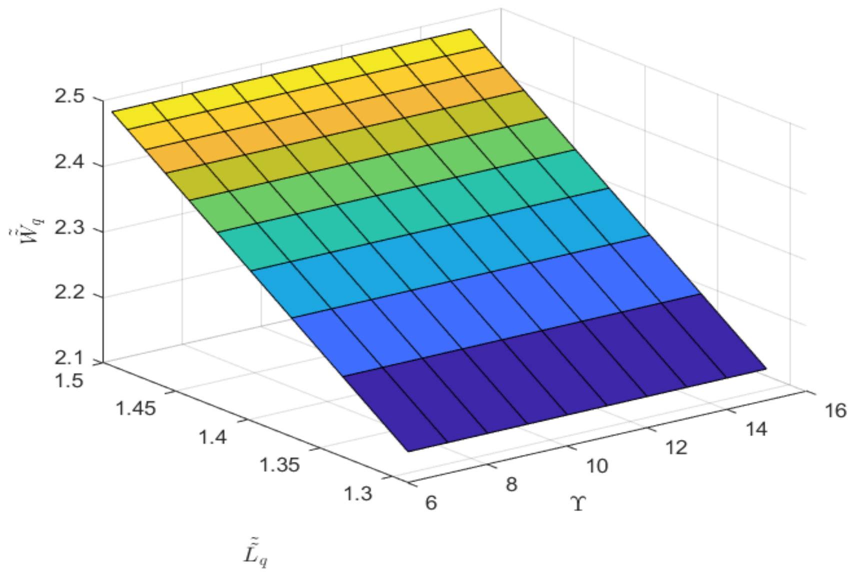

Figure 5: *Υ versus $\tilde{\tilde{L}}_q$ and $\tilde{\tilde{W}}_q$*

7. Conclusion

We discussed a server queue with negative clients in this effort, where customers are serviced one after the other in batches in a system of variable size. Additionally, we presumptively have a general distribution for the service times, delay times, and repair times. For various states, we derived the probability generating functions for the number of customers in the orbit. We have explored a single server queue with batches of reneging or balking clients in a system of variable size in this work. Different performance measures and unique situations have been examined. The outcomes of this work have applications in satellite communication, software-design for various computer-communication system and mailing systems among other things. By including orbit search, starting failure, and working vacation policies, this work can also be expanded.

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