CONSTRUCTION OF DOUBLE SAMPLING INSPECTION PLANS FOR LIFE TESTS BASED ON LOMAX DISTRIBUTION

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Abstract

A life test is a random experiment performed on manufactured products such as electrical and electronic components to estimate their life period based on a randomly chosen components. The lifespan of a component is considered as a random variable that follows a certain continuous-type distribution, called the lifetime distribution. Reliability sampling is one of the decision-making methodologies in product control and deals with inspection procedures for sentencing one or more lots or batches of items submitted for inspection. The concept of sampling plans for life tests involving with two random samples is employed in the present study under the assumption that the lifetime random variable is described by the Lomax distribution. A procedure based on mean / median life criterion is developed for designing the optimum plans with minimum sample sizes when two points on the desired operating characteristic curve are prescribed to ensure protection to the producer and the consumer.

Keywords: Consumer's risk, Double sampling plan, Lomax distribution, Operating characteristic function, Producer's risk, Reliability sampling.

1. Introduction

Sampling inspection is a product control strategy that decides whether a lot should be accepted or rejected based on the information obtained by the inspection of random sample(s) drawn from the submitted lot(s). Sampling inspection procedures are generally classified according to the nature of the quality characteristics, namely, measurable and non-measurable. When the quality characteristics are non-measurable, but are classified into go or no-go basis, such as good or bad, non-conforming or conforming, etc., the sampling inspection procedures are termed as attribute sampling. When the quality characteristics are measurable on a continuous scale, the corresponding sampling inspection procedures are called variables sampling, which are devised under the implicit assumption that the quality characteristic is a continuous random variable following a specific probability distribution. Reliability sampling plans, also termed as life test sampling plans, are operationally attributes sampling procedures, but involve lifetime of the components or items as a random variable which is distributed according to a specific continuous type probability distribution, such as the exponential, Weibull, lognormal, gamma distributions, etc. The lifetime of the components or items is observed by putting the sampled items under the test, called life test, which is defined as the process of evaluating the lifetime of the items through experiments. The literature in product control provides the importance of various continuous probability distributions like exponential, Weibull, lognormal and gamma distributions as well as several compound distributions for modeling lifetime data in the studies relating to the design and evaluation of reliability sampling plans.

The earlier works, which laid the foundation for the expansion of various types of sampling plans, would include the theory of reliability sampling proposed and developed from [1] - [8]. Significant contributions in the development of life test sampling plans employing exponential, Weibull, lognormal and gamma distributions as well as several compound distributions for modeling lifetime data have also been made in the past four decades. A detailed account of such plans was provided in [9]. The recent advances in the theory of life test sampling plans provided in [10] – [28].

Lomax distribution, introduced in [29], is a heavy-tailed probability distribution and is considered as Pareto Type II distribution. It has a wide range of applications in many fields which include business, economics, actuarial, medical and biological sciences. It has been proved to be much useful in reliability and life testing studies and in survival analysis. Properties of Lomax distribution and its extended form can be seen in [30] – [33]. In this paper, a specific life-test sampling plan is devised with reference to the life-time quality characteristic, which is modeled by Lomax distribution. A procedure for the selection of such plans indexed by acceptable and unacceptable mean life ensuring protection to the producer and consumer is described with illustrations. Tables yielding optimum Double sampling plans for life tests are constructed for a set of fixed values of shape parameter of the Lomax distribution.

2. Lomax Distribution

Let T be a random variable representing the lifetime of the components. Assume that T follows Lomax distribution. The probability density function and the cumulative distribution function of T are, respectively, defined by

$$f(t;\theta,\lambda) = \frac{\lambda}{\theta}\left(1+\frac{t}{\theta}\right)^{-(\lambda+1)}, t>0, \theta>0, \lambda>0 \tag{1}$$

and $F(t;\theta,\lambda) = 1-\left(1+\frac{t}{\theta}\right)^{-\lambda}, t>0, \theta>0, \lambda>0,$ (2)

where λ and θ are the shape and scale parameters, respectively.

The mean life, the median life, the reliability function and hazard function for specified time t under Lomax distribution are. Respectively, given by

$$\mu = \frac{\theta}{\lambda-1},\ for\ \lambda>1, \tag{3}$$

$$\mu_d = \theta\left(\sqrt[\lambda]{2}-1\right), \tag{4}$$

$$R(t;\theta,\lambda) = \left(1+\frac{t}{\theta}\right)^{-\lambda}, t>0, \theta>0, \lambda>0 \tag{5}$$

and $Z(t;\theta,\lambda) = \frac{\lambda}{\theta}\left(1+\frac{t}{\theta}\right)^{-1}, t>0, \theta>0, \lambda>0.$ (6)

The reliability life is the life beyond which some specified proportion of items in the lot will survive. The reliability life associated with Lomax Distribution is defined and denoted by

$$\rho(t;\theta,\lambda) = \theta\left(R^{-1/\lambda}-1\right), \tag{7}$$

Where $\underline{R}$ is the proportion of items surviving beyond life ρ.

The proportion, p, of product failing before time t, is defined by the cumulative probability distribution of T and is expressed by

$$p = P(T \leq t) = F(t;\theta,\lambda). \tag{8}$$

3. Operating Characteristic Function of Life Test Sampling Plan

The performance of a single sampling plan adopted in life testing is measured by the associated operating characteristic (OC) function, denoted by $P_a(p)$, which gives the probability of accepting a lot as a function of the failure probability p. Under the conditions for the application of binomial and Poisson models, the expressions for $P_a(p)$ are, respectively, expressed by

$$P_a(p) = \sum_{x:0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \tag{9}$$

and $$P_a(p) = \sum_{x:0}^{c} e^{-np} \frac{(np)^x}{x!}. \tag{10}$$

Associated with a specific value of p, there exists a unique value of t/θ, which can be derived as a function of p and λ from the cumulative distribution function by virtue of expressions (2) and (8) as

$$\frac{t}{\theta} = (1-p)^{-1/\lambda} - 1. \tag{11}$$

The expression for t/μ is then derived using (3) and (11) as

$$\frac{t}{\mu} = (\lambda - 1)\left[(1-p)^{-1/\lambda} - 1\right], \tag{12}$$

which indicates that associated with any specific value of p, there exists a unique value of the dimensionless ratio t/μ. As the value of p is associated with t/μ, the operating characteristic function of a life test sampling plan can be considered as a function of t/μ rather than p, and, hence, the OC curve of the plan could be obtained by plotting the acceptance probabilities against the values of t/μ.

4. Double Sampling Plans for Life Tests with Zero or One Failure

Often in practice sampling inspection plans for life tests are required to be constructed for product characteristics that involve costly or destructive testing. Industrial situations sometimes may warrant small samples to be used for inspection. In such cases, sampling inspection plans allowing either zero failures or a fewer number of failures in the samples are often employed for sentencing the submitted lots. According to [34], a single sampling plan by attributes with zero acceptance number (zero failures) is undesirable as it does not provide protection to the producer and fails to safeguard the primary interests of the producer.

Figure 1 depicts that a single sampling plan for life tests with zero failures or zero acceptance number, designated by $SSP - (n,\ 0)$, is not desirable as it fails to provide protection to the producer against the acceptable mean life of the product. It can be realized in general that the OC curves of any such single sampling plans having zero failures would be uniquely in poor shape, which obviously does not ensure protection to producers, but safeguard the interests of consumers against unacceptable mean life of the product.

It can be demonstrated that single sampling plans allowing one failure or more number of failures in a sample of items do not possess the undesirable properties or characteristics of $SSP - (n,\ 0)$, but would require larger sample sizes rather than small sample sizes. This shortcoming can be prevailed, to some extent, if double sampling plans allowing a maximum of one failure in the random samples drawn from the lot are effectively adopted for sentencing the lot submitted.

In small sample situations, single sampling plans with a fewer number of failures such as $c = 0$and $c = 1$ can be used. But, the OC curves of $c = 0$ and $c = 1$plans reveal the fact that there would always be a conflicting interest between the producer and the consumer as $c = 0$ plans would provide protection to the consumer with lesser amount of risk of accepting the lot against the unacceptable mean life of the product while $c = 1$ plans offer protection to the producer with lesser risk of rejecting the lot having acceptable mean life. Such conflict can be annulled if one is able to design a life test plan having its OC curve lying between the OC curves of $c = 0$ and $c = 1$plans.

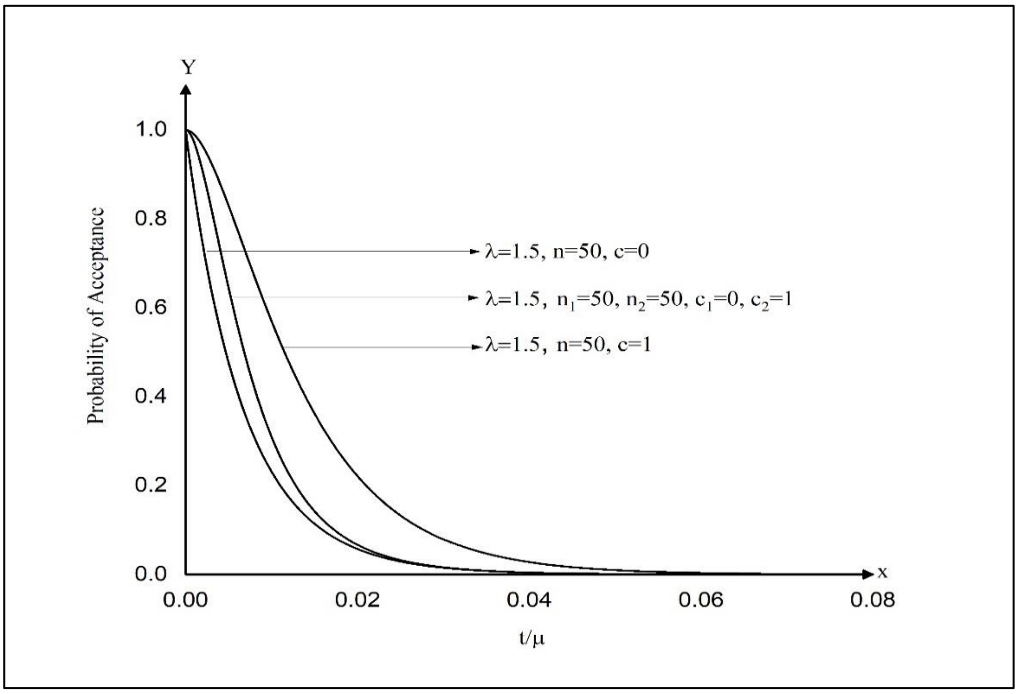

Figure 1: *Operating Characteristic Curves of Single and Double Sampling Plans for Life Tests Based on Lomax Distribution Having smaller acceptance number*$c_1 = 0$ *and*$c_2 = 1$

From Figure 1, it can also be observed that there is a wide gap between the OC curves of $c = 0$ and $c = 1$plans. Hence, it is desirable to bridge the gap by determining a suitable plan such that its OC curve is expected to lie between the OC curves of $c = 0$ and $c = 1$ plans.

A double sampling plan with $c_1 = 0$ and $c_2 = 1$, designated by $DSP - (n_1, n_2)$, overcomes the shortcoming of $c = 0$ plans to a greater extent by providing a desirable shape of the OC curve, which is considered as favorable to both producer and consumer. It can also be shown that the OC curves of $DSP - (n_1, n_2)$ would lie between the OC curves of $c = 0$ and $c = 1$ plans.

One can observe that the OC curve of $DSP - (n_1, n_2)$ coincides with the OC curve of $c = 1$ single sampling plan at the upper portion and coincides with the OC curve of $c = 0$ single sampling plan at the lower portion. This salient feature would be of much help in determining an optimum $DSP - (n_1, n_2)$ providing protection to the producer and consumer against rejection of the lot for the specified acceptable mean life and against acceptance of the lot for the specified unacceptable mean life. Detailed discussion on the significance and construction of sampling plans with the utilization of the conditions c = 0 and c = 1 together as an alternative to single sampling plans with either c = 0 or with c = 1 are found in the literature of acceptance sampling and especially from [35] – [37]. The operating procedure of $DSP - (n_1, n_2)$ is as follows:

Step 1: Draw a random sample of n_1 items from a given lot and put them for a life test.

Step 2: Observe the number,m_1, of failures before reaching the predetermined time t. If $m_1 = 0$, while testing n_1 items, then accept the lot; if $m_1 > 1$, reject the lot; if $m_1 = 1$, draw a second random sample of $n_2$items and put them for a life test.

Step 3: Observe the number, m_2, of failures while testing $n_2$items. If $m_2 = 0$, then accept the lot; if

$m_2 \geq 1$, then reject the lot.

Associated with $DSP-(n_1,n_2)$ are the performance measures, called OC and ASN functions, which are, respectively, expressed by

$$P_a(p) = p(0|n_1,p) + p(1|n_2,p)p(0|n_2,p) \tag{13}$$

And $ASN(p) = n_1 + n_2 p(1|n_1,p),$ (14)

Where p is the proportion,p, of product failing before time t, and $p(0|n_1,p)$, $p(0|n_2,p)$ and $p(1|n_1,p)$ are defined either from the binomial distribution or from the Poisson distribution whose probability functions are given as expressions (9) and (10). Under the conditions of binomial distribution, the expressions for $P_a(p)$ and $ASN(p)$ are, respectively, given by

$$P_a(p) = (1-p)^{n_1} + n_1 p(1-p)^{n_1+n_2-1} \tag{15}$$

and $ASN(p) = n_1 + n_1 n_2 p(1-p)^{n_1-1}$. (16)

Similarly, under the conditions of Poisson distribution, the expressions for $P_a(p)$ and $ASN(p)$ are, respectively, given by

$$P_a(p) = exp(-n_1 p) + n_1 p\, exp(-(n_1+n_2)p) \tag{17}$$

and $ASN(p) = n_1 + n_1 n_2 p\, exp(-n_1 p)$. (18)

It is known that, under the assumption of Lomax distribution for a lifetime quality characteristic, p is defined by the cumulative probability distribution of the lifetime random variable, T, and is expressed by

$$p = P(T \leq t) = F(t;\theta,\lambda)$$

It can be noted that the double sampling plan for life tests allowing a maximum of one failure based on Lomax distribution is specified by the parameters n_1, n_2, θ and λ, where n_1 and n_2 are the sample sizes under the plan, and θ and λ are the parameters of Lomax distribution. As discussed earlier, the failure probability p is associated with t/θ, through the distribution function of Lomax distribution, and the acceptance probabilities can be computed when the sets of values of n_1, n_2 and λ are specified. The probabilities of acceptance of the submitted lot under the double sampling plan for life tests can be computed against the dimensionless ratio μ/μ_0 based on the procedure described in the following Subsection for different combinations of parameters n_1, n_2 and λ, where μ/μ_0 is the ratio of the actual mean life to the assumed mean life. It is to be noted that any change in the values of these parameters would have some impact in the nature of the OC curve.

While selecting a sampling inspection plan for its application, it is the conventional practice to define the *OC* curve in accordance with the desired discrimination and to select the corresponding sampling plan. It is known that the operating ratio, defined as the ratio of the limiting quality level to the acceptable quality level, is one of the widely used measures of discrimination in sampling plans, and is, in general, used to fix the *OC* curve.

Further, a smaller value of μ/μ_0 would indicate that the actual mean life is relatively much smaller than the acceptable mean life whereas a larger value, which is nearer to one would indicate that the difference between μ and μ_0 is less. When the actual mean life is much smaller than the acceptable mean life, smaller the values of λ, greater is the protection to the consumer, whereas protection to the producer is more for larger values of λ. As the actual mean life increases, acceptance probabilities would increase, which indicate that the lots having items with higher mean life that is close to the acceptable mean life will most often have a greater chance of acceptance.

5. Procedure for the Selection of DSP – (n_1, n_2) for Life Tests

Sampling inspection plans for attributes or variables are constructed based on a general approach that the operating characteristic curves of the desired plans should pass through two prescribed points, namely, the acceptable quality level, p_0, and the limiting quality level, p_1, which are associated with the producer's risk, α, and the consumer's risk, β, respectively. The specification of these points is required for the purpose of ensuring protection to the producer as well as the consumer and is considered for fixing the *OC* curve in accordance with a desired degree of discrimination. The operating ratio, *R*, defined as the ratio of p_1 to p_0, is often used as the measure of discrimination.

As discussed in the earlier sections, a specific sampling plan for life tests can be determined by specifying the requirements that the OC curve should pass through two prescribed points, namely, (μ_0, α) and (μ_1, β), where μ_0 and μ_1 are the acceptable and unacceptable mean life associated with the risks α and β, respectively. In such a case, the operating ratio, $R = \mu_0/\mu_1$, which is the ratio of acceptable mean life to unacceptable mean life, can be used as the measure of discrimination just similar to the operating ratio of the limiting quality level to the acceptable quality level. It is obvious to note that $\mu_1 < \mu_0$, and hence, $R > 1$. An optimum double sampling plan for life tests can be determined by satisfying the following two conditions so that the maximum producer's and consumer's risks would be fixed at α and β, respectively:

$$P_a(\mu_0) \geq 1 - \alpha \tag{19}$$

and

$$P_a(\mu_1) \leq \beta. \tag{20}$$

It is to be noted that the *OC* function given as (15) or (17) is not directly related to the mean life; but it can be expressed as a function of t/μ, which corresponds to p, *i.e.*, the proportion of lot failing before time t. The procedure is appropriately used to compute the operating characteristics while searching for the optimum values of the sampling plan satisfying the conditions (19) and (20).

For the specified values of t/μ_0 and t/μ_1, the optimum values of n_1 and n_2 of $\boldsymbol{DSP - (n_1, n_2)}$ under the conditions of Lomax distribution satisfying the conditions (19) and (20) can be determined by using the following procedure:

Step 1: Specify the value of the shape parameter λ or its estimate.

Step 2: Specify the values of t/μ_0 and t/μ_1, with the associated risks $\alpha = 0.05$ and $\beta = 0.10$, respectively, so that the operating ratio is defined by $R = \mu_0/\mu_1$.

Step 4: Using the relationship between p and μ, from (3) and (8), obtain p_0 and p_1 corresponding to t/μ_0 and t/μ_1.

Step 5: Search for the values of n_1 and n_2 for the specified strength $(\mu_0,\ 1-\alpha)$ and $(\mu_1,\ \beta)$ with the values of p_0 and p_1, or equivalently with the values of t/μ_0 and t/μ_1, by using either the expression (15) or the expression (17), such that the conditions (19) and (20) are satisfied.

Based on the above procedure, the optimum double sampling plans for life tests under the assumption of Lomax distribution are obtained for a set of five values of λ, given as 1.25, 1.5, 1.75, 2 and 3, and for various sets of combinations of $R = \mu_0/\mu_1$ and t/μ_0. These plans are provided in Tables 1 through to 5 along with the values of minimum *ASN* at t/μ_0. The optimum plans given in the tables are obtained under the conditions of binomial distribution by a search procedure using the expression (15) for the *OC* function and the expression (8) for the proportion of product failing in an appropriate manner. The parameters of the optimum plans would have a maximum of 5 percent producer's risk and a maximum of 10 percent consumer's risk.

5.1. Numerical Illustration

In an electronic device manufacturing industry, a quality control practitioner wishes to adopt a suitable sampling inspection plan under isolated lot conditions. Though the practitioner is interested to have only zero failures in the random sample items which are placed under the life test, keeping in mind the manufacturer's capabilities of producing long survival items, he wishes to allow a maximum of one failure item under the sampling plan. Hence, he desires to adopt a double sampling plan allowing a maximum of one failure in the randomly sampled items which are to be considered for a life test.

The past history in the industry reveals that the life time random variable is distributed according to Lomax distribution, whose shape parameter is specified to be $\lambda = 1.5$. It is expected that the plan shall provide the desired degree of discrimination, which is measured in terms of the operating ratio, R, ensuring protection to the producer in terms of the acceptable mean life $\mu_0 = 2000$ hours with the associated risk of 5 percent and protection to the consumer against the unacceptable mean life $\mu_1 = 110$ hours with the associated risk of 10 percent.

The practitioner would like to terminate the life test within 1 hour. Based on the given information, one gets $R = \mu_0/\mu_1 = 18.2 \approx 18$, and $t/\mu_0 = 0.0005$. The value of shape parameter $\lambda = 1.5$, with the ratio $R = \mu_0/\mu_1 = 18$ and $t/\mu_0 = 0.0005$, the optimum double sampling plan is chosen with the sample sizes $n_1 = 88$ and $n_2 = 178$, which yield the minimum $ASN = 109$ at t/μ_0. Thus, the desired plan for the given conditions is implemented as given below:

1. Draw a random sample of $n_1 = 88$ items from a submitted lot and place them for life test.
2. Observe the number of failures before reaching the termination time of 1 hour.
3. Terminate the life test once the termination time, *i.e.*, $t = 1$ hour, is reached.
4. If no failures are observed in the 88 items tested or until time t is reached, accept the lot; if one failure is observed in the 88 items tested, select a random sample of $n_2 = 178$ items and place them for a life test.
5. Accept the lot, when no failures are observed while testing 178 items; reject the lot, if one or more failures are observed.
6. Treat the items which survive beyond time $t = 1$ hour as accepted.

5.2. Numerical Illustration

It is assumed that the lifetime of the components in an electronic device follows a Lomax distribution with shape parameter $\lambda = 3.0$. It is desired to implement a double sampling plan for life tests to sentence a submitted lot of manufactured components. The experimenter involved in the decision-making process fixes the test termination time as $t = 75$ hours. The acceptable and unacceptable proportions of the lot failing before time t are, respectively, prescribed as $p_0 = 0.003$ and $p_1 = 0.055$ with the associated risks fixed at the levels $\alpha = 0.05$ and $\beta = 0.10$. The values of t/μ corresponding to $p_0 = 0.003$ and $p_1 = 0.055$ are determined as $t/\mu_0 = 0.002$ and $t/\mu_1 = 0.038$. Hence, the desired operating ratio is obtained as $R = \mu_0/\mu_1 = 19$.

The value of shape parameter $\lambda = 3$, with the ratio $R = 19$ and the index $t/\mu_0 = 0.002$, one obtains the optimum double sampling plan having its parameters specified as $n_1 = 42$ and $n_2 = 120$ which yield the minimum $ASN = 56$ at $t/\mu_0 = 0.002$. These parameters satisfy the conditions (9) and (10). The acceptable mean life and unacceptable mean life are, then, determined as $\mu_0 = t/0.002 = 37500$ hours and $\mu_1 = t/0.038 = 1973.7 \approx 1974$ hours, respectively.

6. Conclusion

A double sampling inspection plans for life-tests which involve two samples and allows a maximum of one failure is proposed when the lifetime quality characteristic is modeled by a Lomax distribution. A procedure for the selection of the proposed plan is discussed through numerical illustrations. The sampling plan which could be derived by the procedure discussed in this paper will ensure protection to the producer and consumer as the plans are indexed by acceptable and unacceptable proportion of product failing before the specified time, t. The practitioners can generate the required sampling plans for various choices of shape parameter λ, adopting the procedure.

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