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IMPROVED DEGRADATION TEST USING INVERSE GAUSSIAN PROCESS FOR SIMPLE STEP-STRESS MODEL

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Abstract

The accelerated Degradation testing (ADT) experiments are important technical methods in reliability studies. Different type of accelerating degradation models has developed with the time and can be used in different types of situations. However, it has become necessary for the manager to test how many numbers of unit should be tested at a particular stress level so that the cost of testing is less. Accelerated Degradation testing (ADT) is preferred to be used in mechanized industries to obtain the required information about the reliability of product components and materials in a short period of time. Accelerated test conditions involve higher than usual pressure, temperature, voltage, vibration or any other combination of them. Data collected at such accelerated conditions are extrapolated through a physically suitable statistical model to estimate the lifetime distribution at design condition stress the life data collected from the high stresses the need to be extrapolated to estimate the life distribution under the normal-use condition. A special class of the ADT is the step-stress testing which regularly increases the stress levels at some pre-fixed time points until the test unit fails. Such experiments allow the experimenter to run the test units at higher-than-usual stress conditions in order to secure failures more quickly. The Inverse Gaussian process is flexible in incorporating random effects and explanatory variables. The different types of models based on IG process are random drift model, random volatility model and random drift-volatility model. In this paper we have considered random drift model for the study on stochastic degradation models for simple step-stress model using inverse Gaussian process observed in degradation problems.

Keywords: Degradation problem, random volatility model, accelerated life testing, inverse Gaussian process, and random drift-volatility model

I. Introduction

In automated industries, Accelerated Degradation Testing (ADT) is the ideal method for quickly obtaining the necessary information regarding the dependability of product components and materials [4]. Higher than normal pressure, temperature, voltage, vibration, etc., or any combination of these, are examples of accelerated test conditions. In order to estimate the lifetime distribution at design condition stress, data collected under such accelerated conditions are extrapolated using a physically appropriate statistical model. The life data collected from the high stresses must also be extrapolated in order to estimate the life distribution under normal-use conditions. Step-stress testing is a unique type of ADT in which the stress level is gradually increased at predetermined intervals until the test unit malfunctions.

Such tests are mostly conducted in order to obtain dependability data as soon as possible or to save both time and money. Since many pressures tend to accelerate the deterioration process, we can employ accelerated degradation tests (ADT) to acquire degradation phenomena more quickly [5]. To assess the life characteristics of interest under use conditions, a basic constant stress ADT experiment allocates a number of units to different stress levels. The deterioration level of these units is then measured, analyzed, and extrapolated to the failure threshold. ADTs have garnered a lot of attention because to their ability to significantly reduce the testing length. For ADT data, there are two types of models [9].

Since Brownian motion's first passage time has an inverse Gaussian distribution, using it as a life time model makes sense. It is helpful for researching the dependability and life testing of a gadget, product, or subcomponent. In order to shorten the product's life or hasten its performance decline, engineers use accelerated testing to estimate the reliability of recently developed products. The items are subjected to severe conditions during this test, including a mix of random vibrations, increases in temperature, voltage, or pressure [11]. The inverse Gaussian process is a helpful model for repair time. Additionally, in the subject of reliability, the inverse Gaussian distribution has been applied in numerous fields, including hydrology, cardiology.

II. Methods

I. Gaussian Process Model Inverse

An inverse Gaussian process $\{Y(t);\ t \geq 0\}$ with mean function $\Lambda(t)$ and scale parameter λ has the following properties:

- Y(t) has independent increments for every pair of disjoint intervals $(t_1, t_2), (t_3, t_4)$ with $t_1 < t_2 < t_3 < t_4$ the random variables $Y(t_2) - Y(t_1)$ and $Y(t_4) - Y(t_3)$ are independent.
- Each increment $Y(t) - Y(s)$ has an inverse Gaussian distribution $IG(\Delta\Lambda(t), \lambda\, \Delta\Lambda(t)^2)$ where $\Delta\Lambda = \Lambda(t) - \Lambda(s)$ and the PDF of an inverse Gaussian distribution random variable $IG(\mu, \lambda)$ with mean μ and variance $\frac{\mu^3}{\lambda}$ has discussed by Chikkara and Folks (1989) is

$$f(x;\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi}}\, x^{-\frac{3}{2}} exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right) X > 0 \tag{1}$$

- $Y(0) = 0$ With probability one. When the amount of degradation reaches a pre-specified critical level D, failure occurs. Let $T = Inf\{t: Y(t) = D\}$ denote the failure time. Since the inverse Gaussian process has a failure time distribution by [16]

$$P(T < t) = P(Y(t) > D) = 1 - G(D;\ \Lambda(t), \lambda\, \Lambda(t)^2)$$
$$= \Phi\left[\sqrt{\frac{\lambda}{D}}(\Lambda(t) - D)\right] - e^{2\lambda\Lambda(t)}\Phi\left[\sqrt{\frac{\lambda}{D}}(\Lambda(t) + D)\right][-\sqrt{\lambda}D(\Lambda(t) + D)] \tag{2}$$

where, $G\,(.;\ \Lambda, \lambda)$ is a cumulative distribution function (CDF) of $IG(\Lambda, \lambda)$ and Φ is the standard normal CDF. From above equation we can write the CDF of the failure time distribution as

$$H_\lambda(t) = \Phi\left[\sqrt{\frac{\lambda}{D}}(t-D)\right] - e^{2\lambda t}\Phi\left[\sqrt{\frac{\lambda}{D}}(t+D)\right] \quad (3)$$

It is an increasing function. Thus, within this class of models, there is a one-to-one relationship between$\Lambda(t)$ and the cdf of the failure time distribution $H_\lambda(t)$ for a fixed scale parameter λ.

$$f(x;\mu,\lambda) = \sqrt{\frac{\lambda}{2\Lambda}}x^{\frac{3}{2}}exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right) \quad (4)$$

Where $\mu > 0$ and $\lambda > 0$ the parameter μ is the mean of the distribution and λ is a scale parameter. (Tweedie) gives three form of above pdf, which he obtained by replace the set of parameters (μ,λ) by $(\propto,\lambda)$ or (μ,ϕ), or (ϕ,λ) using the relationship given by [13]

$$\mu = \frac{\lambda}{\phi} = (2\propto)^{\frac{-1}{2}} \quad (5)$$

Both μ and λ are of the same physical extent as the random variable X itself; but the parameter $\mu = \frac{\lambda}{\phi}$ is invariant under a scale transformation of Xas can be seen from the following relationship:

$$f(x;\ \mu,\lambda) = \mu^{-1}f\left(\frac{x}{\mu};1,\phi\right) = \lambda^{-1}f\left(\frac{x}{\mu};\phi,1\right) \quad (6)$$

The probability density can be numerically computed using any of the three forms in above equation as shown above the cumulative distribution function depends fundamentally on only two variables, which might be taken as $x\mu$ and ϕ. According, the case $\mu = 1$ for the (μ,ϕ) parametric form of above equation could be adopted as a standard form [18]. This has also been obtained as a limiting form of the distribution of the sample size in a Wald's sequential probability ratio test and is sometimes referred to as the standard Wald's distribution of the density function model is

$$\mu\left[\left(1+\frac{9}{4\phi}\right)^{\frac{1}{2}} - \frac{3}{2\phi}\right] \quad (7)$$

II. Random Effects Inverse Gaussian Process

Random effects are needed in Inverse Gaussian process to account for inexplicable heterogeneous degradation rates within the product population. By linking to the Weiner process this investigates different options to incorporate the random effects in the IG process model. Consider the wiener process $W(x) = \mu x + \lambda B(x)$ where $\mu > 0$ is the drift parameter and $\lambda > 0$ is the volatility parameter and $B(x)$ is the standard Brownian motion [12]. Given a fix threshold $\Lambda > 0$, it is well known that the first passage time $T_A = \inf\{x > 0 \mid W(x) \geq \Lambda\}$ follows $IG\left(\frac{\Lambda}{\mu}, \frac{\Lambda^2}{\lambda^2}\right)$ going one step further, we consider a series of the thresholds $\Lambda(t)$ indexed by t with $\Lambda(0) = 0$ and $\Lambda(t)$ increasing in t, and define the first passage time process $Y(t) = T_{\Lambda(t)}$ It is easily verified that the induced $\{Y(t); t > 0\}$ is an IG process with the mean function $\frac{\Lambda(t)}{\mu}$ and variance function $\frac{\Lambda(t)}{\lambda^2}$ by asset of the stationary and independent increment property of the Wiener process $W(x)$.

The inverse relation between the IG and the Wiener processes motivates investigation of the IG process from a new perspective. Existing results on the Wiener processes can let somebody use support to the development of IG process model with the random effects [10].

III. Random Volatility Model

Consider a Wiener process $W(x) = \mu^{-1}x + \lambda^{\frac{-1}{2}}B(x)$ with the induced IG process other way of introducing unit-specific random effects is to assume that each unit possesses a separate realization of the volatility parameter. Accordingly, volatility parameter in the Inverse Gaussian process is random [17]. With the random volatility parameter in the Inverse Gaussian process all units have the same mean degradation path, even though they will have different variance functions. The Inverse Gaussian process with random volatility parameter was originally proposed by Wang and Xu (2010).

Shortcoming of random volatility model is unusual to use the volatility parameter to control heterogeneity in the Weiner process thus application of random volatility model is limited. Thus, random drift model was proposed which overcome inadequacy of random volatility model [13].

IV. Random Drift Model

An effective way to incorporate random effect in the IG process is to let μ be a random variable. To avoid the negative values of μ (Whitmore 1986) and ensure mathematical tractability, we assume $\mu - 1$ follows a truncated normal distribution $TN\ (\omega, k^{-2}), k > 0$ with PDF

$$g(\mu^{-1};\ \omega, k^{-2}) = \frac{k.\phi[k(\mu-1-\omega)]}{1-\Phi(-k\omega)}\mu > 0 \tag{8}$$

Where $(.)$ is a standard normal PDF. In a degradation test, if the degradation of the i^{th} testing unit is observed at time $t_{io} < t_{i1} < \cdots . < t_{ini}$ with observations $Y_i(t_{ij}), j = 0,1,2, \ldots., n_i$ the joint PDF of $Y_i = [Y_i(t_{i1}), Y_i(t_{i2}), \ldots . Y_i(t_{ini})\ ]$ is computed by first conditioning on the random drift parameter μi and then marginalizing it, which yields the following equation is

$$f_{IG}(Y_i) = \frac{1-\phi(-\tilde{\omega}_i\tilde{k}_i)}{1-\phi(-k\omega)}\frac{k}{\tilde{k}_i}\Pi_{j=1}^{ni}\sqrt{\frac{\lambda\Lambda_{ij}{}^2}{2\Lambda yij^3}}\frac{\tilde{k}_i{}^2\tilde{\omega}_i - k^2\omega^2}{2} - \lambda\sum_{j=1}^{ni}\frac{\Lambda ij^2}{2yij} \tag{9}$$

Where,$Y_{ij} = Y_i(t_{ij}) - Y_i(t_{ij} - 1)$ is the observed increment $\Lambda_{ij} = \Lambda(t_{ij}) - \Lambda(t_{ij} - 1)$

$$\tilde{k_{ij}} = \sqrt{\lambda Y_{ij}(t_{ij}k_j) + k^2} \tag{10}$$

$$\tilde{\omega_{ij}} = \frac{[\lambda\Lambda(t_{ij}k_j) + k^2 exp\propto_0 + \propto_1 x_j]}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)} \tag{11}$$

Then the log-likelihood function is given by

$$l(\theta) = \sum_{i=1}^{j}\sum_{j=1}^{N_j}\left[ln\frac{k}{\tilde{k}_{ij}} + \frac{\tilde{k}_{ij}{}^2\tilde{\omega}_{ij}{}^2 - k^2 exp(2\propto_0 + 2\propto_1 x_j)}{2} + \frac{1}{2}\sum_{k=1}^{k_j}\left[In(\lambda\partial\Lambda_{ijk}) - \frac{\lambda\Lambda_{ijk^2}}{y_{ijk}}\right]\right] \tag{12}$$

$l(\theta)$ is the likelihood function up to a constant can be expressed by the above equation. Where θ is a parameter vector include $\propto_0, \propto_1, \lambda, \beta, and\ k$

V. Accelerated Degradation Test Assumptions

Let total N number of units is put into test. Suppose S_0 be the usage stress S_H being the maximum acceptable stress. To collect the degradation data timely we allocate these units J stress level $S_1 < 47\ S_2 < \cdots \ldots \ldots \ldots < S_J$ with $S_0 < S_1$and $S_J = S_H$ consider N_j units to be allocated to j^{th} stress level. $j = 1, 2, 3, \ldots . . J$. The degradation of these units is affected by the stress. Here, we have assumed $\mu_i = h(s)$, and $\lambda$is constant over$s$, where $h(s)$ is a link function reflecting the effect of the stress on the degradation process [17].

Due to the above assumption the degradation speed and drift changes with the stress. Another alternative is that $\lambda = h(s)$ while μ is constant which is not valid for random drift model sinceμ is changing from unit to unit. For simplicity and without loss of generality, the additional assumptions are, the measurement time interval, and the number of measurements $K_j$under the j[th] stress level, where $j = 1,2, \ldots \ldots \ldots J$, are pre-determined and the link function follows one of the following acceleration relations:

- Power law relations $h(s) = \varphi_0 . s^{\propto}$
- Arrhenius relation $h(s) = \varphi_0 . e^{\frac{-\propto}{s}}$
- Exponential relation $h(s) = \varphi_0 . e^{\propto s}$

In real time applications the time approved for the test is often given by the manager and time intervals at which the units are measured are predetermined because of the working time of experimenters [10]. Thus, we assume that τ_j and k_j are given. In our model we delight these two variables as decision variables, and then we optimally determine their values. When the assumed stress-degradation relation i.e., is correct we can use a two-stress ADT, i.e., $J = 2$ in our model. But, in this minimum variance plan we are unable to check the validity of the assumed stress-degradation relationship. Thus, we prefer to use three-stress ADT planning taking $J = 3$ to check the validity of the assumed model. In our settings, the purpose of ADT planning is to optimally determine the stress levels (S_j), and the number of samples for each stress level (N_j) are be investigated in our proposed work [4].

VI. Normalizing the Stress Level

We standardize the stress levels depending on the acceleration relationship of the stress on the rate of degradation as follows:

$$Z_j = \frac{lnS_j - \ln S_0}{lnS_H - \ln S_0} \quad \text{For the power law relation}$$

$$Z_j = \frac{\frac{1}{S_0} - \frac{1}{S_j}}{\frac{1}{S_0} - \frac{1}{S_H}} \quad \text{For the Arrhenius relation}$$

$$Z_j = \frac{S_j - S_0}{S_H - S_0} \quad \text{For the exponential relation}$$

From the above consistency, it is readily seen that $x_0 = 0, x_j = 1$, and $0 < Z_j \leq 1$ for $j = 1,2 \ldots, J$ then.

$$h(x) = exp\,(\propto_0 + \propto_1 Z_j)$$

$$h(x) = \varphi_0 . e^{\frac{-\propto}{s}}$$

$$\ln h(x) = \ln \varphi_0 - \frac{\propto}{s}$$

Were, $\propto_0 = \ln \varphi_0 - \frac{\propto}{S_0}$, $\propto_1 = \propto (\frac{1}{S_0} - \frac{1}{S_H})$ For the Arrhenius function, $\propto_0 = \ln \varphi_0 + \propto lnS_0$, $\propto_1 = \propto (\ln S_H - lnS_0)$ For the power law function and $\propto_0 = \ln \varphi_0 + \propto S_0$, $\propto_1 = \propto (S_H - S_0)$ For the exponential function.

VII. Inferential Procedure

We suppose that the i[th] unit under the j[th] stress level is measured at time $t_{ijk} = k_{\tau j}$ with observations $Y_{ij}\,(t_{ijk}), k = 0,1, \ldots \ldots \ldots, k_j$. Let $Y_{ijk} = Y_{ij}\,(t_{ijk}) - Y_{ij}\,(t_{ij}, k-1)$ be the observed increments, and $\Lambda_{ijk} = \Lambda(t_{ijk}) - \Lambda(t_{ijk}, k-1)$. Now, the log-likelihood function up to a constant can be expressed by the equation above 1. The Fisher information matrix $I(\theta)$ for the element $\propto_0, \propto_1, k, \omega, \Lambda(.)$ canbe developed as below [5]. We assume nonlinear function for $\Lambda(.)$, i.e.,

$\Lambda(t) = t_\beta$ and then $\theta = (k, \omega, \propto_0, \propto_1, \beta)'$ detailed expression for the elements along with the elements of the fisher information matrix can be developed as follows.

$$\frac{\partial l(\theta)}{\partial \omega_j} = \sum_{J=1}^{J} \sum_{i=1}^{N_j} \left[0 + \frac{1}{2} \left\{ \frac{2(\lambda \Lambda(t_{ij}k_j) + k^2 \omega_j) k^2}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)} - 2k\omega_j^{\,2} \right\} + \frac{1}{2} \sum_{k=1}^{k_j} (0 - 0) \right] \tag{13}$$

$$\frac{\partial l(\theta)}{\partial \omega_j} = \sum_{J=1}^{J} \sum_{i=1}^{N_j} \left[\left\{ \frac{k^2(\lambda \Lambda(t_{ij}k_j) + k^2 \omega_j)}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)} 2k^2 \omega_j \right\} \right] \tag{14}$$

$$\frac{\partial^2 l(\theta)}{\partial \omega_{j2}} = \sum_{J=1}^{J} \sum_{i=1}^{N_j} \left[\left\{ \frac{-k^2(0 + k^2)}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)} - k^2 \right\} \right] \tag{15}$$

$$\frac{\partial^2 l(\theta)}{\partial \omega_{j2}} = \sum_{J=1}^{J} \sum_{i=1}^{N_j} \left(\frac{-k^2(\lambda Y_{ij}(t_{ij}k_j))}{\lambda Y_{ij}(t_{ij}k_j) + k^2} - k^2 \right) \tag{16}$$

$$\frac{\partial l(\theta)}{\partial \beta} = \sum_{j=1}^{J} \sum_{i=1}^{N_j} \left[\left(\frac{\Lambda_{ijk}}{\partial \beta} \left\{ \frac{\lambda(\lambda \Lambda_{ijk} + k^2 \omega_j)}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)} \right\} \right) + \sum_{k=1}^{k_j} \left(\frac{1}{\Lambda_{ijk}} - \frac{2\lambda \Lambda_{ijk}}{Y_{ijk}} \right) \frac{\partial \Lambda_{ijk}}{\partial \beta} \right] \tag{17}$$

$$\frac{\partial^2 l(\theta)}{\partial k \partial \beta} = \sum_{J=1}^{J} \sum_{i=1}^{N_j} \left[\frac{\left(\lambda \frac{\partial \Lambda(t_{ij}k_j)}{\partial \beta} \right) \left\{ (2k\omega \lambda Y_{ij}(t_{ij}k_j)) + 2k\omega_j - k\lambda \Lambda(t_{ij}k_j) + k^3 \omega \right\}}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)^2} - \frac{k\lambda \frac{\partial \Lambda(t_{ij}k_j)}{\partial \beta}}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)} \right] \tag{18}$$

$$\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \propto_1} = \sum_{J=1}^{J} \left[Z_j exp(\propto_0 + \propto_1 x_j) \frac{\partial l(\theta)}{\partial \omega_j} + exp(\propto_0 + \propto_1 x_j) \frac{\partial^2 l(\theta)}{\partial \omega_j^{\,2}} x_j \right] \tag{19}$$

$$\frac{\partial^2 l(\theta)}{\partial \propto_0^{\,2}} = \sum_{j=1}^{J} \left[exp(\propto_0 + \propto_1 Z_j) \frac{\partial l(\theta)}{\partial \omega_j} + exp(\propto_0 + \propto_1 Z_j) \frac{\partial^2 l(\theta)}{\partial \omega_j^{\,2}} \right] \tag{20}$$

$$\frac{\partial^2 l(\theta)}{\partial \lambda \partial \beta} = \sum_{J=1}^{J} \sum_{j=1}^{N_j} \left[\frac{1}{2} \left\{ \frac{2(2\lambda \Lambda(t_{ij}k_j)) \frac{\partial \Lambda(t_{ij}k_j)}{\partial \beta} + k^2 \omega \frac{\partial \Lambda(t_{ij}k_j)}{\partial \beta}}{\lambda Y_{ij}(t_{ij}k_j) + k^2} - \frac{Y_{ij}(t_{ij}k_j) \lambda \frac{\partial \Lambda(t_{ij}k_j)}{\partial \beta}}{(\lambda Y_{ij}(t_{ij}k_j) + k^2)^2} \right\} + \frac{1}{2} \sum_{k=1}^{k_j} \left(-\frac{\Lambda_{ijk}}{Y_{ijk}} \frac{\partial \Lambda_{ijk}}{\partial \beta} \right) \right] \tag{21}$$

And then the fisher information matrix can be developed as given below:

$$\begin{matrix}
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0^{\,2}}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \propto_1}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \propto_1}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1^{\,2}}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial k}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k^{\,2}}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \lambda}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \lambda^{\,2}}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \lambda \partial \beta}\right] \\
E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_0 \partial \beta}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \propto_1 \partial \beta}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial k \partial \beta}\right] & \left[-\frac{\partial^2 l(\theta)}{\partial \lambda \partial \beta}\right] & E\left[-\frac{\partial^2 l(\theta)}{\partial \beta^{\,2}}\right]
\end{matrix} \tag{22}$$

The log-likelihood function can be maximized to obtain maximum likelihood estimator MLEs [9]. The direct maximization of log-likelihood function gives equations which are computationally difficult to solve. Under the truncated normal distribution, direct maximization of the likelihood function often yields a solution far away from the MLE.

III. Results

I. Numerical Study

Utilizing the methodology of G. Yang et al. (2007), the suggested process is demonstrated here. In a case study, 30 samples at the electrical connector were found to have failed if the data were collected under one of three temperature levels: 55°C, 75°C, or 100°C. The resistors in the MEMS LAB at the Faculty of Engineering and Technology were all part of a constant stress ADT. The normal use temperature and threshold value for the percent increase in resistance were assumed to be l=6, where observed at different times during the measurement. The samples are tabulated in Table 1 with the 7th point of the second unit under 55°C labeled blank, as suggested by Yang et al. (2007), to maintain the monotone behavior of the stress.

Table 1: *Stress relaxation data under the temperature level*

Temperature	S. No	Stress loss	Mean Time
55^0c	1	2.13, 2.06, 3.43, 4.36, 5.86, 6.24, 6.63, 7.34, 7.58, 8.42, 9.57	7.60
	2	2.34, 3.65, 4.69, 4.85, 5.36, 0, 6.59, 8.48, 9.35, 10.95	
	3	2.8, 3.56, 4.65, 5.89, 6.3, 7.65, 8.95, 9.21, 10.45, 11.32	
	4	2.96, 3.58, 5.38, 5.32, 7.68, 8.27, 8.61, 9.854, 10.97, 11.57	
	5	3.65, 4.55, 5.33, 7.58, 8.39, 9.37, 9.33, 10.24, 11.89, 12.54, 13.59	
	6	3.59, 5.69, 5.87, 6.29, 8.98, 10.25, 11.00, 12.69, 13.69, 15.91	
75^0c	7	2.98, 4.98, 5.87, 6.38, 8.56, 10.21, 11.98, 11.00, 13.24, 15.38	10.65
	8	3.65, 4.27, 6.29, 8.91, 9.54, 10.14, 12.69, 14.32, 16.90	
	9	3.69, 4.28, 6.72, 8.34, 8.64, 10.81, 11.20, 14.57, 16.90, 18.18	
	10	3.58, 4.92, 6.91, 7.34, 9.38, 11.78, 12.98, 13.92, 15.39, 18.29	
	11	3.58, 4.87, 7.96, 8.64, 10.94, 12.61, 13.94, 15.38, 17.82, 19.34	
	12	5.96, 5.89, 8.91, 9.67, 12.67, 13.54, 15.98, 17.51, 20.64, 23.94	
100^0c	13	4.89, 5.91, 8.47, 9.38, 11.84, 13.57, 15.94, 16.97, 18.54, 19.82	14.09
	14	4.94, 6.85, 7.95, 9.64, 10.87, 12.67, 15.47, 16.32, 18.94, 21.98	
	15	5.97, 6.31, 8.57, 10.91, 12.97, 14.51, 16.78, 18.96, 19.49, 21.34	
	16	4.25, 7.58, 9.34, 10.64, 13.95, 15.27, 16.97, 19.84, 20.46, 22.7	
	17	5.94, 6.28, 8.94, 12.73, 14.61, 16.37, 18.39, 21.78, 22.96, 24.75	
	18	4.18, 8.91, 10.94, 12.71, 15.67, 17.64, 19.78, 21.64, 24.97, 28.45	

Table 2: *Measurement time under different temperatures*

Temperature	Measurement time epochs (in hours)
55°C	107, 238, 540, 838, 1063, 1249, 1536, 1789, 2164, 2414, 1812
75°C	45, 109, 247, 411, 641, 758, 1017, 1232, 1621, 249
100°C	44, 110, 204, 322, 457, 684, 847, 1041, 1204

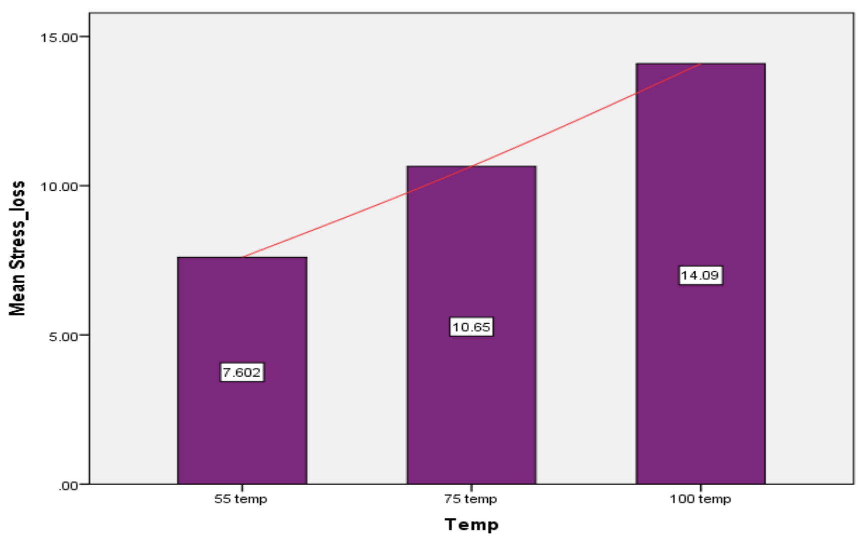

Figure 1: *Measurement temperatures*

In the following, we will determine the optimal ADT plans based on both models. Suppose 10 units are available for the ADT test. In the ADT, we set $\tau_j = 24$, and $k_j = 14$ for all$j = 1,2,\ldots.J$. this setting means that we measure the degradation level once every day, and the test lasts two weeks [19]. Our planning involves selecting the stress level, $(x_1, x_2, \ldots, x_{J-1})$, and the proportion of samples allocated to each testing level, $(N_1, N_2, \ldots, N_{J-1}))$. Consider a two-level ADT plan. Suppose we are interested in minimizing the asymptotic variance of B10, the 0.1-quantile of the failure time distribution at use conditions. When $J = 2$ yields the optimal ADT design

The elements of fisher matrix by solving through mat lab are:

$$\begin{bmatrix} -1.258\times10^{8} & -1.269\times10^{8} & -1.6891\times10^{9} & -20.91\times10^{8} & -1.62\times10^{5} \\ -1.18\times10^{8} & -8.94510\times10^{7} & -1.6541\times10^{9} & -15.7351\times10^{8} & -1.127\times10^{8} \\ -1.26578\times10^{9} & -1.3298\times10^{9} & -6.791\times10^{9} & -8.734\times10^{12} & -1.339\times10^{9} \\ -21.32\times10^{8} & -15.761\times10^{8} & -8.458\times10^{12} & -4.9780\times10^{9} & -1.38\times10^{7} \\ -1.29\times10^{5} & -1.113\times10^{8} & -1.325\times10^{8} & -1.39\times10^{7} & -5.69\times10^{7} \end{bmatrix}$$

Table 3: *Optimization table for random drift model*

Process	x_1	x_2	N_1	N_2	$Std(\varphi p)$
Random drift model	0	1	1	9	4216

The table above displays the ideal ADT design. The fact that 0 is the ideal lower stress value is visually appealing. This outcome is accurate since, even when testing the unit under real-world conditions, the degradation under typical use conditions happens quickly enough to minimize the inaccuracy brought on by extrapolating to the failure threshold.

Table 4: *Optimization table for simple IG process*

Process	x_1	x_2	N_1	N_2	$Std(\varphi p)$
Simple Inverse Gaussian model	0	1	1	9	17450

IV. Discussion

Due to its ability to account for variance in sample product results from unit to unit, the random drift model was chosen for this paper's investigation. With time, many techniques for testing the product are developed. Accelerated deterioration testing, however, is more beneficial in the electronics sector than other approaches. Testing the product quickly is necessary because the corporation creates huge samples of comparable products. To study deterioration performance more effectively, an accelerated degradation test is more appropriate since it increases the stress value during life testing, causing the part to fail faster, and it gathers degradation data to forecast product reliability.

With time, several accelerating degradation models have emerged that can be applied in various contexts. However, to reduce testing costs, it has become imperative for the management to test the number of units that should be tested at a certain stress level. The development of the Simple Stress Accelerated Degradation Test technique considered a number of necessary criteria, including tightening the value of constraints, robustness, and optimality of design. Therefore, the number of units and stress value are optimized using the inverse Gaussian process. This paper presents a proposed model that minimizes the asymptotic variance value to estimate the number of units required for the optimal stress level. A helpful tool for evaluating the value of vectors required to estimate the asymptotic variance is the Fisher information matrix.

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