A TWO NON-IDENTICAL UNIT STANDBY SYSTEM WITH CORRELATED PREVENTIVE MAINTENANCE TIME AND TIME TO PREVENTIVE MAINTENANCE AND INVERSE GAUSSIAN REPAIR TIME DISTRIBUTION

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Abstract

The paper deals with the cost benefit analysis of a two non-identical unit cold standby system model with the implementation of preventive maintenance (PM) on the priority unit after it has operated for a random duration. The objective is to evaluate the economic viability and performance of such system. A single repairman is consistently available within the system, responsible for both PM and repair of each failed unit. The priority in repair is given to priority (p) unit over ordinary (o) unit. The failure time distribution of each unit is assumed to be exponential while the repair time distribution of both the unit is taken as inverse Gaussian. The PM time and time to PM of the priority unit are correlated having their joint distributions as bivariate exponential. By considering the regenerative point technique, various measures of system effectiveness are obtained.

Keywords: Transition probabilities, bivariate exponential distribution, regenerative point, reliability, MTSF, availability, busy period, net expected profit.

I. Introduction

The purpose of reliability engineering is to identify probable failures, implement appropriate actions to enhance reliability and identify the consequences of those failures. The manufacturers as well as consumer of a system always desire a high reliability. High reliability ensures that the system performs its intended function consistently and meets the expectations of its users over time. One way of improving a system's reliability is by incorporating additional or duplicate units into the system. This strategy is known as redundancy. Another crucial way is by providing regular repair and maintenance to the system when they are needed, ensuring its reliability and longevity. Maintenance strategies aim to prevent failures, detect potential issues, and rectify any existing problems to ensure the system operates optimally. Repair and maintenance strategies play a crucial role in improving system reliability, minimizing disruptions and reducing related expenses. These strategies focus on proactive and measures to keep the system in optimal working condition and address potential issues rather than simply responding to problems after they manifest.

Employing redundancies is one of the important aspects of enhancing the system's effectiveness and reliability. Redundant components or resources are intended to serve as backups or fail-safe mechanisms that are ready to take over the functions of primary components if they fail or experience issues. A significant number of authors including [1, 2, 5, 7, 8, 9] have analyzed the two non-identical units cold standby redundant system models due to their vital existence in ensuring uninterrupted operations and minimizing downtime in modern organizations and industries. These system models are particularly relevant in critical systems and industries where the stakes are high and failures can lead to severe consequences, such as aerospace and aviation, healthcare, telecommunications, power distributions, industrial control systems and other mission-critical applications to ensure high reliability and continuity of operations. In practice, planned maintenance activities performed on the system to improve its working capability, prevent potential failures and extend its overall lifespan is called preventive maintenance (PM). PM is a proactive maintenance strategy that involves scheduled inspections, adjustments and repair with the aim of keeping the system in optimal condition and preventing unexpected breakdowns. For example, PM for HVAC (Heating, Ventilation and Air Conditioning) systems. PM tasks may include inspecting electrical connections, calibrating controls, lubricating moving parts and changing filters on a regular basis. By performing these tasks according to a predetermined schedule, potential problems can be found and addressed before they escalate into serious issues, which guarantees the HVAC system will operate effectively and reliably. A number of authors, including [2, 4, 5, 10] have explored the concept of preventive maintenance (PM) i.e. after operating for an arbitrary amount of time, a unit goes for its preventive maintenance. In most of the studies and models related to maintenance and reliability analysis, it's commonly assumed that the working time and PM time of a unit are uncorrelated random variables. However in reality, there is some sort of positive correlation between the failure time and preventive maintenance time of a unit. The concept of correlation between failures times and repair times has been analyzed by various authors including [1, 2, 3, 4, 6].

This paper explores the concept of correlation between time to PM and PM time. The purpose of the present paper is to investigate a two non-identical unit cold standby system model with correlated PM time and time to PM of priority unit having their joint distribution as bivariate exponential. It is also assumed that a single repairman is consistently available with the system for both for PM and repair of each failed unit. Here are some economic related measures of system effectiveness that can be obtained using regenerative point techniques:

- Transition probabilities and sojourn times in various states.
- Reliability analysis and mean time to system failure (MTSF).
- Availability analysis of the system during (0, t).
- Expected busy period of repairman during time interval (0, t) that the repairman is busy in PM and in the repair of p-unit and o-unit.
- Net expected profit earned by the system in the time interval (0, t)

Graphical representations depicting the MTSF and Profit function with respect to different parameters have also been made.

II. System Description and Assumptions

The following are some assumptions about the system model under study:

- The system comprises of two non-identical units. One unit is designated as priority (p) unit while the other is referred as non-priority or ordinary (o) unit.
- Each unit of the system has two possible modes- Normal (N) and Total failure (F).
- Only p-unit is scheduled for preventive maintenance (PM) after working for its random period of time.

- A single repairman is consistently available with the system for PM and repair of a failed unit. The priority in repair and PM is given to p-unit.
- The switching device is used to switch on the standby unit into operation promptly and seamlessly only when the operative unit fails completely. The switching device is assumed to be perfect, independent and instantaneous.
- The failure time distribution of each unit is taken as exponential while the repair time distribution is taken as inverse Gaussian. The time to PM (X) and PM time (Y) are correlated random variables having their joint distribution as bivariate exponential with the density as follows:-

$$f(x,y)=\lambda\mu(1-r)e^{-\lambda x-\mu y}I_0\left(2\sqrt{\lambda\mu rxy}\right)\ ;x,y,\lambda,\mu>0;\ 0\le r<1$$

where,

$$I_0\left(2\sqrt{\lambda\mu rxy}\right)=\sum_{j=0}^{\infty}\frac{(\lambda\mu rxy)^j}{(j!)^2}$$

- Each repaired unit ideally functions as good as new.

III. Notations and States of the System

I. Notations:

E : Set of regenerative states $\equiv\{S_0,S_1,S_2,S_3\}$.

E : Set of non-regenerative states $\equiv\{S_4,S_5\}$.

α_1,α_2 : Constant failure rate of p-unit and o-unit respectively.

$G_i(.)/g_i(.)$: c.d.f./ p.d.f. of time to repair of failed p-unit and o-unit respectively i.e.

$$g_i(t)=\frac{1}{\sqrt{2\pi}}t^{-3/2}\exp\left\{\frac{(t-\beta_i)^2}{2\beta_i^2t}\right\}dt\quad ;\ t>0,\beta_i>0;\{i=1,2\}$$

X : Time to PM of an operating unit when other unit is in standby state.

Y : Time taken in PM of a unit.

f(x,y) : Joint p.d.f. of (X,Y).

$$f(x,y)=\lambda\mu(1-r)e^{-\lambda x-\mu y}I_0\left(2\sqrt{\lambda\mu rxy}\right)\quad ;\ x,y,\lambda,\mu>0;\ 0\le r<1$$

where,

$$I_0\left(2\sqrt{\lambda\mu rxy}\right)=\sum_{j=0}^{\infty}\frac{(\lambda\mu rxy)^j}{(j!)^2}$$

k(y|x) : Conditional p.d.f. of Y given X=x.

$$=\mu e^{-\lambda rx-\mu y}I_0\left(2\sqrt{\lambda\mu rxy}\right)\quad ;\ x,y,\lambda,\mu>0;\ 0\le r<1$$

K(y|x) : Conditional c.d.f. of Y given X=x.

g(x) : Marginal p.d.f. of X i.e.

$$=\lambda(1-r)\exp\{-\lambda(1-r)x\}$$

II. Symbols for the states of the system

N_0^1, N_s^2 : Unit-1/Unit-2 in Normal (N) mode and operative/ standby state.

N_{pm}^1 : Unit-1 in normal mode and under preventive maintenance.

F_r^1, F_w^2 : Unit-1/Unit-2 in failure (F) mode and under repair/waiting for repair.

F_r^2 : Unit-2 in failure (F) mode and under repair.

By considering these symbols according to assumptions stated earlier, we have the following states of the system:

Up states : $S_0 \equiv (N_o^1, N_s^2)$ $S_1 \equiv (N_{pm}^1, N_o^2)$, $S_2 \equiv (F_r^1, N_o^2)$, $S_3 \equiv (N_o^1, F_r^2)$

Down states : $S_5 \equiv (N_{pm}^1, F_w^2)$

Failed states : $S_4 \equiv (F_r^1, F_w^2)$

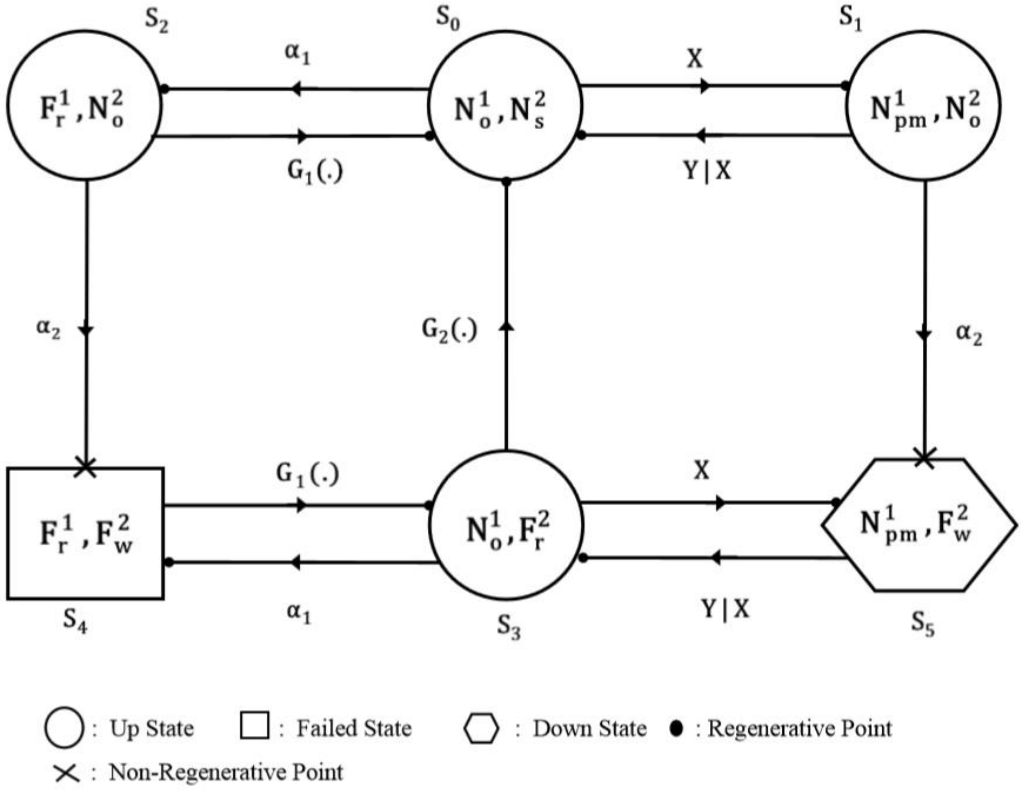

Figure 1: *Transition diagram*

The transition diagram depicting the system model along with failure rates/repair time c.d.f's is shown in Figure 1. From the transition diagram we find that the epochs of transitions into the states S_5 from S_1 and S_4 from S_2 are non-regenerative while all other entrance epochs are regenerative.

IV. Transition Probabilities and Sojourn Times

By using simple probabilistic arguments, the conditional and unconditional transition probabilities are given as:

$$p_{01} = \frac{\lambda(1-r)}{\alpha_1 + \lambda(1-r)} \qquad p_{02} = \frac{\alpha_1}{\alpha_1 + \lambda(1-r)}$$

$$p_{10|x} = \mu' \exp\left\{-\lambda\left(1-\mu'\right)rx\right\} \;;\text{where } \mu' = \frac{\mu}{\mu+\alpha_2} \qquad p_{15} = p_{13|x}^{(5)} = 1-\mu' \exp\left\{-\lambda\left(1-\mu'\right)rx\right\}$$

$$p_{20} = \exp\left[\left\{1-\sqrt{1+2\beta_1^2\alpha_2}\right\}\Big/\beta_1\right] \qquad p_{24} = p_{23}^{(4)} = 1-\exp\left[\left\{1-\sqrt{1+2\beta_1^2\alpha_2}\right\}\Big/\beta_1\right]$$

$$p_{30} = \exp\left[\left\{1 - \sqrt{1+2\beta_2^2\{\alpha_1+\lambda(1-r)\}}\right\}\Big/\beta_2\right]$$

$$p_{34} = \frac{\alpha_1}{\alpha_1+\lambda(1-r)}\left(1-\exp\left[\left\{1 - \sqrt{1+2\beta_2^2\{\alpha_1+\lambda(1-r)\}}\right\}\Big/\beta_2\right]\right)$$

$$p_{35} = \frac{\lambda(1-r)}{\alpha_1+\lambda(1-r)}\left(1-\exp\left[\left\{1 - \sqrt{1+2\beta_2^2\{\alpha_1+\lambda(1-r)\}}\right\}\Big/\beta_2\right]\right)$$

$$p_{43} = \int dG_1(t) = 1 \qquad p_{53|x} = \int dK(t\,|\,x) = 1 \qquad (1\text{-}11)$$

It can be easily verified that

$$p_{01}+p_{02}=1 \qquad p_{10|x}+p_{13|x}^{(5)}=1$$

$$p_{20}+p_{23}^{(4)}=1 \qquad p_{30}+p_{34}+p_{35}=1$$

$$p_{43}=1 \qquad p_{53|x}=1$$

Unconditional transitional probabilities are as follows-

$$p_{10} = \frac{\mu'(1-r)}{(1-r\mu')} \qquad p_{13}^{(5)} = 1-\frac{\mu'(1-r)}{(1-r\mu')}$$

$$p_{53}=1 \qquad (12\text{-}14)$$

Thus, we observe the following relations-

$$p_{01}+p_{02}=1 \qquad p_{10}+p_{13}^{(5)}=1$$

$$p_{20}+p_{23}^{(4)}=1 \qquad p_{30}+p_{34}+p_{35}=1$$

$$p_{43}=1 \qquad p_{53}=1 \qquad (15\text{-}20)$$

Let T_i be the sojourn time in state $S_i \in E$, then the mean sojourn time in state S_i is given by

$$\Theta_i = \int P(T_i > t)dt$$

Therefore

$$\Theta_0 = \frac{1}{\alpha_1+\lambda(1-r)} \qquad \Theta_{1|x} = \frac{1}{\alpha_2}\left[1-\mu'\exp\left\{-\lambda\left(1-\mu'\right)rx\right\}\right]$$

$$\Theta_1 = \frac{\left(1-\mu'\right)}{\alpha_2\left(1-r\mu'\right)} \qquad \Theta_2 = \frac{1}{\alpha_2}\left(1-\exp\left[\left\{1-\sqrt{1+2\beta_1^2\alpha_2}\right\}\Big/\beta_1\right]\right)$$

$$\Theta_3 = \frac{1}{\alpha_1+\lambda(1-r)}\left(1-\exp\left[\left\{1-\sqrt{1+2\beta_2^2\{\alpha_1+\lambda(1-r)\}}\right\}\Big/\beta_2\right]\right)$$

$\Theta_4 = \beta_1$ = mean repair time of p-unit.

$$\Theta_{5|x} = \frac{1+\lambda rx}{\mu} \qquad \Theta_5 = \frac{1}{\mu(1-r)} \qquad (21\text{-}28)$$

V. Analysis of results

I. Reliability and MTSF

Let the random variable T_i be the time to system failure (TSF), when at time t=0, the system starts its operation from state $S_i \in E$. Then, the reliability of the system is given by

$$R_i(t) = \int P(T_i > t)dt$$

To determine $R_i(t)$, we regard the failed state (S_4) of the system as an absorbing state. By employing simple probabilistic arguments, we observe the following relations:

$$R_0(t) = Z_0(t) + q_{01}(t)© R_1(t) + q_{02}(t)© R_2(t)$$
$$R_1(t) = Z_1(t) + q_{15}(t)© Z_5(t) + q_{10}(t)© R_0(t) + q_{13}^{(5)}(t)© R_3(t)$$
$$R_2(t) = Z_2(t) + q_{20}(t)© R_0(t)$$
$$R_3(t) = Z_3(t) + q_{30}(t)© R_0(t) + q_{35}(t)© R_5(t)$$
$$R_5(t) = Z_5(t) + q_{53}(t)© R_3(t) \qquad (29\text{-}32)$$

Where

$$Z_0(t) = \exp\left[-\{\alpha_1 + \lambda(1-r)\}t\right] \qquad Z_1(t) = \int \exp(-\alpha_2 t)\bar{K}(t\,|\,x)g(x)dx$$
$$Z_2(t) = \exp(-\alpha_2 t)\bar{G}_1(t) \qquad Z_3(t) = \exp[-\{\alpha_1 + \lambda(1-r)\}t]\bar{G}_2(t)$$
$$Z_5(t) = \int \bar{K}(t\,|\,x)g(x)dx \qquad (33\text{-}37)$$

Taking the Laplace transform of the relations (29-32) and simplifying the resulting set of equations for $R_0^*(s)$ we obtain;

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad \text{(say)}$$
$$= \frac{\left(Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^* + q_{01}^* q_{15}^* Z_5^*\right)\left(1 - q_{35}^* q_{53}^*\right) + \left(Z_3^* + q_{35}^* Z_5^*\right) q_{01}^* q_{13}^{(5)*}}{\left(1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*\right)\left(1 - q_{35}^* q_{53}^*\right) - q_{01}^* q_{13}^{(5)*} q_{30}^*} \qquad (38)$$

The mean time to system failure (MTSF) can be determined by using the formula;

$$E_0(t) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) \qquad (39)$$
$$= \frac{\left(\Theta_0 + p_{01}\Theta_1 + p_{02}\Theta_2 + p_{01}p_{15}\Theta_5\right)\left(1 - p_{35}\right) + \left(\Theta_3 + p_{35}\Theta_5\right) p_{01} p_{13}^{(5)}}{\left(1 - p_{01}p_{10} - p_{02}p_{20}\right)\left(1 - p_{35}\right) - p_{01} p_{13}^{(5)} p_{30}} \qquad (40)$$

II. Availability Analysis

Let $A_i^p(t)$ and $A_i^o(t)$ be the probabilities that the system is up at epoch 't' due to p-unit and o-unit respectively, when the system initially starts from state $S_i \in E$. By using simple probabilistic laws we get the following relation among $A_i^p(t)$.

$$A_0^p(t) = Z_0(t) + q_{01}(t) © A_1^p(t) + q_{02}(t)© A_2^p(t)$$
$$A_1^p(t) = q_{10}(t) © A_0^p(t) + q_{13}^{(5)}(t)© A_3^p(t)$$
$$A_2^p(t) = q_{20}(t) © A_0^p(t) + q_{23}^{(4)}(t)© A_3^p(t)$$
$$A_3^p(t) = Z_3(t) + q_{30}(t) © A_0^p(t) + q_{34}(t)© A_4^p(t) + q_{35}(t)© A_5^p(t)$$
$$A_4^p(t) = q_{43}(t)© A_3^p(t)$$
$$A_5^p(t) = q_{53}(t)© A_3^p(t) \qquad (41\text{-}46)$$

Where, $Z_0(t)$ and $Z_3(t)$ has already been defined in equations (33) and (36).

Taking the Laplace transform of the relations (41-46) and simplifying the resulting set of equations for $A_0^{p*}(s)$ we obtain;

$$A_0^{p*}(s) = \frac{Z_0^*\left(1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right) + Z_3^*\left(q_{01}^* q_{13}^{(5)*} + q_{02}^* q_{23}^{(4)*}\right)}{\left(1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right)\left(1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*\right) - q_{30}^*\left(q_{01}^* q_{13}^{(5)*} + q_{02}^* q_{23}^{(4)*}\right)} \qquad (47)$$

Similarly, employing the same probabilistic reasoning as in case of $A_i^o(t), (i = 0\text{-}5)$ the recurrence relations among can be determined as follows:-

$$A_0^o(t) = q_{01}(t)©A_1^o(t) + q_{02}(t)© A_2^o(t)$$
$$A_1^o(t) = Z_1(t) + q_{10}(t) ©A_0^o(t) + q_{13}^{(5)}(t)© A_3^o(t)$$
$$A_2^o(t) = Z_2(t) + q_{20}(t) ©A_0^o(t) + q_{23}^{(4)}(t)© A_3^o(t)$$
$$A_3^o(t) = q_{30}(t) ©A_0^o(t) + q_{34}(t)© A_4^o(t) + q_{35}(t)© A_5^o(t)$$
$$A_4^o(t) = q_{43}(t)© A_3^o(t)$$
$$A_5^o(t) = q_{53}(t)© A_3^o(t) \qquad (48\text{-}53)$$

Taking the L.T. of the relations (48-53) and simplifying the resulting sets of algebraic equations for $A_0^{o*}(s)$, we obtain;

$$A_0^{o*}(s) = \frac{\left(1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right)\left(q_{01}^* Z_0^* + q_{02}^* Z_2^*\right)}{\left(1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right)\left(1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*\right) - q_{30}^*\left(q_{01}^* q_{13}^{(5)*} + q_{02}^* q_{23}^{(4)*}\right)} \qquad (54)$$

For brevity, we have omitted the argument's' from $q_{ij}^*(s)$ and $Z_i^*(s)$. Now the steady state availabilities of the system when p-unit and o-unit are operative, respectively given by;

$$A_0^{p*} = \lim_{t\to\infty} A_0^p(t) = \lim_{s\to 0} sA_0^{p*}(s) = N_2/D_2 \qquad (55)$$
$$A_0^{o*} = \lim_{t\to\infty} A_0^o(t) = \lim_{s\to 0} sA_0^{o*}(s) = N_3/D_2 \qquad (56)$$

Where

$$N_2 = p_{30}\Theta_0 + \left\{p_{01}p_{13}^{(5)} + p_{02}p_{23}^{(4)}\right\}\Theta_3 \qquad (57)$$
$$N_3 = p_{30}\left(p_{01}\Theta_1 + p_{02}\Theta_2\right) \qquad (58)$$

We observe that

$$D_2(0) = 0$$

Therefore by using L. Hospital rule, we get

$$A_0^p = \lim_{s\to 0}\frac{N_2(s)}{D_2'(s)} = \frac{N_2}{D_2'} \quad \text{(say)}$$
$$A_0^o = \lim_{s\to 0}\frac{N_3(s)}{D_2'(s)} = \frac{N_3}{D_2'} \quad \text{(say)}$$

Thus, we have

$$\begin{aligned} D_2' = {} & p_{30}\Theta_0 + (1 - p_{01}p_{10} - p_{02}p_{20})\Theta_3 + \{p_{02}p_{30} + (1 - p_{01}p_{10} - p_{02}p_{20})p_{34}\}\Theta_4 \\ & + \{p_{01}p_{30} + (1 - p_{01}p_{10} - p_{02}p_{20})p_{35}\}\Theta_5 \end{aligned} \qquad (59)$$

The mean up time of the system due to p-unit and o-unit during time interval (0, t) are respectively given by;

$$\mu_{up}^p(t) = \int_0^t A_0^p(u)du \quad \text{and} \quad \mu_{up}^o(t) = \int_0^t A_0^o(u)du \qquad (60\text{-}61)$$

Thus

$$\mu_{up}^{p*}(s) = \frac{A_0^{p*}(s)}{s} \quad \text{and} \quad \mu_{up}^{o*}(s) = \frac{A_0^{o*}(s)}{s} \qquad (63\text{-}63)$$

III. Busy period analysis

Let $B_i^1(t), B_i^2(t)$ and $B_i^3(t)$ be the respective probabilities that the repairman is busy in PM, in the repair of a failed p-unit and in the repair of failed o-unit at time 't', when system initially starts from state $S_i \in E$. Using simple probabilistic arguments the system of integral equations for $B_i^1(t), B_i^2(t)$ and $B_i^3(t)$

can be easily developed and by the technique of L.T. the values of $B_0^{1*}(s), B_0^{2*}(s)$ and $B_0^{3*}(s)$ can be easily determined.

The steady state probabilities B_0^1, B_0^2 and B_0^3 are given respectively as follows:

$$B_0^1 = N_4/D_2^{'}, \qquad B_0^2 = N_5/D_2^{'} \qquad \text{and} \qquad B_0^3 = N_6/D_2^{'} \tag{64-66}$$

Where

$$N_4 = p_{01}p_{30}(\Theta_1 + p_{15}\Theta_5) + \left\{p_{01}p_{13}^{(5)} + p_{02}p_{23}^{(4)}\right\}p_{35}\Theta_5$$

$$N_5 = p_{02}p_{30}(\Theta_2 + p_{24}\Theta_4) + \left\{p_{01}p_{13}^{(5)} + p_{02}p_{23}^{(4)}\right\}p_{34}\Theta_4$$

$$N_6 = \left\{p_{01}p_{13}^{(5)} + p_{02}p_{23}^{(4)}\right\}\Theta_3 \tag{67-69}$$

And $D_2^{'}$ is same as in case of availability analysis.

The expected busy periods of the repairman in PM, in repair of failed p-unit and in the repair of failed o-unit respectively, during time interval (0,t) are given by-

$$\mu_b^1(t) = \int_0^t B_0^1(u)du, \qquad \mu_b^2(t) = \int_0^t B_0^2(u)du \qquad \text{and} \qquad \mu_b^3(t) = \int_0^t B_0^3(u)du \tag{70-72}$$

So that

$$\mu_b^{1*}(s) = \frac{B_0^{1*}(s)}{s}, \qquad \mu_b^{2*}(s) = \frac{B_0^{2*}(s)}{s} \qquad \text{and} \qquad \mu_b^{3*}(s) = \frac{B_0^{3*}(s)}{s} \tag{73-75}$$

IV. Profit Function Analysis

The net expected gain incurred in time interval (0,t) is given by-

$$P(t) = K_0\mu_{up}^p(t) + K_1\mu_{up}^o(t) - K_2\mu_b^1(t) - K_3\mu_b^2(t) - K_4\mu_b^3(t) \tag{76}$$

Where

$K_0 =$ revenue per unit time when system is operative due to p-unit.

$K_1 =$ revenue per unit time when system is operative due to o-unit.

$K_2 =$ cost per unit time for PM of p-unit.

$K_3 =$ cost per unit time for repair of failed p-unit.

$K_4 =$ cost per unit time for repair of failed o-unit.

Now the expected profit (gain) per-unit time in steady state is given by-

$$P = K_0A_0^p + K_1A_0^o - K_2B_0^1 - K_3B_0^2 - K_4B_0^3 \tag{77}$$

VI. Graphical Representation

In order to carry out a detailed analysis of the behavior of the system, we plot the MTSF and Profit curves with respect to multiple values of the failure rate (α_1), three distinct values of mean repair time of o-unit (β_2) and two distinct correlation coefficient (r) values.

The MTSF curves w.r.t. "α_1" are displayed in Figure 2 with three distinct values of mean repair time of o-unit (β_2), i.e., 0.25, 0.55, and 0.85, as well as two distinct values of correlation coefficient (r), i.e., 0.2 and 0.7. The other parameters remain constant at β_1 = 0.9, λ = 0.7, μ =0.18, and α_2 = 0.05. From the observations provided in the figure, we observe that MTSF decreases uniformly as the value of failure rate 'α_1' increases. Furthermore, the observation indicates that as the values of the mean repair time of o-unit 'β_2' increase, the expected life of the system decreases. Moreover, with the increase in the value of the correlation coefficient 'r', MTSF tends to increase as well.

From Figure 3, we observe that the profit decreases as failure rate 'α_1' increases with varying three different values of 'β_2' i.e., 0.25, 0.55 and 0.85 and two different values of correlation coefficient 'r' i.e., 0.01 and 0.02, when values of other parameters are kept fixed as $\beta_1 = 0.5$, $\lambda=0.25$, $\mu=0.4$, $\alpha_2 = 0.1$, $K_0 = 60$, $K_1 = 190$, $K_2 = 200$, $K_3 = 300$ and $K_4 = 250$. From the curves, the linear trends in Figure 3 indicate that there is a constant rate of decrease in profit as the values of the failure rate 'α_1' increases.

From Figure 2, the dotted curves depict that to achieve MTSF at least 3000 units, the failure rate 'α_1' of unit-1 must be less than 0.012, 0.016 and 0.021, respectively, for $\beta_2 = 0.25$, 0.55 and 0.85 when r = 0.2. From smooth curves, we observe that to achieve MTSF at least 3500 units, the values of 'α_1' must be less than 0.011, 0.019 and 0.023, respectively, for $\beta_2 = 0.25$, 0.55 and 0.85 when r = 0.7.

From Figure 3, the dotted curves reveal that the system is profitable only if 'α_1' is less than 0.10, 0.18 and 0.28, respectively, for $\beta_2 = 0.25$, 0.55 and 0.85 when r = 0.01. From smooth curves, we conclude that the system is profitable only if 'α_1' is less than 0.11, 0.19 and 0.29, respectively, for $\beta_2 =$ 0.25, 0.55 and 0.85 when r = 0.02.

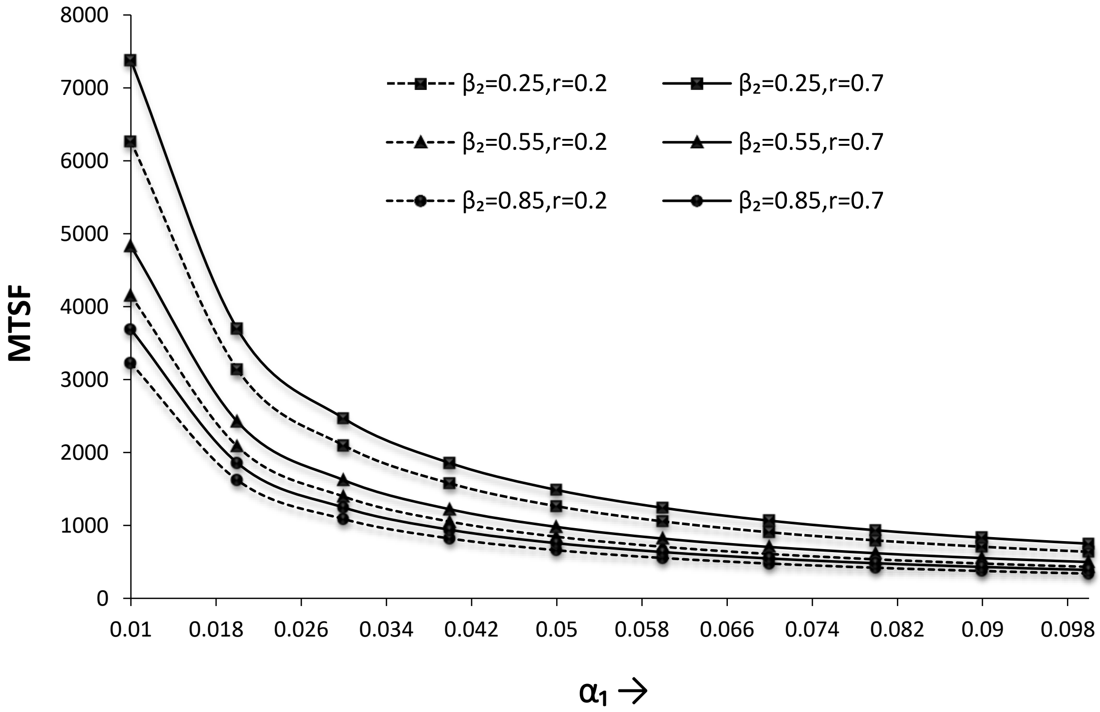

Figure 2: *Behaviour of MTSF with respect to α_1 for different values of β_2 and r*

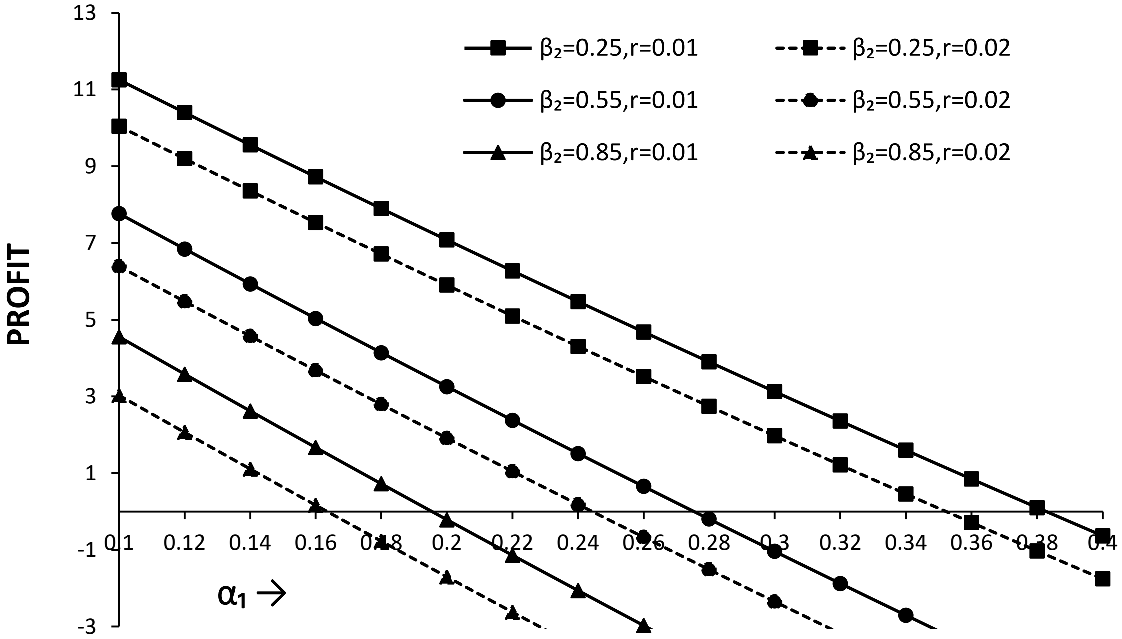

Figure 3: *Behaviour of Profit (P) with respect to α_1 for different values of β_2 and r.*

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