COMPARISON OF SINGLE SERVER RETRIAL QUEUING PERFORMANCE USING FUZZY QUEUING MODEL AND INTUITIONISTIC FUZZY QUEUING MODEL WITH INFINITE CAPACITY

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Abstract

A single server retrial fuzzy queuing model is presented in this study. An unreliable $FM/FM/1$ fuzzy retrial queue with a virtually unlimited retrial orbit and a standard queue is investigated. After an unspecified amount of time has elapsed and the server is workable and inactive, orbit patrons don't rejoin the regular queue, but instead, enter the server momentarily. Customers who arrive and discover the server is engaged or has struggled are placed in the regular queue, whereas customers who are disrupted are always placed in orbit. The model's prosecution proportions are also calculated in a hazy environment. The main goal of this investigation is to compare the efficacy of a single server retrial queuing system based on fuzzy queuing theory and intuitionistic fuzzy queuing theory. The arrival, service, failure, orbit, and repair rates are documented using triangular and triangular intuitionistic fuzzy numbers. The evaluation metrics for the fuzzy queuing theory model are proffered as a range of possible values, whereas the intuitionistic fuzzy queuing theory model encompasses a wide range of values. An approach is conducted to discover quality measures using a design protocol in which the fuzzy values are left alone and not repurposed to crisp values, allowing us to draw research findings in an ambiguous future. Two numerical problems are solved to emphasize the method's protracted survivability.

Keywords: queuing theory, retrial queues, fuzzy numbers, breakdown, repair

I. Introduction

We scrutinize an $FM/FM/1$ fuzzy retrial queue with an undependable server whose retrial orbit and standard queue both have inexhaustible capacity space in this manuscript. People can only access the retrial orbit if their service is thwarted due to an outage. Retrial patrons already don't resume the consistent backlog; instead, they try accessing the server explicitly at random intervals, independent of people arriving and perhaps other retrial clients. These hindered customers, on the other hand, can only regain entry access to the servers when it is fully functional and sedentary, and they just rehash the service until it is efficaciously processed. In the history of queuing systems, a variety of methods for placing fuzzy numbers has been developed. In this paper, we propose a method for solving the single server retrial queuing model in both fuzzy and intuitionistic fuzzy environments while sustaining their essence. Authors and researchers in the literature on fuzzy retrial queuing models used defuzzification methods, whereas here we keep the fuzziness until the end. Our paper is one-of-a-kind in this regard. This method applies to previous methods in that it is straightforward, configurable, and relatable. We can focus on the interplay between the retrial orbit and the standard queue in particular, which is excluded from the overwhelming bulk of retrial concepts which does not include a standard queue with eternal or nontrivial capacity. The random variables are articulated as the combination of two independent random variables, one being a broad sweeping binomial random variable and the other of which conforms to the same estimation for a real-time constructive criticism prototype, that is, one with an unbounded retrial rate. Furthermore, an intriguing stabilization result will be demonstrated, namely that the standard queue can stay constant even though a whole system's (and, specifically, the orbit's) stability condition is contravened. There seem to be different sorts of breakdowns assumed here: engaged breakdowns that happen during a service delivery process and indolent breakdowns that happen when the server is not failing but is sluggish. The time between customer entrants, provider closure, shutdowns, retrials, and refurbishments are assumed to be a random variable with an exponential distribution.

The retrial queuing model with breakages and renovations is a queuing system with a broad array of applications in manufacturing technologies where a server can break down at any time, be repaired, and restarted. Retrial queues and queuing systems with malfunctions have both been intensively investigated in empirical studies. Authors in the antiquity of retrial queue literature considered an innumerable orbit retrial queue and a normal queue, but not a server that is prone to failure. Customers who arrive to seek the server preoccupied can enlist the retrial orbit or the regular queue, according to their model. Customers who arrive to seek the server down (hectic or ceased) are appended to the orbit in retrial concepts without any waiting area and server breakdowns. Some models oblige these customers to join the orbit, whereas others offer them the right to terminate the system. Except for two alternatives, some models also require or enable in-service customers who have been disrupted by a server's inability to enroll in the retrial orbit. Our prescribed concept is unique where orbit consumers need not re-join the standard queue but instead try to enter the server instantaneously after an unidentified amount of time has passed and the server is functional and idle. Customers who arrive and discover the server is overwhelmed or has struggled are placed in the regular queue because customers who are curtailed are always placed in orbit.

Starting failures, vacations, active shutdowns, and both active and idle breakdowns are all taken into account in the retrial fuzzy queue literature. Ramesh et al [1] with the incentre-based sorting method, convert the input rates to crisp numerals. By using retrial queuing models, this article proposes a ruse for perceiving bounteous exploration mission indicators of crisp values for a single server beauty salon using glycolic acid. In a fuzzy environment, the solitary server dual orbit retrial queuing model is probed with customer disparagement. Further, α −cut methodology is used to generate a series of parameterized nonlinear programming for evaluation metrics relying on Zadeh's extension principle, which is then remedied utilizing calculus concepts by S S Sanga et al [2]. Kannadasan et al [3] used hexagonal fuzzy numbers to the input parameters and solved retrial queues with a working vacation. Jain et al [4] looked at the performance of a machine repair system that operates in a fuzzy environment with an admission control F −policy. The steady-state governing equations are constructed using the auxiliary variable correlating to retrial times, and then overt derivations for the queue volume probability distributions are deduced by using the Laplace transform and iterative method, as well as defuzzification. Upadhyaya et al [5] analyzed the $M_x/G/1$ retrial queue with frustrated customers transformed the vacation policy and used Bernoulli feedback. The system size distribution and other key data points are determined using an auxiliary variable approach and the probability-generating function methodology. S S Sanga et al [6] dealt with the admittance control policy for a solo server countable space queuing system with disappointed consumers and dispersed retrial times. By introducing ancillary variables correlating to residual retrial times and interpreting Chapman–Kolmogorov formulations, the steady flow queue size characterization of the system size is reviewed. S S Sanga et al [7] in a dual orbit retrial queuing system with different types of customers, ordinary and premium class customers, the behavior of balking customers was probed. The fuzzified indices are ascertained using a parameterized non-linear optimization framework that relies on the extension principle of Zadeh and the α −cut method is used to determine the fuzzified indices. nonlinear programming approach based on Zadeh's extension principle and α −cut method. Moreover, the performance targets are defuzzified using the ranking index method. Ebenesar Anna Bagyam et al [8] considered the state-dependent batch arrival two-phase retrial queue and used Zadeh's extension principle, the model is further examined in a fuzzy environment. Kalpana et al [9] proposed a numerical method to deduce the membership function of a fuzzy retrial queue with a solo server line model $FM_1, FM_2/FM_1, FM_2/1$ with priority and inequitable service rate. In this paper, fuzzy queues are transmogrified into classical queues using the α −cut methodology and Zadeh's principle. Sherman et al [12] presented several stochastic decomposability results as well as stability conditions where the customers in the retrial queue do not re-join the regular queue; meanwhile, they try to enter the server until it is found to be functional and idle. Mukeba [13] used a method named flexible α −cuts method to quantify the quality metrics of a solo server fuzzy retrial queue with malfunctions and repair work. Kulkarni et al [14] studied the limiting behavior of a solitary server retrial queue where the server is subject to malfunctions and repair work. He used Markov regenerative processes to deduce the convergence criteria and study the system's limiting behavior. Jau-Chuan Ke et al [16] used the α −cut method to turn a fuzzy into a group of traditional retrial queues. A sequence of parameterized non-linear programs is devised to explain the clan of crisp retrial queues using the membership functions of the system components. Artalejo et al [17] are concerned about the balking retrial queue. Using classical mean diffusion characteristics, the ergodicity condition is first researched. A recursive approach based on the theory of regenerative processes is used to ascertain the restricting distribution of the number of clients in the system. Kannadasan et al [18] examined finite capacity retrial queues using hexagonal fuzzy numbers. Rani Shobha et al [19] used ANFIS strategy and set of differential linear equations in the markovian retrial queue with double orbits.

Most previous research on fuzzy queuing models has concentrated on two or three fuzzy variables, with researchers employing ranking techniques or defuzzification processes to repurpose fuzzy variables into crisp. In this paper, we propose a way to collect information about system behavior for retrial queues by using five fuzzy variables. Throughout the paper, we keep the fuzzy values and don't change them to crisp values for the different membership functions (TFN and TIFN).

The remaining part of the article is configured as regards. Prelims and definitions are covered in section 2. The mathematical formalism, as well as the circumstances for stability, are described in Section 3. The layout method for dealing with the current model is detailed in Section 4. Standard queuing relevant factors are discussed in Section 5, and Mathematical descriptions and visual observations are provided in Section 6. This work wraps up with Section 7.

II. Preliminaries

The motive of this division is to give some basic definitions, annotations, and outcomes that are used in our further calculations.

Definition 2.1. [10] A fuzzy set $\tilde{A}$ is defined on R, the set of real numbers is called a **fuzzy number** if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ has the following conditions:

(a) $\tilde{A}$ is convex, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$, such that $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$
(b) $\tilde{A}$ is normal, which means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = \tilde{1}$
(c) $\tilde{A}$ is piecewise continuous.

Definition 2.2. [10] A fuzzy number $\tilde{A}$ is defined on R, the set of real numbers is said to be a **triangular fuzzy number (TFN)** if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ which satisfies the following conditions:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-\tilde{\alpha}_1}{\tilde{\alpha}_2-\tilde{\alpha}_1} \text{ for } \tilde{\alpha}_1 \le x \le \tilde{\alpha}_2 \\ 1 \text{ for } x = \tilde{\alpha}_2 \\ \frac{\tilde{\alpha}_3-x}{\tilde{\alpha}_3-\tilde{\alpha}_2} \text{ for } \tilde{\alpha}_2 \le x \le \tilde{\alpha}_3 \\ 0 \text{ otherwise} \end{cases}$$

The triangular fuzzy number is illustrated in Figure 1.

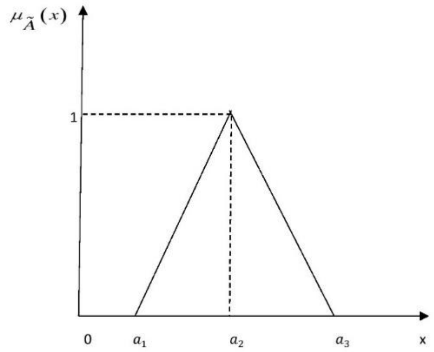

Figure 1: *Triangular fuzzy number*

Definition 2.3. Let the two triangular fuzzy numbers be $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ and then the **arithmetic operations on TFN** be given as follows:

(A)Addition

$$\tilde{P} + \tilde{Q} \approx (\tilde{m}_1 + \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \tag{1}$$

(B)Subtraction

$$\tilde{P} - \tilde{Q} \approx (\tilde{m}_1 - \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \tag{2}$$

(C) Multiplication

$$\tilde{P}.\tilde{Q} \approx (\tilde{m}_1.\tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \tag{3}$$

(D) Division

$$\frac{\tilde{P}}{\tilde{Q}} \approx \left(\frac{\tilde{m}_1}{\tilde{m}_2}, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}\right) \tag{4}$$

Definition 2.4. For every triangular fuzzy number $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3) \in F(R)$ **ranking function** $\Re: F(R) \to R$ is defined by graded mean as

$$\Re(\tilde{P}) = \frac{(\tilde{\alpha}_1 + 4\tilde{\alpha}_2 + \tilde{\alpha}_3)}{6}$$

For any two TFN $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ we have the following comparisons,

$$(a)\tilde{P} \succ \tilde{Q} \Leftrightarrow \Re(\tilde{P}) > \Re(\tilde{Q})$$
$$(b)\tilde{P} \prec \tilde{Q} \Leftrightarrow \Re(\tilde{P}) < \Re(\tilde{Q})$$
$$(c)\tilde{P} \approx \tilde{Q} \Leftrightarrow \Re(\tilde{P}) = \Re(\tilde{Q})$$
$$(d)\tilde{P} - \tilde{Q} \approx 0 \Leftrightarrow \Re(\tilde{P}) - \Re(\tilde{Q}) = 0$$

A triangular fuzzy number $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3) \in F(R)$ is known to be **positive** if $\Re(\tilde{P}) > 0$ and defined by $\tilde{P} \succ 0$

Definition 2.5. [11] Let a non–empty set be X. An **Intuitionistic fuzzy set (IFS)** $\tilde{A}'$ is defined as $\tilde{A}' = \{(x, \mu_{\tilde{A}'}(x), \gamma_{\tilde{A}'}(x)/x \in X)\}$, where $\mu_{\tilde{A}'}: X \to [0,1]$ and $\gamma_{\tilde{A}'}: X \to [0,1]$ denotes the degree of membership and degree of non–membership functions respectively where $x \in X$, for every $x \in X, 0 \leq \mu_{\tilde{A}'}(x) + \gamma_{\tilde{A}'}(x) \leq 1$.

Definition 2.6 [11] An intuitionistic fuzzy set described on R, the real numbers are said to be an **Intuitionistic fuzzy number (IFN)** if its membership function $\mu_{\tilde{A}'}: R \to [0,1]$ and its non–membership function $\gamma_{\tilde{A}'}: R \to [0,1]$ should be agreeable to the following conditions:

i) $\tilde{A}'$ is normal, which means that there exists an $x \in R$, such that $\mu_{\tilde{A}'}(x) = 1, \gamma_{\tilde{A}'}(x) = 0$

ii) $\tilde{A}'$ is convex for the membership functions $\mu_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\mu_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}'}(x_1), \mu_{\tilde{A}'}(x_2)\}$.

iii) $\tilde{A}'$ is concave for the non–membership function $\gamma_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\gamma_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \leq max\{\gamma_{\tilde{A}'}(x_1), \gamma_{\tilde{A}'}(x_2)\}$.

Definition 2.7. [11] A fuzzy number $\tilde{A}'$on R is said to be a **triangular intuitionistic fuzzy number (TIFN)** if its membership function $\mu_{\tilde{A}'}: R \to [0,1]$ and non-membership function $\gamma_{\tilde{A}'}: R \to [0,1]$ has the following conditions:

$$\mu_{\tilde{A}'}(x) = \begin{cases} \frac{x-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1} \text{ for } \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ 1 \text{ for } x = \tilde{a}_2 \\ \frac{\tilde{a}_3-x}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ 0 \text{ otherwise} \end{cases}$$

and

$$\gamma_{\tilde{A}'}(x) = \begin{cases} 1 \text{ for } x < \tilde{a}'_1, x > \tilde{a}'_3 \\ \frac{\tilde{a}_2-x}{\tilde{a}_2-\tilde{a}'_1} \text{ for } \tilde{a}'_1 \leq x \leq \tilde{a}_2 \\ 0 \text{ for } x = \tilde{a}_2 \\ \frac{x-\tilde{a}_2}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}'_3 \end{cases}$$

and is given by $\tilde{A}' = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$.
The triangular intuitionistic fuzzy number is illustrated in Figure 2.
Cases: Let $\tilde{A}' = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ be a TIFN then the following cases arise.
Case:1 If $\tilde{a}'_1 = \tilde{a}_1, \tilde{a}'_3 = \tilde{a}_3$ then $\tilde{A}'$ represent a triangular fuzzy number.
Case:2 If $\tilde{a}'_1 = \tilde{a}_1 = \tilde{a}_2 = \tilde{a}'_3 = \tilde{a}_3 = \tilde{m}$ then $\tilde{A}'$ represent a real number $\tilde{m}$. The parametric form of TIFN $\tilde{A}'$ is represented as $\tilde{A}' = (\tilde{\alpha}, \tilde{m}, \tilde{\beta}; \tilde{\alpha}', \tilde{m}, \tilde{\beta}')$ where $\tilde{\alpha}, \tilde{\alpha}'$&$\tilde{\beta}, \tilde{\beta}'$represents the left spread and right spread of membership functions and non–membership functions respectively.

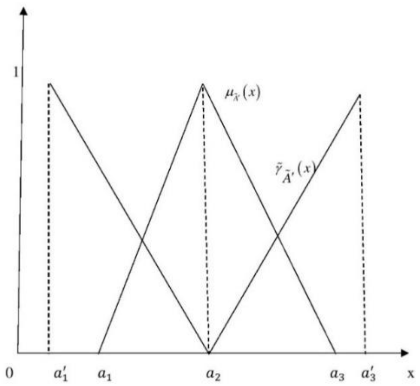

Figure 2: *Triangular intuitionistic fuzzy number*

Definition 2.8. The extension of fuzzy arithmetic operations of Ming Ma et al [10] to the set of triangular intuitionistic fuzzy numbers based upon both location indices and functions of fuzziness indices. The location indices number is taken in the regular arithmetic while the functions of fuzziness indices are assumed to follow the lattice rule which is the least upper bound in the lattice $\tilde{I}'$.

For any two arbitrary TIFN $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ and $* \in \{+,-,\times,\div\}$, then the **arithmetic operations on TIFN** are defined by $\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1' \vee \tilde{\alpha}_2', \tilde{\beta}_1' \vee \tilde{\beta}_2')$

In particular, for any two TIFN $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ the arithmetic operations are defined as

$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1') * (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$
$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 * \tilde{m}_2, max\{\tilde{\alpha}_1', \tilde{\alpha}_2'\}, max\{\tilde{\beta}_1', \tilde{\beta}_2'\})$
$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1' \vee \tilde{\alpha}_2', \tilde{\beta}_1' \vee \tilde{\beta}_2')$

In particular, for any two TIFN $\tilde{P}' \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2', \tilde{a}_3') \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$, $\tilde{Q}' \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1', \tilde{b}_2', \tilde{b}_3') \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ we define:

Addition

$$\tilde{P}' + \tilde{Q}' = (\tilde{m}_1 + \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 + \tilde{m}_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \tag{5}$$

Subtraction

$$\tilde{P}' - \tilde{Q}' = (\tilde{m}_1 - \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 - \tilde{m}_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \tag{6}$$

Multiplication

$$\tilde{P}' \times \tilde{Q}' = (\tilde{m}_1 \times \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 \times \tilde{m}_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \tag{7}$$

Division

$$\tilde{P}' \div \tilde{Q}' = (\tilde{m}_1 \div \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 \div \tilde{m}_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \tag{8}$$

Definition 2.9. Consider an arbitrary TIFN $\tilde{A}' = (a_1, a_2, a_3; a_1', a_2, a_3') = (m, \alpha, \beta; m, \alpha', \beta')$ and the magnitude of TIFN $\tilde{A}'$ is given by

$$mag(\tilde{A}') = \frac{1}{2}\int_0^1 (\tilde{\beta} + \tilde{\beta}' + 6\tilde{m} - \tilde{\alpha} - \tilde{\alpha}')f(r)dr$$

In real-life scenarios, decision-makers select the value of $\tilde{f}(\tilde{r}')$ based on their circumstances. Here for our ease, we choose $f(r) = r^2$

$$\therefore\ mag(\tilde{A}') = \left(\frac{\tilde{\beta} + \tilde{\beta}' + 6\tilde{m} - \tilde{\alpha} - \tilde{\alpha}'}{6}\right)$$

For any two TIFN $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1',)$ & $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_1', \tilde{\beta}_1')$ in $F(R)$, we define

$$(a)\tilde{P}' \geq \tilde{Q}' \Leftrightarrow mag(\tilde{P}') \geq mag(\tilde{Q}')$$
$$(b)\tilde{P}' \leq \tilde{Q}' \Leftrightarrow mag(\tilde{P}') \leq mag(\tilde{Q}')$$
$$(c)\tilde{P}' \approx \tilde{Q}' \Leftrightarrow mag(\tilde{P}') = mag(\tilde{Q}')$$

III. Model Description and Stability Conditions

Presume that a single type of customer enters the queue through a Poisson process with a fuzzy parameter $\tilde{\lambda}$. They form a queue to receive an exponentially distributed service with a fuzzy rate $\tilde{\mu}$ from an unreliable server whose failure times are independent and exponentially distributed with a fuzzy rate $\tilde{\omega}$. When a customer's service is obstructed due to a server failure, the customer can exit the zone and enter the retrial orbit, where a rate is a fuzzy number $\tilde{\theta}$. During this time, the server is delegated to be repaired at a variable rate $\tilde{\psi}$. When the server is functional and idle, orbit consumers do not rejoin the standard queue and instead try to enter the server explicitly after an unspecified period. All processes in the system are hypothesized to be self-contained and distributed uniformly. The queue and orbit sizes are assumed to be infinite, and the service discipline is FIFO (first in first out).

Customers enter the system through a Poisson process with a rate $\tilde{\lambda} > 0; \tilde{\lambda}' > 0$ and response times are independent and identically distributed exponential random variables with rate $\tilde{\mu} > 0; \tilde{\mu}' > 0$. Server faults happen at a stable level $\tilde{\varpi} > 0; \tilde{\varpi}' > 0$, and server repair occurs at a constant rate of $\tilde{\psi} > 0; \tilde{\psi}' > 0$. A customer whose service is disrupted by a server outage joins orbit and spends an accelerating span with a rate $\tilde{\theta} > 0; \tilde{\theta}' > 0$, whereby it arrives service (if available) or persists in orbit for a supplemental period with rate $\tilde{\theta}$ and exponentially distributed time.

The number of clients/messages in the line at the time $\tilde{t}$ is signified by $\tilde{N}_{q\tilde{t}}$. $\tilde{N}_{o\tilde{t}}$ stands for the number of clients/messages in the orbit at the time $\tilde{t}$. The random process $\tilde{X}_{\tilde{t}}$ is the invasion status of the site supplied by

$$\tilde{X}_{\tilde{t}} = \begin{cases} 1 & \text{if the site is overloaded at the time period } \tilde{t} \\ 0 & \text{if the site is not occupied at the time period } \tilde{t} \end{cases}$$

whereas $\tilde{Y}_{\tilde{t}}$ exemplifies the site's operational capability at the time $\tilde{t}$ categorized by

$$\tilde{Y}_{\tilde{t}} = \begin{cases} 1 & \text{if the site is up and running at the time period } \tilde{t} \\ 0 & \text{if the site is down at the time period } \tilde{t} \end{cases}$$

Then $\{(\tilde{N}_{q\tilde{t}}, \tilde{X}_{\tilde{t}}, \tilde{N}_{o\tilde{t}}, \tilde{Y}_{\tilde{t}}): \tilde{t} \geq 0\}$ is a continuous-time Markov process of explaining the system's state at the time $\tilde{t}$. Let $\tilde{N}_{s\tilde{t}}$ symbolised the total number of clients/messages in the system at a time $\tilde{t}$ which means it represents the number in orbit, queue, and in service. The procedure $\{\tilde{N}_{s\tilde{t}}: \tilde{t} \geq 0\}$ exemplifies how the system size varies over time. The server is operational for a proportion of time, and $\frac{\tilde{\psi}}{(\tilde{\psi}+\tilde{\varpi})}$; thus, the excellent service rate is $\frac{\tilde{\psi}\tilde{\mu}}{(\tilde{\psi}+\tilde{\varpi})}$ and $\frac{\tilde{\lambda}(\tilde{\psi}+\tilde{\varpi})}{\tilde{\psi}\tilde{\mu}} < 1$ is a necessary and sufficient condition for stability analysis.[15]

Specifying $\tilde{\pi}_{m,n,o,p}$ as the restricting probability that the system is in the state (m, n, o, p), i.e.,

$$\tilde{\pi}_{m,n,o,p} = \tilde{t} \xrightarrow{\lim} \infty P\left(\tilde{N}_{q\tilde{t}}=m, \tilde{X}_{\tilde{t}}=n, \tilde{N}_{o\tilde{t}}=o, \tilde{Y}_{\tilde{t}}=p\right),$$

Where the index m represents the queue size, the index n represents the invasion status (0 or 1), the index o represents the size of the orbit and the index p represents the operational capability of the server (0 or 1). The orbit and queue size are depicted by the morph variables $\tilde{v}_1$ and $\tilde{v}_2$.

Let the generating function of $\tilde{\pi}_{m,n,o,p}$ concerning the orbit size as follows
$\varepsilon_{m,n,p}(\tilde{v}_1) = \sum_{o=0}^{\infty} \tilde{v}_1^o \, \tilde{\pi}_{m,n,o,p}$ and

Let the generating function of $\tilde{\varepsilon}_{m,n,p}(\tilde{v}_1)$ concerning the queue size as follows

$$\tilde{\chi}_{n,p}(\tilde{v}_1, \tilde{v}_2) = \sum_{m=0}^{\infty} \tilde{v}_2^m \, \varepsilon_{m,n,p}(\tilde{v}_1)$$

Consider the probability-generating function as $\tilde{\varepsilon}_{0,0,1}(\tilde{v}_1)$, $\tilde{\chi}_{0,0}(\tilde{v}_1,\tilde{v}_2)$and $\tilde{\chi}_{1,1}(\tilde{v}_1,\tilde{v}_2)$ when the server is sluggish, ceased, and strenuous respectively. Define

$$E(\tilde{v}_1,\tilde{v}_2)=\sum_{m=0}^{\infty}\sum_{o=0}^{\infty}p(o,m)\,\tilde{v}_1^{o}\tilde{v}_2^{m}=\tilde{\varepsilon}_{0,0,1}(\tilde{v}_1)+\tilde{\chi}_{0,0}(\tilde{v}_1,\tilde{v}_2)+\tilde{\chi}_{1,1}(\tilde{v}_1,\tilde{v}_2)$$

is the joint probability generating function of the orbit and queue size where $p(o,m)$ is the joint probability mass function of $\widetilde{N}_q$ and $\widetilde{N}_o$. And

$$F(\tilde{v})=\sum_{o=0}^{\infty}q(o)\,\tilde{v}^{o}=\tilde{\varepsilon}_{0,0,1}(\tilde{v})+\tilde{\chi}_{0,0}(\tilde{v},\tilde{v})+\tilde{v}\tilde{\chi}_{1,1}(\tilde{v},\tilde{v})$$

is the probability-generating function of system size where $q(o)$ denote the probability mass function of $\widetilde{N}_s$.

IV. Single Server Retrial Queues $(FM/FM/1)$: $(\infty/FIFO)$ in Fuzzy and Intuitionistic Fuzzy Environment

We assume a solitary-server retrial fuzzy queuing system with limitless capacity. The inter-arrival rates $\tilde{\lambda}$, service rate $\tilde{\mu}$, retrial rate $\tilde{\theta}$, failure rate $\tilde{\omega}$ and repair rate $\tilde{\psi}$ are nearly comprehended and depicted by a fuzzy set,

$$\tilde{\lambda}=\{a,\mu_{\tilde{\lambda}}(a)/a\in A\}$$
$$\tilde{\mu}=\{s,\mu_{\tilde{\mu}}(s)/s\in S\}$$
$$\tilde{\theta}=\{o,\mu_{\tilde{\theta}}(o)/o\in O\}$$
$$\tilde{\omega}=\{f,\mu_{\tilde{\omega}}(f)/f\in F\}$$
$$\tilde{\psi}=\{r,\mu_{\tilde{\psi}}(r)/r\in R\}$$

In this, A,S,O,F & R are a traditional universal set of arrival rate, service rate, orbit rate, failure rate, and repair rate respectively and their corresponding membership functions are given as $\mu_{\tilde{\lambda}}(a),\mu_{\tilde{\mu}}(s),\mu_{\tilde{\theta}}(o),\mu_{\tilde{\omega}}(f)\&\mu_{\tilde{\psi}}(r)$ respectively. In addition to that, assume a solitary server retrial intuitionistic fuzzy queuing system with limitless capacity. The inter-arrival rates $\tilde{\lambda}'$, service rate $\tilde{\mu}'$, retrial rate $\tilde{\theta}'$, failure rate $\tilde{\omega}'$ and repair rate $\tilde{\psi}'$ are nearly comprehended and depicted by an intuitionistic fuzzy set,

$$\tilde{\lambda}'=\{a,\mu_{\tilde{\lambda}'}(a),\gamma_{\tilde{\lambda}'}(a)/a\in A\}$$
$$\tilde{\mu}'=\{s,\mu_{\tilde{\mu}'}(s),\gamma_{\tilde{\mu}'}(s)/s\in S\}$$
$$\tilde{\theta}'=\{o,\mu_{\tilde{\theta}'}(o),\gamma_{\tilde{\theta}'}(o)/o\in O\}$$
$$\tilde{\omega}'=\{f,\mu_{\tilde{\omega}'}(f),\gamma_{\tilde{\omega}'}(f)/f\in F\}$$
$$\tilde{\psi}'=\{r,\mu_{\tilde{\psi}'}(r),\gamma_{\tilde{\psi}'}(r)/r\in R\}$$

In this, A,S,O,F & R are a traditional set of arrival, service, orbit, failure, and repair rate respectively and their corresponding membership and non-membership functions are given as $\mu_{\tilde{\lambda}'}(a),\mu_{\tilde{\mu}'}(s),\mu_{\tilde{\theta}'}(o),\mu_{\tilde{\omega}'}(f),\mu_{\tilde{\psi}'}(r)$ & $\gamma_{\tilde{\lambda}'}(a),\gamma_{\tilde{\mu}'}(s),\gamma_{\tilde{\theta}'}(o),\gamma_{\tilde{\omega}'}(f),\gamma_{\tilde{\psi}'}(r)$ respectively.

V. Solo Server Retrial Queuing Model with Infinite Capacity

Let the following assumptions $\tilde{\lambda}$ and $\tilde{\lambda}'$ be the fuzzy and intuitionistic fuzzy arrival rates respectively; $\tilde{\mu}$ and $\tilde{\mu}'$ be the fuzzy and intuitionistic fuzzy service rates respectively; $\tilde{\theta}$ and $\tilde{\theta}'$ be the fuzzy and intuitionistic fuzzy retrial(orbit) rate; $\tilde{\omega}$ and $\tilde{\omega}'$ be the fuzzy and intuitionistic fuzzy failure rates respectively; $\tilde{\psi}$ and $\tilde{\psi}'$ be the fuzzy and intuitionistic fuzzy repair rates respectively. At the steady-state, the $FIFO$ discipline is upheld and the capacity is unlimited.

The following are the fabrication characteristics of the above model:

i) The number of customers in the queue is given as

$$\tilde{N}_q = \frac{\tilde{\lambda}\left[\tilde{\mu}\tilde{\omega}(\tilde{\mu}+\tilde{\omega})+\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})^2\right]}{\tilde{\mu}(\tilde{\psi}+\tilde{\omega})[\tilde{\psi}(\tilde{\mu}+\tilde{\omega})-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} \quad (9)$$

ii) The sojourn time of customers in the queue is given as

$$\tilde{T}_q = \frac{\left[\tilde{\mu}\tilde{\omega}(\tilde{\mu}+\tilde{\omega})+\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})^2\right]}{\tilde{\mu}(\tilde{\psi}+\tilde{\omega})[\tilde{\psi}(\tilde{\mu}+\tilde{\omega})-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} \quad (10)$$

iii) The number of customers in the orbit is given as

$$\tilde{N}_o = \frac{\tilde{\psi}\tilde{\lambda}\tilde{\omega}[\tilde{\mu}(\tilde{\mu}+\tilde{\omega}-\tilde{\lambda})+\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]}{\tilde{\mu}[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})][\tilde{\psi}(\tilde{\mu}+\tilde{\omega})-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} + \frac{\tilde{\lambda}\tilde{\omega}(\tilde{\psi}+\tilde{\omega})}{\tilde{\theta}[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} \quad (11)$$

iv) The sojourn time of customers in the orbit is given as

$$\tilde{T}_o = \frac{\tilde{\psi}\tilde{\omega}[\tilde{\mu}(\tilde{\mu}+\tilde{\omega}-\tilde{\lambda})+\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]}{\tilde{\mu}[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})][\tilde{\psi}(\tilde{\mu}+\tilde{\omega})-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} + \frac{\tilde{\omega}(\tilde{\psi}+\tilde{\omega})}{\tilde{\theta}[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} \quad (12)$$

v) The number of customers in the system is given as

$$\tilde{N}_s = \frac{\tilde{\lambda}\left[\tilde{\mu}\omega+(\tilde{\psi}+\tilde{\omega})^2\right]}{(\tilde{\psi}+\tilde{\omega})[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} + \frac{\tilde{\lambda}\tilde{\omega}(\tilde{\psi}+\tilde{\omega})}{\tilde{\theta}[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} \quad (13)$$

vi) The waiting time of customers in the system is given as

$$\tilde{T}_s = \frac{\left[\tilde{\mu}\omega+(\tilde{\psi}+\tilde{\omega})^2\right]}{(\tilde{\psi}+\tilde{\omega})[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} + \frac{\tilde{\omega}(\tilde{\psi}+\tilde{\omega})}{\tilde{\theta}[\tilde{\psi}\tilde{\mu}-\tilde{\lambda}(\tilde{\psi}+\tilde{\omega})]} \quad (14)$$

VI. Mathematical Description

We considered a communications network with a cohort of streaming server devices linked to an interface message microprocessor as a bandwidth network. Messages arrive in a Poisson stream at the webserver. If the web host wants to transmit information to someone else host controller, one must deliver the data along with the node to the interface message processing unit with which it is hooked up. The message is acknowledged if the processor is free; alternatively, it is assumed to be a failure and the message is returned to the streaming server computer hard disk in a barrier to be transcoded at a later point and is considered a repair rate. In queuing terminology, the buffer in the host controller, the interface processing, and the transcoded policy correlate to the orbit, server, and retrial discipline, respectively. The above system can be modeled using an $FM/FM/1$ retrial queuing model. For consideration of performance and efficiency, the organization wants to learn more about the platform's characteristics, such as the expected wait time and the number of messages in orbit, queue, and system. Interpret the entry, departure, retrial, failure, and repair rate as both TFNs and TIFNs symbolized by $\tilde{\lambda}, \tilde{\lambda}'$; $\tilde{\mu}, \tilde{\mu}'$; $\tilde{\theta}, \tilde{\theta}'$; $\tilde{\omega}, \tilde{\omega}'$ and $\tilde{\psi}, \tilde{\psi}'$ respectively.

6.1 Solo Server Retrial Fuzzy Queuing Model with Unlimited Capability

Let $\tilde{\lambda} = (4,5,6)$, is the arrival rate, $\tilde{\mu} = (26,27,28)$ is the service rate, $\tilde{\theta} = (15,16,17)$ is the retrial rate, $\tilde{\omega} = (37,38,39)$ is the failure rate, $\tilde{\psi} = (47,48,49)$ is the repair rate.

Determine the TFN in the form of $(\tilde{m}, \tilde{\alpha}, \tilde{\beta})$ as $\tilde{\lambda} = (5,1,1)$, $\tilde{\mu} = (27,1,1)$, $\tilde{\theta} = (16,1,1)$, $\tilde{\omega} = (38,1,1)$, and $\tilde{\psi} = (48,1,1)$.

To determine the values of a number of messages and their sojourn time in the queue, orbit as well as a system using suitable formulas among (9), (10), (11), (12), (13) & (14). It is necessary to use the appropriate arithmetic operations described in (1), (2), (3), and (4) for add, sub, multiply, and divide, respectively.

The metrics of performance are calculated and tabulated as follows:

Table 1: *Performance Measures using Triangular Fuzzy Numbers*

Components	Number of Messages$(\tilde{\boldsymbol{N}})$	Waiting Time $(\tilde{\boldsymbol{T}})$
Queue	$\tilde{N}_q = (-0.9171,0.0829,1.0829)$	$\tilde{T}_q = (-0.98342,0.01658,1.01658)$
Orbit	$\tilde{N}_o = (0.4764,1.4764,2.4764)$	$\tilde{T}_o = (-0.7048,0.2952,1.2952)$
System	$\tilde{N}_s = (0.7446,1.7446,2.7446)$	$\tilde{T}_s = (-0.6511,0.3489,1.3489)$

6.2 Solo Server Retrial Intuitionistic Fuzzy Queuing Model with Unlimited Capability

Let $\tilde{\lambda}' = (4,5,6;3,5,7)$, is the arrival rate, $\tilde{\mu}' = (26,27,28;25,27,29)$ is the service rate, $\tilde{\theta}' = (15,16,17;14,16,18)$ is the retrial rate, $\tilde{\omega}' = (37,38,39;36,38,40)$ is the failure rate, $\tilde{\psi}' = (47,48,49;46,48,50)$ is the repair rate.

Determine the TIFN in the form of $(\tilde{m},\tilde{\alpha},\tilde{\beta};\tilde{m},\tilde{\alpha}',\tilde{\beta}')$ as $\tilde{\lambda}' = (5,1,1;5,2,2)$, $\tilde{\mu}' = (27,1,1;27,2,2)$, $\tilde{\theta}' = (16,1,1;16,2,2)$, $\tilde{\omega}' = (38,1,1;38,2,2)$, and $\tilde{\psi}' = (48,1,1;48,2,2)$.

To determine the values of a number of messages and their sojourn time in the queue, orbit as well as a system using suitable formulas among (9), (10), (11), (12), (13) & (14). It is necessary to use the appropriate arithmetic operations described in (5), (6), (7), and (8) for add, sub, multiply, and divide, respectively.

The metrics of performance are calculated and tabulated as follows:

Table 2: *Performance Measures using triangular intuitionistic fuzzy numbers*

Components	Number of Messages$(\tilde{\boldsymbol{N}}')$	Waiting Time $(\tilde{\boldsymbol{T}}')$
Queue	$\tilde{N}_q' = \begin{pmatrix} -0.9171,0.0829,1.0829; \\ -1.9171,0.0829,2.0829 \end{pmatrix}$	$\tilde{T}_q' = \begin{pmatrix} -0.98342,0.01658,1.01658; \\ -1.98342,0.01658,2.01658 \end{pmatrix}$
Orbit	$\tilde{N}_o' = \begin{pmatrix} 0.4764,1.4764,2.4764; \\ -0.5236,1.4764,3.4764 \end{pmatrix}$	$\tilde{T}_o' = \begin{pmatrix} -0.7048,0.2952,1.2952; \\ -1.7048,0.2952,2.2952 \end{pmatrix}$
System	$\tilde{N}_s' = \begin{pmatrix} 0.7446,1.7446,2.7446; \\ -0.2554,1.7446,3.7446 \end{pmatrix}$	$\tilde{T}_s = \begin{pmatrix} -0.6511,0.3489,1.3489; \\ -1.6511,0.3489,2.3489 \end{pmatrix}$

The following figures 3 – 14 depict the visualizations of Tables 1 and 2.

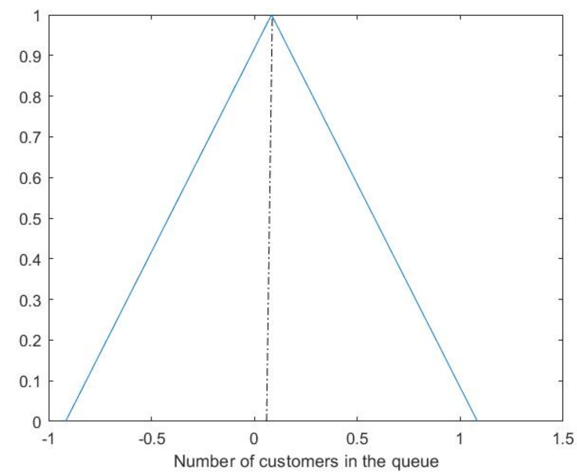

Figure 3: *The number of messages $(\tilde{N}_q)$ in the queue*

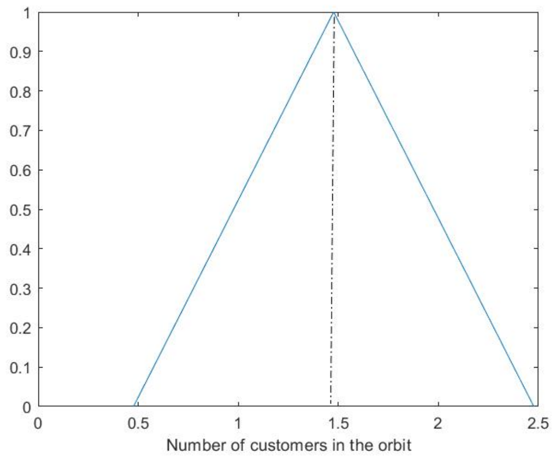

Figure 4: *The number of messages $(\tilde{N}_o)$in the orbit*

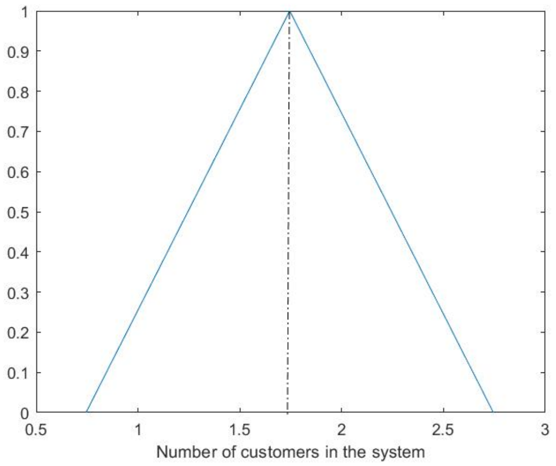

Figure 5: *The number of messages* $(\tilde{N}_s)$ *in the system*

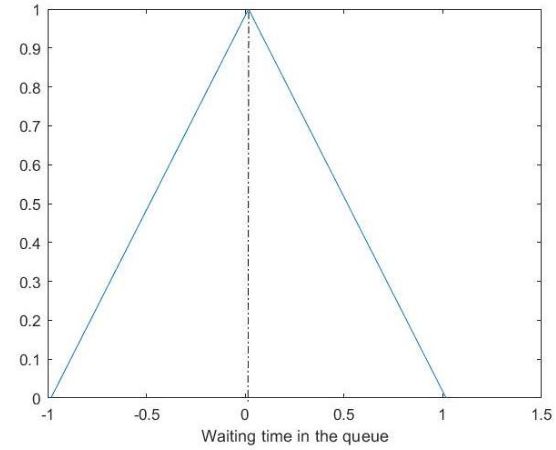

Figure 6: *The waiting time of messages* $(\tilde{T}_q)$*in the queue*

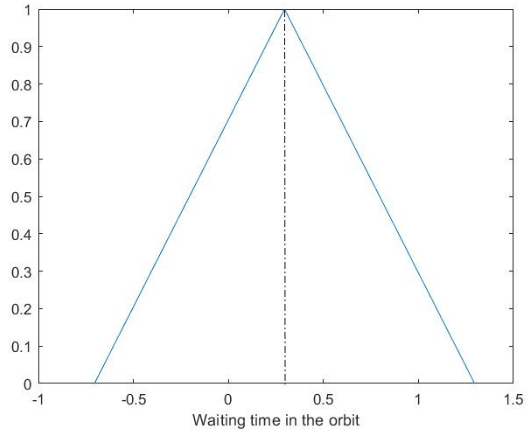

Figure 7: *The waiting time of messages* $(\tilde{T}_o)$ *in the orbit*

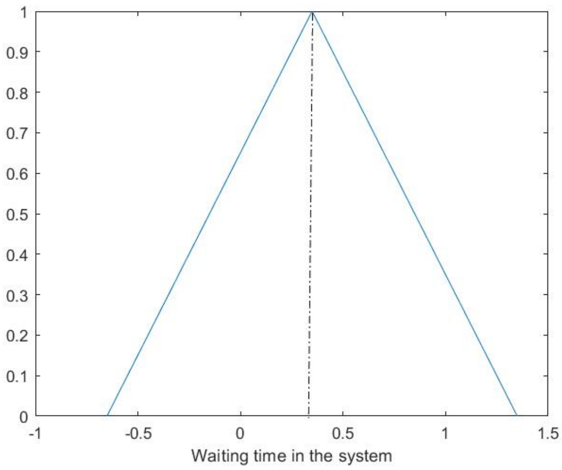

Figure 8: *The waiting time of messages* $(\tilde{T}_s)$ *in the system*

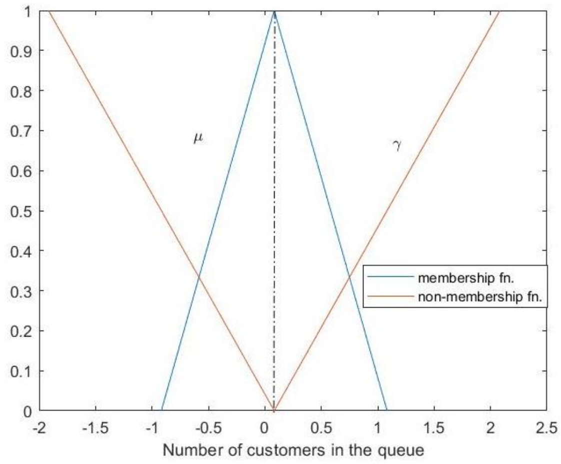

Figure 9: *The membership*$(\tilde{\mu})$ *and the non-membership*$(\tilde{\gamma})$ *functions of the number of messages in the queue* $\tilde{N}_q^{'}$

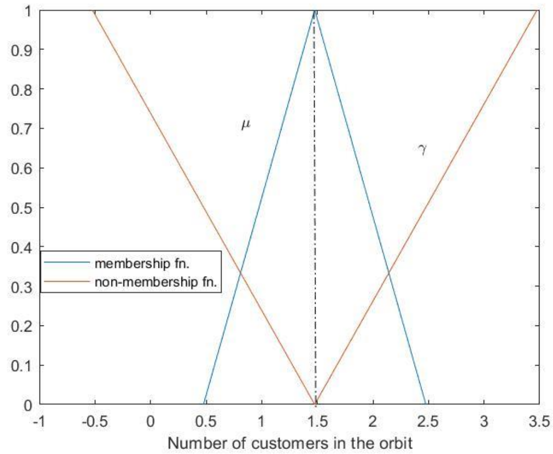

Figure 10: *The membership*$(\tilde{\mu})$ *and the non-membership*$(\tilde{\gamma})$ *functions of the no. of messages in the orbit* $\tilde{N}_o^{'}$

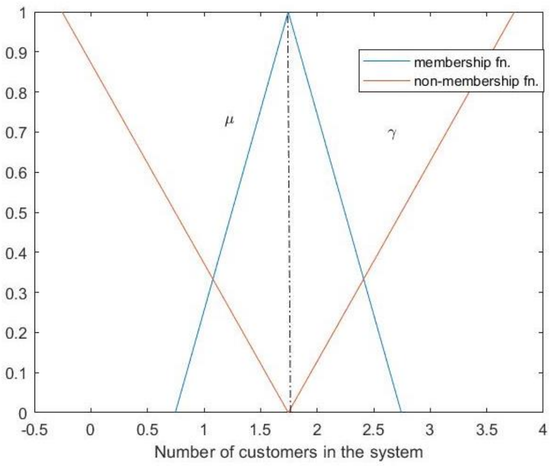

Figure 11: *The membership$(\tilde{\mu})$ and the non-membership$(\tilde{\gamma})$ functions of the number of messages in the system* $\widetilde{N}_s^{'}$

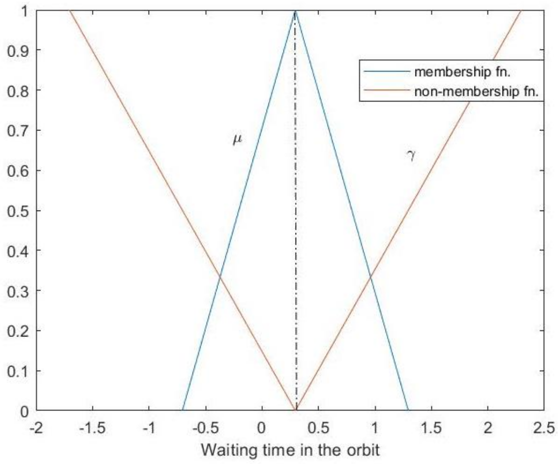

Figure 13: *The membership$(\tilde{\mu})$ and the non-membership$(\tilde{\gamma})$ functions of the waiting time of messages in the orbit* $\tilde{T}_o^{'}$

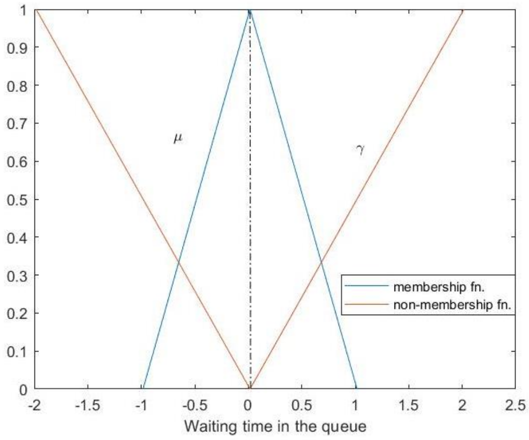

Figure 12: *The membership$(\tilde{\mu})$ and the non-membership$(\tilde{\gamma})$ functions of the waiting time of messages in the queue* $\tilde{T}_q^{'}$

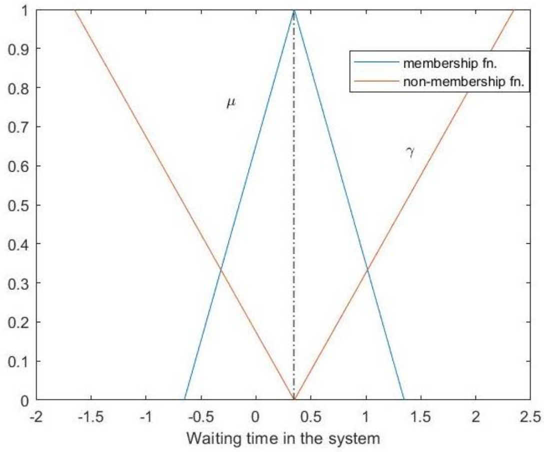

Figure 14: *The membership$(\tilde{\mu})$ and the non-membership$(\tilde{\gamma})$ functions of the waiting time of messages in the system* $\tilde{T}_s^{'}$

VII. Conclusion

The retrial model with breakdowns and repairs has been studied for a large number of fuzzy parameters in the fuzzy queuing theory literature. Out of the existing methods for computing its characteristics, such as nonlinear programming, alpha cut, left-right method, interval arithmetic method, and so on, the present article shows that the suggested method is also suitable for dealing with this model, as evidenced by the example outlined in the previous section. Another advantage of the proposed method is that we solve the problems using the fuzzy value as is, instead of transitioning it to crisp, so it has a broad spectrum of applications in real-world situations. The predicted number of messages and their turnaround time in the queue, orbit, and system are efficaciously tabulated in this example, and the outcome is achieved in both a fuzzified and intuitionistic fuzzy environment. The TFN and TIFN arithmetical representations are used to compare the proposed queuing system's correctness. According to the research findings, the fuzzy queuing model's quality standards are within the range of the intuitionistic fuzzy queuing model's aggregated performance indicators. Because the intuitionistic fuzzy theory is more configurable, the intuitionistic fuzzy queuing model is significantly more efficient and appropriate for evaluating the dimensions of queuing models. As a result, intuitionistic fuzzy queuing, according to this investigation, is one of the most efficient positions of computing evaluation criteria because the evidence gathered from the functionality is simpler to implement and discern. This strategy appears to be more pliable than the others because all estimations are fuzzy. As a result, fuzzy queuing models with a complicated structure benefit from it.

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