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STOCHASTIC ANALYSIS OF THE UTENSIL INDUSTRY SUBJECT TO REPAIR FACILITY

Hanumanolla Indrasena Reddy[1], Mohit Yadav[2] and Hemant Kumar[3*]

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[1,2]Department of Mathematics, University Institute of Sciences
Chandigarh University, Mohali, Chandigarh, India
[3]SOET, Raffles University, Neemrana, Rajasthan
*Corresponding Author
22msm40224@cuchd.in, mohit.e15793@cumail.in, hemantkumar@rafflesuniversity.edu.in

Abstract

The availability and profit values of the utensil industry are analyzed using the regenerative point graphical technique. The utensil industry contains three different units where two units can work with reduced capacity. It is considered that units C and D may be in a complete failed state through partial failure but unit B is in only complete failed state. When a unit is completely failed then the system is in failed state. An expert technician is available to repair the failed unit. Failure and repair times are independent of each other. The distribution of the failure time is general and repair time is exponential. Various parameters such as mean time to system failure, availability, busy period of the server, expected number of server visits and profit values are calculated with the help of tables.

Keywords: Reliability, base-state, mean sojourn time, availability and profit.

I. Introduction

To satisfy the growing demand for products, manufacturers and industrialists must produce products continually which they can accomplish by optimizing their manufacturing processes. This paper discusses the MTSF, availability and profit values of the utensil industry with priority in repair using the regenerative point graphical technique under specified conditions. A large amount of research work has been done on repairable systems such that Kapur and Kapoor [8] described the stochastic nature of a two unit repairable system under one spare unit. Gnedenko and Igor [5] explored reliability and probability measures for engineering purposes. Jack and Murthy [6] discovered the role of limited warranty and extended warranty for the product. Wang and Zhang [16] examined the repairable system of two non identical components under repair facility using geometric distributions. Diaz et al. [4] threw light on the warranty cost management system. Kumar and Goel [12] explored the idea of an imperfect switch on redundant systems in banking industry. Kumar and Goel [11] analyzed the preventive maintenance in two unit cold standby system under general distributions. Malik and Rathee [14] threw light on the two parallel units system under preventive maintenance and maximum operation time. Kashid and Kumar [9] examined the availability of two unit system under degradation and subject to the repair facility. Chaudhary and Tomar [3] examined the behavior of a two unit cold standby system under inspection. Kumar et al. [10] evaluate the effects of washing unit in the paper industry by using the regenerative point graphical technique. Levitin et al. [13] explored the results of optimal preventive replacement of failed units in a cold standby system by using the poisson process. Agarwal et al. [1] analyzed the performance and reliability of water treatment plant under repair facility. Jia et al. [7] explored the two unit system under demand and energy storage techniques. Sengar and Mangey [15] examined the performance of complicated systems under inspection using copula methodology.

II. System Assumptions

There are following system assumptions:

- The utensil industry consists of three distinct units such that cutting system, pressing system, spinning and buffing system.
- Unit *B* consists of a cutting system in which sheets are cut into circular sheets.
- Unit *C* has a pressing system that converts the sheets into the shape of utensils.
- Unit *D* has a spinning and buffing system that gives the final shape and polish to the utensils.
- It is considered that units *C* and *D* may be in a complete failed state through partial failure but unit B is in only complete failed state.
- Failure time follows general distribution whereas repair time follows the exponential distribution.
- A server is always available to repair the failed unit.
- The failed unit works like a new unit after repair.

III. System Notations

There are following system notations:

$i \xrightarrow{Sr} j$	r^{th} directed simple path from state '*i*' to state '*j*' where '*r*' takes the positive integral values for different directions from state '*i*' to state '*j*'.
$\xi \xrightarrow{sff} i$	A directed simple failure free path from state ξ to state '*i*'.
$m-cycle$	A circuit (may be formed through regenerative or non regenerative / failed state) whose terminals are at the regenerative state '*m*'.
$m-\overline{cycle}$	A circuit (may be formed through the unfailed regenerative or non regenerative state) whose terminals are at the regenerative '*m*' state.
$U_{k,k}$	Probability factor of the state '*k*' reachable from the terminal state '*k*' of '*k*' cycle.
$U_{\overline{k,k}}$	The probability factor of state '*k*' reachable from the terminal state '*k*' of $k\ \overline{cycle}$.
μ_i	Mean sojourn time spent in the state '*i*' before visiting any other states.
μ_i'	Total unconditional time spent before transiting to any other regenerative state while the system entered regenerative state '*i*' at t=0.
η_i	Expected waiting time spent while doing a job given that the system entered to the regenerative state '*i*' at *t*=0.
B/b	System first unit is in the operative state/failed state.
$C/\overline{C}/c$	System second unit is in the operative state/reduced state/failed state.
$D/\overline{D}/d$	System third unit is in the operative state/reduced state/failed state.
λ_1 / λ_3	The constant partial failure rate of the unit C/D respectively.

λ_2 / λ_4	The constant complete failure rate of the unit C/D respectively.
λ_5	The constant complete failure rate of unit B.
$f_1(t)/F_1(t)$	PDF/CDF of repair time of unit C from partial failed state.
$f_2(t)/F_2(t)$	PDF/CDF of repair time of unit C from complete failed state.
$f_3(t)/F_3(t)$	PDF/CDF of repair time of unit D from partial failed state.
$f_4(t)/F_4(t)$	PDF/CDF of repair time of unit D from complete failed state.
$f_5(t)/F_5(t)$	PDF/CDF of repair time of unit B from complete failed state.

IV. Transition Diagram and Their Descriptions

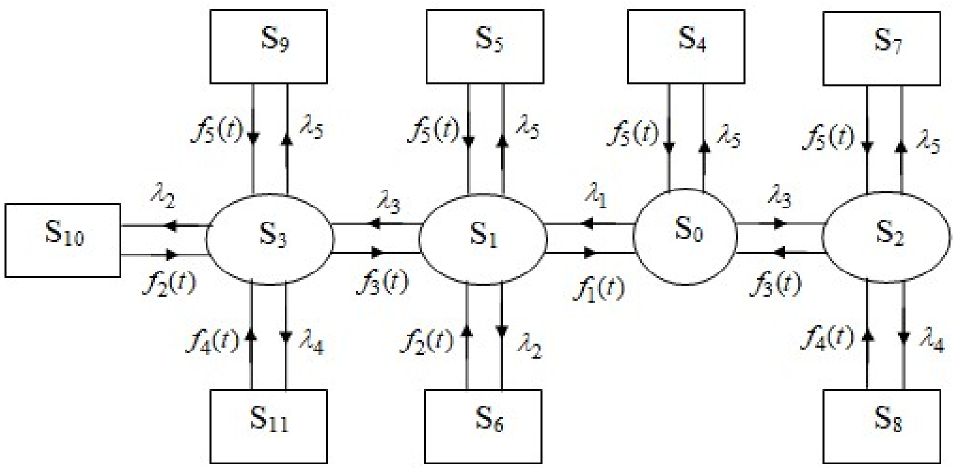

Figure 1: *State Transition Diagram*

In the system transition diagram, there are following states

where, $S_0 = BCD$, $S_1 = B\overline{C}D$, $S_2 = BC\overline{D}$, $S_3 = B\overline{CD}$, $S_4 = bCD$, $S_5 = b\overline{C}D$

$S_6 = BcD$, $S_7 = bC\overline{D}$, $S_8 = BCd$, $S_9 = b\overline{CD}$, $S_{10} = Bc\overline{D}$, $S_{11} = B\overline{C}d$

V. Transition Probabilities

The transition probabilities are following

$p_{0,1} = \lambda_1 /(\lambda_1 + \lambda_3 + \lambda_5)$, $p_{0,2} = \lambda_3 /(\lambda_1 + \lambda_3 + \lambda_5)$

$p_{0,4} = \lambda_5 /(\lambda_1 + \lambda_3 + \lambda_5)$, $p_{1,0} = w_1 /(w_1 + \lambda_2 + \lambda_3 + \lambda_5)$

$p_{1,3} = \lambda_3 /(w_1 + \lambda_2 + \lambda_3 + \lambda_5)$, $p_{1,5} = \lambda_5 /(w_1 + \lambda_2 + \lambda_3 + \lambda_5)$

$p_{1,6} = \lambda_2 /(w_1 + \lambda_2 + \lambda_3 + \lambda_5)$, $p_{2,0} = w_3 /(w_3 + \lambda_4 + \lambda_5)$

$p_{2,7} = \lambda_5 /(w_3 + \lambda_4 + \lambda_5)$, $p_{2,8} = \lambda_4 /(w_3 + \lambda_4 + \lambda_5)$

$p_{3,1} = \lambda_3 /(w_2 + \lambda_3 + \lambda_4 + \lambda_5)$, $p_{3,9} = \lambda_5 /(w_2 + \lambda_3 + \lambda_4 + \lambda_5)$

$p_{3,10} = \lambda_2 /(w_2 + \lambda_3 + \lambda_4 + \lambda_5)$, $p_{3,11} = \lambda_4 /(w_3 + \lambda_2 + \lambda_4 + \lambda_5)$

$$p_{4,0} = p_{5,1} = p_{6,1} = p_{7,2} = p_{8,2} = p_{9,3} = p_{10,3} = p_{11,3} = 1 \quad (1)$$

It has been concluded that

$$p_{0,1} + p_{0,2} + p_{0,4} = 1,\ p_{1,0} + p_{1,3} + p_{1,5} + p_{1,6} = 1$$
$$p_{2,0} + p_{2,7} + p_{2,8} = 1,\ p_{3,1} + p_{3,9} + p_{3,10} + p_{3,11} = 1 \quad (2)$$

VI. Mean Sojourn Time

Let μ_i represents the mean sojourn time. Mathematically, the time taken by a system in a particular state becomes

$$\mu_i = \sum_j m_{i,j} = \int_0^\infty P(T > t)\,dt\,.$$

and $\mu_0 = 1/(\lambda_1 + \lambda_3 + \lambda_5)$, $\mu_1 = 1/(w_1 + \lambda_2 + \lambda_3 + \lambda_5)$, $\mu_2 = 1/(w_2 + \lambda_4 + \lambda_5)$

$$\mu_3 = 1/(w_2 + \lambda_3 + \lambda_4 + \lambda_5),\ \mu_4(t) = \mu_5(t) = 1/(w_5),\ \mu_6 = \mu_{10} = 1/(w_2)$$
$$\mu_7 = \mu_9 = 1/(w_5),\ \mu_8 = \mu_{11} = 1/(w_4) \quad (3)$$

VII. Evaluation of Parameters

All reliability parameters (such as mean time to system failure, availability, busy period of the server and expected number of visits) are determined by using the regenerative point graphical technique.

I. Mean Time to System Failure (MTSF)

The regenerative un-failed states (i=0, 1, 2, 3) to which the system can transit (with initial state 0) before entering to any failed state (using base state ξ=0) then MTSF becomes

$$T_0 = \left[\sum_{i=0}^{3} Sr\left\{\frac{\left\{pr\left(0 \xrightarrow{Sr(sff)} i\right)\right\}.\mu_i}{\Pi_{k_1 \neq 0}\left\{1 - V_{\overline{k_1 k_1}}\right\}}\right\}\right] \div \left[1 - \sum Sr\left\{\frac{\left\{pr\left(0 \xrightarrow{Sr(sff)} 0\right)\right\}}{\Pi_{k_2 \neq 0}\left\{1 - V_{\overline{k_2 k_2}}\right\}}\right\}\right]$$

$$T_0 = [\mu_0 + p_{0,1}\mu_1 + p_{0,2}\mu_2 + p_{0,1}p_{1,3}\mu_3]/[1 - p_{0,1}p_{1,0} - p_{0,2}p_{2,0}] \quad (4)$$

II. Availability of the System

The system is available for use at regenerative states j=0, 1, 2, 3 with ξ=0 then the availability of system is defined as

$$A_0 = \left[\sum_{j=0}^{3} Sr\left\{\frac{\left\{pr\left(0 \xrightarrow{Sr} j\right)\right\}.f_j.\mu_j}{\Pi_{k_1 \neq 0}\left\{1 - V_{\overline{k_1 k_1}}\right\}}\right\}\right] \div \left[\sum_{i=0}^{11} Sr\left\{\frac{\left\{pr\left(0 \xrightarrow{Sr} i\right)\right\}.\mu_i'}{\Pi_{k_2 \neq 0}\left\{1 - V_{\overline{k_2 k_2}}\right\}}\right\}\right]$$

$$A_0 = \frac{[U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3]}{\begin{bmatrix} U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4 + U_{0,5}\mu_5 \\ + U_{0,6}\mu_6 + U_{0,7}\mu_7 + U_{0,8}\mu_8 + U_{0,9}\mu_9 + U_{0,10}\mu_{10} + U_{0,11}\mu_{11} \end{bmatrix}} \tag{5}$$

III. Busy Period of the Server

The server is busy due to repair of the failed unit at regenerative states *j*= 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 with ξ = 0 then the fraction of time for which the server remains busy is defined as

$$B_0 = \left[\sum_{j=1}^{11} Sr \left\{ \frac{\left\{ pr(0 \xrightarrow{Sr} j) \right\} . \eta_j}{\Pi_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{11} Sr \left\{ \frac{\left\{ pr(0 \xrightarrow{Sr} i) \right\} . \mu_i'}{\Pi_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$B_0 = \frac{\begin{bmatrix} U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4 + U_{0,5}\mu_5 + U_{0,6}\mu_6 \\ + U_{0,7}\mu_7 + U_{0,8}\mu_8 + U_{0,9}\mu_9 + U_{0,10}\mu_{10} + U_{0,11}\mu_{11} \end{bmatrix}}{\begin{bmatrix} U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4 + U_{0,5}\mu_5 \\ + U_{0,6}\mu_6 + U_{0,7}\mu_7 + U_{0,8}\mu_8 + U_{0,9}\mu_9 + U_{0,10}\mu_{10} + U_{0,11}\mu_{11} \end{bmatrix}} \tag{6}$$

IV. Estimated Number of Visits Made by the Server

The technician visits at regenerative states *j*= 1, 2, 3 with ξ=0 then the number of visits by the repairman is defined as

$$V_0 = \left[\sum_{j=1}^{3} Sr \left\{ \frac{\left\{ pr(0 \xrightarrow{Sr} j) \right\}}{\Pi_{k_1 \neq 0} \left\{ 1 - V_{\overline{k_1 k_1}} \right\}} \right\} \right] \div \left[\sum_{i=0}^{11} Sr \left\{ \frac{\left\{ pr(0 \xrightarrow{Sr} i) \right\} . \mu_i'}{\Pi_{k_2 \neq 0} \left\{ 1 - V_{\overline{k_2 k_2}} \right\}} \right\} \right]$$

$$V_0 = \frac{[U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3]}{\begin{bmatrix} U_{0,0}\mu_0 + U_{0,1}\mu_1 + U_{0,2}\mu_2 + U_{0,3}\mu_3 + U_{0,4}\mu_4 + U_{0,5}\mu_5 \\ + U_{0,6}\mu_6 + U_{0,7}\mu_7 + U_{0,8}\mu_8 + U_{0,9}\mu_9 + U_{0,10}\mu_{10} + U_{0,11}\mu_{11} \end{bmatrix}} \tag{7}$$

V. Profit Analysis

The profit function may be used to do a profit analysis of the system and it is given by

$$P = E_0 A_0 - E_1 B_0 - E_2 V_0 \tag{8}$$

where, $E_0 = 5000$ (Pay per unit uptime of the system)

$E_1 = 1000$ (Charge per unit time for which technician is busy due to repair)

$E_2 = 500$ (Charge per visit of the technician)

VI. Particular cases

It is considered that

$$f_1(t) = w_1 e^{-w_1 t},\ f_2(t) = w_2 e^{-w_2 t},$$

$$f_3(t) = w_3 e^{-w_3 t},\ f_4(t) = w_4 e^{-w_4 t},\ f_5(t) = w_5 e^{-w_5 t}$$

and $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda,\ w_1 = w_2 = w_3 = w_4 = w_5 = w.$

$$T_0 = \frac{[(w+3\lambda)(w+4\lambda)+\lambda^2](w+2\lambda)}{(w+3\lambda)\left[3\lambda(w+3\lambda)+(w+2\lambda)-\lambda w(w+2\lambda)-\lambda w(w+3\lambda)\right]}$$

$$A_0 = \frac{w^3(w+2\lambda)[(w+3\lambda)^3+\lambda(w+2\lambda)(w+3\lambda)]}{\begin{bmatrix} w^3(w+2\lambda)[(w+3\lambda)^3+\lambda(w+2\lambda)(w+3\lambda)] \\ +\lambda w^2(w+2\lambda)(w+3\lambda)^3(w+2\lambda) \\ +[\lambda^2 w(2w+5\lambda)]w(w+2\lambda)(w+3\lambda)+3\lambda^3 w^2(w+2\lambda)^2 \end{bmatrix}}$$

$$B_0 = \frac{\begin{bmatrix} w^3(w+2\lambda)[(w+3\lambda)^2\lambda+\lambda(w+2\lambda)(w+3\lambda)] \\ +\lambda w^2(w+2\lambda)(w+3\lambda)^3(w+2\lambda) \\ +[\lambda^2 w(2w+5\lambda)]w(w+2\lambda)(w+3\lambda)+3\lambda^3 w^2(w+2\lambda)^2 \end{bmatrix}}{\begin{bmatrix} w^3(w+2\lambda)[(w+3\lambda)^3+\lambda(w+2\lambda)(w+3\lambda)] \\ +\lambda w^2(w+2\lambda)(w+3\lambda)^3(w+2\lambda) \\ +[\lambda^2 w(2w+5\lambda)]w(w+2\lambda)(w+3\lambda)+3\lambda^3 w^2(w+2\lambda)^2 \end{bmatrix}}$$

$$V_0 = \frac{w^3(w+2\lambda)[(w+3\lambda)^2\lambda+\lambda(w+2\lambda)(w+3\lambda)]}{\begin{bmatrix} w^3(w+2\lambda)[(w+3\lambda)^3+\lambda(w+2\lambda)(w+3\lambda)] \\ +\lambda w^2(w+2\lambda)(w+3\lambda)^3(w+2\lambda) \\ +[\lambda^2 w(2w+5\lambda)]w(w+2\lambda)(w+3\lambda)+3\lambda^3 w^2(w+2\lambda)^2 \end{bmatrix}}$$

VIII. Discussion

Table 1 describes the nature of the mean time to system failure of the utensil industry. It has an

Table 1: *MTSF vs. Repair Rate*

w ↓	λ=0.02	λ=0.035	λ=0.05
0.05	4.357262	4.200299	3.696809
0.10	4.600326	4.405797	3.903394
0.15	4.832536	4.599156	4.102564
0.20	5.054602	4.781421	4.29471
0.25	5.267176	4.953519	4.480198
0.30	5.470852	5.116279	4.659367
0.35	5.666179	5.27044	4.832536
0.40	5.853659	5.416667	5.057888
0.45	6.033755	5.555556	5.162037
0.50	6.206897	5.687646	5.318907

increasing trend corresponding to increment in repair rate (w) and has decreasing trend corresponding to an increment in failure rate (λ). In this table, the values of parameters are λ= 0.02, 0.035, 0.05 and w=0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50 respectively. When the value of repair rate enhances then MTSF values are also enhanced. When λ=0.02 changes into λ=0.035, 0.05 then MTSF values are declined.

Table 2 explores the increasing trends of availability with respect to increments in repair rate (w) and has decreasing trends corresponding to increments in failure rate (λ). When the value of the repair rate is enhanced then the availability values are also enhanced. Also, when the failure rate of unit changes λ=0.02 to 0.035, 0.05 then the availability of system declines.

Table 2: *Availability vs. Repair Rate*

$w \downarrow$	λ=0.02	λ=0.035	λ=0.05
0.05	0.665768	0.658036	0.627958
0.10	0.677838	0.668825	0.640662
0.15	0.688592	0.678383	0.652123
0.20	0.698233	0.686909	0.662516
0.25	0.706926	0.694563	0.671982
0.30	0.714804	0.70147	0.680641
0.35	0.721976	0.707736	0.688592
0.40	0.728534	0.713446	0.695917
0.45	0.734553	0.71867	0.702689
0.50	0.742585	0.728578	0.717855

Table 3 explores the trend of profit values with respect to repair rate (w) and its value increase corresponding to increments in repair rate (w) and decrease corresponding to increments in failure rate (λ). It is concluded that when the value of the repair rate enhances then profit values are also enhanced but when the failure rate of the unit changes λ=0.02 to 0.035, 0.05 then the profit of the system declines.

Table 3: *Profit vs. Repair Rate*

$w \downarrow$	λ=0.02	λ=0.035	λ=0.05
0.05	57529.56	56550.96	55338.67
0.10	58611.63	57814.53	56419.58
0.15	59875.52	58249.63	57660.33
0.20	60296.25	59831.35	58504.51
0.25	61854.87	60540.89	59545.69
0.30	62532.78	61361.63	60161.83
0.35	63316.93	62278.88	61875.72
0.40	64193.73	63028.72	62677.66
0.45	65153.92	64361.91	63559.86
0.50	66187.82	65507.87	64513.96

IX. Conclusion

The performance of the utensil industry is discussed using the regenerative point graphical technique. The above tables concluded that when the repair rate increases then the MTSF, availability of the system and profit values also increase but when the failure rate increases then these reliability measures decrease. It is clear that the regenerative point graphical technique is helpful for industries to analyze the behaviour of the products and components of a system.
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