THE TRIANGLE-G FAMILY OF DISTRIBUTIONS: PROPERTIES, SUB-MODELS, ESTIMATION AND APPLICATION IN LIFETIME STUDIES

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Abstract

This paper pictures the importance and the generalization of a new family of distribution developed on Triangle distribution. The new family provides some useful expansions, properties and a suitable alternative to some of existing models with same and higher number of parameters. Exponential distribution (one parameter) and Inverse Weibull distribution (Two parameter) play the role of sub-models. This new family distribution is used as a statistical model to estimate the parameters using the maximum likelihood estimation method. A complete study of Percentage points has been tabled. Two real-world data sets are investigated, demonstrating the suggested model's capacity to fit a variety of data sets along with some other models.

Keywords: G-family of distribution, Maximum Likelihood estimation, Percentage Points, Lifetime data.

I. Introduction

Let us suppose a random variable $T \in (a,b)$ for $-\infty \leq a < b < \infty$ having a probability density function (pdf) $y(t)$ and $W[F(x)]$ be a function of a cumulative distribution function (cdf) of the random variable X which satisfies some statistical conditions such as $W[F(x)] \in (a,b)$, $W[F(x)]$ is differentiable and monotonically non-decreasing and $W[F(x)] \to a$ as $x \to -\infty$ and $W[F(x)] \to b$ as $x \to \infty$. Aljarrah et al. (2014) defined the T-X family cdf by $G(x) = \int_a^{W[F(x)]} y(t)dt = Y\{W[F(x)]\}$, where $W[F(x)]$ satisfied all the conditions. The corresponding pdf of T-X family of distribution is $g(x) = \left\{\frac{d}{dx} W[F(x)]\right\} y\{W[F(x)]\}$.

This study suggests a new distribution family that is inspired by the Triangle-G family. Below is a quick explanation of the Trianlge-G family. The pdf and cdf of Triangle distribution is as follows

$$g(x;p) = \frac{2x}{p};\ x \in \mathbb{R} \tag{1.1}$$

$$G(x;p) = \frac{x\char94 2}{p};\ x \in \mathbb{R} \tag{1.2}$$

The simple form (putting $p = 1$) of the pdf and cdf of Triangle distribution is defined as

$$g(x) = 2x; x \in \mathbb{R} \tag{1.3}$$

$$G(x) = x^2; x \in \mathbb{R} \tag{1.4}$$

This distribution has a number of advantages, such as its simplicity and capacity for enhancing the flexibility of PDF and CDF while introducing new flexible models. Researchers may have more options when it comes to these distributions along with trigonometric functions.

Here a table of chronological review has been added for the recent G-families based on the trigonometric functions and their inverses techniques.

Table 1*: Literature reviews of some recent trigonometric functions and G-families*

Sl. No.	Authors	Years	Contributions in distribution family
1	Souza et al.	2022	Sec-G class of probability distribution
2	Sakthivel et al.	2022	transmuted Sin-G class of probability distribution
3	Mahmood et al.	2022	extended cosine-G class of probability distribution
4	Rahman M.	2021	Arcsine-G class of probability distribution
5	Eghwerido et al.	2021	Teissier-G class of probability distribution
6	Chesneau et al.	2021	distribution based on the arccosine function
7	Muhammad et al.	2021	exponentiated sine-G class of probability distribution
8	Ahmad et al.	2021	exponential T-X class of probability distribution
9	Liang Tung et al.	2021	arcsine-X class of probability distribution
10	Alkhiary et al.	2021	ArcTan Lomax distribution
11	Souza et al.	2021	Tan-G class of probability distribution
12	Muhammad et al.	2021	A New Extended Cosine—G distributions
13	He et al.	2020	arcsine exponentiated- X class of probability distribution
14	Al-Babtain et al.	2020	Sine Topp-Leone-G class of probability distribution
15	Chesneau and Jamal	2019	Sine Kumaraswamy-G class of probability distribution
16	Mahmood et al.	2019	A New Sine-G Family of Distributions: Properties and Applications
17	Chesneau et al.	2019	new class of probability distributions via cosine and sine functions
18	Mahmood et al.	2019	sine-G class of probability distribution

The Triangle-G family of distribution was introduced in this study. The Tr-G family's key benefit is that practitioners will have a one-parameter class that is adaptable to actual data in relevant disciplines. It may be a good substitute for other distributions with one, two, three, or four parameters. In some real-world circumstances, nevertheless, it might also exceed other kinds of distributions in terms of model fit, although this is not always assured. Additionally, a full account of some of its mathematical properties is provided.

The outline of rest of the paper is as follows. The derivation of the form for the Tr-G density function described in Section 2. Some of the general mathematical aspects of the proposed family that are included in Section 3. In Section 4, one unique model of this family is presented, along with various plots of their pdfs and hrfs. The proposed model's percentage point results are discussed in Section 5.

In Section 6, we use two particular models of the proposed family on real data sets to demonstrate their applicability. In Section 7, some concluding remarks are presented.

II. Triangle- G (TR-G) family of distribution

The derivation of pdf and cdf of Triangle-G family of distribution is discussed in this section. Let us consider a random variable X that belongs to the Triangle-G family, the cdf and pdf can be written in the following form

$$G_{Triangle-G}(x;p;\phi) = \frac{F(x;\phi)^2}{p}; x \in \mathbb{R} \quad (2.1)$$

$$g_{Triangle-G}(x;p;\phi) = \frac{2F(x;\phi)f(x;\phi)}{p}; x \in \mathbb{R} \quad (2.2)$$

The simplest form of TR-G family of distribution is formed by putting $p=1$. The cdf and pdf are as follows

$$G_{Triangle-G}(x;\phi) = F(x;\phi)^2; x \in \mathbb{R} \quad (2.3)$$

$$g_{Triangle-G}(x;\phi) = 2F(x;\phi)f(x;\phi); x \in \mathbb{R} \quad (2.4)$$

Here $f(x;\Phi)$ and $F(x;\Phi)$ are considered as the pdf and cdf of baseline (or parent) random variable depending on the parameter vector. The complementary cdf (or survival function (srf)), instantaneous failure rate (or hazard rate function (hrf), retro hazard (or reversed hazard rate function), integrated hazard rate (or cumulative hazard rate function) can be written as below

$$S_{Triangle-G}(x;p;\phi) = 1 - \left[\frac{F(x;\phi)^2}{p}\right]; x \in \mathbb{R} \quad (2.5)$$

$$h_{Triangle-G}(x;p;\phi) = \frac{2F(x;\phi)f(x;\phi)}{p-[F(x;\phi)^2]}; x \in \mathbb{R} \quad (2.6)$$

$$r_{Triangle-G}(x;p;\phi) = \frac{2f(x;\phi)}{F(x;\phi)}; x \in \mathbb{R} \quad (2.7)$$

$$H_{Triangle-G}(x;p;\phi) = -\log\left[1 - \left\{\frac{F(x;\phi)^2}{p}\right\}\right]; x \in \mathbb{R} \quad (2.8)$$

III. Some Properties

I. Quantile function, Median, Bowley skewness and Moors kurtosis

The quantile function (also known as the inverse cdf) of the Triangle-G family follows by inverting the Triangle-G distribution function. Let us consider $u \sim U(0,1)$, the u^{th} quantile function of TR-G is defined as $Q_F(u)$ is the solution of $Q(u) > 0$. It may be written as follows in terms of the tangent trigonometric function as

$$x = Q_F(u) = G^{-1}(u) = F^{-1}\left[(pu)^{\frac{1}{2}}\right] \quad (3.1)$$

where $u \in (0,1)$. The quantile function expression may be used to generate random numbers from

TR-G distributions. The median of the TR-G family can be obtained by setting $u = 0.5$. The effects of the shape parameters on the skewness and kurtosis can be studied by using (3.1).
The Bowley skewness (S) and Moors kurtosis(K) can be formulated as

$$(S) = \frac{Q\left(\frac{3}{4}\right)+Q\left(\frac{1}{4}\right)-2Q\left(\frac{1}{2}\right)}{Q\left(\frac{3}{4}\right)-Q\left(\frac{1}{4}\right)} \text{ and } (K) = \frac{Q\left(\frac{3}{8}\right)-Q\left(\frac{1}{8}\right)+Q\left(\frac{7}{8}\right)-Q\left(\frac{5}{8}\right)}{Q\left(\frac{6}{8}\right)-Q\left(\frac{2}{8}\right)}$$

where $Q(.)$ represents the quantile function. When the distribution is symmetric, $S = 0$ and when the distribution is right (or left) skewed, $S > 0$(or $S > 0$). The tail of the distribution gets thicker as K expands. These metrics exist even for distributions without moments and are less subject to outliers.

II. Critical Points and Asymptotes

The critical points of $f(x)$ are the solution x_0 of the nonlinear equation $f'(x_0) = 0$ i.e.,

$$\frac{2f(x)^2 + F(x)f'(x)}{p} = 0$$

The critical points of $h(x)$ are the solution x_* of the nonlinear equation $h'(x_*) = 0$ i.e.,

$$\frac{2f(x_*)^2[F(x_*)^2 + p] + 2F(x_*)f'(x_*)[p - F(x_*)^2]}{[p - F(x_*)^2]^2} = 0$$

By identifying the sign of the second derivative of the function taken at this point, we are able to identify the type of the critical point.

IV. Ordinary and Incomplete moments, Moment generating Function and Mean Deviation

Moments are crucial in the fields of actuarial and financial science, especially in applications. It assists the researcher in taking important features and characteristics of the suggested distribution under perspective. The r^{th} moment of the TR-G family of distribution is given by

$$\mu_r' = \int_{-\infty}^{\infty} x^r g_{Tr-G}(x; p, \Phi)\, dx \tag{3.4}$$

Using the pdf of TR-G family of distribution (2.1) in equation number (3.4), we have

$$\mu_r' = \int_{-\infty}^{\infty} \frac{x^r F(x)^2}{p} dx$$

Using Binomial Expansions

$$\mu_r{}' = \sum_{k=0}^{\infty} \alpha_k \psi_{2k+1}$$

$$\text{Where } \alpha_k = \sum_{k=0}^{\infty} \frac{2(-1)^k}{p(2k+1)!} \text{ and } \psi_{2k+1} = \sum_{k=0}^{\infty} G(x)^{4k+2}$$

The i^{th} incomplete moment is defined as $I(x; p, \Phi)$ and is given by

$$I(x, \theta, \Phi) = \int_{-\infty}^{t} x^i f(x, \theta, \Phi)\, dx$$

$$I(x) = \sum_{k=0}^{\infty} \alpha_k \Psi_{i,2k+1}$$

Getting the mean deviations is important for lifetime models as well. The following are the possible ways to express the mean deviations from the mean and median for a random variable $X \sim TR - G$.

$$\varepsilon_1 = \int_0^{\infty} |x - \mu_1'|\, g_{TR-G}(x, p, \Phi) dx = 2\mu_1' G(\mu_1') - 2I_{(1)}(\mu_1').$$

where $I_1(\mu_1)$ is the first incomplete moment of TR-G family.

$$\varepsilon_2 = \int_0^{\infty} |x - Q(0.5)|\, g_{TR-G}(x, p, \Phi) dx = \mu_1' - 2I_{(1)}\big(Q(0.5)\big)$$

The moment-generating function and cumulant-generating function for the TR-G family can be expressed in a general form as follows

$$M_x(t) = \sum_{r,n=0}^{\infty} \frac{t^r}{r!} \alpha_k \Psi_{r,2k+1}.$$

$$\Phi_x(t) = M_x(it) = \sum_{r,n=0}^{\infty} \frac{(it)^r}{r!} \alpha_k \Psi_{r,2k+1}.$$

V. Reliability function for parallel and series systems

Let us Consider an independent system with $n*$ TR-G family-equipped components. The reliability of the parallel system (P) and reliability of the series system (S) are provided by

$$R_p(x; p, \Phi) = \left[1 - \left\{\frac{2F(x)f(x)}{p}\right\}\right]^{\theta n*}.$$

$$R_s(x; p, \Phi) = \left[\left\{1 - \left\{\frac{2F(x)f(x)}{p}\right\}\right\}^{\theta}\right]^{n*}.$$

VI. Mean time to failure (MTTF), mean time between failure (MTBF) and availability (AvB)

The reliability signs MTTF, MTBF, and AvB are based on techniques and procedures for predicting a product's longevity. A failure rate and the subsequent time frame of expected performance may be quantified using metrics such as MTTF, MTBF, and AvB, which are techniques of delivering a numerical number based on a compilation of data.

If $X \sim TR - G(p_1, \Phi_1)$ then the MTBF is given as

$$MTBF = \frac{-x}{\ln\left(1 - G(x; p_1, \Phi_1)\right)};\ x > 0.$$

If $X \sim TR - G(p_2, \Phi_2)$ then the MTTF is given as

$$MTTF = E(X) = \mu_1' | (p_2, \Phi_2);\ x > 0.$$

The AvB is consider the probability that the component is successful at time X, i.e.

$$AvB = MTTF/MTBF = -\mu_1' | (p_2, \Phi_2) \frac{\ln\left(1 - G(x, p_1, \Phi_1)\right)}{x}.$$

VII.Bonferroni and Lorenz curves

Bonferroni and Lorenz curves defined for a given probability π is given by

$$B(\pi) = I_1(q)/\pi\mu_1' \text{ and } L(\pi) = I_1(q)/\mu_1'.$$

Where $q = Q(\pi)$ is the quantile function of X at π.

IV. Special Members of TR-G family of distribution

This section carries certain cases of the intended family of distributions by using different base cumulative distribution functions.

I. TR-Inverse Weibull Distribution

Let us consider the cdf and pdf of Inverse Weibull distribution with positive parameter (α, β) given by $\alpha\beta^{\alpha} x^{-\alpha-1} e^{-\left(\frac{\beta}{x}\right)^{\alpha}}$ and $e^{-\left(\frac{\beta}{x}\right)^{\alpha}}$ respectively with the random variable X. Considering that $F(x; \alpha, \beta)$ and $f(x; \alpha, \beta)$ are the cdf and pdf of the two-parameter Inverse Weibull distribution.
The cdf of the three parameter TR-IW distribution (substituting in (2.2)), for $x > 0$, can be expressed as

$$G_{TR-IW}(x; \alpha, \beta, p) = \frac{e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}}{p};\quad x \in \mathbb{R}, p > 0. \tag{4.1}$$

The corresponding pdf and the complementary cdf (or survival function (srf)), instantaneous failure rate (or hazard rate function (hrf), retro hazard (or reversed hazard rate function), integrated hazard rate (or cumulative hazard rate function) (three parameter) can be written as below

$$g_{TR-IW}(x; \alpha, \beta, p) = \frac{2\alpha\beta^{\alpha} x^{-\alpha-1} e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}}{p};\quad x \in \mathbb{R}, p > 0. \tag{4.2}$$

$$S_{TR-IW}(x; \alpha, \beta, p) = 1 - \left[\frac{e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}}{p}\right]; x \in \mathbb{R} \tag{4.3}$$

$$h_{TR-IW}(x; \alpha, \beta, p) = \frac{4\alpha\beta^{\alpha} x^{-\alpha-1} e^{-\left(\frac{\beta}{x}\right)^{4\alpha}}}{p\left[p - \left\{e^{-\left(\frac{\beta}{x}\right)^{4\alpha}}\right\}\right]}; x \in \mathbb{R} \tag{4.4}$$

$$r_{TR-IW}(x;\alpha,\beta,p) = 4\alpha\beta^{\alpha}x^{-\alpha-1};\, x \in \mathbb{R} \tag{4.5}$$

$$H_{TR-IW}(x;\alpha,\beta,p) = -\log\left[1-\left\{\left[\frac{e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}}{p}\right]\right\}\right];\, x \in \mathbb{R} \tag{4.6}$$

By substituting $p = 1$, the two parameter TR-IW exists with the pdf and cdf

$$g_{TR-IW}(x;\alpha,\beta) = 2\alpha\beta^{\alpha}x^{-\alpha-1}e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}; \quad x \in \mathbb{R},\ (\alpha,\beta) > 0. \tag{4.7}$$

$$G_{TR-IW}(x;\alpha,\beta) = e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}; \quad x \in \mathbb{R},\ (\alpha,\beta) > 0. \tag{4.8}$$

$$S_{TR-IW}(x;\alpha,\beta) = 1 - e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}; \quad x \in \mathbb{R},\ (\alpha,\beta) > 0. \tag{4.9}$$

$$h_{TR-IW}(x;\alpha,\beta) = \frac{2\alpha\beta^{\alpha}x^{-\alpha-1}e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}}{1-e^{-\left(\frac{\beta}{x}\right)^{2\alpha}}}; \quad x \in \mathbb{R},\ (\alpha,\beta) > 0. \tag{4.10}$$

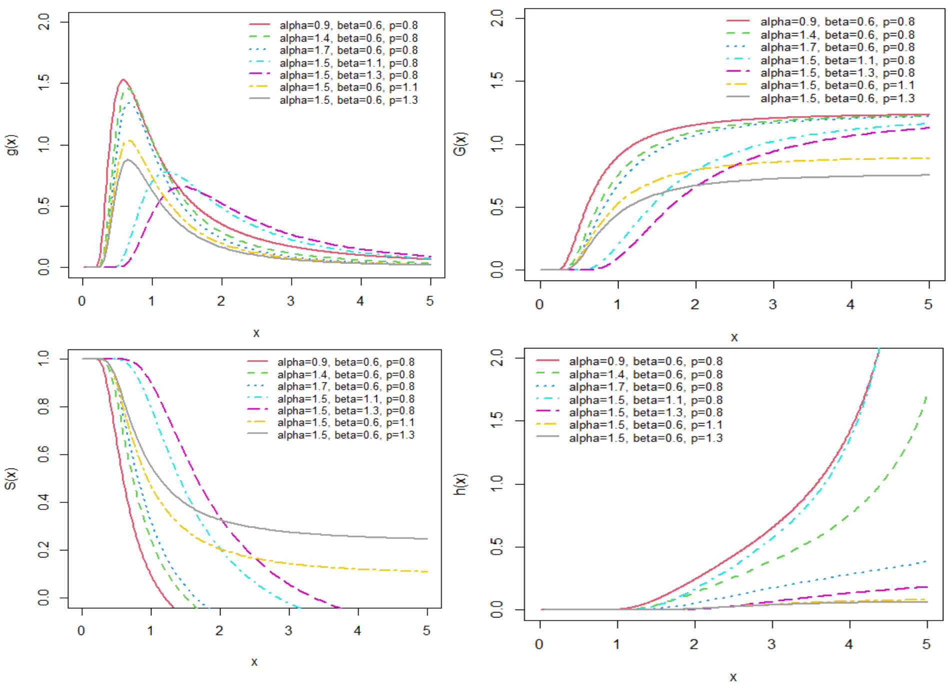
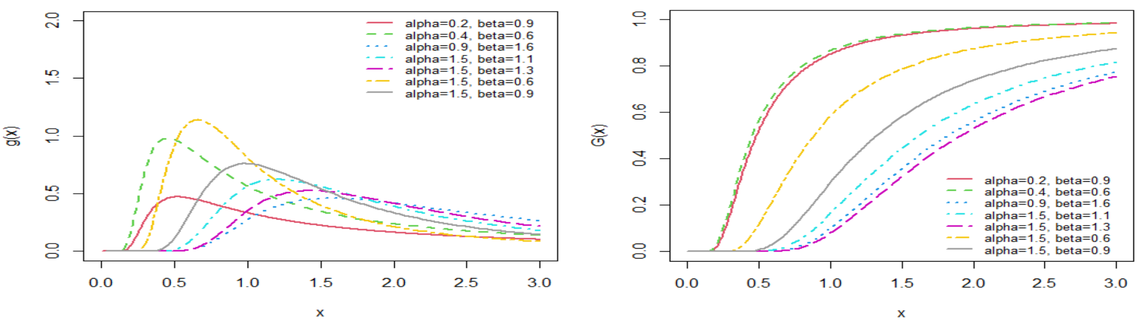


Figure 1: *pdf, cdf, survival and hazard plot of Tr-IW (three parameter) distribution*



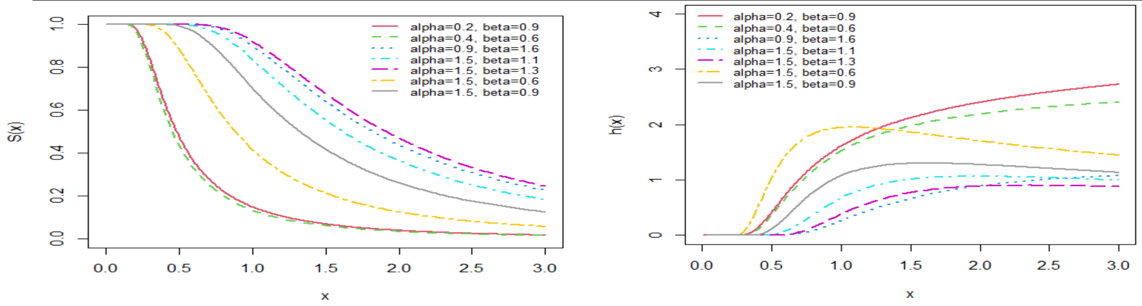


Figure 2: *pdf, cdf, survival and hazard plot of Tr-IW (two parameter) distribution*

The PDF in figures 1 and 2 can have different forms based on the values of the parameters. The shape of the proposed distribution is closed to bell shape by increasing the shape parameter. Furthermore, the hrf can be increasing or unimodal-bell shape, it increases the distribution's adaptability to fit various sets of lifespan data, as seen in figure 4.

Table 2: *New contributed special cases of the Triangle-G family*

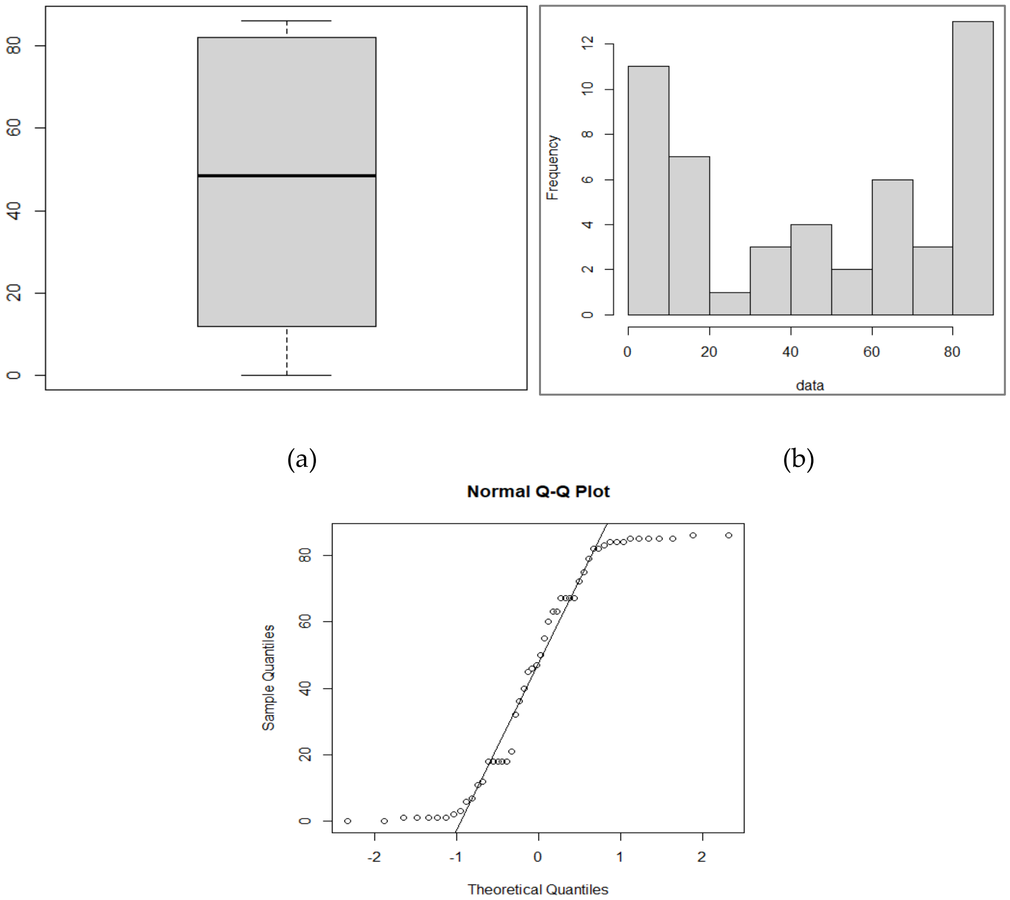
Sl. No.	Baseline mode	CDF form	Generated Model	Support
1	Exponential	$\frac{(1-\exp(-\lambda x))^2}{p}$	Tr-E	$x \in \mathbb{R}^*$
2	Rayleigh	$\frac{\left(1-\exp\left(\frac{-x^2}{2\sigma^2}\right)\right)^2}{p}$	Tr-R	$x \in \mathbb{R}^*$
3	Frechet	$\frac{\exp\left(\frac{-x-m}{s}\right)^{-2\alpha}}{p}$	Tr-F	$x \in \mathbb{R}^*$
4	Gamma	$\frac{\left(\left(\frac{1}{\Gamma(\alpha)}\right)\gamma(\alpha,\beta x)\right)^2}{p}$	Tr-G	$x \in \mathbb{R}^*$
5	Lomax	$\frac{\left(1-\left(1+\frac{x}{\lambda}\right)\right)^{-2\alpha}}{p}$	Tr-L	$x \in \mathbb{R}^*$

V. Practical Illustration

This section discusses the theoretical significance of the Tr-G model utilizing two applications to complete real data. The competitive distributions' best-fitting capabilities are determined using certain analytical metrics. To choose the most suited ones, the values of the Akaike Information Criterion (AIC),
Hannan-Quinn Information Criterion (HQIC), Corrected Akaike Information Criterion (CAIC), and Bayesian Information Criterion (BIC) were taken into consideration. Other goodness-of-fit tests, such

as the Cramer-von Mises (W) distance value test, the Kolmogorov-Smirnov (K-S) statistic with accompanying p values, and the loglikelihood function, are also recorded in addition to discriminating tests. The AIC, BIC, CAIC, and HQIC values as well as the W and K-S tests are consistently should be lowest for the ideal model. To compare the competing distributions, the model with the highest p values for the K-S statistics is used. Two data sets have been taken into consideration.

Dataset 1: The Tr-IW (three parameter) distribution is analysed using the dataset contained the lifetimes of fifty devices. They were given by: 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67, 72, 75, 83, 84, 84, 84, 85, 85, 85, 85, 85, 86, 86, 0.1, 0.2, 1, 1, 79, 82, 82, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, and 18.



(a) (b)

(c)

Figure 3: *Boxplot (a), Histogram (b) and Normal QQ plot (c) for Data set 1*

The competing models included the generalized modified Extended Cosine Power (ECSP) model [29], Weibull–Poisson (GMWP) model, generalized modified Weibull-Geometric (GMWG) model, generalized modified Weibull-logarithmic (GMWL) model [7], Poisson-odd generalized uniform (POGE-U) model [28], exponentiated generalized linear exponential (EGLE) model [35], gamma-uniform (GU) model [41], generalized linear failure rate (GLFR) model [36], beta Weibull (BW) model [21], generalized modified Weibull (GMW) model [9], modified Weibull distribution (MW) model [20], generalized linear exponential (GLE) model [25], beta-modified Weibull (BMW) model [37], power (P) model.

Table 3: *MLE's and other statistics value for dataset 1*

Model	Estimated Parameter					Model Comparison Method			
	$\hat{p}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$	$-L$	AIC	BIC	K-S (p-value)
Tr-IW3	0.10	0.23	0.44	-	-	128.30	262.60	268.33	0.05 (0.97)
ECSP	0.21	86.01	0.35	-	-	202.59	411.91	416.92	0.08 (0.86)
GU	0.27	51.94	0.09	86.71	-	207.33	418.65	426.30	0.15 (0.20)
TUq	-0.19	0.10	86.0	0.93	-	212.86	433.72	441.37	0.12 (0.42)
BMW	2.4×10^{-4}	0.05	0.20	0.17	1.4	220.28	450.56	460.12	0.13 (0.36)
BW	1.0×10^{-5}	0.13	0.07	3.32	-	223.11	454.22	461.87	0.12 (0.42)
MW	0.06	0.02	0.36	-	-	226.16	460.31	466.05	0.14 (0.33)
EGLE	3.3×10^{-3}	1.7×10^{-4}	4.56	0.11	-	224.34	456.67	464.32	0.15 (0.21)
GLE	9.9×10^{-3}	4.5×10^{-4}	0.73	-	-	235.93	477.85	483.59	0.16 (0.14)
GLFR	3.8×10^{-3}	3.1×10^{-4}	0.53	-	-	233.15	472.29	478.03	0.16 (0.13)
POGE-U	0.02	0.37	1.77	87.01	-	206.68	419.34	425.08	0.09 (0.75)
GMWP	5.4×10^{-8}	0.13	0.08	2.13	-	220.88	451.75	461.31	0.14 (0.28)
GMWL	2.13	2.68	0.01	0.28	1.00	217.77	445.53	455.09	0.13 (0.36)
GMWG	9.4×10^{-8}	0.12	0.08	2.23	0.46	220.78	451.55	461.11	0.13 (0.33)
GMW	1.0×10^{-5}	0.07	0.22	1.37	-	221.40	452.81	460.46	0.15 (0.23)
P	86.01	0.73	-	-	-	219.89	433.78	447.60	0.99 8.9×10^{-16}

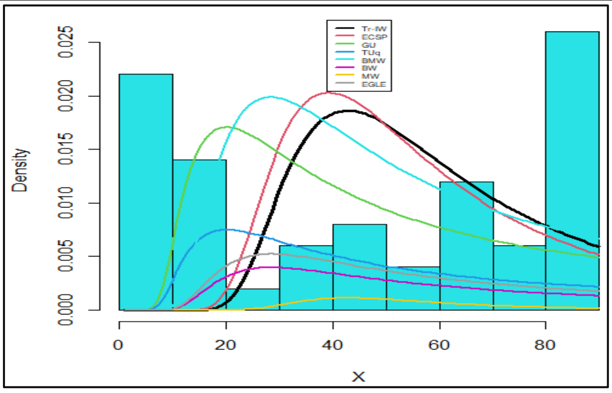
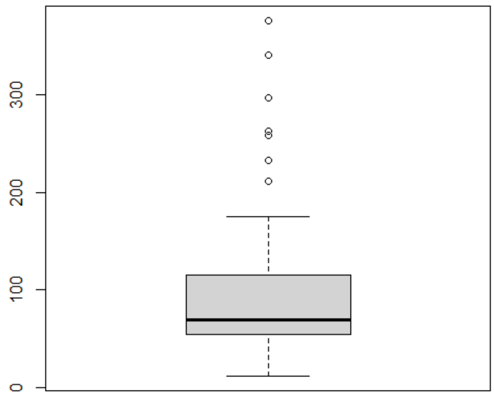
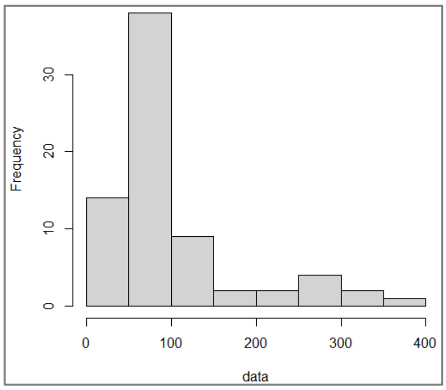


Figure 4: *Plots of the estimated pdfs for dataset 1*

Dataset 2: The dataset of 72 survival times in days of guinea pigs, voluntary contaminated with different doses of tubercle bacilli [8] is used for analysed the Tr-IW (two parameter). The data are listed as 12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67, 68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 376.



(a)



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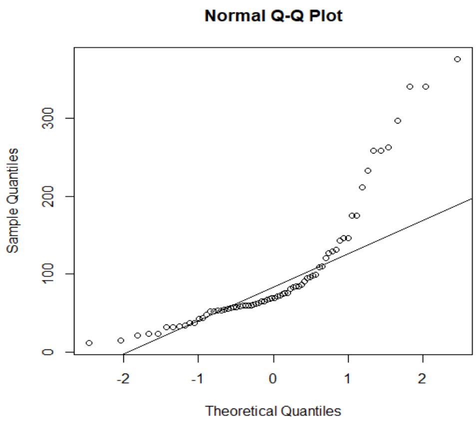

(c)

Figure 5: *Boxplot (a), Histogram (b) and Normal QQ plot (c) for Data set 2*

The comparative distributions are the New Sine Inverse Weibull (NSIW) model [23], sine inverse Weibull model (SIW) [19], inverse Weibull model (IW) [17], inverse Nadarajah-Haghighi model (INH) [40], inverse exponential model (IED) [18] and inverse Rayleigh model (IRD) [42].

Table 4: *MLE's and other statistics value for dataset 2*

Model	Estimated Parameter		Model Comparison			
	$\hat{\alpha}$	$\hat{\beta}$	$-L$	AIC	BIC	$KS(p-value)$
Tr-IW2	0.74	33.07	356.25	716.51	721.06	0.09 (0.85)
NSIW	1.19	59.28	391.11	786.23	790.78	0.12 (0.25)
SIW	1.09	78.68	391.82	787.66	792.21	0.13 (0.20)
IW	1.42	54.15	395.65	795.47	799.85	0.15 (0.07)
INH	1.84	25.78	400.47	804.94	809.49	0.14 (0.11)
IED	60.09	-	402.67	807.34	809.62	0.18 (0.01)
IRD	2124.00	-	406.77	815.53	817.81	0.26 (0.0001)

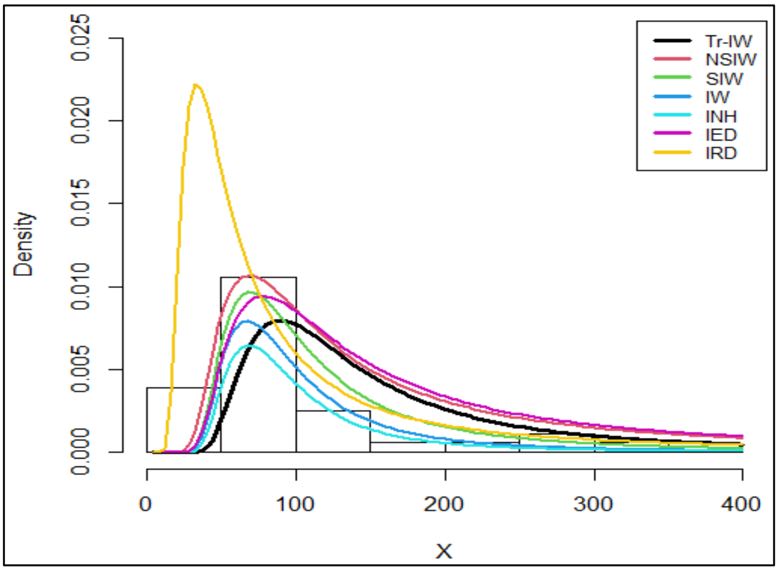

Figure 6: *Plots of the estimated pdfs for dataset 2*

VI. Results

The three parameter Tr-IW distribution is applied in Dataset1 and compared with three, four and more parameter distribution models. The Tr-IW model is fitted comfortably more flexible than other models with more parameters. The two parameter Tr-IW model is fitted better than the other equal parameter models.

VII. Conclusion and Remarks

The triangle family of distribution is introduced. Some of the important properties are discussed. Inverse Weibull model is taken as sub model distribution. The paper introduced two types of distribution with two and three parameters. Both of the distributions are discussed with various properties and real-life data fitting. Both of the distributions are fitted consistently better than the other models with equal and more parameter. This paper introduced one created family and two generated model with a hope that it will attract wider applications in several areas such as reliability engineering, insurance, hydrology, economics and survival analysis.

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