RELIABILITY MODELING OF TWO-UNIT GAS TURBINE SYSTEM CONSIDERING THE EFFECT OF HUMIDITY

Pinki
•
Department of Mathematics, Maharshi Dayanand University Rohtak, India
pinkichahar95@gmail.com

Dalip Singh
•
Department of Mathematics, Maharshi Dayanand University Rohtak, India
dsmdur@gmail.com

Abstract

Aim. The purpose of this paper is to find the reliability measures and profit of a two-unit gas turbine power generating system incorporated with one gas turbine and one steam turbine. Effects of different humidity condition (humidity ≤/> 50%) are taken into consideration by fixing the range of temperature (5°C-25°C) for developing the model. At initial stage, both units (gas turbine and steam turbine) are in operative mode. If steam turbine fails, gas turbine remains in operative mode but if gas turbine fails, system goes to down state and when both unit fails, system fails. In this system we assume that failure time distribution is exponentially distributed while repair time distribution is arbitrary. Methods. In this paper we use the Laplace transform for mathematical analysis, and semi-markov process and regenerative point technique to investigate reliability measures and profit of the system. Findings. The system is analysed in steady state and different reliability measures such as mean time to system failure, availability for different cycles and for different humidity conditions, busy period, down time of the system etc. are calculated and the graphs have been drawn to see the effect of different transition rates such as failure rate and repair rate of the units for different humidity conditions on reliability measures and the profit for particular case is evaluated using the information/data collected from gas turbine power generating system located at Bawana, Delhi, India. Conclusion. Our finding shows that mean time to system failure and availability when both turbines are working decreases with increase in any one of failure rate while availability when only gas turbine is working increases with increase in steam turbine failure rate and profit for plant decreases with increase in failure rates. From this we concluded that availability for the fixed range of temperature (5°C-25°C) is higher when humidity is >50% as compared to when humidity is ≤50%. Thus, a comprehensive study of gas turbine system may be helpful to those who are involved in power generating industry.

Keywords: Gas Turbine, Steam Turbine, Failure Rate, Reliability Measures

1. Introduction

In this era of high competition, reliability has been widely progressed over the years to attain the needs and requirements of global competition. Reliability analysis of system plays pivotal role in deciding the productivity and profitability of the system. Gas turbine power systems are regarded as key element in industrial production and any deficiency in power supplying may lead to significant financial detriment as large capital investments are required for industrial production. Several studies have been conducted so far for gas turbine power system by assuming different failure and repair polices. A number of researcher have studied the system of two or more dissimilar/similar units including [1-3] in which one unit is operation mode and another one is standby state by using different repairs. El-Berry [4] discussed reliability based on failure of data of a gas turbine power plant. Singh and Taneja [5-6] introduced a situation of gas turbine power plant where units are dissimilar but nature of output is same by considering effect of random/scheduled inspection. Farouk and Sheng [7] proposed a situation for power plant to study effect of ambient temperature on gas turbine. Abigail et al. [8] studied the combined cycle power plant considering effect of ambient temperature with post-combustion CO_2 capture. Saleh et al. [9] discussed the performance of gas turbine in Saudi weather condition. Rajesh et al. [10-11] focused on reliability analysis of gas turbine power plant with effect of ambient temperature with different repair polices. Fernandez et al. [12] studied the effect of temperature in tropical climate on performability of gas turbine. Bird and Grabe [13] discussed humidity effects on gas turbine performance. Hanachi et al. [14] focused on the effects of intake air humidity on monitoring of gas turbine system but they did not find reliability measures like Mean time to system failure, Availability, Down time, etc.

Taking all above into consideration, in present paper we discuss stochastic modeling of gas turbine system (One Gas Turbine and One Steam Turbine). Here we fix the range of temperature (5°C-25°C) and develop model for two different humidity conditions (i.e. humidity less than or equal to 50% and humidity greater than 50%) are taken into consideration while developing model. Initially, system works with full capacity that is both the gas and the steam turbine are in operative state. If steam turbine fails, gas turbine continues to work then system is said to be working in single cycle but if gas turbine fails then steam turbine is put to down mode and system is said to be in down state. When both the gas and the steam turbine fails, the system is claimed to be in failed state. Different system effectiveness measures have been obtained by using semi-markov process and regenerative point technique. We obtained system effectiveness measures like mean time to system failure, availability analysis for full capacity as well as for single cycle for different humidity conditions. Interesting conclusions have been drawn by using assessed values on the basis of information/data gathered from a gas turbine power plant situated at Bawana, Delhi, India.

2. System Description and Assumptions

2.1. Assumptions

A two-unit gas turbine system is developed under some reasonable assumptions which are as follows:

- The failure time distribution is taken exponentially while repair time distribution is arbitrary.
- After each repair unit is claimed to be good as new unit.
- Repair pattern of the system rely on first come first serve.
- Complete failure of system is claimed on failure of both the units.

2.2. System Description of the Model

State transition diagram for gas turbine system consisting of a gas and a steam turbine is shown in Figure 1. States 0,1,2,3,5, and 6 are the regenerative points and hence 0,1,2,3,5 and 6 are regenerative states. States 4 and 7 corresponds to complete failure of the system. States 0 and 1 are the states representing production in combined cycle when humidity is less than equal to 50% and greater than 50% respectively. States 2 and 5 are the states at which point only gas turbine is working, representing production in single cycle when humidity is less than equal to 50% and greater than

50% respectively. At state 3 and 6, it is required to put steam turbine in down mode as on failure of gas turbine it can't work and hence we say state 3 and 6 are down state.

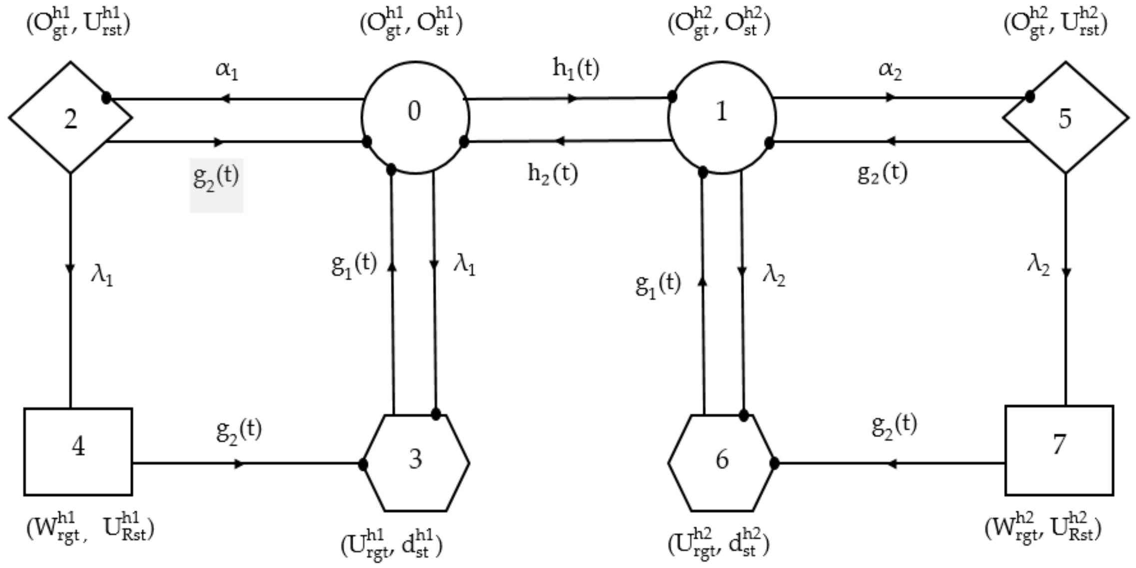

Figure 1: *State Transition Diagram of the System*

2.3. Notations

$O_{gt}^{h_1}/O_{st}^{h_1}$: Gas Turbine/Steam Turbine is operative when humidity ≤ 50%

$O_{gt}^{h_2}/O_{st}^{h_2}$: Gas Turbine/Steam Turbine is operative when humidity > 50%

$U_{rgt}^{h_1}/U_{rst}^{h_1}$: Gas Turbine/Steam Turbine is under repair when humidity ≤ 50%

$U_{rgt}^{h_2}/U_{rst}^{h_2}$: Gas Turbine/Steam Turbine is under repair when humidity > 50%

$U_{Rst}^{h_1}/U_{Rst}^{h_2}$: Continuing repair from previous state of Steam Turbine when humidity is ≤/> 50%

$d_{st}^{h_1}/d_{st}^{h_2}$: Steam Turbine is put to down mode when humidity is ≤/> 50%

$W_{rgt}^{h_1}/W_{rgt}^{h_2}$: Gas Turbine waiting for repair when humidity is ≤/> 50%

λ_1/λ_2 : Failure rate of Gas Turbine when humidity is ≤/> 50%

α_1/α_2 : Failure rate of Steam Turbine when humidity is ≤/> 50%

$g_1(t)/g_2(t)$: Probability density function of repair time of Gas Turbine/Steam Turbine

$G_1(t)/G_2(t)$: Cumulative distribution function of repair time of Gas Turbine/Steam Turbine

$h_1(t)/h_2(t)$: Probability density function for changing the humidity from ≤ 50% to > 50% / from > 50% to ≤ 50%

$q_{ij}(t), Q_{ij}(t)$: Probability density function/Cumulative distribution function of first passage time from regenerative state I to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]

$q_{ij}^{(k)}(t)/Q_{ij}^{(k)}$: Probability density function/Cumulative distribution function of first passage time from regenerative state i to a regenerative state j or visiting state k one time in (0, t]

©/ Ⓢ : Laplace convolution/ Laplace Stieltjes convolution

3. State Transition Probabilities and Mean Sojourn Time

Based on state transition diagram, expression $dQ_{ij} = q_{ij}(t)dt$ for all essential combinations of i and j are derived and the transition probabilities p_{ij} are obtained by using Laplace transform and using $p_{ij} = \lim_{s\to 0} q^*_{ij}(s)$. Table 1 represents the state transition probabilities of the system.

Table 1: *State Transition Probabilities*

$dQ_{01} = e^{-(\alpha_1+\lambda_1)t}h_1(t)dt$	$dQ_{02} = \alpha_1 e^{-(\alpha_1+\lambda_1)t}\bar{H}_1(t)dt$	$dQ_{03} = \lambda_1 e^{-(\alpha_1+\lambda_1)t}\bar{H}_1(t)dt$
$dQ_{10} = e^{-(\alpha_2+\lambda_2)t}h_2(t)dt$	$dQ_{15} = \alpha_2 e^{-(\alpha_2+\lambda_2)t}\bar{H}_2(t)dt$	$dQ_{16} = \lambda_2 e^{-(\alpha_2+\lambda_2)t}\bar{H}_2(t)dt$
$dQ_{20} = e^{-\lambda_1 t}g_2(t)dt$	$dQ_{24} = \lambda_1 e^{-\lambda_1 t}\bar{G}_2(t)dt$	$dQ^{(4)}_{24} = (1-e^{-\lambda_1 t})g_2(t)dt$
$dQ_{30} = g_1(t)dt$	$dQ_{51} = e^{-\lambda_2 t}g_2(t)dt$	$dQ_{57} = \lambda_2 e^{-\lambda_2 t}\bar{G}_2(t)dt$
$dQ^{(7)}_{56} = (1-e^{-\lambda_1 t})g_2(t)dt$	$dQ_{61} = g_1(t)dt$	$p_{01} = h^*_1(\alpha_1+\lambda_1)$
$p_{02} = \frac{\alpha_1}{\alpha_1+\lambda_1}[1- h^*_1(\alpha_1+\lambda_1)]$	$p_{03} = \frac{\lambda_1}{\alpha_1+\lambda_1}[1- h^*_1(\alpha_1+\lambda_1)]$	$p_{10} = h^*_2(\alpha_2+\lambda_2)$
$p_{15} = \frac{\alpha_2}{\alpha_2+\lambda_2}[1- h^*_2(\alpha_2+\lambda_2)]$	$p_{16} = \frac{\lambda_2}{\alpha_2+\lambda_2}[1- h^*_2(\alpha_2+\lambda_2)]$	$p_{20} = g^*_2(\lambda_1)$
$p_{24} = 1-g^*_2(\lambda_1)$	$p^{(4)}_{23} = 1-g^*_2(\lambda_1)$	$p_{30} = 1 = p_{61}$
$p_{51} = g^*_2(\lambda_2)$	$p_{57} = 1-g^*_2(\lambda_2)$	$p^{(7)}_{56} = 1-g^*_2(\lambda_2)$

Mean Sojourn Time (μ_i) is the amount of time expected to spend in state i by the system. The expressions for μ_i are obtained by using $\mu_i = \int_0^\infty P[T_i>t]dt$ where T_i represents stay time of the system in state i. Table 2 represents means sojourn time of the system.

Table 2: *Mean Sojourn Time*

$\mu_0 = \frac{1}{\alpha_1+\lambda_1}[1- h^*_1(\alpha_1+\lambda_1)]$	$\mu_1 = \frac{1}{\alpha_2+\lambda_2}[1- h^*_2(\alpha_2+\lambda_2)]$	$\mu_2 = \frac{1}{\lambda_1}[1- g^*_2(\lambda_1)]$
$\mu_3 = \int_0^\infty \overline{G_1}(t)dt$	$\mu_5 = \frac{1}{\lambda_2}[1- g^*_2(\lambda_2)]$	$\mu_6 = \int_0^\infty \overline{G_1}(t)dt$
$\mu'_2 = \int_0^\infty \overline{G_2}(t)dt$		

4. Mean Time to System Failure

$\phi_i(t)$ denotes the cumulative distribution function of first passage time to a failed state from regenerative state i. Recursive relations to obtain mean time to system failure of the system are:

$$\phi_0(t)= Q_{01}(t) Ⓢ \phi_1(t)+Q_{02}(t) Ⓢ \phi_2(t)+Q_{03}(t) Ⓢ \phi_3(t) \quad (1)$$

$$\phi_1(t)= Q_{10}(t) Ⓢ \phi_0(t)+Q_{15}(t) Ⓢ \phi_5(t)+Q_{16}(t) Ⓢ \phi_6(t) \quad (2)$$

$$\phi_2(t)= Q_{20}(t) Ⓢ \phi_0(t)+Q_{24}(t) \quad (3)$$

$$\phi_3(t)= Q_{30}(t) Ⓢ \phi_0(t) \quad (4)$$

$$\phi_5(t)= Q_{51}(t) Ⓢ \phi_1(t)+Q_{57}(t) \quad (5)$$

$$\phi_6(t)= Q_{61}(t) Ⓢ \phi_1(t) \quad (6)$$

Applying Laplace Stieltjes Transform on both sides of above relations and solve them using Cramer's Rule, we get

$$MTSF = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D} \quad (7)$$

Where, $N = \mu_1 p_{01}+\mu_3 p_{01}p_{16}+\mu_5 p_{01}p_{15}+\mu_0 p_{15}p_{57}+\mu_2 p_{02}p_{15}p_{57}+\mu_3 p_{03}p_{15}p_{57}+\mu_0 p_{10}+ \mu_2 p_{02}p_{10}+\mu_3 p_{03}p_{10}$

$D = p_{02}p_{24}p_{15}p_{57}+p_{02}p_{10}p_{24}+p_{01}p_{15}p_{57}$

5. Availability in System when Humidity is ≤ 50%

$AH_i^1(t)$/ $AH_i^{1s}(t)$ denotes the probability that system available in full capacity/single cycle when humidity is ≤ 50% at any instant of time t provided it has entered regenerative state i at time t=0. By analyzing probabilistic arguments, we derive the expressions for availability in combined cycle as well as for single. The expression for combined cycle are:

$$AH_0^1(t) = MH_0(t)+Q_{01}(t)©AH_1^1(t) + Q_{02}(t)©AH_2^1(t)+Q_{03}(t)©AH_3^1(t) \quad (8)$$

$$AH_1^1(t) = Q_{10}(t)©AH_0^1(t) + Q_{15}(t)©AH_5^1(t)+Q_{16}(t)©AH_6^1(t) \quad (9)$$

$$AH_2^1(t) = Q_{20}(t)©AH_0^1(t) + Q_{23}^{(4)}(t)©AH_3^1(t) \quad (10)$$

$$AH_3^1(t) = Q_{30}(t)©AH_0^1(t) \quad (11)$$

$$AH_5^1(t) = Q_{51}(t)©AH_1^1(t) + Q_{56}^{(7)}(t)©AH_6^1(t) \quad (12)$$

$$AH_6^1(t) = Q_{61}(t)©AH_1^1(t) \quad (13)$$

Solving them we get,

$$AH_0^1(t) = \lim_{s\to 0} sAH_0^{1^*}(s)= \frac{U_1}{V} \quad (14)$$

Where, $MH_0(t)= e^{-(\alpha_1+\lambda_1)t}\bar{H}_1(t)$ and $U_1= \mu_0 p_{10}$

$V= \mu_0 p_{10}+\mu_1 p_{01}+\mu_2^{'}(p_{02}p_{10}+p_{01}p_{15})+\mu_3(p_{03}p_{10}+p_{02}p_{10}p_{23}^{(4)}+p_{01}p_{16}+p_{01}p_{15}p_{56}^{(7)})$

Similarly, we derive expressions for single cycle availability as:

$$AH_0^{1s}(t) = Q_{01}(t)©AH_1^{1s}(t) + Q_{02}(t)©AH_2^{1s}(t)+Q_{03}(t)©AH_3^{1s}(t) \quad (15)$$

$$AH_1^{1s}(t) = Q_{10}(t)©AH_0^{1s}(t) + Q_{15}(t)©AH_5^{1s}(t)+Q_{16}(t)©AH_6^{1s}(t) \quad (16)$$

$$AH_2^{1s}(t) = MH_2(t)+Q_{20}(t)©AH_0^{1s}(t) + Q_{23}^{(4)}(t)©AH_3^{1s}(t) \quad (17)$$

$$AH_3^{1s}(t) = Q_{30}(t)©AH_0^{1s}(t) \quad (18)$$

$$AH_5^{1s}(t) = Q_{51}(t)©AH_1^{1s}(t) + Q_{56}^{(7)}(t)©AH_6^{1s}(t) \quad (19)$$

$$AH_6^{1s}(t) = Q_{61}(t)©AH_1^{1s}(t) \quad (20)$$

Solving them we get,

$$AH_0^{1s}(t) = \lim_{s\to 0} sAH_0^{1s^*}(s)= \frac{U_2}{V} \quad (21)$$

Where, $MH_2(t)= e^{-\lambda_1 t}\bar{G}_2(t)$, $U_2= \mu_2 p_{10}p_{02}$ and V = as defined above

6. Availability of the System when Humidity is > 50%

$AH_i^2(t)$/ $AH_i^{2s}(t)$ denotes the probability that is available in full capacity/single cycle when humidity is > 50% at any instant of time t provided it has entered regenerative state i at time t=0. By analyzing probabilistic arguments, we subsequently derive the expression for availability in combined as well as for single cycle. The following are expression for combined cycle:

$$AH_0^2(t) = Q_{01}(t)©AH_1^2(t) + Q_{02}(t)©AH_2^2(t)+Q_{03}(t)©AH_3^2(t) \quad (22)$$

$$AH_1^2(t) = MH_1(t)+Q_{10}(t)©AH_0^2(t) + Q_{15}(t)©AH_5^2(t)+Q_{16}(t)©AH_6^2(t) \quad (23)$$

$$AH_2^2(t) = Q_{20}(t)©AH_0^2(t) + Q_{23}^{(4)}(t)©AH_3^2(t) \quad (24)$$

$$AH_3^2(t) = Q_{30}(t)©AH_0^2(t) \quad (25)$$

$$AH_5^2(t) = Q_{51}(t)©AH_1^2(t) + Q_{56}^{(7)}(t)©AH_6^2(t) \quad (26)$$

$$AH_6^2(t) = Q_{61}(t)©AH_1^2(t) \quad (27)$$

Solving them we get,

$AH_0^2(t) = \lim_{s\to 0} sAH_0^{2*}(s) = \frac{U_3}{V}$ (28)

Where, $MH_1(t) = e^{-(\alpha_2+\lambda_2)t}\bar{H}_2(t)$, $U_3 = \mu_1 p_{01}$ and V = already defined

Similarly, we derive the recursive relations for availability in single cycle as:

$AH_0^{2s}(t) = Q_{01}(t)©AH_1^{2s}(t) + Q_{02}(t)©AH_2^{2s}(t) + Q_{03}(t)©AH_3^{2s}(t)$ (29)

$AH_1^{2s}(t) = Q_{10}(t)©AH_0^{2s}(t) + Q_{15}(t)©AH_5^{2s}(t) + Q_{16}(t)©AH_6^{2s}(t)$ (30)

$AH_2^{2s}(t) = Q_{20}(t)©AH_0^{2s}(t) + Q_{23}^{(4)}(t)©AH_3^{2s}(t)$ (31)

$AH_3^{2s}(t) = Q_{30}(t)©AH_0^{2s}(t)$ (32)

$AH_5^{2s}(t) = MH_5(t) + Q_{51}(t)©AH_1^{2s}(t) + Q_{56}^{(7)}(t)©AH_6^{2s}(t)$ (33)

$AH_6^{2s}(t) = Q_{61}(t)©AH_1^{2s}(t)$ (34)

Solving them we get,

$AH_0^{2s}(t) = \lim_{s\to 0} sAH_0^{2s*}(s) = \frac{U_4}{V}$ (35)

Where, $MH_5(t) = e^{-\lambda_2 t}\bar{G}_2(t)$, $U_4 = \mu_5 p_{15} p_{01}$ and V = already defined

7. System's Expected Down Time

$DH_i^1(t)/DH_i^2(t)$ denotes that the system is in down state at specific instant of time t when humidity is ≤ /> 50%. The recursive expressions for $DH_i^1(t)/DH_i^2(t)$ are given below:

$DH_0^1(t) = Q_{01}(t)©DH_1^1(t) + Q_{02}(t)©DH_2^1(t) + Q_{03}(t)©DH_3^1(t)$ (36)

$DH_1^1(t) = Q_{10}(t)©DH_0^1(t) + Q_{15}(t)©DH_5^1(t) + Q_{16}(t)©DH_6^1(t)$ (37)

$DH_2^1(t) = Q_{20}(t)©DH_0^1(t) + Q_{23}^{(4)}(t)©DH_3^1(t)$ (38)

$DH_3^1(t) = NH_3(t) + Q_{30}(t)©DH_0^1(t)$ (39)

$DH_5^1(t) = Q_{51}(t)©DH_1^1(t) + Q_{56}^{(7)}(t)©DH_6^1(t)$ (40)

$DH_6^1(t) = Q_{61}(t)©DH_1^1(t)$ (41)

$DH_0^1(t) = \lim_{s\to 0} sDH_0^{1*}(s) = \frac{U_5}{V}$ (42)

Where, $NH_3(t) = \bar{G}_1(t)$, $U_5 = \mu_3 p_{10}(p_{03} + p_{02} p_{23}^{(4)})$ and V = already defined

$DH_0^2(t) = Q_{01}(t)©DH_1^2(t) + Q_{02}(t)©DH_2^2(t) + Q_{03}(t)©DH_3^2(t)$ (43)

$DH_1^2(t) = Q_{10}(t)©DH_0^2(t) + Q_{15}(t)©DH_5^2(t) + Q_{16}(t)©DH_6^2(t)$ (44)

$DH_2^2(t) = Q_{20}(t)©DH_0^2(t) + Q_{23}^{(4)}(t)©DH_3^2(t)$ (45)

$DH_3^2(t) = Q_{30}(t)©DH_0^2(t)$ (46)

$DH_5^2(t) = Q_{51}(t)©DH_1^2(t) + Q_{56}^{(7)}(t)©DH_6^2(t)$ (47)

$DH_6^2(t) = NH_6(t) + Q_{61}(t)©DH_1^2(t)$ (48)

$DH_0^2(t) = \lim_{s\to 0} sDH_0^{2*}(s) = \frac{U_6}{V}$

Where, $NH_6(t) = \bar{G}_1(t)$, $U_5 = \mu_3 p_{01}(p_{16} + p_{15} p_{56}^{(7)})$ and V = already defined

8. Time Period for which Repairman is Busy

$BH_i^1(t)/BH_i^2(t)$ denotes the probability that repairman is busy at an instant t when humidity is ≤/> 50%. By analyzing probabilistic arguments, the recursive expressions for $BH_i^1(t)/BH_i^2(t)$ are given below:

$BH_0^1(t) = Q_{01}(t)©BH_1^1(t) + Q_{02}(t)©BH_2^1(t) + Q_{03}(t)©BH_3^1(t)$ (49)

$BH_1^1(t) = Q_{10}(t)©BH_0^1(t) + Q_{15}(t)©BH_5^1(t) + Q_{16}(t)©BH_6^1(t)$ (50)

$BH_2^1(t) = RH_2(t) + Q_{20}(t)©BH_0^1(t) + Q_{23}^{(4)}(t)©BH_3^1(t)$ (51)

$BH_3^1(t) = RH_3(t) + Q_{30}(t)©BH_0^1(t)$ (52)

$BH_5^1(t) = Q_{51}(t)©BH_1^1(t) + Q_{56}^{(7)}(t)©BH_6^1(t)$ (53)

$BH_6^1(t) = Q_{61}(t)©BH_1^1(t)$ (54)

$BH_0^1(t) = \lim_{s\to 0} sBH_0^{1*}(s) = \frac{U_6}{V}$ (55)

Where, $RH_2(t)=e^{-\lambda_1 t}\bar{G}_2(t)$, $RH_3(t)=\bar{G}_1(t)$,

$U_6 = p_{10}[p_{02}\mu_2 + (p_{03}+p_{02}p_{23}^{(4)})\mu_3]$ and V = already defined

$BH_0^2(t) = Q_{01}(t) \copyright BH_1^2(t) + Q_{02}(t) \copyright BH_2^2(t) + Q_{03}(t) \copyright BH_3^2(t)$ (56)

$BH_1^2(t) = Q_{10}(t) \copyright BH_0^2(t) + Q_{15}(t) \copyright BH_5^2(t) + Q_{16}(t) \copyright BH_6^2(t)$ (57)

$BH_2^2(t) = Q_{20}(t) \copyright BH_0^2(t) + Q_{23}^{(4)}(t) \copyright BH_3^2(t)$ (58)

$BH_3^2(t) = Q_{30}(t) \copyright BH_0^2(t)$ (59)

$BH_5^2(t) = RH_5(t) + Q_{51}(t) \copyright BH_1^2(t) + Q_{56}^{(7)}(t) \copyright BH_6^2(t)$ (60)

$BH_6^2(t) = RH_6(t) + Q_{61}(t) \copyright BH_1^2(t)$ (61)

$BH_0^2(t) = \lim_{s\to 0} sBH_0^{2*}(s) = \frac{U_7}{V}$ (62)

Where, $RH_5(t) = e^{-\lambda_2 t}\bar{G}_2(t)$, $RH_6(t) = \bar{G}_1(t)$

$U_7 = p_{01}p_{15}\mu_5 + p_{01}\mu_6(p_{16}+p_{15}p_{56}^{(7)})$ and V = already defined

9. Number of Visits to be Expected by Repairman

$VH_i^1(t)/VH_i^2(t)$ denotes the expected number of visits by the repairman when humidity is ≤/> 50%. By analyzing probabilistic arguments, the recursive expressions for $VH_i^1(t)/VH_i^2(t)$ are given below:

$VH_0^1(t) = Q_{01}(t) \circledS VH_1^1(t) + Q_{02}(t) \circledS [1+VH_2^1(t)] + Q_{03}(t) \circledS [1+VH_3^1(t)]$ (63)

$VH_1^1(t) = Q_{10}(t) \circledS VH_0^1(t) + Q_{15}(t) \circledS VH_5^1(t) + Q_{16}(t) \circledS VH_6^1(t)$ (64)

$VH_2^1(t) = Q_{20}(t) \circledS VH_0^1(t) + Q_{23}^{(4)}(t) \circledS VH_3^1(t)$ (65)

$VH_3^1(t) = Q_{30}(t) \circledS VH_0^1(t)$ (66)

$VH_5^1(t) = Q_{51}(t) \circledS VH_1^1(t) + Q_{56}^{(7)}(t) \circledS VH_6^1(t)$ (67)

$VH_6^1(t) = Q_{61}(t) \circledS VH_1^1(t)$ (68)

$VH_0^1(t) = \lim_{s\to 0} sVH_0^{1*}(s) = \frac{U_8}{V}$ (69)

Where, $U_8 = p_{10}(p_{02}+p_{03})$ and V = already defined

$VH_0^2(t) = Q_{01}(t) \circledS VH_1^2(t) + Q_{02}(t) \circledS VH_2^2(t) + Q_{03}(t) \circledS VH_3^2(t)$ (70)

$VH_1^2(t) = Q_{10}(t) \circledS VH_0^2(t) + Q_{15}(t) \circledS [1+VH_5^2(t)] + Q_{16}(t) \circledS [1+VH_6^2(t)]$ (71)

$VH_2^2(t) = Q_{20}(t) \circledS VH_0^2(t) + Q_{23}^{(4)}(t) \circledS VH_3^2(t)$ (72)

$VH_3^2(t) = Q_{30}(t) \circledS VH_0^2(t)$ (73)

$VH_5^2(t) = Q_{51}(t) \circledS VH_1^2(t) + Q_{56}^{(7)}(t) \circledS VH_6^2(t)$ (74)

$VH_6^2(t) = Q_{61}(t) \circledS VH_1^2(t)$ (75)

$VH_0^2(t) = \lim_{s\to 0} sVH_0^{2*}(s) = \frac{U_9}{V}$ (76)

Where, $U_9 = p_{01}(p_{15}+p_{16})$ and V = already defined

10. Profit Analysis of the System

Expected profit expressions induced per unit time in steady state for the system are:

$P = CA_1{*}AH_0^1 + CA_2{*}AH_0^2 + CA_{1s}{*}AH_0^{1s} + CA_{2s}{*}AH_0^{2s} - CB_1{*}BH_0^1 - CB_2{*}BH_0^2 - CV_1{*}VH_0^1 - CV_2{*}VH_0^2 - C_0$

CA_1/CA_2 : Revenue generated per unit uptime when humidity is ≤ 50%, system works in Combined Cycle/Single Cycle

CA_{1s}/CA_{2s} : Revenue generated per unit uptime when humidity is > 50%, system works in Combined Cycle/Single Cycle

CB_1/CB_2 : Cost per unit time when humidity is ≤/> 50% while repairman is busy in doing repair

CV_1/CV_2 : Cost per visit by repairman when humidity is ≤/> 50%

C_0 : Other expenses in plant operation

11. Results and Graphical Representation of Reliability Measures

Now, by considering the particular cases $g_1(t)=\beta_1 e^{-\beta_1 t}$, $g_2(t)=\beta_2 e^{-\beta_2 t}$, $h_1(t)=\gamma_1 e^{-\gamma_1 t}$, $h_2(t)=\gamma_2 e^{-\gamma_2 t}$ and using information collected from gas turbine power plant we get $\alpha_1=\lambda_1=0.001$, $\alpha_2=\lambda_2=0.0014$, $\beta_1=\beta_2=0.028$, $\gamma_1=0.333$, $\gamma_2=0.317$, $CA_1 = 1372000$, $CA_2 = 1250000$, $CA_{1s} = 882000$, $CA_{2s} = 800000$, $CB_1 = 12000$, $CB_2 = 20500$, $CV_1 = 8000$, $CV_2 = 12500$, $C_0 = 650000$ and plotted different graphs according to these esteemed values and studied various measures of reliability through graphs as explained below:

11.1. Mean Time to System Failure Vs Different Failure Rates

Figure 2 illustrates the behavior of MTSF with different failure rates α_1, α_2, λ_1, λ_2.

- MTSF decreases with increase in any one of the failure rates.
- MTSF increases with increase in repair rates and when both repair rates (β_1, β_2) are increased by same amount MTSF is higher in case of β_2 as compared to β_1.

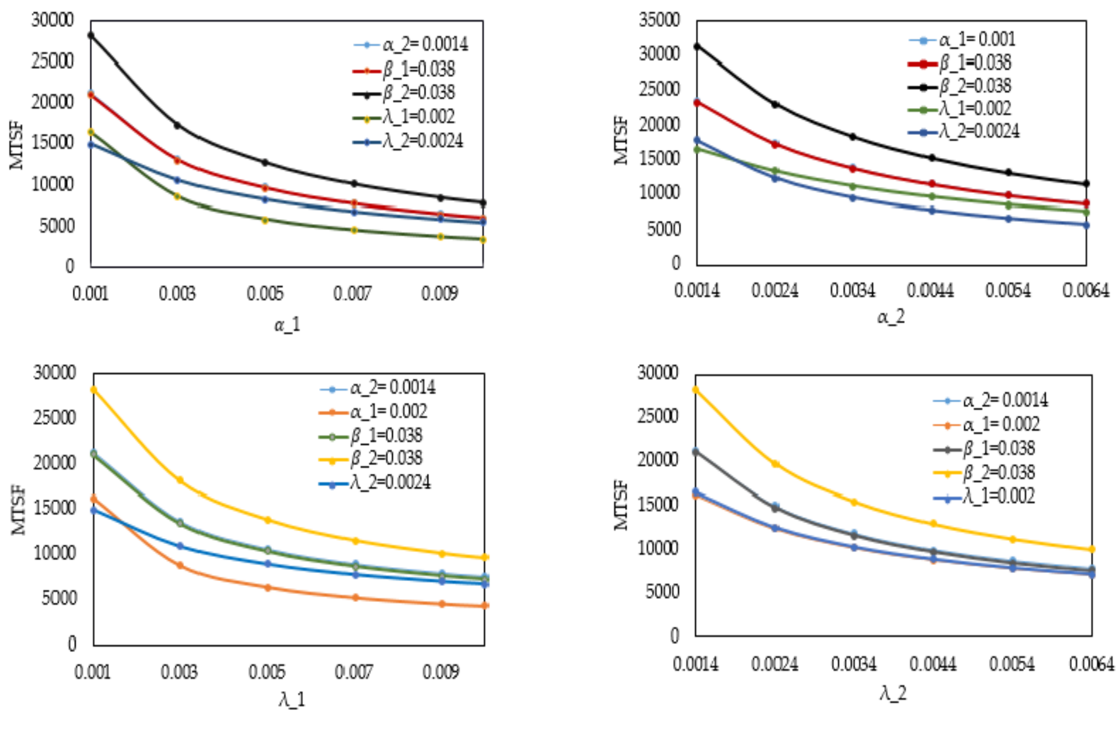

Figure 2: *MTSF Vs Failure rates* α_1, α_2, λ_1, λ_2

11.2. Availability in Combined Cycle Vs Failure Rate of Steam Turbine when Humidity is ≤ 50%

Figure 3 demonstrates the availability in combined cycle when humidity is ≤ 50% and when humidity is > 50%

- Both availabilities (when humidity is ≤/> 50%) of combined cycle decreases as we increase in any one of failure rate.
- Both availabilities (when humidity is ≤/> 50%) in combined cycle increases with increase in repair rates.
- Availability in Combined Cycle when humidity is > 50% is higher than Availability in Combined Cycle when Humidity is ≤ 50%.

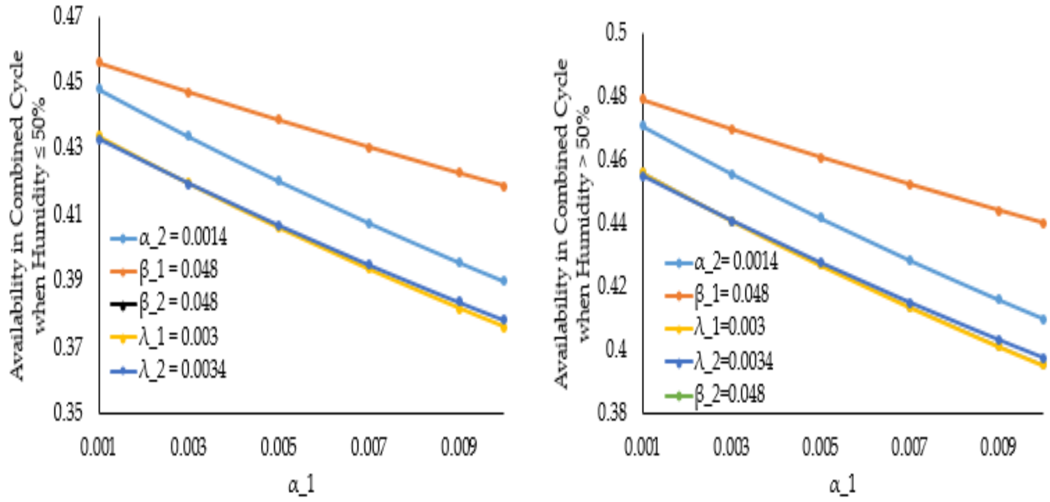

Figure 3: *Availability in combined cycle Vs Failure rate* $\propto_1$

11.3. Availability in Single Cycle Vs Failure Rate of Steam Turbine when Humidity is ≤ 50%

Figure 4 demonstrates the availability in single cycle when humidity is ≤ 50% and when humidity is > 50%

- Availability in single cycle when humidity is ≤ 50% increases with increase in α_1 but decreases with increase in any other failure rate while availability in single cycle when humidity is > 50% decreases with increase in α_1, λ_1, λ_2 but increases with increase in α_2.
- Availability in single cycle when humidity is ≤ 50% decrease with increase in β_1 while availability in single cycle when humidity is > 50% increases with increase in β_1.

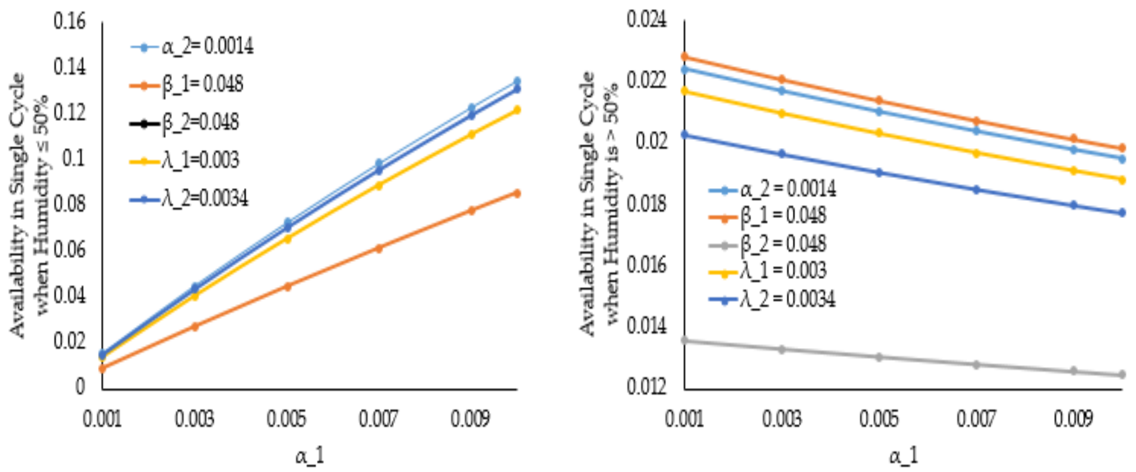

Figure 4: *Availability in single cycle Vs Failure rate* $\propto_1$

11.4. Profit Vs Failure Rate of Steam Turbine when Humidity is ≤ 50%

Figure.5 illustrates the behavior of Profit of plant with respect to failure rate α_1.

- Profit decreases with increase in any one failure rates $\alpha_1, \alpha_2, \lambda_1, \lambda_2$.

- Profit increases as we increase in repair rates and when both repair rates (β_1, β_2) are increased by same amount, Profit in case of β_2 is higher than Profit in case of β_1 (when α_1 = 0.001 to 0.006) and Profit in case of β_1 is higher than Profit in case of β_2 (when α_1 = 0.006 to 0.01).

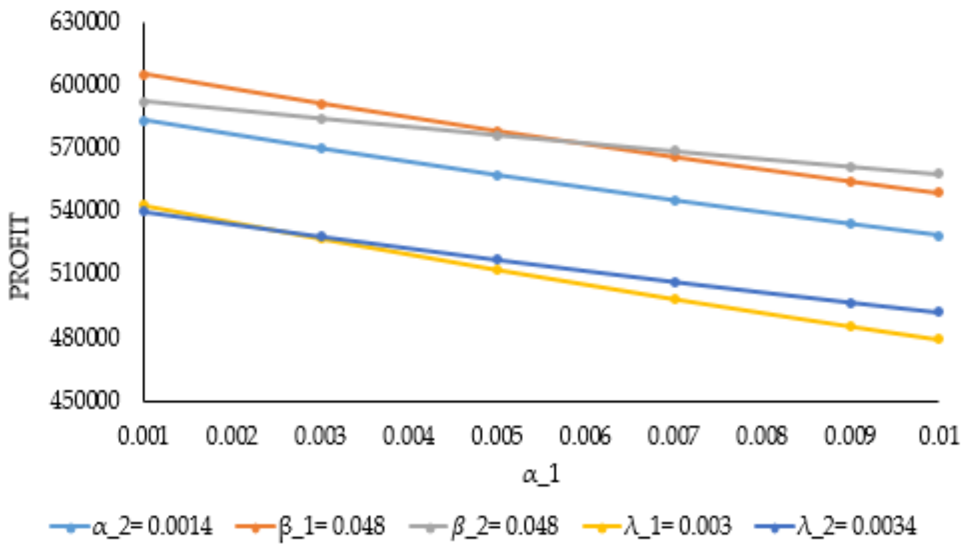

Figure 5: *Profit Vs Failure rate* $\propto_1$

12. Conclusion

A stochastic model for a two-unit gas turbine system has been developed by fixing the range of temperature and using the idea of different humidity conditions. Various reliability measures like mean time to system failure, availability for combined as well as for single cycle when humidity is ≤/> 50% have been obtained for particular cases using information gathered from gas turbine power plant. Simultaneous effects of failure rates of gas and steam turbines when humidity is ≤/> 50% on mean time to system failure have been graphically analyzed and from them we concluded that mean time to system failure decreases as any failure rate increases. Trends in the availability for both cycles and different humidity conditions i.e. when humidity is ≤/> 50% has been illustrated with respect to failure rate of steam turbine and various interesting results have been obtained regarding availability. At last, profit for plant is also depicted which decreases with increase in failure rates. Here we see that for this fixed range of temperature availability is higher when humidity is >50% as compared to when humidity is ≤50% which further impacts on profit of the plant. Furthermore, a comprehensive examination of gas turbine system may be helpful to those who are involved in power generating industry.

References

[1] Baohe, Su (1997). On a two-dissimilar-unit system with three modes and random check. Microelectron Reliab, 37:1233-1238.
[2] Chow, D.K. (1973). Reliability of some redundant systems with repair. IEEE Transactions on Reliability, 22:223-228.
[3] Tuteja, R.K., Arora R.T. and Taneja, G. (1991). Analysis of a two-unit system with partial failures and three types of repairs. Reliability Engineering and System Safety, 33:199-214.

[4] El-Berry, A. (2020). Reliability analysis of gas turbine power plant based on failure data. International Journal of Mechanical and Mechatronics Engineering, 2:13-25.
[5] Singh, D. and Taneja, G. (2012). Reliability analysis of a power generating system through gas and steam turbines with scheduled inspection. Aryabhatta Journal of Mathematics and Informatics, 5:373-380.
[6] Singh, D. and Taneja, G. (2014). Reliability and economic analysis of a power generating system comprising one gas and one steam turbine with random inspection. International Journal of Mathematics and Statistics, 4:30-37.
[7] Farouk, N., Sheng, L. and Hayat, Q. (2013). Effects of ambient temperature on performance of gas turbines power plant. International Journal of Computer Science, 10:30-37.
[8] Abigail, G.D., Agustin, M.A.C., Maria, O.G.D., Angel, M.A., Mathieu, L. and Jose, M.G.S., (2017). Effect of the ambient conditions on gas turbine combined cycle power plants with post-combustion CO_2 capture. Elsevier Ltd., 134:221-233.
[9] Saleh, S., Baakeem, J.O. and Hany, A. (2015). Performance of a typical simple gas turbine unit under Saudi weather conditions. International Journal of Energy and Power Engineering, 1:59-71.
[10] Rajesh, Taneja G. and Prasad, J. (2018). Reliability of a gas turbine system with change in weather and optimization of electricity price when working in single cycle. International Journal of Agricultural and Statistical Sciences. 14:119-128.
[11] Rajesh, Taneja G. and Prasad, J. (2018). Reliability and availability analysis for a three-unit gas turbine power generating system with seasonal effect and FCFS repair pattern. International Journal of Applied Engineering Research. 13:10948-10964.
[12] Fernandez, D.A.P., Foliaco, B., Padilla, R.V., Bula, A. and Quiroga, A.G. (2021). High ambient temperature effects on the performance of a gas turbine-based cogeneration system with supplementary fire in a tropical climate. Case Studies in Thermal Engineering, 26:101206-101219.
[13] Bird, J. and Grabe, W. (1991). Humidity effects on gas turbine performance. The American Society of Mechanical Engineers, 329.
[14] Hanachi, H., Liu, J., Banerjee, A. and Chen, Y., (2015). Effects of the intake air humidity on gas turbine performance monitoring. Proceedings of ASME Turbo 6.