Maintenance policy costs considering imperfect repairs

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Abstract

Objective: This paper extends the analysis of imperfect preventive maintenance interspaced with minimal repairs. The aim is to find the intervals of future scheduled maintenance actions considering different recovery factors and costs. Methods: The optimal preventive maintenance scheduling are obtained by minimizing the overall maintenance costs. Minimal repairs interspersed with scheduled imperfect preventive maintenance actions are considered. The model parameters of the power law process are estimated using the maximum likelihood estimation method and a Differential Evolution algorithm is used to solve the maximization problem. Results: The optimal preventive maintenance periods for different levels of maintenance restoration with respect to corrective and preventive maintenance costs are found. Graphs are drawn to highlight the effect of future maintenance costs and the hazard function paths. It is shown that the preventive maintenance becomes more frequent as the equipment ages and the hazard function increases. Also, it is perceived that the scheduled maintenance intervals become shorter as the corrective maintenance becomes more expensive. Conclusion: A hazard rate model which considers minimal repairs interspersed with scheduled imperfect preventive maintenance provides a useful tool for defining the optimal maintenance policy. The results obtained in this paper show that maintenance cost varies widely according to the recovery factor of the maintenance action and that the optimal interval of two consecutive preventive maintenance actions strongly depends on the costs.

Keywords: Reliability, imperfect maintenance, proportional age reduction model, maintenance costs, power law model.

1. Introduction

Recently statisticians and engineers have paid a lot of attention to reliability centered maintenance and its cost assessment. As stated by Löfsten (2000), the overall costs of maintenance, estimated to be between 15% and 40% of production costs, and the trend toward industry automation has forced engineers and managers to pay more attention to maintenance policies.

There are several papers in the recent literature that have attempted to estimate the failure probability distributions implied by different maintenance policies. Researchers have developed a wide variety of models to deal with maintenance policy optimization. Performance and condition-based maintenance models can be found in Dui et al. (2023), Azizi and Salari (2023) and Chen et al. (2022). Preventive maintenance policies with degradation models can be found in Wei et al. (2023) and Li et al. (2023). Predictive maintenance models can be found in Huynh et al. (2022) and Guo and Liang (2022) and new probability distributions have been studied, such as in S. and Sebastian (2022). Also, maintenance models are under constantly development, as can be found in Tijjani A. Waziri (2022) for a replacement policy, in Naveen Kumar (2022), Shanti Parkash (2022) and Neetu Dabas (2022) for priority repair policies and in Nse Udoh (2022) studying maintenance policies for non-repairable products. Even modern techniques, such as Artificial-intelligence-based model can be found in Nguyen et al. (2022).

In the present study, it is analyzed the consequence of minimal corrective repairs interspersed with imperfect schedulued preventive maintenance. Once a scheduled maintenance is performed on an equipment, it will be restored to a state that is between as good as new and as bad as old. Under this assumption, the Proportional Age Reduction model was proposed in Malik (1979). The paper's objective is to refine the study made by Shin et al. (1996) including the cost analysis and the possibility to change the level of the equipment's regeneration after maintenance actions.

The remaining part of this paper is organized as follows. In Section 2 it is made a review of the Reliability theory, it is shown the maximum likelihood estimation of the power law process under the PAR model and it is presented the expected cost of the maintenance policy. In Section 3 it is described the failure and maintenance actions data. Also, the maximum likelihood estimators are calculated. In Section 4 the results is extended. It is analyzed the optimal preventive maintenance interval under different recovery parameters. The overall cost is also analyzed. Section 5 concludes the paper.

2. Reliability Theory

The purpose of the use of reliability theory is to assist management in decision making by using known quantitative facts effectively and by reducing the reliance on subjective judgement (Löfsten (2000)).

A formal definition of reliability is given by Elsayed (2021): *"Reliability is the probability that a product will operate or a service will be provided properly for a specified period of time (design life) under the design operating conditions (such as temperature, load, volt…) without failure."*

As the probability theory is the foundation of the reliability engineering and of the reliability centered maintenance, we review the following definitions that can be found in William Q. Meeker (2021) and in in Elsayed (2021).

Let $f(t)$ be a real function such that

$$f(t) \geq 0 \quad \forall t \geq 0$$

and

$$\int_0^\infty f(s)ds = 1.$$

Then, $f(t)$ is a failure probability density function. The probability of failure up to time t is given by

$$F(t) = \int_0^t f(s)ds, \tag{1}$$

so that the reliability function is given by

$$C(t) = 1 - F(t) = \int_t^\infty f(s)ds. \tag{2}$$

The following function

$$h(t) = \lim_{\Delta t \to 0} \frac{C(t) - C(t + \Delta t)}{\Delta t C(t)} = \frac{1}{C(t)} \left[-\frac{d}{dt} C(t) \right] = \frac{f(t)}{C(t)} \tag{3}$$

is called the hazard function. Considering that only minimal repairs are performed when the equipment fails, the expected number of failures in $[0, t]$ is given by

$$H(t) = \int_0^t h(s)ds. \tag{4}$$

The reliability function can be also calculated by

$$C(t) = \mathrm{e}^{\int_0^t h(s)ds}, \tag{5}$$

and the expectation of T is defined as the mean time to failure (MTTF):

$$MTTF = \int_0^\infty C(s)ds = \int_0^\infty sf(s)ds. \tag{6}$$

As an example, the Weibull distribution is given by the following probability density function

$$f(t;\alpha,\beta) = \alpha\beta t^{\beta-1}\exp\{-\alpha t^{\beta}\}1_{\{t>0\}}, \tag{7}$$

for fixed parameters α and β, scale and shape, respectively. A plot of the probability density function $f(t)$, the reliability function 5, the hazard function 3 and cumulative distribution function 1 for the Weibull distributions with different parameters are shown in Figure 1.

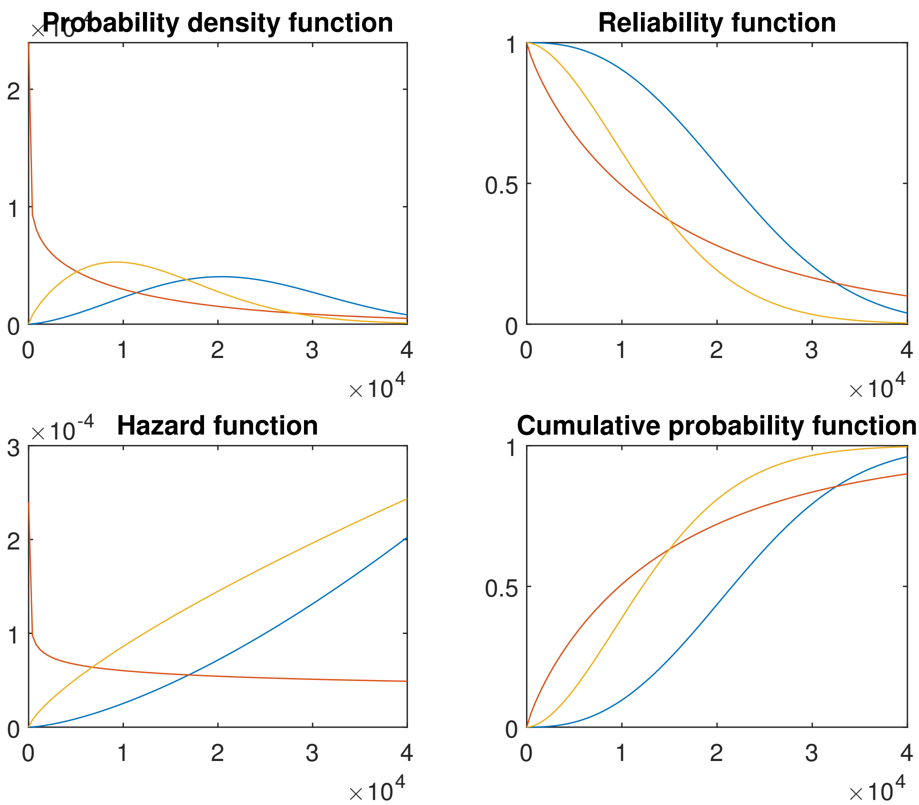

Figure 1: *Probability functions for the Weibull distribution*

2.1. Maintenance policy cost

Following Pham (2003), maintenance can be defined as actions to control the deterioration process leading to failure of a system - called preventive maintenance, and to restore the system to its operational state through corrective actions after a failure - called corrective maintenance. The behavior of the equipment after a repair depends on the type of repair carried out.

As stated by Huynh et al. (2022), maintenance is an effective solution to improve not only the system availability, but also the system safety, the product quality, as well as the customer satisfaction. An appropriate preventive maintenance policy is an effective way to save cost by reducing the probability of failure (Li et al. (2023)).

Unscheduled or corrective maintenance refers to maintenance actions carried out after the occurrence of component's failure. Suppose that the equiment are minimally repaired at failure and the effect of imperfect preventive maintenance is modeled according to the Proportional Age Reduction (PAR) criterion. In the case of perfect maintenance, the action restores the equipment to be as new. The PAR approach introduced by Malik (1979), assumes that each preventive maintenance action reduces the age of the equipment by a quantity proportional to the operating time elapsed form the most recent scheduled maintenance.

The failure pattern in each maintenance cycle is described by a Non-homogeneous Poisson process in which the age of the equipment in the k-th maintenance cycle is reduced by a fraction ρ of the most recent scheduld maintenance action τ_{k-1}. The hazard function (3) at a time t is

$$h(t) = h(t - \rho\tau_{k-1}), \quad \tau_{k-1} < t < \tau_k. \tag{8}$$

The following are the hypotheses assumed by Shin et al. (1996) to model minimal repairs interspersed with scheduled imperfect preventive maintenance actions:

1. Suppose that l units are observed until T_i, $i = 1, ..., l$.
2. Suppose that each equipment i is subjected to m_i scheduled maintenance actions at $\tau_{i,1} < \tau_{i,m_i} \leq T_i$.
3. The i-th equipment experiences $r_{i,k}$ failures during the k-th preventive maintenance cycle $(k = 1, ..., m_i + 1)$
4. Let $t_{i,k,j}$ be the time of the j-th failure of the i-th equipment that occurs in the k-th maintenance cycle.

Pham and Wang (1996) presents another interesting approach to imperfect maintenance via quasi-renewal process.

The maintenance model here adopted considers that the preventive action is executed periodically at a prespecified times and different policies for treat the failures may be employed.

Let

- c_p be the preventive maintenance cost;
- c_m be the corrective maintenance cost.

Adapting the maintenance policy for repairable equipments of Pham (2003), the maintenance expected cost per unit time for the period $[t_{m+1}, t_{m+2}]$ is given by

$$V_1(t_{m+1}, t_{m+2}) = \frac{c_m H(t_{m+1}, t_{m+2}) + c_p}{t_{m+2} - t_{m+1}}, \tag{9}$$

where $H(t_{m+1}, t_{m+2}) = \int_{t_{m+1}}^{t_{m+2}} h(s)ds$.

2.2. Maximum Likelihood Estimation

The maximum likelihood estimation (MLE) is a method of estimating the parameters of an assumed probability density function given some observed data by maximizing a likelihood function so that, under the assumed statistical model, the observed data is most probable. The MLE of the model (8) is given by

$$\begin{aligned} L = \quad & \prod_{i=1}^{l} \left\{ \prod_{k=1}^{m_i+1} \left[\prod_{j=1}^{r_{i,k}} h(t_{i,k,j} - \rho\tau_{i,k-1}) \right] \times \right. \\ & \left. \exp\left[- \sum_{k=1}^{m_i+1} \int_{\tau_{i,k-1}}^{\tau_{i,k}} h(x - \rho\tau_{i,k-1}) dx \right] \right\}, \end{aligned} \tag{10}$$

where $\tau_{i,0} = 0$ and $\tau_{i,m_i+1} = T_i$ (see more details in Shin et al. (1996)).

The most frequently used non-homogeneous Poisson process (NHPP) is the power law process (Pham (2003)), whose intensity, applied in (8), is given by

$$h(t) = \frac{\beta}{\alpha} \left(\frac{t - \rho\tau_{k-1}}{\alpha} \right)^{\beta - 1}. \tag{11}$$

Using (11) in (10) we have

$$L = \left(\frac{\beta}{\alpha}\right)^n \left[\prod_{i=1}^{l}\prod_{k=1}^{m_i+1}\prod_{j=1}^{r_{i,k}}\left(\frac{t_{i,k,j}-\rho\tau_{i,k-1}}{\alpha}\right)^{\beta-1}\right] \times \exp\left\{-\sum_{i=1}^{l}\sum_{k=1}^{m_i+1}\left[\left(\frac{t_{i,k}-\rho\tau_{i,k-1}}{\alpha}\right)^{\beta}-\left(\frac{t_{i,k-1}-\rho\tau_{i,k-1}}{\alpha}\right)^{\beta}\right]\right\}, \tag{12}$$

where n is the total number of failures for l units during the whole observation period. The maximum likelihood estimation for α is given by the following analytical solution:

$$\alpha = \left\{\sum_{i=1}^{l}\sum_{k=1}^{m_i+1}\frac{\left[(t_{i,k}-\hat{\rho}\tau_{i,k-1})^{\hat{\beta}}-(t_{i,k-1}-\hat{\rho}\tau_{i,k-1})^{\hat{\beta}}\right]}{n}\right\}^{\frac{1}{\hat{\beta}}}. \tag{13}$$

The estimators for β e ρ are found by maximizing the modified two-parameter loglikelihood function:

$$\begin{aligned} l(\beta,\rho) = \quad & n\ln\beta \\ & -n\ln\left\{\sum_{i=1}^{l}\sum_{k=1}^{m_i+1}\left[(t_{i,k}-\rho\tau_{i,k-1})^{\beta}-(t_{i,k-1}-\rho\tau_{i,k-1})^{\beta}\right]\right\} \\ & +n\ln n+(\beta-1) \\ & \times\left[\sum_{i=1}^{l}\sum_{k=1}^{m_i+1}\sum_{j=1}^{r_{i,k}}\ln(t_{i,k,j}-\rho\tau_{i,k-1})\right]-n \end{aligned} \tag{14}$$

In order to solve the maximization problem (14), which does not have analytical solution, it is applied a Differential Evolution method. As it is described in Kienitz and Wetterau (2012), the Differential Evolution method is a population-based search algorithm which belongs to the class of genetic algorithms. It mimics the process of Darwinian evolution using techniques such as inheritance, mutation, recombination, selection and crossover. The algorithm is designed to converge to the global optimal solution. The algorithm is described in Chapter 9 of Kienitz and Wetterau (2012).

3. Failure Data

In this paper it is considered the failure data of a central cooler system of a nuclear power plant analyzed first by Shin et al. (1996) and discussed in Pham (2003). The data consist of $n = 15$ failure times and $m = 3$ preventive maintenance epochs - highlighted by $(^*)$, observed over 612 days. The data are given in Table 1.

Table 1: *Failure data*

116	151	154*	213	263*	386	387	395	407
463	492	494	501	512*	537	564	590	609

Minimal repairs are performed at failures. No information are given with respect to the level of recovery of the preventive maintenance actions.

The Differential Evolution method described in Kienitz and Wetterau (2012) was used to solve the modified two-parameter loglikelihood function (14). The parameters shown in the Table 7 of Shin et al. (1996) were exactly recovered, namely

$$\alpha = 141, \quad \beta = 2.91, \quad \rho = 0.77.$$

The parameters suggest that the equipment is restored to a level corresponding to 33% of the actual age. Also, as the shape parameter β is greater that 2, it is known that the hazard rate increases in a convex form towards infinite.

The objective is to find the scheduled maintenance interval that minimizes the overall cost given by (9). This work also aims to analyze the impact of the recovery parameter ρ for future maintenance actions.

4. Results

Let t_{m+1} be the actual time, when the last preventive maintenance action was performed with $\rho = 0.77$. In order to calculate the optimal preventive maintenance interval which minimizes the cost (9), we need to find

$$\begin{aligned}
\frac{\partial V_1(t_{m+1}, t_{m+2})}{\partial t_{m+2}} &= \frac{\left[c_m h(t_{m+1}, t_{m+2})\right](t_{m+2} - t_{m+1}) - \left[c_m H(t_{m+1}, t_{m+2}) + c_p\right]}{(t_{m+2} - t_{m+1})^2} = 0 \\
&= c_m (t_{m+2} - t_{m+1}) \left(\frac{\beta}{\alpha}\right) \left[\left(\frac{t_{m+2} - \rho t_{m+1}}{\alpha}\right)^{(\beta-1)}\right] - \\
&\quad \left\{ c_m \left[\left(\frac{t_{m+2} - \rho t_{m+1}}{\alpha}\right)^{\beta} - \left(\frac{(1-\rho) t_{m+1}}{\alpha}\right)^{\beta}\right] + c_p \right\}. \qquad (15)
\end{aligned}$$

Applying the Power Law process (11), we cannot find the solution for (15) analytically. We find the root of the nonlinear function (15) using the fzero function of Matlab, which is a combination of bisection, secant, and inverse quadratic interpolation methods (Forsythe G. E. (1976)).

Suppose that $c_m = 1.25c_p$, the next maintenance epoch which minimizes the overall maintenance cost under the hypotheses that a major overhaul was performed at $t_{m+1} = 612$ days is in $t_{m+2} = 678$ days. The next optimal scheduled maintenance actions is shown in Table 2. We observe that the first optimal scheduled maintenance interval is 66 days. We can note that the preventive maintenance becomes more frequent as the equipment ages and the hazard function increases. The next intervals are the following: 64, 63, 61, 59 and 58 days. Figure 2 compares the time between scheduled maintenance actions for $c_c = 0.75c_p$, $c_c = c_p$ and $c_c = 1.25c_p$. As expected, the scheduled maintenance interval is shorter as the corrective maintenance becomes more expensive.

It can be analyzed in Figure 3 that higher level of recovery results in larger scheduled maintenance intervals. It is only true for $\beta > 2$, which is the actual case. For $\beta = 2$ the optimal preventive maintenance interval would be the same for any value of ρ. The opposite behavior is found for $1 < \beta < 2$. It can also be seen in Figure 3 that as the preventive maintenance cost becomes greater than the corrective maintenance costs, that is, $\frac{CM}{CP} < 1$, the scheduled maintenance interval increases fastly.

Table 2: *Tempos de parada programada*

t_{m+3}	742
t_{m+4}	805
t_{m+5}	866
t_{m+6}	925
t_{m+7}	983

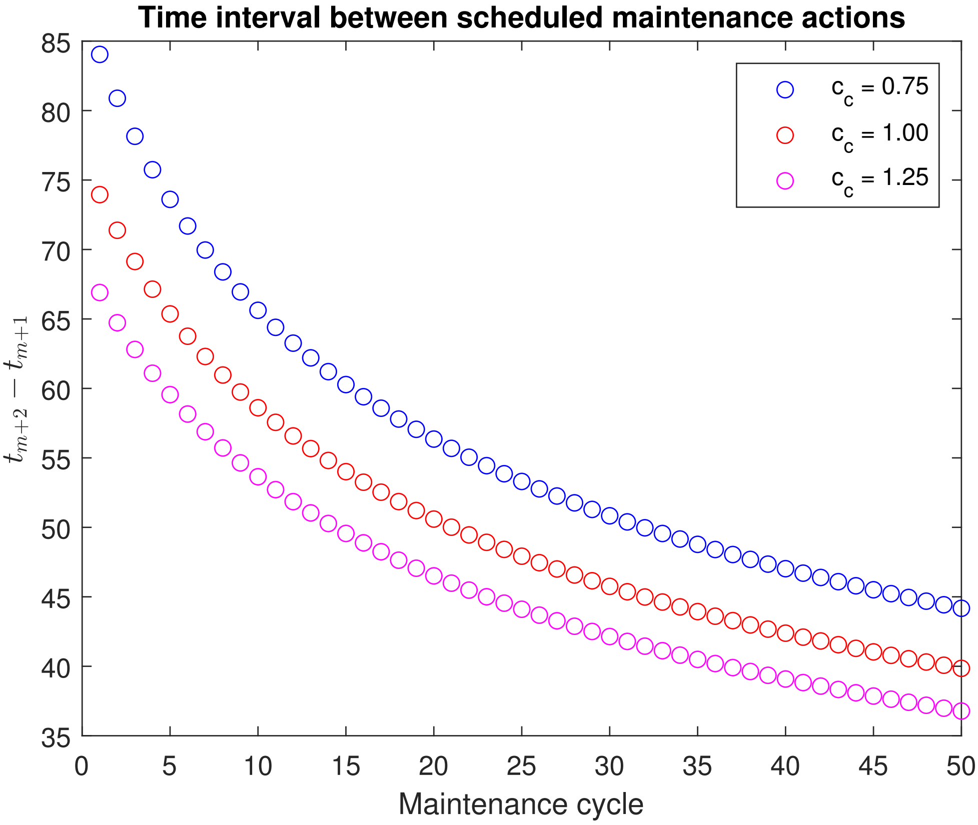

Figure 2: *Time period of the next maintenance actions*

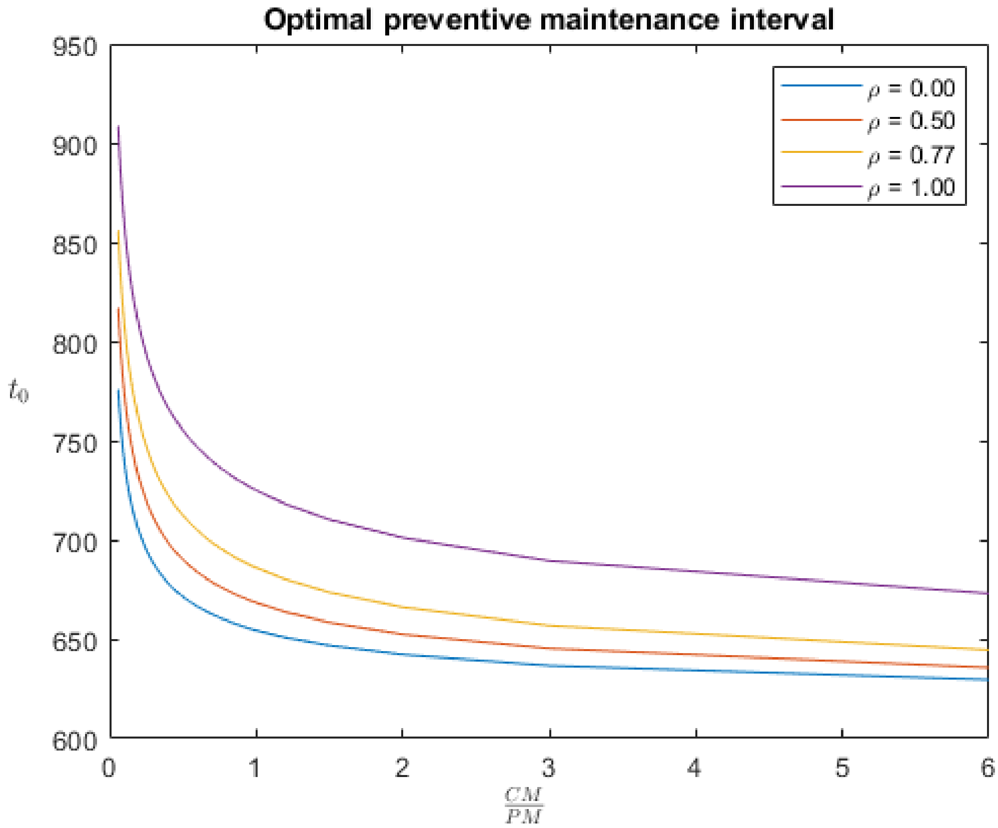

Figure 3: *Optimal preventive maintenance next interval as a function of the corrective repair cost*

In Figure 4 it is exhibited the hazard rate paths. In the left panel, it is shown the in-sample path. We can note an unorganized maintenance epochs in the original data which results in a high value for the hazard function before the third action. In the right panel it is shown the out of sample predicted hazard function. It is considered that the scheduled maintenance actions are performed at the optimal interval, that is, at each 58 days. It is noteworthy that, respecting the historical recovery factor of $\rho = 0.77$, in the 11-th action the hazard function finds in a lower level than it was observed in the third maintenance interval.

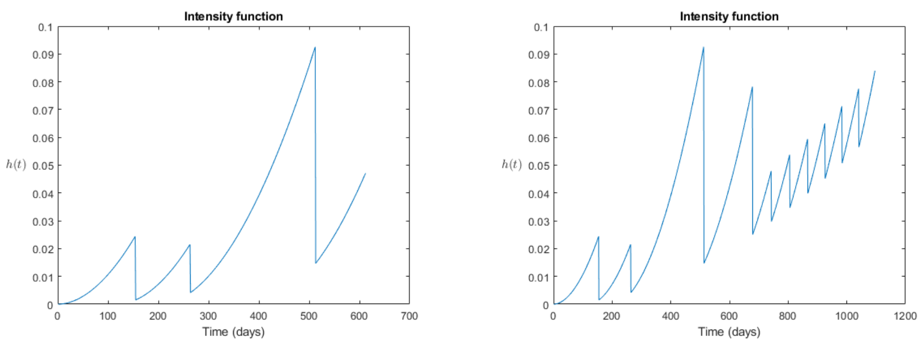

Figure 4: *In-sample and out of sample intensity function*

In Figure 5 we fix $c_m = 1$ and analyze the cost of the next maintenance cycle. We note that cost varies widely according to the recovery parameter ρ.

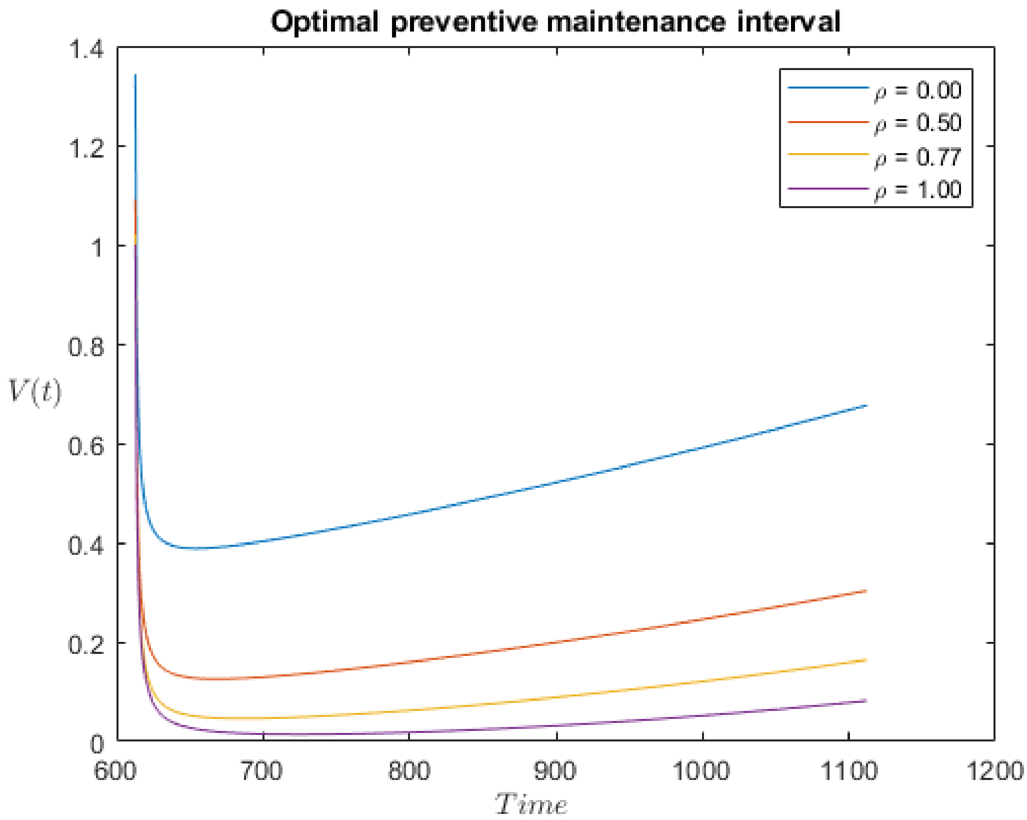

Figure 5: *Maintenance cost*

In Figure 6 it is exhibited the coupled in-sample and out of sample hazard function for different recovery parameters ρ after the observed period of 612 days. With $t_{m+2} = 678$, and t_{m+n} according to Table 2, we can compare the hazard function for $\rho = 0$ (minimal repairs), $\rho = 0.5$ (intermediary repairs), $\rho = 0.77$ (actual policy) and $\rho = 1$ (perfect repairs).

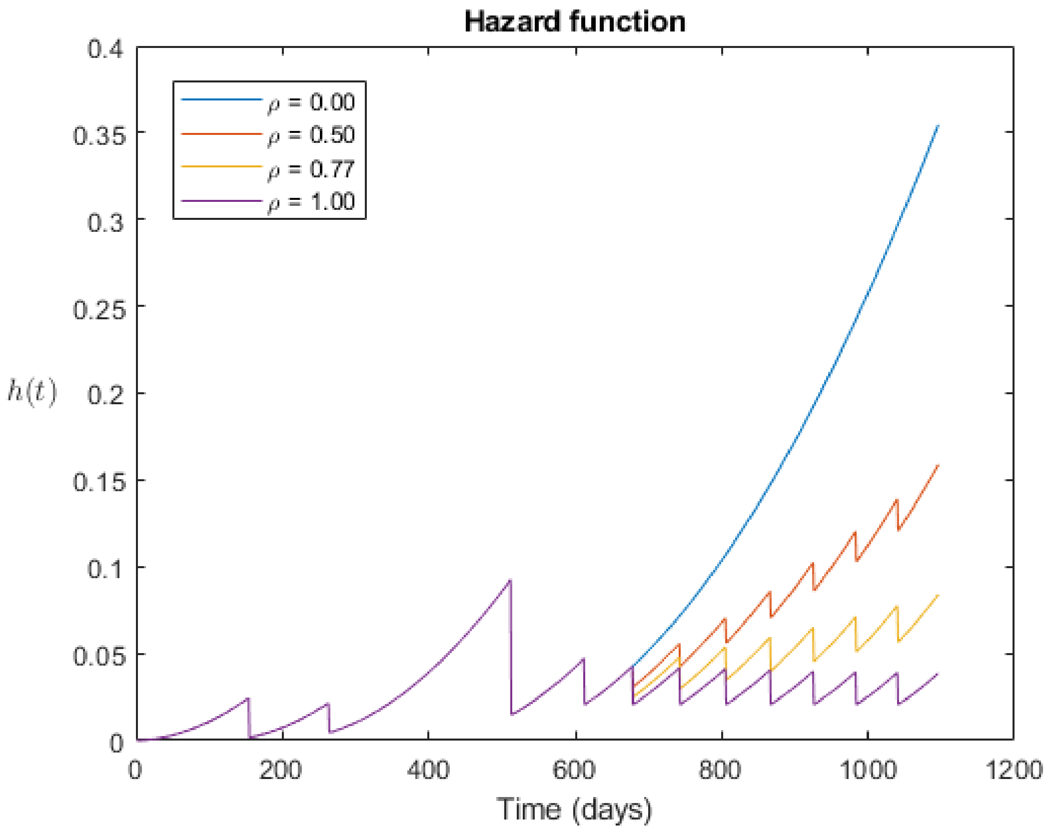

Figure 6: *Hazard function*

Finally,

5. Conclusion

A maintenance model under the assumption of imperfect preventive maintenance interspaced with minimal repairs was considered. This study analyzed a real failure database available in the literature consisting of 15 failure times and 3 unequally-spaced preventive maintenance actions. The model parameters of the power law process were estimated using the MLE and a Differential Evolution algorithm. The results obtained in this paper showed that maintenance cost varies widely according to the recovery parameter. It is also clear from the results here obtained that the optimal interval of two consecutive preventive maintenance actions strongly depends on the costs. Hence, a proper estimation of repairs expenditures are needed. The proposed cost model can be extended to consider random unscheduled repair costs.

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