A NONPARAMETRIC CONTROL CHART FOR JOINT MONITORING OF LOCATION AND SCALE

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Abstract

Traditional control charts are based on the assumption that the process observations are normally distributed. However, in many applications, there is insufficient information to justify this assumption. Thus, nonparametric control charts have been designed in literature to monitor location parameter and scale parameter of a process. In this paper, a single nonparametric control chart based on modified Lepage test is proposed for simultaneously monitoring of location and scale parameters of any continuous process distribution. The charting statistic combines two nonparametric test statistics namely Baumgartner test for location and Ansari-Bradely test for scale. The performance of the proposed chart is examined through simulation studies in terms of the mean, the standard deviation, the median and some percentiles of the run length distribution. The average run length (ARL) performance of the proposed chart is compared with that of the existing nonparametric Shewhart-Cucconi (SC) and Shewhart-Lepage (SL) charts for joint monitoring of location and scale.

Keywords: Control chart; average run length; joint monitoring; nonparametric tests; location parameter; scale parameter.

1. Introduction

Control charts are the most important statistical process control tool used to monitor manufacturing processes with the objective of detecting any change in process parameters that may affect the quality of the output. Shewhart $\bar{X}$ and Ror S control charts are most popular control charts for monitoring process mean and process variability. These control charts are easy to implement but are based on the fundamental assumption that the distribution of quality characteristic is normal. In real applications, there are many situations in which process data come from non-normal distribution. In such situations, it is desirable to use nonparametric control charts. The main advantage of nonparametric control chart is that it does not assume any probability distribution for the characteristic of interest. A formal definition of nonparametric or distribution-free control chart is given in terms of its run-length distribution. The number of samples that need to be collected before the first out-of-signal is given by a chart is a random variable called the run-length; the probability distribution of the run-length is referred to as run-length distribution. If the in-control run-length distribution is same for every continuous distribution then the chart is called as distribution-free or nonparametric (Chakraborti and Eryilmaz [1]). The location and scale of a process are the two main parameters most often monitored in nonparametric control charts. The problem of monitoring the location of a process is important in many applications. The location parameter could be the mean or the median or some percentiles of the distribution. Many authors have developed nonparametric control charts to monitor location parameter of the process some of these includes Bakir [2-3], Chakraborti and Eryilmaz [1], Khilare and Shirke [4], Human et al. [5]. These charts are based on sign and/or rank statistics. Chakraborti et al. [6] and Chakraborti and Graham [7] presented an extensive overview of literature on nonparametric control charts and discussed their advantages.

The problem of monitoring the scale parameter of a process is also important in many applications. For monitoring scale parametric of a process very few nonparametric are available in literature. Amin et al. [8] proposed a sign chart for process variation based on quartiles. Das [9] proposed a nonparametric control chart for controlling variability based on squared rank test. Das [10] developed a nonparametric control chart based on rank test. Das and Bhattacharya [11] proposed a control chart for controlling variability based on some nonparametric tests. Murakami and Matsuki [12] developed a nonparametric control chart based on Mood statistic for dispersion. Khilare and Shirke [13] developed a nonparametric synthetic control chart for process variability based on sign statistic. Zombade and Ghute [14] provided nonparametric control charts for process variation based on Sukhatme's test and Mood's test. Shirke and Barale [15] proposed a nonparametric cumulative sum control chart for process dispersion using in-control deciles.

The existing nonparametric control charts are designed for monitoring location and scale by using separate control charts. Using two separate charts for monitoring location and scale can sometimes be difficult in practice for the interpretation of signals because the effect of changes in one of the parameters can affect the changes in other one. The joint monitoring scheme with single chart has received more attention in the recent literature due to simplicity and clarity. A single control chart uses a statistic that is a combination of two separate statistics one each for mean and variance. Joint monitoring of a process involves two parameters, the mean (location) and variance (scale) and typically uses an efficient statistic for monitoring each parameter. The control charts currently available for jointly monitoring the mean and variance are focused on parametric control chart. Cheng and Thaga [16] provided a review of literature on joint monitoring of control charts up to 2005. McCracken and Chakraborti [17] presented an overview of literature on joint monitoring control charts. They also discussed some of the joint monitoring schemes for multivariate processes, autocorrelated data, and individual observations. Most of the parametric control charts for joint monitoring the mean and variability of a process are based on the assumption that process distribution is normal. However, in many applications there is not always enough knowledge or information to support the assumption that process distribution is of specific shape or form such as normal. In such cases nonparametric control charts can be useful. The literature in the area of nonparametric joint monitoring schemes is currently very limited. A few nonparametric joint monitoring schemes are available in the literature. Zou and Tsung [18] developed EWMA control chart based on goodness-of-fit test. It has been shown that the proposed chart is effective for detecting changes in location, scale and shape. Mukherjee and Chakraborti [19] developed a single distribution-free control chart for joint monitoring of location and scale. The chart is based on nonparametric test for location-scale by Lepage [20] which combines the Wilcoxon rank sum (WRS) location statistic and with Ansari-Bradely scale statistic. Chowdhury et al. [21] proposed distribution-free chart based on Cucconi statistic, for joint monitoring of location and scale parameters of continuous distribution. Nonparametric joint monitoring scheme is an important area for research and literature in this area is currently very limited and thus presents a great opportunity for further research. The purpose of this paper is to contribute the research on nonparametric joint monitoring scheme.

In this paper, a single nonparametric Shewhart-type control chart is developed for joint monitoring of location and scale parameters of a continuous process distribution. The proposed chart is based on nonparametric two sample modified Lepage-type test proposed by Neuhäuser [22]. The test combines the Baumgartner statistic and Ansari-Bradely statistic for jointly detecting location and scale changes. The in-control and out-of-control performance of the proposed control chart is evaluated through average run length for the normal and double exponential distributions. The rest of the paper is organized as follows. The nonparametric Baumgartner and Ansari-Bradely tests for location and scale respectively are modified Lepage-type test proposed by Neuhäuser [22] for joint location and scale is are discussed in Section 2. A single nonparametric control chart for simultaneously monitoring the location parameter and the scale parameter of a process based on modified Lepage-type test statistic is presented in Section 3. In-control and out-of-control performance of the proposed control chart is studied in detail in Section 4. Performance of the proposed control chart is compared with the existing nonparametric charts in Section 5. Some conclusions are given in Section 6.

2. Nonparametric Tests for Location and Scale

In this section, we briefly discuss the nonparametric tests for location parameter, scale parameter and jointly location scale parameters.

2.1 Baumgartner two sample test for location

Baumgartner test is a two-sample test can be applied for location and scale parameters. Let $(X_{1,}X_2,\dots,X_n)$ and $(Y_{1,}Y_2,\dots,Y_m)$ denote two random samples. The observations within each sample are independent and identically distributed, and we assume independence between two samples. Let F and G be continuous distribution functions corresponding two populations1 and2 respectively. In location shift, model considered first the distribution functions are same except perhaps for change in their location; that is$F(x) = F(x-\theta)$. The null hypothesis is $H_0{:}\,\theta = 0$, whereas alternative is $H_1{:}\,\theta \neq 0$. Baumgartner et al. [23] proposed a distribution-free two-sample rank test for general alternative. For combined samples, let $R_1 < R_2 < \dots < R_n$ and $H_1 < H_2 < \dots < H_m$ denote the ranks of the $X-$values and $Y-$values in increasing order of magnitude, respectively. Baumgartner et al. [23] defined a nonparametric two-sample rank statistic B as follows:

$$B = \frac{B_X+B_Y}{2} \tag{1}$$

where $B_X = \frac{1}{n}\sum_{i=1}^{n}\frac{\left(R_i - \frac{N}{n}i\right)^2}{\frac{i}{n+1}\left(1-\frac{i}{n+1}\right)\left(\frac{m\,N}{n}\right)}$ and $B_Y = \frac{1}{m}\sum_{j=1}^{m}\frac{\left(H_j - \frac{N}{m}j\right)^2}{\frac{j}{m+1}\left(1-\frac{j}{m+1}\right)\left(\frac{n\,N}{m}\right)}$

The larger value of statistic B gives evidence to reject the null hypothesis. Baumgartner et al. [23] also provided asymptotic distribution of test statistic B.

2.2 Ansari-Bradely test for scale

The Ansari-Bradely test is a two-sample rank test applied for scale parameter. The test statistic is defined as follows: In the combined samples, the observations less than or equal to the median are replaced by their ranks in the increasing order and those larger than the median are replaced by their ranks in descending order. The statistic is the sum of these ranks for the Y sample. The corresponding test statistic is defined as (Gibbons and Chakraborti [24]),

$$AB = \sum_{k=1}^{n}\left(k - \frac{N+1}{2}\right)Z_k \tag{2}$$

The mean and variance of statistic AB is given by,

$$E(AB) = \begin{cases}\frac{m\,(N+1)}{4}, & \text{when N is even}\\ \frac{m\,(N+1)^2}{4N}, & \text{when N is odd}\end{cases} \quad \text{and} \quad V(AB) = \begin{cases}\frac{m\,n\,(N^2-4)}{48\,(N-1)}, & \text{when N is even}\\ \frac{m\,n\,(N+1)\,(N^2+3)}{48\,N^2}, & \text{when N is odd}\end{cases}$$

2.3 Modified Lepage-type test for location and scale

After Lepage statistic was proposed, various Lepage-type statistics have been proposed and discussed by many authors in the literature. One of the most famous and powerful modified Lepage-type statistic proposed by Neuhäuser [22] is a combination of the Baumgartner and Ansari-Bradely statistic given as:

$$L_M = \left(\frac{B-E_0(B)}{\sqrt{Var_0(B)}}\right)^2 + \left(\frac{AB-E_0(AB)}{\sqrt{Var_0(AB)}}\right)^2 \tag{3}$$

where B is Baumgartner statistic for location shift and AB is Ansari-Bradely statistic for scale shift. In this paper, we useL_M test statistic as a charting statistic for detecting simultaneous location and scale shifts in a continuous process distribution.

3. Control chart based on modified Lepage-type statistic

In this Section, we develop a nonparametric control chart based on modified Lepage-type test statistic proposed by Neuhäuser [22] for simultaneously monitoring the location and the scale parameters of a continuous process. The single plotting statistic for the joint monitoring of location and scale is given by L_M in Eq. (3) and chart is called LM chart. To adopt the idea of two sample test for control chart implementation, m independent observations $(X_1, X_2, \ldots, X_m)$ from an in-control process are used as reference sample and compared to future sample subgroups of n independent observations $(Y_1, Y_2, \ldots, Y_n)$ an arbitrary test sample.

The proposed LM control chart for joint monitoring of location and scale is constructed as follows:

Step1: Collect Phase-I reference sample $X = (X_1, X_2, \ldots, X_m)$ of size$m$ from an in-control process.
Step2: Let $Y = (Y_1, Y_2, \ldots, Y_n)$ be j^{th} Phase-II (test) sample of size n , $j = 1, 2, 3, \ldots$
Step 3: Calculate B_j and$(AB)_j$ using (1) and (2) for j^{th} test sample.
Step 4: Compute means and standard deviations of B andAB statistics respectively
Step 5: Calculate the standardized B andABstatistics as

$$T_{1j} = \left(\frac{B - E_0(B)}{\sqrt{Var_0(B)}}\right) \text{ and } T_{2j} = \left(\frac{AB - E_0(AB)}{\sqrt{Var_0(AB)}}\right) \text{ respectively.}$$

Step 6: Calculate the control chart statistic LM chart as$T_j = T_{1j}^2 + T_{2j}^2$, $j = 1, 2, 3, \ldots$
Step 7: Plot T_j against an upper control limit$(UCL), H > 0$.
Step 8: If T_j exceed H, the process is out-of-control at the j^{th} test sample. If not, the process is thought to be in-control and testing continues to the next sample.

4. Performance evaluation and analysis of LM chart

Implementation of the proposed LM chart requires the upper control limit H. Typically, in practice, it is determined for some specified in-control average run length (ARL_0), say, 370 or 500. A Monte-Carlo simulation approach based on sufficiently large number of possible samples is used to determine H. For a given pair of (m, n) values, a search is conducted with different values of H, and that value of H is obtained for which $ARL_0$is equal to nominal (target) value. We choose $m = 30, 50, 100$ for the reference sample size and $n = 5, 11, 25$ as the test sample size and target values $ARL_0 = 200, 370, 500$. The results are presented in Table 1.

Table 1: *Charting constant H for the LM chart for some standard (target) values of ARL_0*

Reference sample size (m)	Test sample size (n)	Upper control limit (H)		
		$ARL_0 = 200$	$ARL_0 = 370$	$ARL_0 = 500$
30	5	29.540	35.242	37.960
30	11	25.050	33.128	37.312
30	25	16.985	22.089	24.820
50	5	14.510	19.510	22.390
50	11	15.389	18.712	20.752
50	25	15.798	19.123	20.910
100	5	20.020	29.050	32.800
100	11	20.740	27.490	31.305
100	25	18.540	24.450	28.023

The performance of a control chart is generally studied through its runlength distribution. If the runlength distribution is skewed to the right, it is useful to come across at various measures such as average run length (ARL), the standard deviation of run length (SDRL) and several percentiles including the first and third quartiles to characterize the distribution. We study the performance of the proposed LM chart both under in-control and out-of-control setup. For the in-control setup, we simulate both the reference and the test sample from standard normal distribution. We choose $m = 30, 50, 100$ and $n = 5, 11, 25$. For a given pair of (m, n) values, we obtain upper control limits H for nominal (target) $ARL_0 = 500$ and simulate different characteristics of the in-control run-length distribution. The results of simulation are shown in Table 2.

It indicates that the target $ARL_0 = 500$ is much larger than the median (Q_2) for all (m, n) combinations. Hence, in-control run-length distribution of the LM chart is highly skewed to the right.

In order to investigate the out-of-control performance of the proposed LM chart, we consider the underlying process distributions as normal and double exponential. The double exponential distribution is considered as process distribution to study the effect of heavy tailed distribution on the performance of the LM chart. The distribution of observations from the process is considered to have mean zero and variance one for both the process distributions under study.

Table 2: *In-control performance characteristics of the LM chart forARL*$_0$ $= 500$.

m	n	H	ARL_0	$SDRL_0$	P_5	Q_1	Q_2	Q_3	P_{95}
30	5	37.960	501.0	500.5	26	146	350	694	1484
30	11	37.312	499.7	499.2	27	145	346	692	1493
30	25	24.820	500.4	499.9	26	144	347	695	1481
50	5	22.390	499.5	499.0	27	143	344	691	1508
50	11	20.752	501.4	500.9	26	144	351	698	1499
50	25	20.910	501.3	500.8	26	143	348	692	1502
100	5	32.800	500.6	500.1	26	144	348	696	1506
100	11	31.305	500.1	499.6	26	145	345	694	1499
100	25	28.023	502.9	502.4	26	147	349	695	1508

4.1 Performance analysis of LM chart under normal distribution

In order to investigate the out-of-control performance of the proposed LM chart, we consider the underlying process distribution as normal; samples are taken from $N(\theta,\lambda)$ distribution, with in-control samples coming from $N(0,1)$distribution. To examine the effects of shifts in process parameters, 30 combinations of (θ,λ)values are considered with $\theta = 0, 0.25, 0.5, 1.0, 1.5, 2.0$ and $\lambda = 1.0, 1.25, 1.5, 1.75, 2.0$.

Tables 3 and 4 present the performance characteristics of the LM chart when underlying process distribution is normal with combinations of the reference and test sample sizes $m = 50, 100$ and $n = 5$.

The results in Table 3 and Table 4 indicate that the out-of-control run-length distributions are also skewed to right. It is observed that, for a fixed m, n and a given ARL_0, the out-of-control ARL values as well as the percentiles all decrease sharply with increasing shift in the location and also with the increasing shift in the scale. It indicates that the proposed LM chart is effective in detecting shifts in location and/or in the scale. The proposed LM chart detect shift in the scale more quickly than that in the location. For example, from Table 3, we observe that for 25% increase in location when scale is in-control, the ARL decreases by 68%, whereas for a 25% increase in a scale when the location is in-control, ARL decreases by 78%. Finally, when location and scale increases by 25% the ARL decreased by 88%. The pattern is same for SDRL; it decreases for an increase in the shift in both parameters, but decreases more for a shift in scale. For example, from Table 3, for 25% increase in location, the SDRL decreases by 68% but for 25% increase in scale, the SDRL decreases by 78%.

Table 3: *Performance characteristics of the LM chart for the normal distribution.*
$(ARL_0 = 500, m = 50$ *and* $n = 5)$.

θ	λ	ARL	SDRL	P_5	Q_1	Q_2	Q_3	P_{95}
0.0	1.0	499.5	499.0	27	143	344	691	1508
0.25	1.0	160.3	159.8	9	46	111	223	478
0.5	1.0	42.4	41.9	3	13	30	59	126
1.0	1.0	5.9	5.4	1	2	4	8	16
1.5	1.0	1.9	1.3	1	1	1	2	5
2.0	1.0	1.2	0.5	1	1	1	1	2
0.0	1.25	108.1	107.6	6	31	76	150	322
0.25	1.25	58.4	57.9	3	17	41	81	175
0.5	1.25	24.1	23.6	2	7	17	33	72
1.0	1.25	5.7	5.1	1	2	4	8	16
1.5	1.25	2.2	1.7	1	1	2	3	6
2.0	1.25	1.3	0.7	1	1	1	2	3
0.0	1.5	43.0	42.5	3	13	30	60	127

0.25	1.5	30.6	30.1	2	9	21	42	90
0.5	1.5	16.6	16.1	1	5	12	23	48
1.0	1.5	5.5	4.9	1	2	4	7	15
1.5	1.5	2.5	1.9	1	1	2	3	6
2.0	1.5	1.5	0.9	1	1	1	2	3
0.0	1.75	22.8	22.3	2	7	16	31	68
0.25	1.75	18.8	18.2	1	6	13	26	55
0.5	1.75	12.7	12.2	1	4	9	17	37
1.0	1.75	5.3	4.8	1	2	4	7	15
1.5	1.75	2.7	2.2	1	1	2	4	7
2.0	1.75	1.7	1.1	1	1	1	2	4
0.0	2.0	14.5	14.0	1	5	10	20	42
0.25	2.0	13.1	12.6	1	4	9	18	39
0.5	2.0	9.9	9.4	1	3	7	13	29
1.0	2.0	5.1	4.6	1	2	4	7	14
1.5	2.0	2.9	2.3	1	1	2	4	8
2.0	2.0	1.9	1.3	1	1	1	2	4

Table 4: *Performance characteristics of the LM chart for normal distribution.*
($ARL_0 = 500,\ m = 100\ and\ n = 5$).

θ	λ	ARL	SDRL	P_5	Q_1	Q_2	Q_3	P_{95}
0.0	1.0	500.6	500.1	26	144	348	696	1506
0.25	1.0	273.6	273.1	15	79	191	380	814
0.5	1.0	66.4	65.9	4	20	46	91	199
1.0	1.0	7.5	7.0	1	3	5	10	21
1.5	1.0	2.1	1.5	1	1	2	3	5
2.0	1.0	1.2	0.5	1	1	1	1	2
0.0	1.25	119.9	119.4	7	35	83	166	355
0.25	1.25	83.8	83.3	5	25	58	116	251
0.5	1.25	33.3	32.8	2	10	23	46	98
1.0	1.25	6.8	6.3	1	2	5	9	19
1.5	1.25	2.4	1.9	1	1	2	3	6
2.0	1.25	1.4	0.7	1	1	1	2	3
0.0	1.5	48.6	48.1	3	14	34	67	145
0.25	1.5	39.6	39.1	2	12	27	55	118
0.5	1.5	21.5	21.0	2	7	15	30	63
1.0	1.5	6.4	5.9	1	2	5	9	18
1.5	1.5	2.7	2.2	1	1	2	4	7
2.0	1.5	1.6	1.0	1	1	1	2	4
0.0	1.75	26.3	25.8	2	8	18	36	78
0.25	1.75	23.3	22.8	2	7	16	32	69
0.5	1.75	15.5	15.0	1	5	11	21	45
1.0	1.75	6.1	5.6	1	2	4	8	17
1.5	1.75	2.9	2.4	1	1	2	4	8
2.0	1.75	1.8	1.2	1	1	1	2	4
0.0	2.0	16.5	16.0	1	5	12	23	49
0.25	2.0	15.4	14.9	1	5	11	21	45
0.5	2.0	11.8	11.3	1	4	8	16	35
1.0	2.0	5.8	5.2	1	2	4	8	16

1.5	2.0	3.1	2.5	1	1	2	4	8
2.0	2.0	2.0	1.4	1	1	1	2	5

4.2 Performance analysis of LM chart under double exponential distribution

To study the effect of heavy tailed distribution on the performance of the proposed LM chart, double exponential distribution is included in the study as heavy tailed process distribution. We conduct simulation study with data from double exponential distribution. The performance characteristics of the run-length are evaluated when the in-control sample is from double exponential with mean 0 and variance 1, and test samples are generated from the double exponential distribution with mean θ and standard deviation λ.

To examine the effects of shifts in location and scale, as in normal case, we studied 30 combinations of (θ, λ)values. Table 5 and Table 6 presents the performance characteristics of the proposed LM chart when underlying process distribution is double exponential with combinations of reference and test samples of size $m = 50, 100$and $n = 5$.

Table 5: *Performance characteristics of LM chart for double exponential distribution.* $(ARL_0 \; = 500, m = 50 \; and \; n = 5)$.

θ	λ	ARL	SDRL	P_5	Q_1	Q_2	Q_3	P_{95}
0.0	1.0	499.6	499.1	25	143	345	693	1505
0.25	1.0	91.5	91.0	5	27	64	127	272
0.5	1.0	18.2	17.7	1	6	13	25	53
1.0	1.0	3.0	2.4	1	1	2	4	8
1.5	1.0	1.5	0.8	1	1	1	2	3
2.0	1.0	1.1	0.4	1	1	1	1	2
0.0	1.25	222.2	221.7	12	64	155	307	663
0.25	1.25	59.7	59.2	4	18	42	82	178
0.50	1.25	17.0	16.5	1	5	12	23	50
1.0	1.25	3.6	3.0	1	1	3	5	10
1.5	1.25	1.7	1.1	1	1	1	2	4
2.0	1.25	1.3	0.6	1	1	1	1	2
0.0	1.5	128.6	128.1	7	38	90	177	381
0.25	1.5	45.5	45.0	3	14	32	63	136
0.5	1.5	16.1	15.6	1	5	11	22	47
1.0	1.5	4.2	3.6	1	2	3	6	11
1.5	1.5	2.0	1.5	1	1	2	3	5
2.0	1.5	1.4	0.8	1	1	1	2	3
0.0	1.75	87.1	86.6	5	26	61	121	261
0.25	1.75	37.0	36.5	2	11	26	51	110
0.50	1.75	15.8	15.3	1	5	11	22	46
1.0	1.75	4.7	4.2	1	2	3	6	13
1.5	1.75	2.3	1.8	1	1	2	3	6
2.0	1.75	1.6	1.0	1	1	1	2	4
0.0	2.0	65.0	64.5	4	19	45	90	193
0.25	2.0	31.8	31.3	2	10	22	44	94
0.50	2.0	15.4	14.9	1	5	11	21	45
1.0	2.0	5.2	4.7	1	2	4	7	14
1.5	2.0	2.6	2.1	1	1	2	3	7
2.0	2.0	1.8	1.2	1	1	1	2	4

Table 6: *Performance characteristics of LM chart for double exponential distribution* $(ARL_0 \; = 500, \quad m = 100 \; and \; n = 5)$.

θ	λ	ARL	SDRL	P_5	Q_1	Q_2	Q_3	P_{95}
0.0	1.0	497.9	497.4	25	142	344	691	1502
0.25	1.0	789.9	789.4	42	225	546	1098	2376
0.5	1.0	194.6	194.1	10	56	134	269	583
1.0	1.0	10.5	10.0	1	3	7	14	30
1.5	1.0	2.2	1.6	1	1	2	3	5
2.0	1.0	1.2	0.5	1	1	1	1	2
0.0	1.25	179.6	179.1	10	52	125	249	536
0.25	1.25	246.2	245.7	13	71	170	341	739
0.5	1.25	91.5	91.0	5	27	64	126	272
1.0	1.25	9.6	9.1	1	3	7	13	27
1.5	1.25	2.5	2.0	1	1	2	3	6
2.0	1.25	1.4	0.7	1	1	1	2	3
0.0	1.5	91.1	90.6	5	26	64	127	272
0.25	1.5	111.4	110.9	6	32	77	154	332
0.5	1.5	55.5	55.0	3	16	38	77	164
1.0	1.5	9.3	8.7	1	3	7	13	27
1.5	1.5	2.8	2.3	1	1	2	4	7
2.0	1.5	1.5	0.9	1	1	1	2	3
0.0	1.75	54.4	53.9	3	16	38	75	162
0.25	1.75	63.3	62.8	4	19	44	88	188
0.5	1.75	37.8	37.3	2	11	26	52	112
1.0	1.75	8.7	8.2	1	3	6	12	25
1.5	1.75	3.1	2.5	1	1	2	4	8
2.0	1.75	1.7	1.1	1	1	1	2	4
0.0	2.0	36.7	36.2	2	11	26	51	109
0.25	2.0	41.2	40.7	3	12	29	57	123
0.5	2.0	27.9	27.4	2	8	20	39	83
1.0	2.0	8.3	7.8	1	3	6	11	24
1.5	2.0	3.3	2.7	1	1	2	4	9
2.0	2.0	1.8	1.2	1	1	1	2	4

From Tables 5 and 6, it is observed that when underlying process distribution is doubling exponential, the general pattern remains the same as in the case of normal distribution. However, the out-of-control ARL values for detecting a shift in the mean and/or variance under double exponential distribution are larger than that of the ARL values under normal process distribution. For example, from Table 6, mean shift is 50% ($\theta = 0.50$) and dispersion shift is 50% ($\lambda = 1.5$), The ARL is 194.6 which is larger than 66.4 in the normal case of Table 4. It indicates that the proposed LM chart detects shifts in process location and scale slower under heavy tailed distribution. Moreover, the percentiles as well as SDRL all increase under double exponential distribution as compared with normal distribution.

5. Performance comparison with existing control charts

In this section, the performance of the proposed LM chart is compared with that of the SL chart by Mukherjee and Chakraborti [19] and SC chart by Chowdhury et al. [21] when underlying process distributions are normal and double exponential. Table 7 presents the ARL performance of SC chart, SL chart and LM chart for normal distribution with reference sample size $m = 50, 100$ and test sample of size $n = 5$.

Table7: *Performance comparisons between SC, SL and LM charts for the normal distribution with $ARL_0 = 500$.*

θ	λ	$m = 50, n = 5$			$m = 100, n = 5$		
		SC chart	SL chart	LM chart	SC chart	SL chart	LM chart
0.0	1.0	497.3	499.6	499.5	509.4	513.0	500.6
0.5	1.0	92.2	94.7	42.4	68.6	66.5	66.4
1.0	1.0	8.5	9.3	5.9	7.7	7.7	7.5
1.5	1.0	2.2	2.3	1.9	2.1	2.1	2.1
2.0	1.0	1.2	1.3	1.2	1.2	1.2	1.2
0.0	1.25	71.1	106.2	108.1	74.5	102.9	119.9
0.5	1.25	27.6	35.4	24.1	26.2	30.9	33.3
1.0	1.25	6.6	7.4	5.7	6.2	6.7	6.8
1.5	1.25	2.4	2.6	2.2	2.4	2.5	2.4
2.0	1.25	1.4	1.4	1.3	1.3	1.4	1.4
0.0	1.5	22.8	36.82	43.0	24.3	37.5	48.6
0.5	1.5	13.3	19.0	16.6	13.4	17.8	21.5
1.0	1.5	5.2	6.5	5.5	5.3	6.1	6.4
1.5	1.5	2.4	2.8	2.5	2.4	2.7	2.7
2.0	1.5	1.5	1.6	1.5	1.5	1.6	1.6
0.0	1.75	10.9	18.5	22.8	11.7	19.1	26.3
0.50	1.75	8.1	12.1	12.7	8.4	12.1	15.5
1.0	1.75	4.4	5.7	5.3	4.4	5.5	6.1
1.5	1.75	2.5	2.9	2.7	2.4	2.8	2.9
2.0	1.75	1.6	1.8	1.7	1.6	1.8	1.8
0.0	2.0	6.6	11.3	14.5	7.1	11.5	16.5
0.5	2.0	5.5	8.5	9.9	5.8	8.6	11.8
1.0	2.0	3.7	4.9	5.1	3.8	4.8	5.8
1.5	2.0	2.4	2.9	2.9	2.4	2.9	3.1
2.0	2.0	1.7	1.9	1.9	1.7	1.9	2.0

Examination of Table 7 that for normal distribution leads the following findings:

- For location shifts only when the scale parameter is in-control, the proposed LM chart performs better than the SL and SC charts.
- For scale shifts only when the location parameter is in-control, the proposed LM chart is not as much better as the SL and SC charts.
- For reference sample of size $m = 50$, for any given shift in location parameterθ with a fixed shift in scale parameter as $\lambda = 1.25$, the proposed LM chart performs better than the SL and SC charts. As shift in scale parameter λincreases to 1.5 with any given shift in location parameter θ, the proposed LM chart is efficient than the SL chart only. For scale shift of size $\lambda = 1.25$ and location shift $\theta = 1.5$ and $2.0$the proposed LM chart is efficient than the SL chart only. For scale shift $\lambda = 2.0$ and location shift $\theta = 1.5$ and $2.0$the proposed LM chart is equally efficient to the SL chart only.

- For reference sample of size $m = 100$, for detecting shift in location parameter as $\theta = 1.5$ and $2.0$and shift in scale parameter $\lambda = 1.5$ and 1.5 the proposed LM chart is equally efficient to the SL chart only.

Table 8: *Performance comparison between the SC, SL and LM charts for the double exponential distribution with $ARL_0 = 500$.*

θ	λ	$m = 50, n = 5$			$m = 100, n = 5$		
		SC chart	SL chart	LM chart	SC chart	SL chart	LM chart
0.0	1.0	492.7	493.2	499.6	509.6	508.3	497.9
0.5	1.0	240.0	235.2	18.2	191.0	159.2	194.6
1.0	1.0	41.4	36.1	3.0	26.5	19.9	10.5
1.5	1.0	7.2	5.93	1.5	4.8	4.1	2.2
2.0	1.0	2.1	2.0	1.1	1.8	1.7	1.2
0.0	1.25	118.0	156.8	222.2	124.5	153.2	179.6
0.5	1.25	69.7	79.8	17.0	61.7	66.19	91.5
1.0	1.25	20.1	19.9	3.6	14.6	14.0	9.6
1.5	1.25	5.1	5.2	1.7	4.4	4.2	2.5
2.0	1.25	2.1	2.2	1.3	2.0	2.0	1.4
0.0	1.5	43.3	65.9	128.6	47.8	66.8	91.1
0.5	1.5	29.3	42.1	16.1	29.6	36.8	55.5
1.0	1.5	12.0	14.2	4.2	10.7	11.1	9.3
1.5	1.5	4.5	4.7	2.0	4.0	4.1	2.8
2.0	1.5	2.2	2.3	1.4	2.1	2.2	1.5
0.0	1.75	22.8	35.6	87.1	24.4	36.4	54.4
0.5	1.75	16.7	24.5	15.8	16.9	23.2	37.8
1.0	1.75	8.5	10.4	4.7	7.9	9.2	8.7
1.5	1.75	4.0	4.5	2.3	3.7	4.0	3.1
2.0	1.75	2.2	2.4	1.6	2.1	2.3	1.7
0.0	2.0	13.8	22.1	65.0	14.5	22.9	36.7
0.5	2.0	11.1	17.0	15.4	11.3	16.6	27.9
1.0	2.0	6.5	8.6	5.2	6.3	7.9	8.3
1.5	2.0	3.5	4.3	2.6	3.5	3.9	3.3
2.0	2.0	2.2	2.5	1.8	2.1	2.3	1.8

Examination of Table 8 that for double exponential distribution leads the following findings:

- For reference sample of size $m = 50$, for location shifts only when the scale parameter is in-control, the proposed LM chart performs better than the SL and SC charts. For scale shifts only when the location parameter is in-control, the proposed LM chart is not as better as the SL and SC charts. For any given shift in location parameter θ with any shift in scale parameter λ, the proposed LM chart performs better than the SL and SC charts.
- For reference sample of size $m = 100$, for detecting shift in location parameter as $\theta = 1.0$ and 2.0 and shift in scale parameter $\lambda = 1.25, 1.5$ and 1.75, the proposed LM chart is efficient than the to the SL and SC charts. For detecting shift in location parameter as $\theta = 1.0$ and 2.0 and shift in scale parameter $\lambda = 2.0$, the proposed LM chart is efficient than the to the SL and SC charts.

6. Conclusions

In this paper, a single nonparametric control chart based on modified Lepage-type test statistic is developed for joint monitoring of location and scale parameters of a continuous process distribution. Both in-control and out-of-control performance of the chart are studied under normal and heavy tailed double exponential distributions. The various performance characteristics such as mean, median and some percentiles of the run-length distribution are examined. It is observed that the proposed LM chart maintain its designed in-control ARL under the considered process distributions. The chart is found to be more efficient under normal distribution as compared to double exponential distribution. The performance of the proposed chart is compared with SL chart by Mukherjee and Chakraborti [19] and SC chart by Chowdhury [21]. It is observed that the proposed LM chart for joint monitoring of location and scale performs better than the SL and SC charts in some situations.

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