A DISCRETE PARAMETRIC MARKOV-CHAIN SYSTEM MODEL OF A TWO-UNIT STANDBY SYSTEM WITH TWO TYPES OF REPAIR

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Abstract

The aim of the present paper is to deal with the cost-benefit analysis of a two identical unit cold standby system model. There are two modes of a unit say Normal(N) and Total failure(F). When a unit operates then any of the two types of failure minor or major may occur some fixed known probabilities. Two repairmen are always available with the system to repair a unit failed with minor or major fault respectively. Upon failure of an operative unit the cold standby unit starts operations instantaneously with a perfect switching device. After minor and major repair of a failed unit it becomes as good as new. The distributions of failure times of minor and major faults and each type of repair time are assumed to follow geometric distributions with different parameters. Using regenerative point technique with the basic tools of probabilistic argument and Laplace Transform various important measures of system effectiveness useful to system designers and operations managers have been obtained.

Keywords: Cold standby, transition probabilities, mean sojourn time, regenerative pint, MTSF, geometric distribution.

1. Introduction

The stochastic models pertaining to two-unit standby redundant systems have been frequently analysed in the field of reliability theory due to their wide applicability in modern business and industrial units. The consideration of repair is one of the important criteria to enhance the system effectiveness such as reliability, expected life and availability of the system etc. Various authors during past many decades have analysed the two identical and non-identical unit standby system models by using different repair policies. Some of the authors have analysed models by assuming two types of failure say minor and major in an operating unit and so accordingly they have considered two types of repair with different repair time distributions.

Levitin et al. [7] studied a series-parallel reparable system model with two types of failure states-failure safe and failure dangerous. Ram and Singh [8] performed the stochastic analysis of a complex system model assuming that a unit can fail in n-mutually exclusive ways of total failure or common cause failure. Choudhary and Kumar [1] analysed a system model consisting of two units-one is main unit and other is supporting unit. The main unit passes through two types of failure-partial failure and total failure. A single repairman is always available with the system for the repair of each type of failure. Gupta and Vinodiya [6] have analysed a two non-identical unit cold standby system model assuming two types of failure in one of the unit. They have considered a single repairman for the repair of each type of failures. Most recently Chaudhary and Tyagi [2] analysed a two non-identical unit parallel system model assuming that one of the unit can fail either due to hardware or due to human error. All the above system models are based on the continuous parametric Markov-chain.

The purpose of the present study is to analyse a stochastic model based on discrete parametric Markov-chain system composed of two identical units in cold standby configuration. Two types of failure (minor and major) have been considered in an operating unit. Two different repairmen are always available with the system. One is considered to attend a failed unit due to minor fault and other is to attend a failed unit due to major fault. Some authors including [3-5] infact analysed the system models by taking geometric distributions of failure and repair times but not much work is done in this direction. By using regenerative point technique the following economic related measures of system effective are obtained -

- Transition probabilities and mean sojourn times.
- Reliability and mean time to system failure.
- Point-wise and steady-state availabilities of the system and expected up time of the system by the epoch (t-1).
- Expected busy period of the repairman by the epoch (t-1).
- Net expected profit incurred by the system by the epoch (t-1) and in steady state.

2. Model Description and Assumptions

The system under study is based on the following assumptions

i. The system is composed of two identical units. Initially one unit is operative and other is kept as cold standby. The cold standby never lose its operational ability in its standby state.

ii. Each unit has two modes: Normal (N) and Total failure (F).

iii. Two types of failure minor and major may occur in an operating unit with respective probabilities a and b. (a+b=1).

iv. Two different repairmen are always available with the system. One is ordinary repairman for minor fault and other is skilled repairman for major fault in an operating unit.

v. Upon either type of failure in an operating unit, the standby unit starts working immediately with a perfect and instantaneous switching device.

vi. After each type of repair, a unit becomes as good as new.

vii. The time to failure and each type of repair time follow geometric distribution with different parameters.

3. Notations and States of the System

3.1 Notations used in the paper :

pq^x : p.m.f. of failure time of a unit $(p+q=1)$.

$r_i s_i^x$: p.m.f. of repair time of minor and major types respectively for i=1 and 2.

a, b : probability that the failed unit requires minor and major repairs. $(a+b=1)$.

$q_{ij}(\cdot), Q_{ij}(\cdot)$: p.m.f. and c.d.f. of one step or direct transition time from state S_i to S_j.

p_{ij} : Steady state transition probability from state S_i to S_j.

$$p_{ij} = Q_{ij}(\infty)$$

$Z_i(t)$: Probability that the system sojourn in state S_i up to the cycles 0,1,2,……..t-1.

ψ_i : Mean sojourn time in state S_i.

$*, h$: Symbol and dummy variable used in geometric transform e. g.

$$GT\left[q_{ij}(t)\right] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$$

© : Symbol of ordinary convolution i.e.,

$$A(t)©B(t) = \int_0^t A(u)B(t-u)du$$

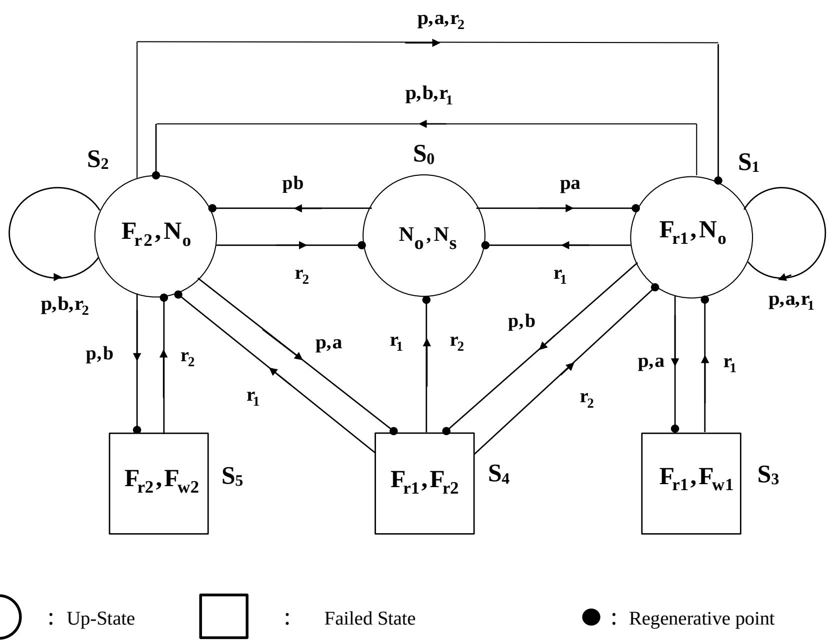

Figure.1 *Transition Diagram*

3.2 Symbols used for the states of the system:

N_o , N_s : Unit in normal mode and operative/cold standby.

F_{r1}, F_{w1} : Unit in total failure mode and under minor repair/waiting for minor repair.

F_{r2}, F_{w2} : Unit in total failure mode and under major repair/waiting for major repair.

With the help of above symbols and keeping into consideration the stated assumptions, the possible states of the system are as follows-

Up States: $S_0 \equiv (N_0, N_s)$, $S_1 \equiv (F_{r1}, N_o)$, $S_2 \equiv (F_{r2}, N_o)$

Failed States: $S_3 \equiv (F_{r1}, F_{w1})$, $S_4 \equiv (F_{r1}, F_{r2})$, $S_5 \equiv (F_{r2}, F_{w2})$

The transition diagram of the system model alongwith the transition rates is shown in fig.1. We observe that all the entrance epochs into the states are regenerative epochs.

4. Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By considering the elementary probabilistic arguments we have,

$$Q_{01}(t) = \sum_{u=0}^{t} apq^u = a\left(1-q^{t+1}\right) \tag{1}$$

Similarly,

$$Q_{02}(t) = \sum_{u=0}^{t} bpq^u = b\left(1-q^{t+1}\right) \tag{2}$$

$$Q_{10}(t) = \frac{r_1 q}{(1-qs_1)}\left[1-(qs_1)^{t+1}\right] \tag{3}$$

$$Q_{11}(t) = \frac{apr_1}{(1-qs_1)}\left[1-(qs_1)^{t+1}\right] \tag{4}$$

$$Q_{12}(t) = \frac{bpr_1}{(1-qs_1)}\left[1-(qs_1)^{t+1}\right] \tag{5}$$

$$Q_{13}(t) = \frac{aps_1}{(1-qs_1)}\left[1-(qs_1)^{t+1}\right] \tag{6}$$

$$Q_{15}(t) = \frac{bps_1}{(1-qs_1)}\left[1-(qs_1)^{t+1}\right] \tag{7}$$

$$Q_{20}(t) = \frac{r_2 q}{(1-qs_2)}\left[1-(qs_2)^{t+1}\right] \tag{8}$$

$$Q_{21}(t) = \frac{apr_2}{(1-qs_2)}\left[1-(qs_2)^{t+1}\right] \tag{9}$$

$$Q_{22}(t) = \frac{bpr_2}{(1-qs_2)}\left[1-(qs_2)^{t+1}\right] \tag{10}$$

$$Q_{24}(t) = \frac{bps_2}{(1-qs_2)}\left[1-(qs_2)^{t+1}\right] \tag{11}$$

$$Q_{25}(t) = \frac{aps_2}{(1-qs_2)}\left[1-(qs_2)^{t+1}\right] \tag{12}$$

$$Q_{31}(t) = \left[1-(s_1)^{t+1}\right] \tag{13}$$

$$Q_{42}(t)=\left[1-(s_2)^{t+1}\right] \tag{14}$$

$$Q_{50}(t)=\frac{r_1 r_2}{(1-s_1 s_2)}\left[1-(s_1 s_2)^{t+1}\right] \tag{15}$$

$$Q_{51}(t)=\frac{r_2 s_1}{(1-s_1 s_2)}\left[1-(s_1 s_2)^{t+1}\right] \tag{16}$$

$$Q_{52}(t)=\frac{r_1 s_2}{(1-s_1 s_2)}\left[1-(s_1 s_2)^{t+1}\right] \tag{17}$$

From the transient-state transition probabilities (1-17), the steady state t.p.m can be obtained as follows

$$\left(\left(p_{ij}\right)\right)=\begin{pmatrix} p_{00} & p_{01} & p_{02} & p_{03} & p_{04} & p_{05} \\ p_{10} & p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{20} & p_{21} & p_{22} & p_{23} & p_{24} & p_{25} \\ p_{30} & p_{31} & p_{32} & p_{33} & p_{34} & p_{35} \\ p_{40} & p_{41} & p_{42} & p_{43} & p_{44} & p_{45} \\ p_{50} & p_{51} & p_{53} & p_{54} & p_{55} & p_{56} \end{pmatrix}$$

$$p_{01}=\lim_{t\to\infty} Q_{01}(t)=a$$

Similary,

$p_{02}=b$, $p_{10}=\frac{r_1 q}{(1-qs_1)}$, $p_{11}=\frac{apr_1}{(1-qs_1)}$, $p_{12}=\frac{bpr_1}{(1-qs_1)}$

$p_{13}=\frac{aps_1}{(1-qs_1)}$, $p_{15}=\frac{bps_1}{(1-qs_1)}$, $p_{20}=\frac{r_2 q}{(1-qs_2)}$, $p_{21}=\frac{apr_2}{(1-qs_2)}$

$p_{22}=\frac{bpr_2}{(1-qs_2)}$, $p_{24}=\frac{bps_2}{(1-qs_2)}$, $p_{25}=\frac{aps_2}{(1-qs_2)}$, $p_{31}=1$

$p_{42}=1$, $p_{50}=\frac{r_1 r_2}{(1-s_1 s_2)}$, $p_{51}=\frac{r_2 s_1}{(1-s_1 s_2)}$, $p_{52}=\frac{r_1 s_2}{(1-s_1 s_2)}$

We observe that the following relations hold-

$$p_{01}+p_{02}=1 \tag{18}$$

$$p_{10}+p_{11}+p_{12}+p_{13}+p_{15}=1 \tag{19}$$

$$p_{20}+p_{21}+p_{22}+p_{24}+p_{25}=1 \tag{20}$$

$$p_{31}=p_{42}=1 \tag{21}$$

$$p_{50}+p_{51}+p_{52}=1 \tag{22}$$

5. Mean Sojourn Times

Let T_i be the sojourn time in state S_i (i=0,1,2,3,4,5,6) then mean sojourn time ψ_i in state S_i is given by

$$\psi_i=\sum_{t=1}^{\infty} P\left[T_i > t-1\right]=\sum_{t=1}^{\infty} P\left[T_i \ge t\right]$$

In particular,

$$\psi_0=\frac{q}{p} \tag{23}$$

$$\psi_1=\frac{qs_1}{(1-qs_1)} \tag{24}$$

$$\psi_2 = \frac{qs_2}{(1-qs_2)} \tag{25}$$

$$\psi_3 = \frac{s_1}{r_1} \tag{26}$$

$$\psi_4 = \frac{s_2}{r_2} \tag{27}$$

$$\psi_5 = \frac{s_1 s_2}{(1-s_1 s_2)} \tag{28}$$

6. Methodology For Developing Equations

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows-

6.1 Reliability of the system-

Let us define $R_i(t)$ as the probability that the system does not fail up to the epochs (t-1) when it is initially starts from state S_i. To determine it, we assume the failed states S_3, S_4 and S_5 as absorbing state. Using the simple probabilistic reasoning in regenerative point technique we have the following set of convolution equations $R_i(t); i = 0,1,2$.

$$\begin{aligned} R_0(t) &= q^t + \sum_{u=0}^{t-1} q_{01}(u) R_1(t-1-u) + \sum_{u=0}^{t-1} q_{02}(u) R_2(t-1-u) \\ &= Z_0(t) + q_{01}(t-1) © R_1(t-1) + q_{02}(t-1) © R_2(t-1) \end{aligned} \tag{29}$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t-1) © R_0(t-1) + q_{11}(t-1) © R_1(t-1) + q_{12}(t-1) © R_2(t-1) \tag{30}$$

$$R_2(t) = Z_2(t) + q_{20}(t-1) © R_0(t-1) + q_{21}(t-1) © R_1(t-1) + q_{22}(t-1) © R_2(t-1) \tag{31}$$

Where,

$Z_0(t) = q^t$, $Z_1(t) = q^t s_1^t$ and $Z_2(t) = q^t s_2^t$

6.2 Availability of the system-

Let $A_i(t)$ be the probability that the system is up (operative) during the t[th] cycle (t-1, t), when it initially started from state S_i. Using elementary probabilistic arguments as in case of reliability, we have the following recurrence relations-

$$A_0(t) = Z_0(t) + q_{01}(t-1) © A_1(t-1) + q_{02}(t-1) © A_2(t-1) \tag{32}$$

$$\begin{aligned} A_1(t) = Z_1(t) &+ q_{10}(t-1) © A_0(t-1) + q_{11}(t-1) © A_1(t-1) + q_{12}(t-1) © A_2(t-1) \\ &+ q_{13}(t-1) © A_3(t-1) + q_{15}(t-1) © A_5(t-1) \end{aligned} \tag{33}$$

$$\begin{aligned} A_2(t) = Z_2(t) &+ q_{20}(t-1) © A_0(t-1) + q_{21}(t-1) © A_1(t-1) + q_{22}(t-1) © A_2(t-1) \\ &+ q_{24}(t-1) © A_4(t-1) + q_{25}(t-1) © A_5(t-1) \end{aligned} \tag{34}$$

$$A_3(t) = q_{31}(t-1) © A_1(t-1) \tag{35}$$

$$A_4(t) = q_{42}(t-1) © A_2(t-1) \tag{36}$$

$$A_5(t) = q_{50}(t-1)©A_0(t-1) + q_{51}(t-1)©A_1(t-1) + q_{52}(t-1)©A_2(t-1) \quad (37)$$

Where, the values of $Z_i(t)$; i=0,1,2 are same as given in section 6.1.

6.3 Busy period of Repairman-

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the repairman is busy in the minor and major repair of a failed unit during the t[th] cycle (t-1, t), when it initially started from state S_i. Then, by using simple probabilistic arguments as in case of reliability, the following recurrence relations can be easily developed for $B_i^j(t)$; i= 0 to 5. The dichotomous variable δ takes value 1 and 0 respectively for j=1 and 2.

$$B_0^j(t) = q_{01}(t-1)©B_1^j(t-1) + q_{02}(t-1)©B_2^j(t-1) \quad (38)$$

$$B_1^j(t) = \delta Z_1(t) + q_{10}(t-1)©B_0^j(t-1) + q_{11}(t-1)©B_1^j(t-1) + q_{12}(t-1)©B_2^j(t-1) + q_{13}(t-1)©B_3^j(t-1) + q_{15}(t-1)©B_5^j(t-1) \quad (39)$$

$$B_2^j(t-1) = (1-\delta)Z_2(t) + q_{20}(t-1)©B_0^j(t-1) + q_{21}(t-1)©B_1^j(t-1) + q_{22}(t-1)©B_2^j(t-1) + q_{24}(t-1)©B_4^j(t-1) + q_{25}(t-1)©B_5^j(t-1) \quad (40)$$

$$B_3^j(t) = \delta Z_3(t) + q_{31}(t-1)©B_1^j(t-1) \quad (41)$$

$$B_4^j(t) = Z_4(t) + q_{42}(t-1)©B_2^j(t-1) \quad (42)$$

$$B_5^j(t-1) = (1-\delta)Z_5(t) + q_{50}(t-1)©B_0^j(t-1) + q_{51}(t-1)©B_1^j(t-1) + q_{52}(t-1)©B_2^j(t-1) \quad (43)$$

Where, $Z_3(t) = s_1^t$, $Z_4(t) = s_2^t$ and $Z_5(t) = s_1^t s_2^t$.

7. Analysis of Reliability and MTSF

Taking geometric transform of (29-31) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h) = \frac{N_1(h)}{D_1(h)} \quad (44)$$

Where,

$$N_1(h) = \left[\left(1-hq_{11}^*\right)\left(1-hq_{22}^*\right) - h^2 q_{12}^* q_{21}^*\right] Z_0^* + \left[hq_{01}^*\left(1-hq_{22}^*\right) + h^2 q_{02}^* q_{21}^*\right] Z_1^* + \left[h^2 q_{01}^* q_{12}^* + hq_{02}^*\left(1-hq_{11}^*\right)\right] Z_2^*$$

and

$$D_1(h) = \left(1-hq_{11}^*\right)\left(1-hq_{22}^*\right) - h^2 q_{12}^* q_{21}^* - hq_{10}^*\left[hq_{01}^*\left(1-hq_{22}^*\right) + h^2 q_{02}^* q_{21}^*\right] - hq_{20}^*\left[h^2 q_{01}^* q_{12}^* + hq_{02}^*\left(1-hq_{11}^*\right)\right]$$

Taking the inverse geometric transform of (44). one can get the expression of reliability of the system.

The MTSF is given by

$$E(T) = \sum_{t=1}^{\infty} R(t) = \lim_{h \to 1} \sum_{t=1}^{\infty} h^t R(t)$$

$$= \lim_{h \to 1} R^*(h) - R(0) = \frac{N_1(1)}{D_1(1)} - 1 \quad (45)$$

Where observing $q_{ij}^{*}(1)=p_{ij}$ and using results (23-28), we have

$$N_1(1)=\left[(1-p_{11})(1-p_{22})-p_{12}p_{21}\right](1+\psi_0)+\left[p_{01}(1-p_{22})+p_{02}p_{21}\right](1+\psi_1)$$
$$+\left[p_{01}p_{12}+p_{02}(1-p_{11})\right](1+\psi_2)$$

and

$$D_1(1)=(1-p_{11})(1-p_{22})-p_{12}p_{21}-p_{10}\left[p_{01}(1-p_{22})+p_{02}p_{21}\right]-p_{20}\left[p_{01}p_{12}+p_{02}(1-p_{11})\right]$$

8. Availability Analysis

On taking geometric transform of (32-37) and simplifying the resulting equations for $A_0^{*}(h)$, we get

$$A_0^{*}(h)=\frac{N_2(h)}{D_2(h)} \tag{46}$$

Where,

$$N_2(h)=\Big[\left(1-hq_{11}^{*}-h^2q_{13}^{*}q_{31}^{*}\right)\left\{1-hq_{22}^{*}-h^2q_{24}^{*}q_{42}^{*}-h^2q_{25}^{*}q_{52}^{*}\right\}$$
$$-hq_{21}^{*}\left\{hq_{12}^{*}+h^2q_{15}^{*}q_{51}^{*}\right\}-hq_{15}^{*}\left\{h^2q_{12}^{*}q_{25}^{*}+hq_{51}^{*}\left(1-hq_{22}^{*}-h^2q_{24}^{*}q_{42}^{*}\right)\right\}\Big]Z_0^{*}$$
$$+\Big[hq_{01}^{*}\left\{1-hq_{22}^{*}-h^2q_{24}^{*}q_{42}^{*}-h^2q_{25}^{*}q_{52}^{*}\right\}$$
$$+hq_{02}^{*}\left\{hq_{21}^{*}+h^2q_{25}^{*}q_{51}^{*}\right\}\Big]Z_1^{*}+\Big[hq_{01}^{*}\left\{hq_{12}^{*}+h^2q_{15}^{*}q_{52}^{*}\right\}$$
$$+hq_{02}^{*}\left\{1-hq_{11}^{*}-h^2q_{13}^{*}q_{31}^{*}-h^2q_{15}^{*}q_{51}^{*}\right\}\Big]Z_2^{*}$$

and

$$D_2(h)=\Big[\left(1-hq_{11}^{*}-h^2q_{13}^{*}q_{31}^{*}\right)\left\{1-hq_{22}^{*}-h^2q_{24}^{*}q_{42}^{*}-h^2q_{25}^{*}q_{52}^{*}\right\}$$
$$-hq_{21}^{*}\left\{hq_{12}^{*}+h^2q_{15}^{*}q_{51}^{*}\right\}-hq_{15}^{*}\left\{h^2q_{12}^{*}q_{25}^{*}+hq_{51}^{*}\left(1-hq_{22}^{*}-h^2q_{24}^{*}q_{42}^{*}\right)\right\}\Big]$$
$$-hq_{10}^{*}\Big[hq_{01}^{*}\left\{1-hq_{22}^{*}-h^2q_{24}^{*}q_{42}^{*}-h^2q_{25}^{*}q_{52}^{*}\right\}+hq_{02}^{*}\left\{hq_{21}^{*}+h^2q_{25}^{*}hq_{51}^{*}\right\}\Big]$$
$$-hq_{20}^{*}\Big[hq_{01}^{*}\left\{hq_{12}^{*}+h^2q_{15}^{*}q_{52}^{*}\right\}+hq_{02}^{*}\left\{1-hq_{11}^{*}-h^2q_{13}^{*}q_{31}^{*}-h^2q_{15}^{*}q_{51}^{*}\right\}\Big]$$
$$-hq_{50}^{*}\Big[hq_{01}^{*}\left\{h^2q_{12}^{*}q_{25}^{*}+hq_{15}^{*}\left(1-hq_{22}^{*}-h^2q_{24}^{*}q_{42}^{*}\right)\right\}$$
$$+hq_{02}^{*}\left\{hq_{25}^{*}\left(1-hq_{11}^{*}-h^2q_{13}^{*}q_{31}^{*}\right)+h^2q_{21}^{*}q_{15}^{*}\right\}\Big]$$

The steady-state availability of the system is given by

$$A_0=\lim_{t\to\infty}A_0(t)=\lim_{h\to 1}(1-h)\frac{N_2(h)}{D_2(h)}$$

Observing $q_{ij}^{*}(1)=p_{ij}$ and using relations (18-22), we get $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$A_0=-\frac{N_2(1)}{D_2'(1)}$$

Where,

$$N_2(1)=u_0(1+\psi_0)+u_1(1+\psi_1)+u_2(1+\psi_2)$$
$$u_0=(1-p_{11}-p_{13})(1-p_{22}-p_{24}-p_{25}p_{52})-p_{21}(p_{12}+p_{15}p_{51})-p_{15}\left\{p_{12}p_{25}+p_{51}(1-p_{22}-p_{24})\right\}$$
$$u_1=p_{01}(1-p_{22}-p_{24}-p_{25}p_{52})+p_{02}(p_{21}+p_{25}p_{51})$$

$$u_2 = p_{01}(p_{12}+p_{15}p_{52})+p_{02}(1-p_{11}-p_{13}-p_{15}p_{51})$$

and

$$D_2'(1) = -\left[u_0(1+\psi_0)+u_1\{(1+\psi_1)+p_{13}(1+\psi_3)\}+u_2\{(1+\psi_2)+p_{24}(1+\psi_4)\}+u_5(1+\psi_5)\right]$$

Where

$$u_5 = p_{01}\{p_{12}p_{25}+p_{51}(1-p_{22}-p_{24})+p_{02}p_{25}(1-p_{11}-p_{13})+p_{02}p_{21}p_{15}\}$$

Now the expected-up time of the system by the epoch (t-1) is given by

$$\mu_{up}(t) = \sum_{x=0}^{t-1} A_0(x)$$

So that,

$$\mu_{up}^*(h) = \frac{A_0^*(h)}{(1-h)}$$

9. Busy Period Analysis of Repairman

On taking geometric transforms of relation (38-43) and simplifying the resulting equations for minor and major repair i.e., for $\delta = 0,1$ we get,

$$B_0^{1*}(h) = \frac{N_3(h)}{D_2(h)} \quad \text{and} \quad B_0^{2*}(h) = \frac{N_4(h)}{D_2(h)}$$

Where,

$$N_3(h) = \left[hq_{01}^*\{1-hq_{22}^*-h^2q_{24}^*q_{42}^*-h^2q_{25}^*q_{52}^*\}+hq_{02}^*\{hq_{21}^*+h^2q_{25}^*q_{51}^*\}\right](Z_1^*+hq_{13}^*Z_3^*)$$
$$+\Big[hq_{01}^*\{h^2q_{12}^*q_{25}^*+hq_{15}^*(1-hq_{22}^*-h^2q_{24}^*q_{42}^*)\}$$
$$+h^2q_{02}^*q_{25}^*(1-hq_{11}^*-h^2q_{13}^*q_{31}^*)+h^3q_{02}^*q_{21}^*q_{15}^*\Big]Z_5^*$$

$$N_4(h) = \left[hq_{02}^*\{hq_{12}^*+h^2q_{15}^*q_{52}^*\}+hq_{02}^*\{1-hq_{11}^*-h^2q_{13}^*q_{31}^*-h^2q_{15}^*q_{51}^*\}\right](Z_2^*+hq_{24}^*Z_4^*)$$
$$+\Big[hq_{01}^*\{h^2q_{12}^*q_{25}^*+hq_{15}^*(1-hq_{22}^*-h^2q_{24}^*q_{42}^*)\}$$
$$+h^2q_{02}^*q_{25}^*(1-hq_{11}^*-h^2q_{13}^*q_{31}^*)+h^3q_{02}^*q_{21}^*q_{15}^*\Big]Z_5^*$$

and $D_2(h)$ is same as in availability analysis.

In the long run the respective probabilities that the repairman is busy in the minor and major repair of a failed unit are respectively given by-

$$B_0^1 = \lim_{t\to\infty} B_0^1(t) = \lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)}$$

$$B_0^2 = \lim_{t\to\infty} B_0^2(t) = \lim_{h\to 1}(1-h)\frac{N_4(h)}{D_2(h)}$$

But $D_2(h) = 0$ at h=1, therefore by applying L. Hospital rule, we get

$$B_0^1 = -\frac{N_3(1)}{D_2'(1)}, \quad B_0^2 = -\frac{N_4(1)}{D_2'(1)} \tag{47}$$

Where,

$$N_3(1) = u_1\{(1+\psi_1)+p_{13}(1+\psi_3)\}+u_5(1+\psi_5)$$

$$N_4(1) = u_2(\psi_2+p_{24}\psi_4)+u_5\psi_5$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period duration of the repairman in minor repair and major repair of a failed unit by the epoch (t-1) are respectively given by -

$$\mu_b^1(t)=\sum_{x=0}^{t-1}B_0^1(x),\qquad \mu_b^2(t)=\sum_{x=0}^{t-1}B_0^2(x)$$

So that,

$$\mu_b^{1*}(h)=\frac{B_0^{1*}(h)}{(1-h)},\qquad \mu_b^{2*}(h)=\frac{B_0^{2*}(h)}{(1-h)} \tag{48}$$

10. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred by the epoch (t-1) by considering the characteristics obtained in earlier section.

Let us consider,

K_0 = revenue per-unit time by the system due to its operation.

K_1 = cost per-unit time when repairman is busy in the minor repair

K_2 = cost per-unit time when repairman is busy in the minor repair

Then, the net expected profit incurred by the epoch (t-1) is given by

$$P(t)=K_0\mu_{up}(t)-K_1\mu_b^1(t)-K_2\mu_b^2(t) \tag{49}$$

The expected profit per-unit time in steady state is given by

$$P=\lim_{t\to\infty}\frac{P(t)}{t}=\lim_{h\to1}(1-h)^2P^*(h)$$

$$=K_0\lim_{h\to1}(1-h)^2\frac{A_0^*(h)}{(1-h)}-K_1\lim_{h\to1}(1-h)^2\frac{B_0^{1*}(h)}{(1-h)}-K_2\lim_{h\to1}(1-h)^2\frac{B_0^{2*}(h)}{(1-h)}$$

$$=K_0A_0-K_1B_0^1-K_2B_0^2 \tag{50}$$

11. Graphical Representation and conclusions

The curves for MTSF and profit function have been drawn for different values of failure parameters. Fig. 2 depicts the variation in MTSF with respect to failure rate (p) for different values of repair rate r_1 and r_2 when values of other parameters are kept fixed as $a=0.8$. From the curves we conclude that expected life of the system decrease with increase in p. Further, it increases with the increase of the values of r_1 and r_2. Also to achieve at least MTSF at 475 units, we conclude from smooth curves that the value of p must be less than 0.044, 0.050, 0.059 for $r_1=0.6,0.75,0.95$ where $r_2=0.25$. Whereas from dotted curves we conclude that the value of p must be less than 0.041, 0.046, 0.053 for $r_1=0.6,0.75,0.95$ when $r_2=0.20$.

Similarly, Fig. 3 reveals the variations in profit (P) with respect to p for varying values of r_1 and r_2, when other parameters are kept fixed as $a=0.01$, $K_0=175$, $K_1=195$ and $K_2=180$. From the figure, it is clearly observed from the smooth curves, that the system is profitable if the value of parameter p is less than 0.038, 0.056 and 0.077 respectively for $r_1=0.6,0.75,0.95$ when $r_2=0.24$. From dotted curves, we conclude that system is profitable only if value of parameter p is less than 0.029, 0.044 and 0.062 respectively for $r_1=0.6,0.75,0.95$ when $r_2=0.22$.

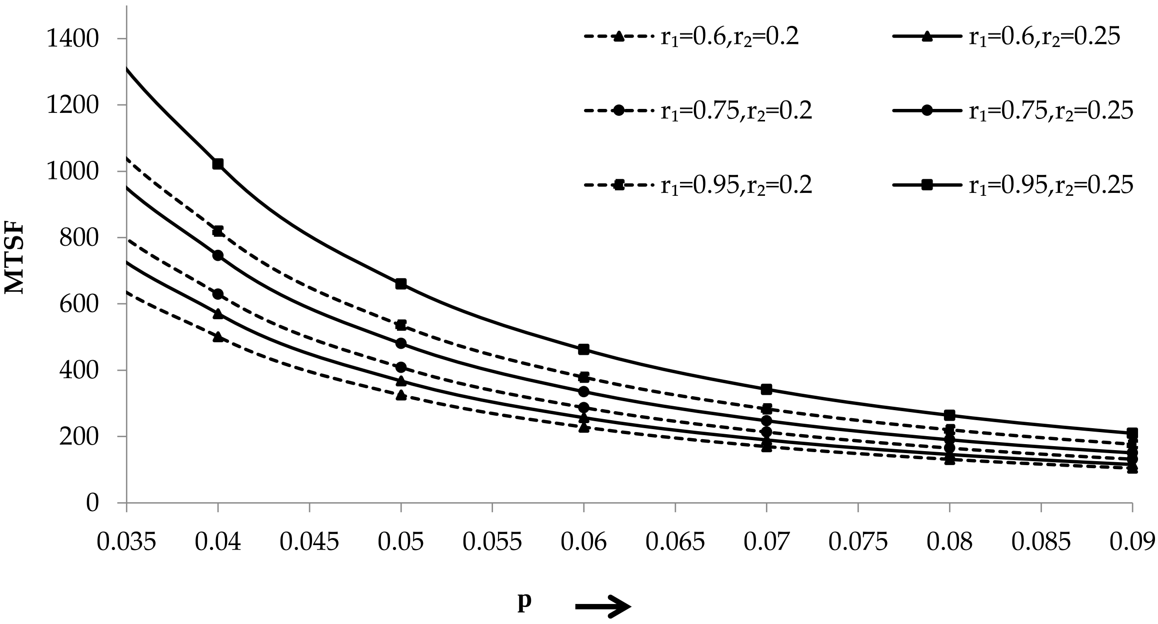

Figure.2 *Behavior of MTSF with respect to* p, r_1 *and* r_2

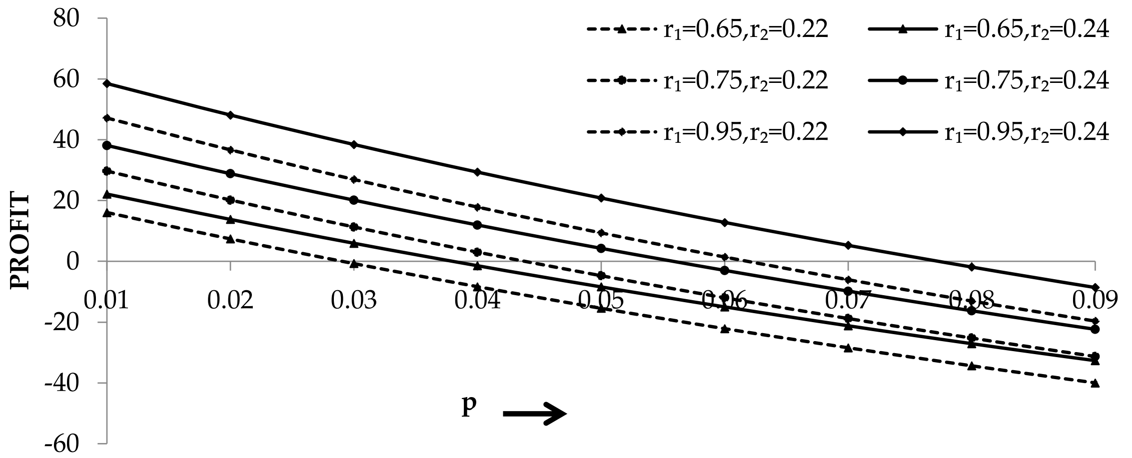

Figure.3 *Behavior of Profit (P) with respect to* p, r_1 *and* r_2

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