STOCHASTIC ANALYSIS OF DISCRETE PARAMETRIC MARKOV CHAIN SYSTEM MODEL

Manoj Kumar¹, Shiv Kumar²

¹D.A.V. College, Muzaffarnagar, U.P., (India) ²J.V. College Baraut (Baghpat), U.P., (India) <u>manojdu26@gmail.com</u> <u>s.shiv@gmail.com</u>

Abstract

The present paper deals with the behavior of the parallel model system of two non-identical units, warm standby models have been developed by in view of all random variables are independent. Initially priority unit is working and the non-priority unit is warm standby. Two repairmen are always available with the system to carry out the system operation as soon as possible, skilled repairmen carry out phase-1 repair while ordinary repairmen carry out a phase-2 repair. The main unit is take two phases for his repair while the repair of the ordinary unit is completed in one phase. The statistical measures of the model are analyzed probabilistically by applying the regenerative point technique the distribution of failure and repair time of the system taken as a geometric distribution with different parameters.

Keywords: Geometric distribution, Steady state transition probability, MTSF, Availability, Busy period, and Cost-benefit analysis.

1. Introduction

The configuration of the stochastic model is very complex with the development of modern system models, minimizing the high maintenance cost and increasing the system efficiency by reducing the frequency of failures. The design and model of industrial systems such as communication systems, satellite systems, power plant systems mechanical engineering, aeronautical engineering software engineering, and gaming systems are more complex to design in the current scenario. Using the different probabilistic measures of a two-unit system model with various kinds of repair policy deals with the system model involving various general human failures. Kumar and Kadyan [1] analyzed a non-identical parallel unit system with a single repairman visit whenever the original unit requires a repair facility, to repair the original unit with immediate effect and the duplicate unit is replaced by a similar new one. The various reliability characteristic such as study state availability, MTSF, and busy period and profit analysis of the system model are estimated by applying the semi-Markov approach. Sureria at el. [2] analyzed a computer system model whenever a system failed, priority is given to software replacement against hardware repair purpose to determine a mean sojourn time, reliability, availability, and busy period of a computer system of two similar units, initially one is active and the other is kept into cold standby whenever operative unite is failed, the cold unit is operative. The failure rate of the computer system is independent having an exponential distribution with different parameters while the repair and replacement rates distribution are taken as common. Each unit has hardware and software components that may have independent complete failure from the normal mode.

There is always a possibility that any system model during its operative condition to failure condition by two or more kinds of failure with single repair, post-repair, or waits for repair facility has been analyzed under some common assumption. Various other reliability characteristics models have been discussed of two identical or non-identical system models applying various types of repair facilities. Using discrete distribution, Bhatti at el. [3] introducing the concept of inspection to detect the major or minor failure, the repairman perform dual role of inspection and repair of the system after detect the type of failure of dissimilar operative cold standby systems. Ahmed at el. [4] studies a two non-identical parallel cold standby redundant unit system models each unit has two possible mode normal (N) and total failure (F). A repairman is always available to repair the system whenever it's required for preventive maintenance, priority to repair the failed unit is by given initially operative unit after the repair of a unit works as well as new. The one parametric geometric distribution with different random variables is taken for failure and repair rate of the each unit. Malik [5] studied a repairable system under different weather conditions. Singh at el. [6] applying a probabilistic assessment of parallel system with correlated lifetime under different inspection method. Kumar at el. [7] analyzed a redundant system with priority and weibull distribution for failure and repair rate. Kumar at el. [8] introduce a repairable system of non-identical units with priority and conditional failure of repairman.

2. Methods System description and assumptions

The aim of the present paper deals with priority (unit-I) and non-priority (unit-II) parallel unit systems, each unit has two achievable modes normal (N) and total failure (F), in the beginning one unit is operative and another unit is reserved in warm standby. Two repairmen are always available with the system to repair the failed unit. A master repairman carries out the phase-I repair while an assistant repairman is present to take out the phase-II repair. Initially, the failure unit-I goes to phase-I repair while completing phase-I repair it enters into phase-II for its final repair by the assistant repairman, and the repair of a non-priority unit is completed in one phase (phase-I) repaired by the master repairman. The operation priority is given to unit-I and repair priority is first come first serve (FCFS) bases. All the random variables are independent and uncorrelated under this study. The distributions of failure and repair times are taken as a discrete nature having a geometric distribution with different parameters. The system model is derived using the Markov-chain approach and using the regenerative point technique for various probabilistic analyses of the system effect such as mean sojourn time, reliability, availability, mean time to system failure (MTSF), a busy period in the different repair facility and cost-benefit function have been derived. The system consists of the following assumptions:

- The system consists of priority and non-priority units, and they are connected in parallel. Initially, one unit is operative (unit-I) and the other is kept on warm standby (non-priority unit-II).
- Both units have two possible modes, normal (*N*) when the unit is operative and total failure (*F*) when the unit is in failure mode.
- Two repairmen are always with the system to carry out the repair facility, the repair of unit-I is completed in two phases while the repair of unit-II is completed in one phase. The master repairman perform phase-I repair while the assistant repairman perform phase-II repair.
- The priority unit failed than non-priority unit is loading warm standby unit into operation using switching device to be perfect, the repair of priority unit is completed in two phases (phase-I and phase-II) i.e., a failed unit first enters in phase-I for its repair and after the completion of phase-I repair it enters phase-II for finishing repair, and the repair of a non-priority unit is done in one phase (phase-I). After repair of a unit is work as well as a new one.

- The system transition rate from state S_i to S_j is independent having a one parametric discrete geometric distribution.
- The repair priority is first come first serve bases while the operation priority is given to unit-I.

3. Notations and states of the system

N_0^1/N_0^2	The unit-I/ unit-II is in normal-mode and operative.
$\frac{F_r^1/F_r^2}{F_w^1/F_w^2}$	The unit-I/unit-II is in failure-mode and under repair by master repairman. The unit-I/unit-II is in failure-mode and waiting for repair.
F_R^1 N_s^2	The unit-I is in F-mode and under repair by assistant repairman. The unit-II is in normal-mode and kept into standby.
pq ^k /rs ^k ab ^k /cd ^k mn ^k	Probability mass function of failure rate of unit-I/unit-II. Probability mass function of repair rate of unit-I in phase-I/phase-II. Probability mass function of repair rate of unit-II.
q_{ij}, Q_{ij}	Probability mass function and cumulative density function of one step transition time from state $S_i toS_j$.
\mathbf{p}_{ij}	Steady state transition probability from state S _i to S _j .
Ψ _i Z _i (k)	Mean sojourn time in regenerative state S _i . Probability that the system is operational, initially in state sojourns S _i up to time k.
h,*	Dummy variable used in geometric transformation and sign.
U	Symbol for ordinary convolution.



Figure 1: Transitions Probability Diagram

Operative States $S_0 = (N_0^1, N_s^2); S_1 = (F_r^1, N_0^2); S_2 = (F_R^1, N_0^2) \text{ and } S_5 = (N_0^1, F_r^2)$ Failed States $S_3 = (F_r^1, F_w^2); S_4 = (F_R^1, F_r^2) \text{ and } S_6 = (F_w^1, F_r^2)$

4. Transition probabilities and mean sojourn times

Using simple probabilistic arguments that the system transits from state S_i to S_j within time interval (0, k), then $Q_{ij}(k)$ maybe obtain the following approach:

$$\begin{array}{ll} Q_{ij}(k) = [K_{n+1} = j, K_{n+1} - K_n < k | K_n = i] \\ Q_{01}(k) = 1 - q^{k+1} & Q_{12}(k) = \frac{as[1 - (bs)^{k+1}]}{1 - bs} \\ Q_{13}(k) = \frac{rb[1 - (bs)^{k+1}]}{1 - bs} & Q_{14}(k) = \frac{ar[1 - (bs)^{k+1}]}{1 - bs} \\ Q_{20}(k) = \frac{cs[1 - (ds)^{k+1}]}{1 - ds} & Q_{24}(k) = \frac{rd[1 - (sd)^{k+1}]}{1 - sd} \\ Q_{25}(k) = \frac{rc[1 - (sd)^{k+1}]}{1 - dn} & Q_{34}(k) = 1 - b^{k+1} \\ Q_{40}(k) = \frac{cm[1 - (dn)^{k+1}]}{1 - dn} & Q_{42}(k) = \frac{md[1 - (dn)^{k+1}]}{1 - dn} \\ Q_{45}(k) = \frac{cn[1 - (nq)^{k+1}]}{1 - nq} & Q_{50}(k) = \frac{mn[1 - (nq)^{k+1}]}{1 - nq} \\ Q_{51}(k) = 1 - n^{k+1} \end{array}$$

Similarly, using $p_{ij} = \lim_{k \to \infty} Q_{ij}(k)$, the steady state transition probability is:

$$p_{01} = p_{34} = p_{61} = 1, \ p_{12} = \frac{as}{1 - bs}, \ p_{13} = \frac{rb}{1 - bs}, \ p_{14} = \frac{ar}{1 - bs}, \ p_{20} = \frac{cs}{1 - ds}, \ p_{24} = \frac{rd}{1 - ds}, \ p_{25} = \frac{rc}{1 - ds}, \ p_{40} = \frac{cm}{1 - dn}, \ p_{42} = \frac{md}{1 - dn}, \ p_{45} = \frac{cn}{1 - dn}, \ p_{50} = \frac{mq}{1 - nq}, \ p_{51} = \frac{mp}{1 - nq} \ and \ p_{56} = \frac{pn}{1 - nq}$$

We can easily verify that $p_{12} + p_{13} + p_{14} = 1$, $p_{20} + p_{24} + p_{25} = 1$, $p_{40} + p_{42} + p_{45} = 1$ and $p_{50} + p_{51} + p_{56} = 1$

5. Mean sojourn time

The expected time a system spends in one state before moving onto another state is known as the mean sojourn time Ψ_i in state S_i ; *i*=0,1,2,3,4,5,6 is defined as:

$$\begin{split} \Psi_{i} &= E[K_{i}] = \sum_{k=1}^{n} P[K_{i} \geq K] \\ \text{So that} \\ \Psi_{0} &= \frac{p}{q}, \Psi_{1} = \frac{bs}{1-bs}, \Psi_{2} = \frac{ds}{1-ds}, \Psi_{3} = \frac{b}{a}, \Psi_{4} = \frac{dn}{1-dn}, \Psi_{5} = \frac{qn}{1-qn} \text{ and } \Psi_{6} = \frac{n}{m} \end{split}$$

6. Reliability of the system and mean time to system failure (MTSF)

The system originally starts operational from state $S_i \in E$. Then the system reliability, $R_i(k)$; i = 0, 1, 2, 5; have the following set of convolution equations is given by:

 $\overline{R_0(k) = q^k + \sum_{u=0}^{k-1} q_{01}(u) \mathbb{O}R_1(k-1-u)}$ $R_0(k) = Z_0(k) + q_{01}(k-1) \odot R_1(k-1)$ Similarly, $R_1(k) = Z_1(k) + q_{12}(k-1) \otimes R_2(k-1)$ $R_{2}(k) = Z_{2}(k) + q_{20}(k-1) \otimes R_{0}(k-1) + q_{25}(k-1) \otimes R_{5}(k-1)$ $R_5(k) = Z_5(k) + q_{50}(k-1) \odot R_0(k-1)$ where, $Z_1(k) = b^k s^k$; $Z_2(k) = d^k s^k$ and $Z_5(k) = q^k n^k$ Using the geometric transformation of the above set of equations, get the algebraic solutions for $R_0^*(h)$. We get $R_0^*(h) = \frac{N_1(h)}{D_1(h)}$ where, $N_{1}(h) = Z_{0}^{*}(h) + hq_{01}^{*}Z_{1}^{*}(h) + h^{2}q_{01}^{*}q_{12}^{*}Z_{2}^{*}(h) + h^{3}q_{01}^{*}q_{12}^{*}q_{25}^{*}Z_{5}^{*}(h)$ $D_1(h) = 1 - h^3 q_{01}^* q_{12}^* q_{20}^* - h^4 q_{01}^* q_{12}^* q_{25}^* q_{50}^*$ The MTSF is given by: $E(K_0) = \lim_{s \to 0} R_0^s(h) = \frac{N_1(0)}{D_1(0)}$ To determine $N_1(0)$ and $D_1(0)$, we apply the results $Z_{i}^{*}(0) = \Psi_{i}$ and $q_{ij}(0) = p_{ij}$ We get, MTSF = $\frac{\Psi_0 + \Psi_1 + p_{12}\Psi_2 + p_{12}p_{25}\Psi_5}{1 - p_{12}p_{20} - p_{12}p_{25}p_{50}}$

7. Availability analyses

Let $A_i(k)$; *i*=0,1,2,3,4,5,6 be the probability that the system will be normal at epoch time *k*, when at the system start function from state $S_i \in E$. We observe the following recurrence relations can be easily developed for $A_i(k)$, using similar probabilistic arguments:

$$\begin{split} A_{0}(k) &= q^{k} + \sum_{u=0}^{k-1} q_{01}(u) \odot A_{1}(k-1-u) \\ A_{0}(t) &= Z_{0}(k) + q_{01}(k-1) \odot A_{1}(k-1) \\ \text{Similarly,} \\ A_{1}(k) &= Z_{1}(k) + q_{12}(k-1) \odot A_{2}(k-1) + q_{13}(k-1) \odot A_{3}(k-1) + q_{14}(k-1) \odot A_{4}(k-1) \\ A_{2}(k) &= Z_{2}(k) + q_{20}(k-1) \odot A_{0}(k-1) + q_{24}(k-1) \odot A_{4}(k-1) + q_{25}(k-1) \odot A_{5}(k-1) \\ A_{3}(k) &= q_{34}(k-1) \odot A_{4}(k-1) \\ A_{4}(k) &= q_{40}(k-1) \odot A_{0}(k-1) + q_{42}(k-1) \odot A_{2}(k-1) + q_{45}(k-1) \odot A_{5}(k-1) \\ A_{5}(k) &= Z_{5}(k) + q_{50}(k-1) \odot A_{0}(k-1) + q_{51}(k-1) \odot A_{1}(k-1) + q_{56}(k-1) \odot A_{6}(k-1) \\ A_{6}(k) &= q_{61}(k-1) \odot A_{1}(k-1)(17-23) \\ \end{split}$$

 $Z_1(k)$; $Z_2(k)$; and $Z_5(k)$ same as in reliability.

After solving the set of algebraic equations that emerge from applying geometric transforms to the equations above, we have

 $A_0^*(s) = \frac{N_2(s)}{D_2(s)}$

where,

$$\begin{split} N_2(s) &= \{1 - q_{24}^* q_{42}^* - (q_{51}^* + q_{56}^* q_{61}^*) [q_{45}^* (q_{13}^* q_{34}^* + q_{14}^*) + q_{25}^* q_{42}^* (q_{13}^* q_{34}^* + q_{14}^*) + q_{12}^* q_{25}^* + q_{12}^* q_{42}^* q_{45}^*] \} Z_0^* - (q_{01}^* q_{24}^* q_{42}^* - q_{01}^*) Z_1^* + [q_{01}^* q_{12}^* + q_{01}^* q_{42}^* (q_{13}^* q_{34}^* + q_{14}^*)] Z_2^* + [q_{01}^* q_{12}^* (q_{24}^* q_{45}^* + q_{25}^*) + q_{01}^* q_{45}^* (q_{13}^* q_{34}^* + q_{14}^*) + q_{01}^* q_{25}^* q_{42}^* (q_{13}^* q_{34}^* + q_{14}^*)] Z_2^* + [q_{01}^* q_{12}^* (q_{24}^* q_{45}^* + q_{25}^*) + q_{01}^* q_{45}^* (q_{13}^* q_{34}^* + q_{14}^*) + q_{01}^* q_{25}^* q_{42}^* (q_{13}^* q_{34}^* + q_{14}^*)] Z_5^* \\ \text{and} \\ D_2(s) &= 1 - q_{01}^* q_{12}^* (q_{20}^* + q_{24}^* q_{40}^* + q_{24}^* q_{45}^* q_{50}^* + q_{25}^* q_{50}^*) - q_{01}^* (q_{13}^* q_{34}^* + q_{14}^*) (q_{20}^* q_{42}^* + q_{25}^* q_{42}^* q_{50}^* + q_{40}^* + q_{45}^* q_{50}^*) - q_{12}^* (q_{51}^* + q_{56}^* q_{61}^*) (q_{24}^* q_{45}^* + q_{25}^*) - (q_{13}^* q_{34}^* + q_{14}^*) (q_{51}^* + q_{56}^* q_{61}^*) (q_{25}^* q_{42}^* + q_{45}^*) - q_{13}^* q_{13}^* q_{14}^* + q_{14}^*) q_{15}^* q_{16}^* q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*) - q_{13}^* q_{13}^* q_{14}^* + q_{14}^*) q_{16}^* q_{16}^* q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*) - q_{13}^* q_{14}^* q_{14}^*] q_{16}^* q_{16}^* q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*) - q_{12}^* q_{16}^* q_{16}^* q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*) - q_{12}^* q_{16}^* q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*) - q_{16}^* q_{16}^* q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*) - q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*) - q_{16}^*] (q_{25}^* q_{42}^* + q_{45}^*$$

$q_{24}^*q_{42}^*$

Now, the steady state availability i.e. the probability that the system will be active in long run is known as:

$$\begin{split} A_0 &= \lim_{k \to \infty} A_0(k) = \lim_{s \to 0} A_0^*(s) = \lim_{s \to 0} \frac{N_2(s)}{D_2(s)} \\ \text{Since, } D_2(0) &= 0, \text{ therefore by applying L-Hospital rule;} \\ A_0 &= \lim_{s \to 0} \frac{N_2(s)}{D_2(s)} = \frac{N_2(0)}{D_2(0)} \\ \text{where,} \\ N_2(0) &= [1 - p_{24}p_{42} - (p_{13} + p_{14})(p_{51} + p_{56})(p_{45} + p_{25}p_{42}) - p_{12}(p_{51} + p_{56})(p_{25} + p_{24}p_{45})]\Psi_0 + \\ (1 - p_{24}p_{42})\Psi_1 + [p_{12} + p_{42}(p_{13} + p_{14})]\Psi_2 + [p_{12}(p_{24}p_{45} + p_{25}) + (p_{13} + p_{14})(p_{25}p_{42} + p_{45})]\Psi_5 \\ \text{and} \\ D_2'(0) &= [(p_{13} + p_{14})(p_{20}p_{42} + p_{25}p_{42}p_{50} + p_{40} + p_{45}p_{50}) + p_{12}(p_{20} + p_{25}p_{50}) + p_{12}p_{24}(p_{40} + p_{45}p_{50})]\Psi_0 + [p_{42}(p_{20} + p_{25}) + p_{40} + p_{45}]\Psi_1 + [p_{12} + p_{42}(p_{13} + p_{14})]\Psi_2 + [p_{13}p_{42}(p_{20} + p_{25}) + p_{13}(p_{40} + p_{45})]\Psi_3 + (p_{12}p_{24} + p_{13} + p_{14})\Psi_4 + [(p_{13} + p_{14})(p_{25}p_{42} + p_{45}) + p_{12}(p_{24}p_{45} + p_{25})]\Psi_6 \end{split}$$

8. Busy period for master repairman

Let $B_i^r(k)$; *i*=0,1,2,3,4,5,6 be the probability that the master repairman is busy repairing the failed unit in phase-I at epoch time *k* when the system operational from the state $S_i \in E$. Now for $B_0^r(k)$, we have the sum of the probabilities of the following contingencies:

$$\begin{split} B_0^r(k) &= \sum_{u=0}^{k-1} q_{01}(u) \odot B_1^k(k-1-u) \\ B_0^r(k) &= q_{01}(k-1) \odot B_1^r(k-1) \\ \text{Similarly,} \\ B_1^r(k) &= Z_1^r(k) + q_{12}(k-1) \odot B_2^r(k-1) + q_{13}(k-1) \odot B_3^r(k-1) + q_{14}(k-1) \odot B_4^r(k-1) \\ B_2^r(k) &= q_{20}(k-1) \odot B_0^r(k-1) + q_{24}(k-1) \odot B_4^r(k-1) + q_{25}(k-1) \odot B_5^r(k-1) \\ B_3^r(k) &= Z_3^r(k) + q_{34}(k-1) \odot B_4^r(k-1) \\ B_3^r(k) &= Z_4^r(k) + q_{40}(k-1) \odot B_0^r(k-1) + q_{42}(k-1) \odot B_2^r(k-1) + q_{45}(k-1) \odot B_5^r(k-1) \\ B_5^r(k) &= Z_6^r(k) + q_{50}(k-1) \odot B_0^r(k-1) + q_{51}(k-1) \odot B_1^r(k-1) + q_{56}(k-1) \odot B_6^r(k-1) \\ B_5^r(k) &= Z_6^r(k) + q_{61}(k-1) \odot B_1^r(k-1) \\ \text{where,} \\ Z_1^r(k) &= b^k s^k; \ Z_3^r(k) &= b^k; \ Z_4^r(k) &= d^k n^k; \ Z_5^r(k) &= q^k n^k \text{ and } Z_6^r(k) &= n^k \\ \text{Using the inverse Laplace transform of } B_0^{r*}(s), \text{ we get:} \\ B_0^{r*} &= \lim_{s\to 0} \frac{N_3(s)}{D_2(s)} \\ \text{here,} \\ D_2(0) &= 0 \\ \text{Therefore, by L-hospital rule, we have} \\ B_0^{r*} &= \lim_{s\to 0} \frac{N_3(s)}{D_2(s)} &= \frac{N_3(0)}{D_2(0)} \\ \text{where,} \\ N_3(0) &= (1 - p_{24}p_{42})(\Psi_1 + \Psi_3) + [p_{12}p_{24} + (p_{13} + p_{14})]\Psi_4 + [p_{12}(p_{24}p_{45} + p_{25}) + (p_{13} + p_{14})(p_{25}p_{42} + p_{45})](\Psi_5 + p_{56}\Psi_6) \\ \end{split}$$

9. Busy period for assistant repairman

Let $B_0^R(k)$ *i*=0,1,2,3,4,5,6 be the probability that the master repairman is busy repairing the failed unit in phase-I at epoch time *k* when the system operational from the state $S_i \in E$. Now for $B_0^R(k)$, we have the sum of the probabilities of the following contingencies:

$$B_0^R(k) = \sum_{u=0}^{K-1} q_{01}(u) OB_1^R(k-1-u)$$

 $B_0^R(k) = q_{01}(k-1) \odot B_1^R(k-1)$ Similarly, $B_1^R(k) = q_{12}(k-1) \odot B_2^R(k-1) + q_{13}(k-1) \odot B_3^R(k-1) + q_{14}(k-1) \odot B_4^R(k-1)$ $B_{2}^{R}(k) = Z_{2}^{R}(k) + q_{20}(k-1) \otimes B_{0}^{R}(k-1) + q_{24}(k-1) \otimes B_{4}^{R}(k-1) + q_{25}(k-1) \otimes B_{5}^{R}(k-1)$ $B_{\underline{3}}^{R}(k) = q_{\underline{3}4}(k-1) \odot B_{4}^{R}(k-1)$ $B_{4}^{R}(k) = Z_{4}^{R}(k) + q_{40}(k-1) \otimes B_{0}^{R}(k-1) + q_{42}(k-1) \otimes B_{2}^{R}(k-1) + q_{45}(k-1) \otimes B_{5}^{R}(k-1)$ $B_5^{R}(k) = q_{50}(k-1) \odot B_0^{R}(k-1) + q_{51}(k-1) \odot B_1^{R}(k-1) + q_{56}(k-1) \odot B_6^{R}(k-1)$ $B_6^R(k) = q_{61}(k-1) \odot B_1^R(k-1)$ where, $Z_2^R(k) = d^k s^k$ and $Z_4^R(k) = d^k n^k$ Using the inverse Laplace transform of $B_0^{R*}(s)$ we get: $B_0^{R*} = \lim_{s \to 0} \frac{N_4(s)}{D_2(s)}$ here, $D_2(0) = 0$ Therefore, by L-hospital rule, we have $B_0^{R*} = \lim_{s \to 0} \frac{N_4(s)}{D_2'(s)} = \frac{N_4(0)}{D_2'(0)}$ where, $N_4(0) = [p_{12} + p_{42}(p_{13} + p_{14})]\Psi_2 + [p_{12}p_{24} + (p_{13} + p_{14})]\Psi_4$

10. Profit analysis

The system model net-expected profit during the time interval (0, k) is given below: P(k) = Expected total revenue in (0, k) - Expected cost of repair in (0, k) P(k) = $C_0 \mu_{up}(k) - C_1 \mu_b^r(k) - C_2 \mu_b^R(k)$

Where C_0 per-unit up time revenue by the system due to the operation of unit-I and unit-II, C_1 and C_2 are the repair cost per-unit of time when unit is repair by master repairman and assistant repairman respectively.

11. Conclusion

The expected total cost per-unit time in steady state is given by:

$$\begin{split} P &= \lim_{k \to \infty} \frac{P(k)}{k} \\ &= C_0 A_0 - C_1 B_0^r - C_2 B_0^R \\ Where A_0 , B_0^r \text{ and } B_0^R \text{ have been already defined.} \end{split}$$

This paper concludes with an analysis of stochastic modeling of various reliability measures such as MTSF, availability and busy period for a master repairman, assistant repairman, and profit analysis by different levels of performance. Let us suppose that the random variables follow a geometric distribution with dissimilar probability mass functions. The numerical analysis of MTSF, availability, and profit analysis have been studied at various levels of failure rate (*q*) of unit-I, and failure rate (*s*) of unit-II by fixing the values of certain parameters *a*=0.8, *b*=0.2, *c*=0.6, *d*=0.4, *m*=0.4 and *n*=0.6. Table 1 and Figure 2 a show the variation in MTSF is decries by increasing the failure rate of unit-II. The availability is linearly falling shown in Table 2 and Figure 3, for various values of the failure rate of unit-I and unit-II. Also putting the other parameters *C*₀=10000, *C*₁=2000, and *C*₂=1000 the profit analysis concerning various values of failure rate (*q*) of unit-I, failure rate (*s*) of unit-II, and the fixing value of *a*, *b*, *c*, *d*, *m*, and *n* showed in a smooth curve in Figures 4 and Table 3.

References

- [1] Kumar, J. and Kadyan, M.S., (2012). Profit analysis of a system of non-identical units with degradation and replacement, International journal of computer application, Vol. 40 (3): 19-25.
- [2] Sureria, J.K., Malik, S.C. and Anand, J., (2012). Cost benefit analysis of a computer system with priority to software replacement over hardware repair, Applied Mathematical Sciences, Vol. 6 (75): 3723-3734.
- [3] J. Bhatti, A. K. Chitkara, M. K. Kakkar, (2016). Stochastic analysis of dis-similar standby system with discrete failure, inspection and replacement policy, Demonstratio Mathematica, Vol. 49(2): 224-235.
- [4] M.A. El-Damcese, N. H. El-Sodany, (2015). Discrete Time Semi-Markov Model of a Two Non-Identical Unit Cold Standby System with Preventive Maintenance with Three Modes, American Journal of Theoretical and Applied Statistics, Vol. 4 (4): 277-290.
- [5] Malik, S.C., (2016). Stochastic Modeling of a Repairable System under Different Weather Conditions, Recent Advances in Mathematics Statistics and Computer Science, 155-163.
- [6] Singh, V. V., Poonia, P.K., (2019). Probabilistic Assessment of Two-Unit Parallel System with Correlated Lifetime under Inspection Using Regenerative Point Technique, International Journal of Reliability, Risk and Safety: Theory and Application, Vol. 2 (1): 5-14.
- [7] Kumar, A., Saini, M., Devi, K., (2016). Analysis of a redundant system with priority and weibull distribution for failure and repair, Cogent Mathematics, Vol. 3 (1).
- [8] Kumar, N., Malik, S.C. and Nandal, N. (2022). Stochastic analysis of a repairable system of non-identical units with priority and Conditional failure of repairman, Reliability Theory & Application, No 1 (67), Vol. 17: 123-133.

٦

Appendix

Table 1: Effect of a, b, c, d, m	and n on system performance w	vith respect to various failure r	ate of unit-I and unit-II

		<i>a</i> =0.8, <i>b</i> =0.2, <i>c</i> =0.6, <i>d</i> =0.4, <i>m</i> =0.4 and <i>n</i> =0.6		
Failure rate of unit-I (<i>p</i>)	Failure rate of unit-II (r)	MTSF	Availability	Profit Analysis
0.02	0.01	47.66470	0.99	8117.19246
0.04	0.02	23.79169	0.98	8044.72539
0.06	0.03	15.86021	0.97	7966.79472
0.08	0.04	11.91403	0.96	7885.26352
0.10	0.05	9.562128	0.95	7801.60347
0.12	0.06	8.007633	0.94	7716.98608
0.14	0.07	6.909151	0.93	7632.35110
0.16	0.08	6.096083	0.92	7548.45847
0.18	0.09	5.473738	0.91	7465.92800
0.20	0.10	4.985377	0.90	7385.27013

1	T able 2: Effect of a, b, c, d, m an	d n on system performance	e with respect to var	rious failure rate of	funit-I and unit-II

		a=0.8, b=0.2, c=0.6, d=0.4, m=0.4 and n=0.6		
Failure rate of unit-I (<i>p</i>)	Failure rate of unit-II (r)	MTSF	Availability	Profit Analysis
0.03	0.02	23.81516	0.98	8030.99201
0.05	0.04	11.88649	0.97	7855.23332
0.07	0.06	7.916658	0.95	7677.4412
0.09	0.08	5.938082	0.93	7499.77873
0.11	0.10	4.757167	0.91	7323.78275
0.13	0.12	3.976017	0.90	7150.55723
0.15	0.14	3.424099	0.88	6980.90371
0.17	0.16	3.016157	0.86	6815.41137
0.19	0.18	2.704848	0.85	6654.52024
0.21	0.20	2.461794	0.83	6498.56645

Table 3: Effect of a, b, c, d, m and n on system performance with respect to various failure rate of unit-I and unit-II

		<i>a</i> =0.8, <i>b</i> =0.2, <i>c</i> =0.6, <i>d</i> =0.4, <i>m</i> =0.4 and <i>n</i> =0.6		
Failure rate of unit-I (<i>p</i>)	Failure rate of unit-II (r)	MTSF	Availability	Profit Analysis
0.04	0.04	11.86965	0.97	7846.51242
0.06	0.08	5.897579	0.94	7493.47397
0.08	0.12	3.903218	0.91	7159.24093
0.10	0.16	2.905934	0.88	6841.64064
0.12	0.20	2.309357	0.85	6539.07996
0.14	0.24	1.914567	0.82	6250.37274
0.16	0.28	1.636242	0.79	5974.63725
0.18	0.32	1.431696	0.77	5711.23236
0.20	0.36	1.277211	0.74	5459.71658



Figure 2: MTSF vs failure rate of unit-I (p) and unit-II (r)



Figure 3: Availability analysis vs failure rate of unit-I (p) and unit-II (r)



Figure 4: Profit analysis vs failure rate of unit-I (p) and unit-II (r)