# RELIABILITY ANALYSIS OF PARALLEL SYSTEM USING PRIORITY TO PM OVER INSPECTION

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**Abstract**

*Reliability optimization of a system is an extant problem. By solving these problems, new methodologies are obtained that have invent new engineering technology and changes the management perspective. Aim of the reliability analysis is to study the failure mechanisms of a system and and outcomes of the analysis serve to identify design solutions and maintenance actions for preventing the failures from occurring. So, it is used to evaluate and improve the quality of products, processes, and systems. Measurement, planning, and improvement in the reliability are the things which are well do in any business but only when efforts are focused on important problems which are highlighted by monetary values, improve reliability, reduce unreliability costs, generate more profit, and get more business. To serve this purpose, present study investigates a parallel system of two identical units which is based on several assumptions like, the system is served by one serviceman who is immediately available for service when it will call. System failure rate is fix and the failure type (repairable or replaceable) is known by inspection. The failure and repair activities are stochastically independent, and their rates are exponentially distributed. Priority to PM over inspection is given in the system. Several measures of reliability effectiveness like MTSF, availability and cost-benefit analysis of the system are obtained by semi-Markov and regenerative point approach. Reliability characteristics parameters are random variables, and results are obtained in the form of graphs and tables by changing the values of these variables one by one, while keeping other variables constant at that point. From the results we conclude how to make the given system more profitable. Findings of present system model shows that when the failure rate is low then the system obtained more profit by increasing preventive maintenance rate. On the other hand, when failure rate going high then we make the system more profitable by increasing inspection rate. These insights of modelling and analysis helps the system developers and managers to make good choices of action against specified criteria that managing engineered products and industrial plants safely and reliably. This leads to more profit and making a business more growing.*

**Keywords:** Parallel system, priority, inspection, semi-Markov, regenerative points, repair activities, profit.

# 1. Introduction

The objectives of any equipment or system manufactured is to design it in such a way that it achieves its goals in terms of production. A reliable equipment is the one which works satisfactorily for a given time period under given environmental conditions without any interruptions. Hence the reliability is the key point in the manufacturing/ production industries. Although ever-increasing urge of the society is making the design of system more complicated and to control the strength and effectiveness of failure of such system reliability experts frame a model which is more productive and profitable. Various researcher has done a lot of research work in the reliability theory to improve the repair techniques. Several authors such as Gupta and Agarwal [1], Dhillon and Yang [2] extensively discussed complex systems by considering various failure and repair disciplines. Ram [3] gave the summery of various reliability approaches. Parallel redundancy is highly used by the researcher to enhance the system reliability. Hitomi [4] investigates the reliability of a manufacturing system in which two machines are arranged in parallel. Termoto et al. [5] studied an optimal inspection policy for an n-unit parallel system which is checked at successive times and PM is carried out after the failure of a certain number of units at each inspection. Gupta et al. [6] analyzed a two non-identical unit parallel system. They have taken the joint distribution of lifetimes of both units as bivariate negative exponential. Malik et al. [7] considered two reliability models with two parallel units in which one is original and other is duplicate. Priority to repair of the original unit is given in model I and no priority is given in model II. Yu and Khambadkone [8] derived a parallel inverter system to analyses the reliability and cost optimization using sensitivity analysis. Rathee and Malik [9] observed a parallel system under the aspect of priority to PM over repair and replacement. Sivakumar and Jayanth [10] studied the reliability of a Mobile network system during communication. Kakkar et.al. [11] worked on the reliability and profit analysis of a parallel industrial system. Bhardwaj and Parasher [12] analyzed a cold standby system with geometric failure and repair rates. Pundir et.al. [13] studied a two non-identical units' parallel system with priority in repair to first unit. Chopra and Ram [14] investigate the reliability measures of the system, which has two dissimilar units in the parallel network under copula. Li et al. [15] developed a two similar component parallel degradation repairable system. They considered that when the repairman is on vacation then the failed component is not repaired as good as new. Dabas and Rathee [16,17] derived a parallel system with the idea of priority to preventive maintenance over replacement and inspection for repair activities. Sherbeny [18] studied the impact of some system parameters on an industrial system consisting parallel units with one repairer and optional vacations under Poisson shocks.

In this paper we consider a two identical units parallel system using priority to PM over inspection. All repair activities and inspection is done by a single serviceman. Units' failure rate is taken to be constant, and inspection is done to find the type of failure. Time taken in repair activities is distributed arbitrarily and their rates are exponentially distributed. The failure and repair activities are stochastically independent. To meet the objective of reliability evolution we derive some suitable measure like MTSF, availability and cost-benefit analysis of the system using semi-Markov and regenerative point approach. Graphs are plotted to observe the change in the behaviour of these measures with failure rate for particular cases of various rates included in the system.

## 2. Notations for System Model

| | | |
|---|---|---|
| $\lambda$ | : | Constant Failure Rate |
| $a/b/\alpha_0$ | : | Rate by which system goes for Repair / Replacement / Preventive Maintenance respectively |
| $\alpha/\beta/\gamma/\theta$ | : | Repair / Replacement / Inspection / Preventive Maintenance rate respectively done by the server |
| h(t)/f(t)/r(t)/g(t) | : | pdf of the Inspection / Repair / Replacement / Preventive Maintenance time respectively |
| H(t)/F(t)/R(t)/G(t) | : | cdf of the Inspection / Repair / Replacement / Preventive Maintenance time respectively |
| $p_{ij}$ | : | Transition probability from state $S_i$ to state $S_j$ |
| $p_{ij.kr}$ | : | Transition probability from state $S_i$ to state $S_j$ via state $S_k$, $S_r$ |
| $Q_{ij}(t)/q_{ij}(t)$ | : | Cdf/pdf of passage time from regenerative state $S_i$ to a regenerative state $S_j$ or to a failed state Sj without visiting any other regenerative state in (0, t] |
| $Q_{ij.kr}(t)/q_{ij.kr}(t)$ | : | pdf/cdf of direct transition time from regenerative state $S_i$ to a regenerative state $S_j$ or to a failed state $S_j$ visiting state $S_k$, $S_r$ once in (0, t] |
| $\mu_i$ | : | Mean sojourn time in state $S_i$ |
| $m_{ij}$ | : | Contribution to mean sojourn time in state $S_i$ when the system transits directly to state $S_j$ |
| */** | : | Symbol for Laplace transformation/ Laplace Stieltjes Transformation |
| ©/Ⓢ | : | Symbol for Laplace transformation/Laplace Stieltjes convolution |

## 3. System Description and Assumptions

To evaluate the effectiveness of the present research, the numerical data is examined in order to establish essential assumptions to the system model. The Semi-markov and re-generative point process are used to provide formulations of system dependability measures such as reliability, mean time to system failure (MTSF), availability, and profit function. Numerical examples are provided to demonstrate the acquired conclusions. Results are obtained in tabular and graphical form to investigate the influence of various system features.

Assumptions

- Initially both the units are in working mode
- Units are failed with constant rate
- System is served by single serviceman
- All the times associated with all events are random and independent.
- All the repair activities follow exponential distribution

**Table 1:** *Description of the states*

| States | Description |
|---|---|
| $S_0$ | Both the units are operative |
| $S_1$ | One unit is operative and other is failed under inspection |
| $S_2$ | Resume for PM |
| $S_3$ | One is working and other is failed under replacement |
| $S_4$ | One unit is continuously under inspection from previous state and other is waiting for inspection |
| $S_5$ | One is working and other is failed under repair |
| $S_6$ | One unit is waiting for inspection from previous state and other is under PM |
| $S_7$ | One unit is working and other under PM |
| $S_8$ | One unit is continuously under replacement from previous state and other is waiting |

| | |
|---|---|
| | for inspection |
| $S_9$ | One unit is under repair and other is continuously waiting for inspection from previous state |
| $S_{10}$ | One unit is under replacement and other is continuously waiting for inspection from previous state |
| $S_{11}$ | One unit is continuously under repair from previous state and other is waiting for inspection |
| $S_{12}$ | One unit is continuously under repair from previous state and other is waiting for PM |
| $S_{13}$ | One unit is continuously under replacement from previous state and other is waiting for PM |
| $S_{14}$ | One unit is continuously under PM from previous state and other is waiting for inspection |

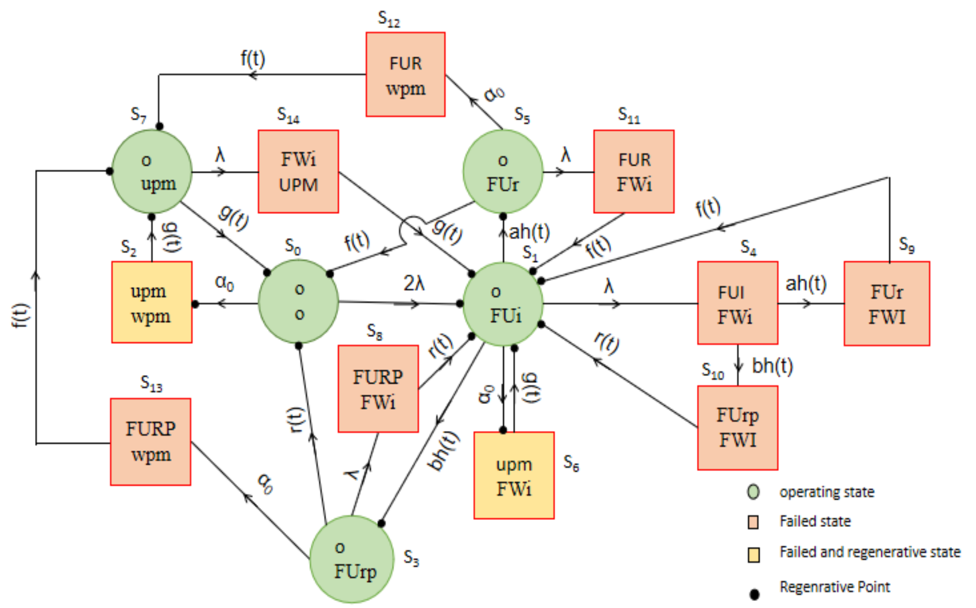


**Figure 1:** *Transition State Diagram*

## 4. Formulation and Stochastic Analysis of the Model

### 4.1. Transition Probabilities & Mean Sojourn Times ($\mu_i$)

Steady- state transition probabilities from regenerative state i to state j are given by the formula

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$

$p_{01} = \frac{2\lambda}{2\lambda+\alpha_0}$, $p_{02} = \frac{\alpha_0}{2\lambda+\alpha_0}$, $P_{13} = bh^*(\lambda+\alpha_0)$, $p_{14} = \frac{\lambda}{\lambda+\alpha_0}(1-h^*(\lambda+\alpha_0))$,

$p_{15} = ah^*(\lambda+\alpha_0)$, $p_{16} = \frac{\alpha_0}{\lambda+\alpha_0}(1-h^*(\lambda+\alpha_0))$, $p_{30} = r^*(\lambda+\alpha_0)$, $p_{38} = p_{31.8} = \frac{\lambda}{\lambda+\alpha_0}(1-r^*(\lambda+\alpha_0))$,

$p_{3,13} = p_{37.13} = \frac{\alpha_0}{\lambda+\alpha_0}(1-r^*(\lambda+\alpha_0))$,

$p_{49} = a$, $p_{4,10} = b$, $p_{50} = f^*(\lambda+\alpha_0)$, $p_{5,11} = p_{51.11} = \frac{\lambda}{\lambda+\alpha_0}(1-f^*(\lambda+\alpha_0))$,

$p_{5,12} = p_{57.12} = \frac{\alpha_0}{\lambda+\alpha_0}(1-f^*(\lambda+\alpha_0))$, $p_{70} = g^*(\lambda)$, $p_{7,14} = p_{71.14} = 1-g^*(\lambda)$,

$p_{11.49} = \frac{\lambda a}{\lambda+\alpha_0}(1-h^*(\lambda+\alpha_0))$, $p_{11.4,10} = \frac{\lambda b}{\lambda+\alpha_0}(1-h^*(\lambda+\alpha_0))$,

$p_{1,14.6} = \frac{\alpha_0 b}{\lambda+\alpha_0}(1-h^*(\lambda+\alpha_0))$, $p_{17.6,13} = \frac{\alpha_0 a}{\lambda+\alpha_0}(1-h^*(\lambda+\alpha_0))$,

$p_{27} = p_{61} = p_{81} = p_{91} = p_{10,1} = p_{11,1} = p_{12,7} = p_{13,7} = p_{14,1} = 1$

It is noticed that the $\sum_j p_{ij} = 1$ for all possible values of 'i'.
Further mean sojourn times ($\mu_i$) is the expected time taken by the system in a particular state before transiting to any other state. If $T_i$ is the sojourn time in state 'i' ,then
$\mu_i = E(t) = \int_0^\infty P(T_i > t)\, dt = \sum_j m_{ij}$ and $m_{ij} = \frac{d[Q_{ij}^{**}(s)]}{ds} |s = 0.$
Expressions for $\mu_i$ are given as
$\mu_0 = \frac{1}{2\lambda + \alpha_0}$ , $\mu_1 = \frac{1}{\lambda + \alpha_0}(1 - h^*(\lambda + \alpha_0)), \mu_3 = \frac{1}{\lambda + \alpha_0}(1 - r^*(\lambda + \alpha_0))$
$\mu_5 = \frac{1}{\lambda+\alpha_0}(1 - f^*(\lambda + \alpha_0), \mu_7 = \frac{1}{\lambda}(1 - g^*(\lambda))$
$\mu_1' = [\frac{1}{\lambda + \alpha_0} + \frac{\lambda}{\lambda + \alpha_0}(\frac{b}{\beta} + \frac{1}{\gamma} + \frac{a}{\alpha})](1 - h^*(\lambda + \alpha_0))$
$\mu_3' = \frac{1}{\beta}$ , $\mu_5' = \frac{1}{\propto}, \mu_7' = \frac{1}{\theta}$

4.2. Reliability & Mean Time to System Failure (MTSF)

Let cdf of first transition time from the state $S_i$ to the state in which failure occur is represented by $\Phi_i(t)$ . We take absorbing state as the failed state. So, the expressions for $\Phi_i(t)$ from which MTSF of discussed system is obtained as

$$\Phi_i(t) = \sum_{i,j} Q_{ij}(t) \circledS \Phi_j(t) + \sum_{i,k} Q_{ik}(t) \quad (1)$$

Where i is the operating state from which transition takes place to j (operating & regenerative state) and k (failed state).
If we take LST of above relation (1) and solved them for $\Phi_0^{**}(s)$, we have

$$R^*(s) = \frac{1 - \Phi^{**}(s)}{s} \quad (2)$$

By taking Inverse Laplace transform of (2), we get system reliability.
And MTSF is obtained as: $\mathrm{MTSF} = \lim_{s\to 0} \frac{1 - \Phi^{**}(s)}{s} = \frac{N}{D}$
Where, $N = \mu_0 + \mu_1 p_{01} + \mu_3 p_{01} p_{13} + \mu_5 p_{01} p_{15}$ and $D = 1 - p_{01} p_{13} p_{30} - p_{01} p_{15} p_{50}$

4.3. Analysis of Availability

Let $A_i(t)$ be the probability of the system availability at an instant 't' given that system goes to regenerative state $S_i$ at t = 0. So the expressions for $A_i(t)$ as

$$A_i(t) = M_i(t) + \sum_{i,j} q_{ij}^{(n)}(t) © A_j(t) \quad (3)$$

Where i is regenerative state from which transition takes place to j (successive regenerative state) through n transitions.
$M_i(t)$ be the probability of the system in up state Si up to the time 't' without visiting to any other regenerative state.
$M_0(t) = e^{-(2\lambda + \alpha_0)t}$ , $M_1(t) = e^{-(\lambda+\alpha_0)t}\overline{H(t)},$
$M_3(t) = e^{-(\lambda+\alpha_0)t}\overline{R(t)}, M_5(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)}$ , $M_7(t) = e^{-(\lambda t)}\overline{G(t)}$
Now, if we use LT of (3) and solved it for $A_0^*(s)$.We get the result for steady state availability as

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1} \quad (4)$$

Where
$N_1 = \mu_0 X + (\mu_1 + \mu_3 p_{13} + \mu_5 p_{15})Y + \mu_7 Z$ and
$D_1 = (\mu_0 + \mu_2 p_{02})X + (\mu_1' + \mu_3' p_{13} + \mu_5' p_{15} + \mu_6 p_{16})Y + \mu_7' Z$

4.4. Busy Period Analysis for Server

Let $B_i^I(t), B_i^R(t), B_i^{Rp}(t), B_i^P(t)$be the probability of busy period of server during inspection, repair, replacement and PM at instant 't' with the given condition that the system go to regenerative state Si at t=0. The expressions for $B_i^I(t), B_i^R(t), B_i^{Rp}(t), B_i^P(t)$ are as follows:

$B_i^I(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t)©B_j^I(t)$ , $B_i^R(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t)©B_j^R(t)$

$B_i^{Rp}(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t)©B_j^{Rp}(t)$ , $B_i^P(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t)©B_j^P(t)$ (5)

Where i is regenerative state from which transition takes place to j (successive regenerative state) through n transitions.

Wi(t) be the probability of server busyness at state Si due to repair activities at time t without making any transition to any other regenerative state or returning to the same via one or more non regenerative state.

Here,

$W_1(t) = e^{-(\lambda+\alpha_0)t}\overline{H(t)} + (\lambda e^{-(\lambda+\alpha_0)t}©1)\overline{H(t)}$

$W_5(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)} + (\lambda e^{-(\lambda+\alpha_0)t}©1)\overline{F(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t}©1)\overline{F(t)}$

$W_3(t) = e^{-(\lambda+\alpha_0)t}\overline{R(t)} + (\lambda e^{-(\lambda+\alpha_0)t}©1)\overline{R(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t}©1)\overline{R(t)}$

$W_2(t) = \overline{G(t)} = W_6(t), W_7(t) = e^{-(\lambda)t}\overline{G(t)} + (\lambda e^{-(\lambda)t}©1)\overline{G(t)}$

Take LT of (5) and solving it for $B_0^{I^*}(s), B_0^{R^*}(s), B_0^{Rp^*}(s), B_0^{P^*}(s)$ .The busy time in inspection, repair, replacement and PM for server is given by

$B_0^I(\infty) = \lim_{s\to 0} sB_0^{I^*}(s) = \frac{N_2}{D_1}$, $B_0^R(\infty) = \lim_{s\to 0} sB_0^{R^*}(s) = \frac{N_3}{D_1}$

$B_0^{Rp}(\infty) = \lim_{s\to 0} sB_0^{Rp^*}(s) = \frac{N_4}{D_1}$, $B_0^P(\infty) = \lim_{s\to 0} sB_0^{P^*}(s) = \frac{N_5}{D_1}$

Here,

$N_2 = W_1^*(0)Y, N_3 = W_5^*(0)p_{15}Y, N_4 = W_3^*(0)p_{13}Y$ and

$N_5 = W_2^*(0)p_{02}X + W_6^*(0)p_{16}Y + W_7^*(0)Z$ and D1 is mentioned above.

4.5. Expected Number of Visits by The Server

Consider $I_i(t), R_i(t), Rp_i(t), Pm_i(t)$ as the expected number of visits make by the server for inspection, repair, replacement and PM in (0, t] .We have the following recursive relations for $I_i(t), R_i(t), Rp_i(t), Pm_i(t)$ are

$I_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t)Ⓢ(C + I_j(t))$ , $R_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t)Ⓢ(C + R_j(t))$

$Rp_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t)Ⓢ(C + Rp_j(t))$ , $Pm_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t)Ⓢ(C + Pm_j(t))$ (6)

Where i is regenerative state from which transition takes place to j (successive regenerative state) through n transitions and and C = 1 server does the job afresh at j, otherwise C = 0.

Take LST of (6) and solving it for $I_0^{**}(s), R_0^{**}(s), Rp_0^{**}(s), Pm_0^{**}(s)$.The expected number of inspections, repairs, replacements and PM by the server is given by (per unit time)

$I_0(\infty) = \lim_{s\to 0} sI_0^{**}(s) = \frac{N_6}{D_1}$ , $R_0(\infty) = \lim_{s\to 0} sR_0^{**}(s) = \frac{N_7}{D_1}$

$Rp_0(\infty) = \lim_{s\to 0} sRp_0^{**}(s) = \frac{N_8}{D_1}$ , $Pm_0(\infty) = \lim_{s\to 0} sPm_0^{**}(s) = \frac{N_9}{D_1}$

Where,

$N_6 = (1 - p_{16})Y, N_7 = (p_{11.49} + p_{15})Y, N_8 = (p_{11.4,10} + p_{13})Y,$

$N_9 = p_{02}X + p_{16}Y + Z$ and $D_1$ is already mentioned.

Here X, Y & Z are

$X = p_{13}p_{30} + p_{15}p_{50} + p_{70}(p_{13}p_{37.13} + p_{15}p_{57.12})$ , $Y = (1 - p_{02}p_{70})$ and

$Z = p_{13} + p_{15} - p_{13}p_{31.8} - p_{15}p_{51.11} - p_{01}p_{13}p_{30} - p_{01}p_{15}p_{50}$

### 4.6. Profit Analysis

In steady state the profit function of the system model can be obtained as

$$P = k_0A_0 - k_1B_0^I - k_2B_0^R - k_3B_0^{Rp} - k_4B_0^{Pm} - k_5I_0 - k_6R_0 - k_7Rp_0 - k_8Pm_0$$

Here,

$P$ = Profit function of system model
$k_0$ = Revenue per unit up − time of the system
$k_1, k_2, k_3, k_4$ = Cost per unit time of the server when it is busy in inspection, repair, replacement, preventive maintenance
$k_5, k_6, k_7, k_8$ = Cost per unit time for inspection, repair, replacement, preventive maintenance

## 5. Analytical Study of the Model

To make the study more practical we draw the results in the form of tables and graphs. Tables 1, 2, 3 and figures 2, 3, 4 show the behaviour of MTSF, availability and profit with respect to failure rate for different values of the given parameters by assuming that the rate of repair activities follow exponential distribution. Table 1 and figure 2 talks about the values of MTSF goes decreasing when failure rate increases. If we increase the values of repair rate ($\alpha$=4.1), replacement rate ($\beta$=5) and inspection rate ($\gamma$=3) one by one and keep all other parameters fix we find that the MTSF is increases. But if we increase $\alpha_0$=3.1 (rate by which system goes for PM) the MTSF is decreases.

Table 2 and figure 3 shows the effect of various parameters on availability with respect the failure rate and we observe that the availability of the system decreases with increase in the failure rate. Similarly if we increase $\alpha$, $\beta$, $\gamma$ and $\theta$ (rate by which system do preventive maintenance) then the availability is also increases but availability decreases when we increase in $\alpha_0$.

From table 3 and figure 4 we conclude about the profit of the system. We see that if values of $\alpha$, $\beta$, $\gamma$ and $\theta$ increases then the profit is also increases but if we increase $\alpha_0$ then the profit goes decline.

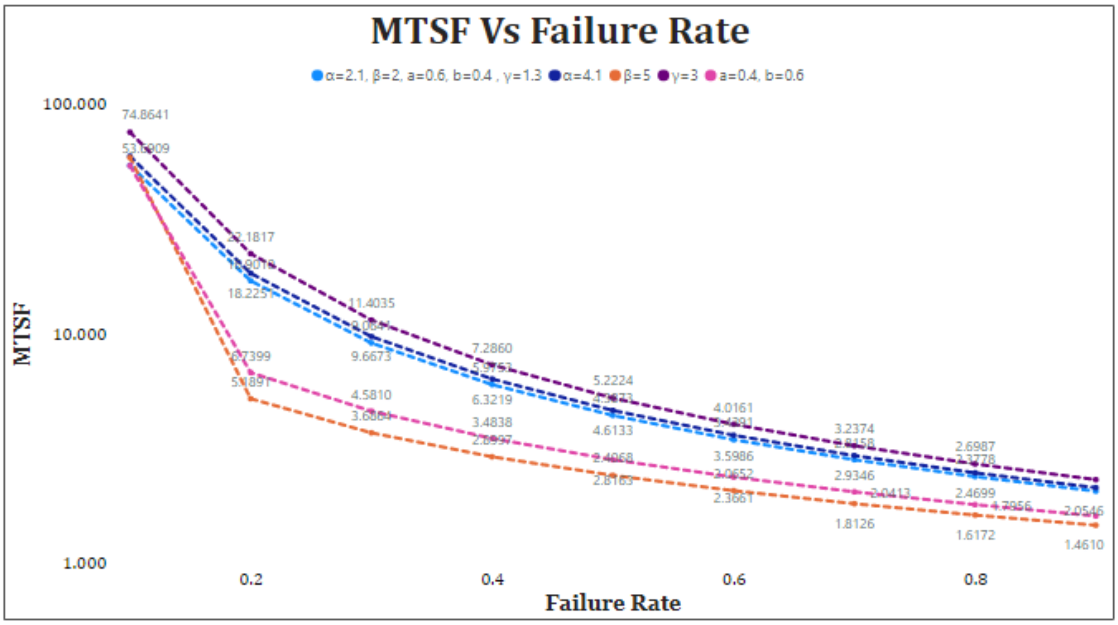


**Figure 2:** *MTSF VS Failure Rate*

**Table 2:** *MTSF w.r.t various parameters*

| Failure rate | α=2.1,β=2,a=0.6,b=0.4,γ =1.3,α0=3 | α= 4.1 | β=5 | γ= 3 | α0= 3.1 | a= 0.4, b= 0.6 |
|---|---|---|---|---|---|---|
| 0.1 | 0.33274 | 0.33275 | 0.33275 | 0.33279 | 0.32204 | 0.33274 |
| 0.2 | 0.33113 | 0.33119 | 0.33119 | 0.33130 | 0.32056 | 0.33113 |
| 0.3 | 0.32875 | 0.32887 | 0.32886 | 0.32908 | 0.31838 | 0.32875 |
| 0.4 | 0.32577 | 0.32596 | 0.32594 | 0.32627 | 0.31563 | 0.32577 |
| 0.5 | 0.32235 | 0.32260 | 0.32258 | 0.32302 | 0.31247 | 0.32234 |
| 0.6 | 0.31859 | 0.31890 | 0.31887 | 0.31943 | 0.30899 | 0.31858 |
| 0.7 | 0.31458 | 0.31495 | 0.31492 | 0.31558 | 0.30527 | 0.31457 |
| 0.8 | 0.31040 | 0.31082 | 0.31079 | 0.31153 | 0.30138 | 0.31039 |
| 0.9 | 0.30610 | 0.30657 | 0.30653 | 0.30736 | 0.29737 | 0.30609 |

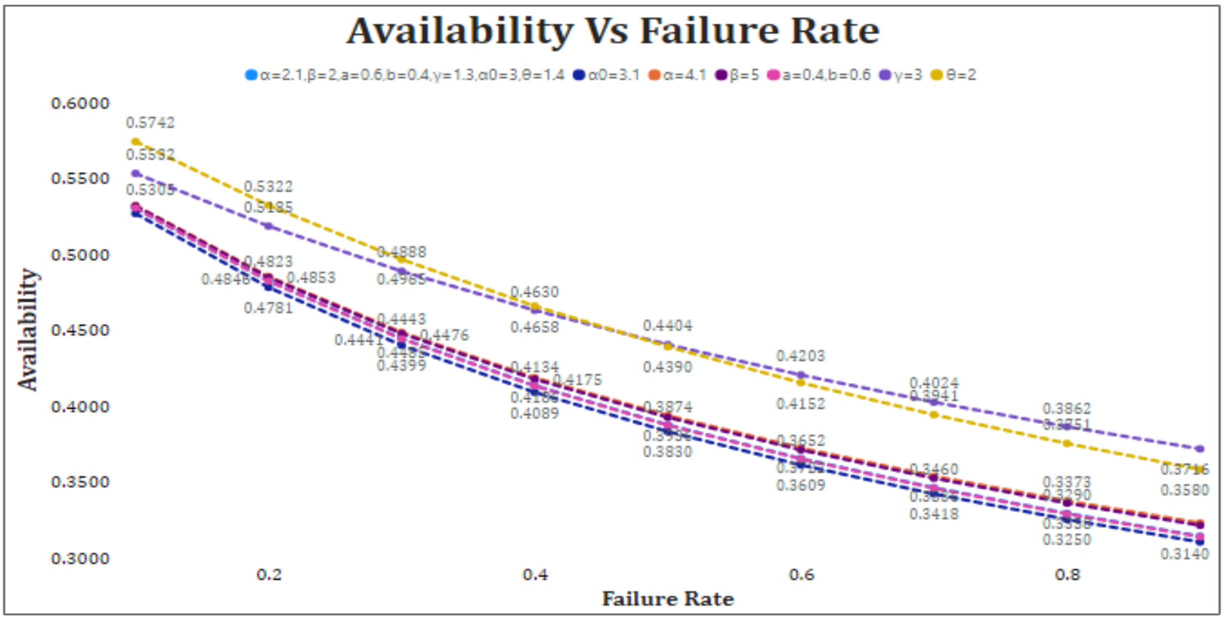


**Figure 3:** *Availability VS Failure Rate*

**Table 3:** *Availability w.r.t various parameters*

| Failure Rate | α=2.1,β=2,a=0.6,b=0.4,γ=1.3,α0=3,θ=1.4 | α=4.1 | β=5 | a=0.4,b=0.6 | γ=3 | α0=3.1 | θ=2 |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.53052 | 0.53208 | 0.53170 | 0.53043 | 0.55323 | 0.52682 | 0.57425 |
| 0.2 | 0.48233 | 0.48528 | 0.48462 | 0.48218 | 0.51847 | 0.47809 | 0.53218 |
| 0.3 | 0.44435 | 0.44852 | 0.44764 | 0.44414 | 0.48875 | 0.43990 | 0.49653 |
| 0.4 | 0.44435 | 0.41859 | 0.41754 | 0.41312 | 0.46298 | 0.40888 | 0.46580 |
| 0.5 | 0.38741 | 0.41859 | 0.39237 | 0.38714 | 0.44036 | 0.38300 | 0.43895 |
| 0.6 | 0.36524 | 0.37219 | 0.37087 | 0.36494 | 0.42031 | 0.36094 | 0.41524 |
| 0.7 | 0.34599 | 0.35362 | 0.35221 | 0.34567 | 0.40238 | 0.34183 | 0.39411 |
| 0.8 | 0.34599 | 0.33727 | 0.33578 | 0.32871 | 0.38622 | 0.32505 | 0.37514 |
| 0.9 | 0.34599 | 0.33727 | 0.32115 | 0.31363 | 0.37156 | 0.31014 | 0.35799 |

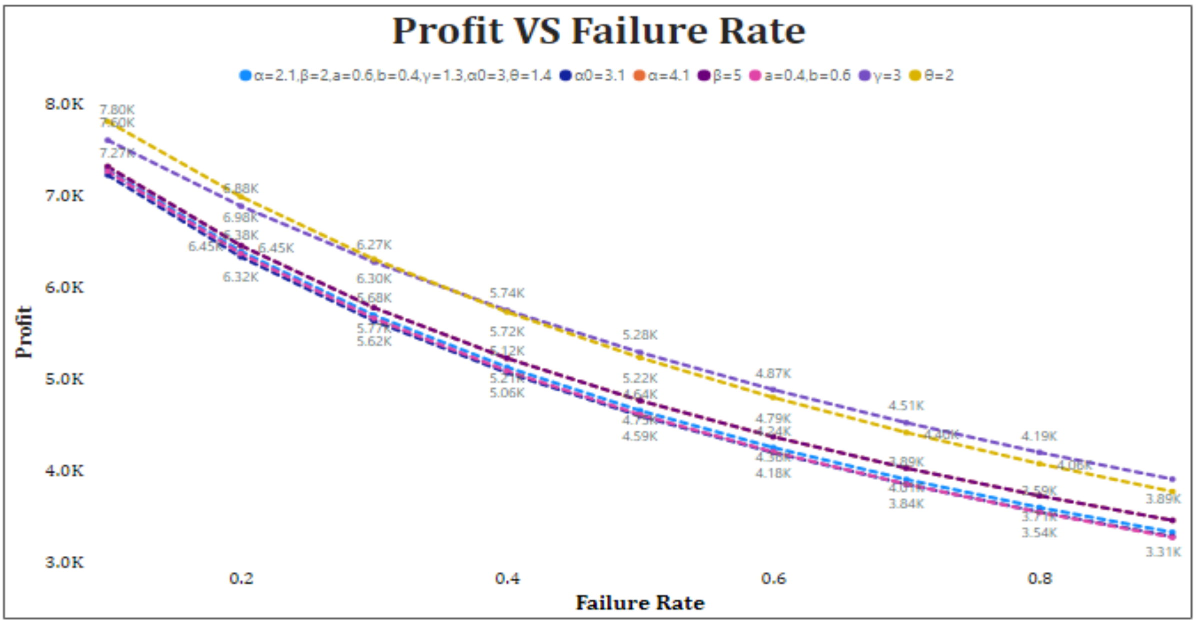


**Figure 4:** *Profit VS Failure Rate*

**Table 4:** *Profit w.r.t various parameters*

| Failure Rate | α=2.1,β=2,a=0.6,b=0.4,γ=1.3,α0=3,θ=1.4 | α=4.1 | β=5 | a=0.4,b=0.6 | γ=3 | α0=3.1 | θ=2 |
|---|---|---|---|---|---|---|---|
| 0.1 | 7271.45 | 7307.88 | 7308.10 | 7256.59 | 7596.16 | 7216.20 | 7800.53 |
| 0.2 | 6382.77 | 6445.78 | 6446.11 | 6357.37 | 6877.03 | 6321.97 | 6980.11 |
| 0.3 | 5684.64 | 5767.44 | 5767.80 | 5651.40 | 6265.67 | 5622.79 | 6297.90 |
| 0.4 | 5117.12 | 5214.77 | 5215.12 | 5077.84 | 5738.43 | 5056.42 | 5719.43 |
| 0.5 | 4643.64 | 4752.40 | 4752.75 | 4599.59 | 5278.18 | 4585.18 | 5221.28 |
| 0.6 | 4240.52 | 4357.50 | 4357.87 | 4192.61 | 4872.23 | 4184.81 | 4786.85 |
| 0.7 | 3891.68 | 4014.63 | 4015.04 | 3840.60 | 4510.94 | 3838.93 | 4404.02 |
| 0.8 | 3585.81 | 3712.90 | 3713.39 | 3532.09 | 4186.87 | 3536.04 | 4063.65 |
| 0.9 | 3314.65 | 3444.43 | 3445.03 | 3258.70 | 3894.18 | 3267.80 | 3758.75 |

## 6. Practical Implication

Redundancy is a useful method of increasing reliability and optimizing the balance between operation effectiveness and expenditure. Arranging elements of the system in parallel provide alternative paths of operation. The parallel structure in the reliability engineering is widely used in many industrial systems such as power generation systems, pump systems, production systems and computing systems. One of the examples is in the commercial boiler market. Packaged boilers installed in multiple boiler cascade systems offer long-term energy savings and reliability. L. Vorsteveld [19] gives that the preferred control scheme is parallel cascading, and it leads to high turn down ratio.


**Figure 5:** *Parallel Boiler Cascade System*

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