Reliability Measures of a 2-out-of-3: G System with Priority and Failure of Service Facility during Repair

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Abstract

Aim. The objective of this paper is to describe a particular case of the k-out-of-n: G system for k=2 and n=3 with different repair policies and to discuss the application of the proposed in a toxic waste incinerator. Methods. The system has all the three units identical in nature. The system model is developed using semi-Markov process and regenerative point technique. The preventive maintenance and repair activities of the units are carried out immediately by a single service facility whenever desires. The service facility is subjected to failure during repair of the units while it does preventive maintenance of the units without any problem. The failed service facility undergoes for treatment to restore its efficiency to perform the remaining jobs with full capacity. The provision of priority to preventive maintenance of the units has been made over the repair in order to avoid the earlier failure of the system. Findings. The measures that can affect and enhance the performance of the system have been discussed for arbitrary values of the rates which follow some arbitrary distributions including the negative exponential. The system is analysed in steady state and the graphs have been drawn to see the effect of different transition rates such as failure rate, preventive maintenance rate, treatment rate, and repair rate of the units on reliability measures and the profit. The study reveals that there is a decline in these measures with the increase of the rate by which unit undergoes for preventive maintenance, failure rates of the units and service facility. However, the values of reliability measures MTSF, availability and profit function keep on increasing with the increase of treatment rate, repair rate of the unit and preventive maintenance completion rate. The profit increases if the rate with which a unit completes its preventive maintenance. Hence, implementing the preventive maintenance repair policy for a 2-out-of-3 system is beneficial as it increases the availability and hence the profit of the system.

Keywords: 2-out-of-3 System, Preventive Maintenance, Treatment Rate, Priority, Failure of Service Facility and Reliability Measures

1. Introduction

Now a day, complex systems are used in almost all the areas of science and technology specially in the field of industries. Complex systems are made of multiple dependent and independent components. This kind of formulation may affect the efficient and accurate evaluation of reliability measures. So, for the accurate analysis of the system performance, it is obligatory to understand the nature of components, their failure rates, preventive maintenance rates and their interactions. The k -out-of-n: G system is one such type of complex system defined as the system which has 'n' identical units, all are working initially and at least k are required to be good to make the system work. k-out-of-n system is one of the most useful partial redundancy types in complex systems, which is often used in various areas including software and hardware engineering for the purpose of providing a proper level of redundancy during the operation of a system to increase the availability and hence the profit. There are many systems in our day-to-day life which are based upon k-out-of-n configuration for example multi engine system of an airplane, multi pump system in a hydraulic control system, networking and communication systems, toxic waste incinerator etc.

Several studies have been conducted so far for arbitrary k-out-of-n systems by assuming that system can perform nonstop work without requiring any maintenance. But in real life this assumption seems hard to believe as continuous operation deteriorates the performance and hence reliability of the system. To avoid such problems preventive maintenance can be considered as an alternative solution conducted after a specific operation time. Preventive maintenance also helps in slow down the deterioration rate of the process. Malik and Bhardwaj [1] have analysed 2-out-of-3 redundant system with general repair and waiting time distribution. They have also studied the same system with feasibility to repair. Malik and Singh [2] have studied a 2-out-of-3 redundant system with a single service facility for inspection and repair of the units by using the concept of degradation of unit after repair. Kishan and Kumar [3] studied a parallel system with preventive maintenance. Jain et al. [4] have analysed a Load Sharing m-out-of-n: G System with Non -Identical Components Subject to Common Cause Failure. Further, the concept of priority in repair policies has also been suggested by the researchers to make the system more profitable. Yang et al. [5] have discussed the reliability analysis of load-sharing k-out-of-n system considering component degradation. Poonia et al. [6] has performed the cost analysis of a repairable warm standby k-out-of-n: G and 2-out-of-4: G systems in series configuration under catastrophic failure using copula repair. Anuradha et al. [7] have analysed the profit of a 1-out-of-2: G system with the concept of server failure and priority to repair. Singh et al. [8] have done the performance assessment of the complex repairable system with n-identical units under (k-out-of-n: G) scheme and copula linguistic repair approach. Abdullahi et. al [9] have performed the cost analysis of 2-out-of-4 system connected to two units parallel supporting device for operation However, the idea of priority to preventive maintenance over repair along with the failure of service facility (all the three repair policies together) has not been introduced while analysing system reliability models of three or more identical units.

So, here a particular case of k-out-of-n: G system for k=2 and n=3 has been considered. Initially all the three units are operative out of which two units are required to be good for the proper functioning of the system. The system undergoes for preventive maintenance with an arbitrary rate in order to restore the efficiency of the working units. A single service facility performs the preventive maintenance and repair of the units whenever desires. The service facility may fail and therefore, undergoes for treatment to restore the efficiency to carry out the remaining repair activities. Preventive maintenance of the units is kept as priority over repair in order to avoid the earlier failure of the system. The well-known semi-Markov process and regenerative point technique are used to develop the system model under certain assumptions. The measures that can affect and enhance the performance of the system have been discussed for arbitrary values of the rates which follow some arbitrary distributions including the negative exponential. The system is analysed in steady state and the graphs have been drawn to see the effect of different transition rates such as failure rate, preventive maintenance rate, treatment rate, and repair rate of the units on reliability measures and the profit. Thus, the focus of the present paper is to analyse reliability measures and profit of a 2-out-of-3 system.

2. System description and Notations

2.1) Notations

Table 1: *Symbol Description*

Symbol	Description
O	Unit is operative
λ	Failure Rate of the Unit
μ	Failure Rate of the Server
WO	Unit is Waiting for Operation
$p(t)/P(t)$	pdf/cdf of the preventive maintenance time of the unit
$p_m(t)/P_m(t)$	pdf/cdf of the preventive maintenance completion time of the unit
$r_s(t)/R_s(t)$	pdf/cdf of the treatment time of the server
$r(t)/R(t)$	pdf/cdf of the repair time of the unit
Fur/FUR	Unit is failed and under repair/under repair continuously from previous state
FWr /FWR	Unit is failed and waiting for repair/waiting for repair continuously from previous state
UPm /UPM	Unit is failed and under preventive maintenance/under preventive maintenance continuously from previous state
SFut	Server failed under treatment
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of first passage time from regenerative state S_i to a regenerative state S_i or to a failed state S_j without visiting anyother regenerative state in (0, t]
$q_{ij.k}(t)/Q_{ij.k}(t)$	pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k once in (0, t]
$M_i(t)$	Probability that the system up initially in state S_i and is up at time t without visiting any regenerative state
$W_i(t)$	Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states
μ_i	The mean sojourn time in state S_i given by: $\mu_i=E(t)=\int_0^\infty P(T>t)dt=\sum_j m_{ij}$, where T is the time to system failure
m_{ij}	Contribution to mean sojourn time(μ_i) in state S_i when the system directly transits to state S_j so that $\mu_i=\sum_j m_{ij}$ and $m_{ij}=\int tdQ_{ij}(t)dt=-q_{ij}^{*'}(0)$
Ⓢ/©	Symbols for Laplace Stieltjes convolution/Laplace convolution
*/**	Symbols for Laplace transformation/Laplace Stieltjes transformation

2.2) The state transition diagram of the system model

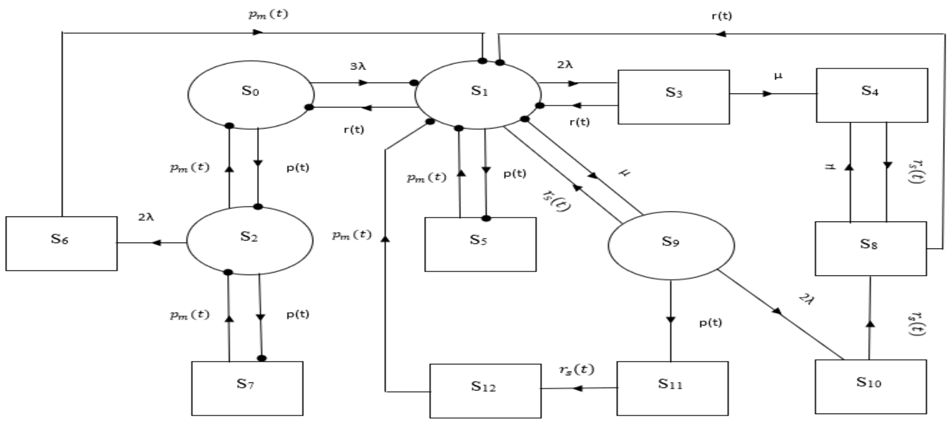

Figure 1: *State Transition Diagram*

3. State Transition Probabilities and Mean Sojourn Time

Table 1: *Transition State Description*

Transition state	Description
S_0(O,O,O)	All three units are in operation
S_1 (Fur ,O,O)	Two units are in operation and one unit is under repair
S_2 (O ,O ,UPm)	Two units are in operation and one unit is under preventive maintenance
S_3 (WO ,FUR ,Fwr)	One unit is waiting for operation. One unit is waiting for repair and one failed unit is under repair continuously from previous state.
S_4(WO ,Fwr ,FWR ,SFut)	One unit is waiting for operation. One unit is waiting for repair and third unit is waiting for repair from previous state. Server is under treatment.
S_5(WO ,Fwr , UPm)	One unit is waiting for operation, one is waiting for repair and one unit is under preventive maintenance.
S_6(WO ,Fwr ,UPM)	One unit is waiting for operation, one is waiting for repair and one unit is under preventive maintenance from previous state.
S_7 (WO, UPM, WPm)	One unit is waiting for operation, one unit is under preventive maintenance from its previous state and one unit is waiting for preventive maintenance for first time.
S_8 (WO, Fur, FWR)	One unit is waiting for operation, failed unit is under repair and one unit is waiting for repair from previous state.
S_9 (O,O, Fwr, SFut)	Two units are in operation. Failed unit is waiting for repair and failed server is under rectification.
S_{10}(WO, FWR ,Fwr ,SFUT)	Failed unit is waiting for repair from previous state another one is waiting for repair for the first time. Server is under rectification from previous state and one unit is waiting for operation.
S_{11}(WO ,WPm ,FWR,SFUT)	Failed unit is waiting for repair from previous state. One unit is waiting for preventive maintenance. Server is under treatment.
S_{12} (WO, FWR, UPm)	One unit is waiting for operation, failed unit is waiting for repair and one unit is under preventive maintenance.

4. Reliability Measures

4.1) Transition Probabilities

Using the formula given below transition probabilities from any state i to j are obtained as follows:

$p_{ij}= Q_{ij}(\infty)= \int_0^\infty q_{ij}(t)dt$

$p_{01}=\frac{\lambda}{\lambda+\alpha}$, $p_{02}=\frac{\alpha}{\lambda+\alpha}$,$p_{10}=\frac{b}{2(\lambda+\alpha)+\mu+b}$, $p_{15}=\frac{2\alpha}{2(\lambda+\alpha)+\mu+b}$, $p_{13}=\frac{2\lambda}{2(\lambda+\alpha)+\mu+b}$, $p_{19}=\frac{\mu}{2(\lambda+\alpha)+\mu+b}$

$p_{20}=\frac{\eta}{2(\lambda+\alpha)+\eta}$, $p_{27}=\frac{2\alpha}{2(\lambda+\alpha)+\eta}$, $p_{26}=\frac{2\lambda}{2(\lambda+\alpha)+\eta}$,$p_{31}=p_{81}=\frac{b}{\mu+b}$, $p_{3,8}=p_{84}=\frac{\mu}{\mu+b}$

$p_{4,8}=p_{51}=p_{61}= p_{72}= p_{10,12}=p_{11.12}=p_{12,1}=1$,$p_{91}=\frac{a}{2(\lambda+\alpha)+a}$, $p_{9,10}=\frac{2\lambda}{2(\lambda+\alpha)+a}$, $p_{9,11}=\frac{2\alpha}{2(\lambda+\alpha)+a}$

It is verified that:

$p_{01}+p_{02}=1$,$p_{10}+p_{15}+p_{19}+p_{13}=1$,$p_{20}+p_{22.7}+p_{21.6}=1$,$p_{31}+p_{3,8}=1$,$p_{81}+p_{84}=1$,$p_{91}+p_{9,10}+p_{9,11}=1$,$p_{4,8}=p_{51}=p_{61}=p_{72}=$ $p_{10,12}=p_{11.12}=p_{12,1}=1$

4.2) Mean Sojourn Time

μ_i in state S_i are given as:

$\mu_0=\frac{1}{3(\lambda+\alpha)}$, $\mu_1=\frac{1}{(2\lambda+2\alpha+\mu+b)}$,$\mu_2=\frac{1}{(2\lambda+2\alpha+\eta)}$, $\mu_3=\mu_8=\frac{1}{\mu+b}$,$\mu_9=\frac{1}{2\lambda+2\alpha+a}$, $\mu_4=\mu_{10}=\mu_{11}=\frac{1}{a}$,$\mu_5=\mu_6=\mu_7=\mu_{12}=\frac{1}{\eta}$

$\mu_1^{'}=m_{10}+m_{15}+m_{19}+m_{11.3}+m_{11.3(4,8)^n}$, $\mu_2^{'}=m_{20}+m_{22.7}+m_{21.6}$,$\mu_9^{'}=m_{91}+m_{91.(11,12)}+m_{91.(10,8)}+m_{91.10(8,4)^n}$

4.3) Mean Time to System Failure

Let $\Phi_i(t)$- cdf of first passage time from regenerative state S_i to a failed state.

$\Phi_0(t)= Q_{01}(t)ⓈΦ_1(t) + Q_{02}(t)ⓈΦ_2(t)$ (1)

$\Phi_1(t) =Q_{10}(t)ⓈΦ_0(t) +Q_{19}(t)ⓈΦ_9(t)+ Q_{15}(t)+Q_{13}(t)$ (2)

$\Phi_2(t)=Q_{20}(t)ⓈΦ_0(t) +Q_{26}(t) + Q_{27}(t)$ (3)

$\Phi_9(t)=Q_{91}(t)ⓈΦ_1(t)+Q_{9,10}(t) + Q_{9,11}(t)$ (4)

Solving the above equations for $\Phi_0^{**}(s)$ by taking Laplace Stieltjes Transformation, We get,

$R^*(s)=\frac{1-\Phi_0^{**}(s)}{s}$

MTSF= $\lim_{n\to\infty}\frac{1-\Phi_0^{**}(s)}{s}=\frac{N_1}{D_1}$

Where $N_1=(1-p_{19}p_{91})(\mu_0+p_{02}\mu_2)+ p_{01}(\mu_1 +p_{19}\mu_9)$ and $D_1= (1-p_{02}p_{20})(1-p_{19}p_{91})-p_{01}p_{10}$

4.4) Long Run Availability of the System

Define, $A_i(t)$- Probability that the system is available at any instant time t given that it has entered regenerative state S_I at time t =0

$M_i(t)$- Probability that the system is up initially as well as at time t in state S_i without making any transition to regenerative state

The following expressions are obtained:

$A_0(t)= M_0(t)+q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$ (6)

$A_1(t)= M_1(t)+q_{10}(t)©A_0(t) + (q_{11.3}+q_{11.3(4,8)^n})©A_1(t)+q_{15}(t)©A_5(t)+q_{19}©A_9(t)$ (7)

$A_2(t)= M_2(t)+q_{20}(t)©A_0(t) + q_{22.7}©A_2(t) +q_{21.6}©A_1(t)$ (8)

$A_9(t)= M_9(t)+(q_{91}(t) + q_{91.(11,12)}+q_{91.10(8,4)^n}+q_{91.(10,8)}©A_1(t)$ (9)

$A_5(t)= q_{51}©A_1(t)$ (10)

Where, $M_0(t)=e^{-3\lambda t}\overline{P(t)}$,$M_1(t)=e^{-(2\lambda+\mu)t}\overline{P(t)R(t)}$, $M_2(t)=e^{-2\lambda t}\overline{P_m(t)P(t)}$, $M_9(t)=e^{-2\lambda t}\overline{P(t)R_s(t)}$

Solving for $A_0^{**}(s)$ using LST

We have,

$A_0(\infty)=\lim_{n\to\infty} sA_0^{**}(s)=\frac{N_2}{D_2}$ (11)

$N_2=p_{10}(1-p_{27})\mu_0+\mu_2p_{02}p_{10}+(p_{01}p_{20}+p_{26})(p_{19}\mu_9+\mu_1)$

$D_2=p_{10}(1-p_{27})\mu_0+\mu'_2p_{02}p_{10}+(p_{01}p_{20}+p_{26})(\mu_1^{'}+\mu_5p_{15}+p_{19}\mu_9^{'})$

4.4) Busy Period of the Server Due to Repair

$BP_i^r(t)$- Probability that the server is busy in repairing at an instant 't'.

The recursive expressions for $BP_i^r(t)$ are given below:

$BP_0^r(t)= q_{01}(t)©BP_1^r(t)+ q_{02}(t)©BP_2^r(t)$ (12)

$BP_1^r(t)= W_1^r(t)+q_{10}(t)©BP_0^r(t) + (q_{11.3}+q_{11.3(4,8)^n})©BP_1^r(t)+q_{15}©BP_5^r(t)+q_{19}©BP_9^r(t)$ (13)

$BP_2^r(t)= q_{20}(t)©BP_0^r(t) + q_{22.7}©BP_2^r(t)+q_{21.6}©BP_1^r(t)$ (14)

$BP_9^r(t)= (q_{91}(t) + q_{91.(11,12)}+q_{91.10(8,4)^n}+q_{91.(10,8)})©BP_1^r(t)$ (15)

$BP_5^r(t)= q_{51}©BP_1^r(t)$ (16)

$W_1^r(t)=e^{-(2\lambda+\mu)t}\overline{P(t)R(t)}$

Solving for $BP_0^{r**}(s)$, We have:

$BP_0^r(\infty)=\lim_{n\to\infty} sBP_0^{r**}(s)=\frac{N_5}{D_2}$,$N_5=W_1^*(0)(p_{20}p_{01}+p_{26})$ and D_2 is already specified.

4.5) Busy Period of the Server due to Preventive Maintenance

$BP_i^{pm}(t)$- Probability that the server is busy in preventive maintenance at an instant 't' Expressions for $BP_i^{pm}(t)$ are given below:

$BP_0^{pm}(t)= q_{01}(t)©BP_1^{pm}(t)+ q_{02}(t)©BP_2^{pm}(t)$ (17)

$BP_1^{pm}(t)=q_{10}(t)©BP_0^{pm}(t) + (q_{11.3}+q_{11.3(4,8)^n})©BP_1^{pm}(t)+q_{15}©BP_5^{pm}(t)+q_{19}©BP_9^{pm}(t)$ (18)

$BP_2^{pm}(t)= W_2^{pm}(t)+q_{20}(t)©BP_0^{r}(t) + q_{22.7}©BP_2^{r}(t)+q_{21.6}©BP_1^{r}(t)$ (19)

$BP_9^{pm}(t)= (q_{91}(t) + q_{91.(11,12)}+q_{91.10(8,4)^n}+q_{91.(10,8)})©BP_1^{pm}(t)$ (20)

$BP_5^{pm}(t)=W_5^{pm}(t)+q_{51}©BP_2^{pm}(t)$ (21)

Where, $W_2^{pm}(t)=e^{-2\lambda t}\overline{P(t)P_m(t)}, W_2^{pm}(t)=\overline{P_m(t)}$

Solving for $BP_0^{pm**}(s)$

We get,

$BP_0^{pm}(\infty)= \lim_{n\to\infty} sBP_0^{pm**}(s)=\frac{N_4}{D_2}$

$N_4=W_2^{pm*}(0)p_{02}p_{10}+W_5^{pm*}(0)p_{02}p_{26}p_{15}$ and D_2 is already specified.

4.6) Expected Number of Repairs (ENR) of the Unit

Let $Rp_i(t)$be the expected number of repairs by the server in (0, t].

The recursive relations for $Rp_i(t)$are given as:

$Rp_0(t)=Q_{01}(t)ⓈRp_1(t) + Q_{02}(t)ⓈRp_2(t)$ (22)

$Rp_1(t)=Q_{10}(t)Ⓢ(1+Rp_0(t)) + (Q_{11.3}+Q_{11.3(4,8)^n})Ⓢ(1+Rp_1(t))+Q_{11.5}ⓈRp_5(t) +Q_{19}ⓈRp_9(t)$ (23)

$Rp_2(t)=Q_{20}(t)ⓈRp_0(t) + Q_{22.7}ⓈRp_2(t) +Q_{21.6}ⓈRp_1(t)$ (24)

$Rp_9(t)=(Q_{91}(t) + Q_{91.(11,12)})ⓈRp_1(t)+(Q_{91.(10,8)}+ Q_{91.10(8,4)^n}))Ⓢ(1+Rp_1(t))$ (25)

$Rp_5(t)= Q_{51}ⓈRp_1(t)$ (26)

Solving for $Rp_0^{**}(s)$ We have,

$Rp_0(\infty)= \lim_{n\to\infty} sRp_0^{**}(s)=\frac{N_5}{D_2}$, $N_5= \left((p_{20}p_{01}+p_{26})(p_{10}+p_{13}+p_{19}p_{9,10})\right)$

4.7) Expected Number of Preventive Maintenance (PM) of the Unit

Let $Pm_i(t)$be the expected number of repairs by the server in (0, t].

The expressions for $Pm_i(t)$are given as:

$Pm_0(t)=Q_{01}(t)ⓈPm_1(t) + Q_{02}(t)ⓈPm_2(t)$ (27)

$Pm_1(t)=Q_{10}(t)ⓈPm_0(t) + (Q_{11.3}+Q_{11.3(4,8)^n})ⓈPm_1(t)+Q_{15}ⓈPm_5(t)) +Q_{19}ⓈPm_9(t)$ (28)

$Pm_2(t)=Q_{20}(t)Ⓢ(1+Pm_0(t)) + Q_{22.7}Ⓢ(1+Pm_2(t)) +Q_{21.6}Ⓢ(1+Pm_1(t)$ (29)

$Pm_9(t)=(Q_{91}(t)+Q_{91.(10,8)}+Q_{91.10(8,4)^n})ⓈPm_1(t)+(Q_{32}+Q_{91.(11,12)})Ⓢ(1+Pm_2(t))$ (30)

$Pm_5(t)= Q_{51}Ⓢ(1+Pm_1(t))$ (31)

Solving for $Pm_0^{**}(s)$(by taking LST)

$Pm_0(\infty)= \lim_{n\to\infty} sPm_0^{**}(s)=\frac{N_6}{D_2}$, $N_6=p_{02}p_{10}+(p_{26}+p_{01}p_{20})\left(p_{15}+p_{9,11}p_{19}\right)$

4.8) Expected Number of Visits of the Server

Let $V_i(t)$be the expected number of visits by the server.

The equations for $V_i(t)$ are as follow:

$V_0(t)=Q_{01}(t)Ⓢ(1+V_1(t)) + Q_{02}(t)Ⓢ(1+V_2(t))$ (32)

$V_1(t)=Q_{10}(t)ⓈV_0(t) + (Q_{11.3}+Q_{11.3(4,8)^n})ⓈV_1(t)+Q_{15}ⓈV_5(t)) +Q_{19}ⓈV_9(t)$ (33)

$V_2(t)=Q_{20}(t)ⓈV_0(t) + Q_{22.7}ⓈV_2(t) +Q_{21.6}ⓈV_1(t)$ (34)

$V_9(t)=(Q_{31}(t) + Q_{91.(11,12)}+Q_{91.10(8,4)^n}++Q_{91.10,8})\{V_1 t\}$ (35)

$V_5(t)= Q_{51}ⓈV_1(t)$ (36)

Solving for $V_0^{**}(s)$

$V_0(\infty)= \lim_{n\to\infty} sV_0^{**}(s)=\frac{N_7}{D_2}$,$N_7=p_{10}\left(p_{20}+p_{26}\right)$

5. Profit Analysis

The profit analysis of the model can be represented as : $P_C=C_0A_0-C_1BP_0^r-C_2BP_0^{pm}-C_3V_0$

Here,C_0, C_1, C_2&C_3 are respectively the revenue per unit time, cost per unit time the service facility is busy for repair, busy for preventive maintenance and costs per unit time visit by the service facility The particular case $p(t)=\alpha e^{-\alpha t}$,$p_m(t)=\eta e^{-\eta t}$, $r_s(t)=ae^{-at}$and $r(t)=be^{-bt}$, $C_0=10000$,$C_1=1000$,$C_2=500$,$C_3=100$ has been considered to obtain the reliability measures MTSF, availability and profit function. The values of these measures have been evaluated for arbitrary values of the parameters since there is no reliable source of information which tells about the actual values of the parameters.

6. Results and Graphical Representation of Reliability Measures

Reliability measures such as MTSF, Availability and Profit have been studied for different values of parameters. The graphs have been plotted for a range (1.1-1.8) of values of preventive maintenance completion rate(η) and corresponding effects have been explained.

6.1) MTSF Vs Rate of Preventive Maintenance (η)

a: Treatment rate of the server
b: Repair rate of the unit
α: Rate by which unit undergoes for preventive maintenance

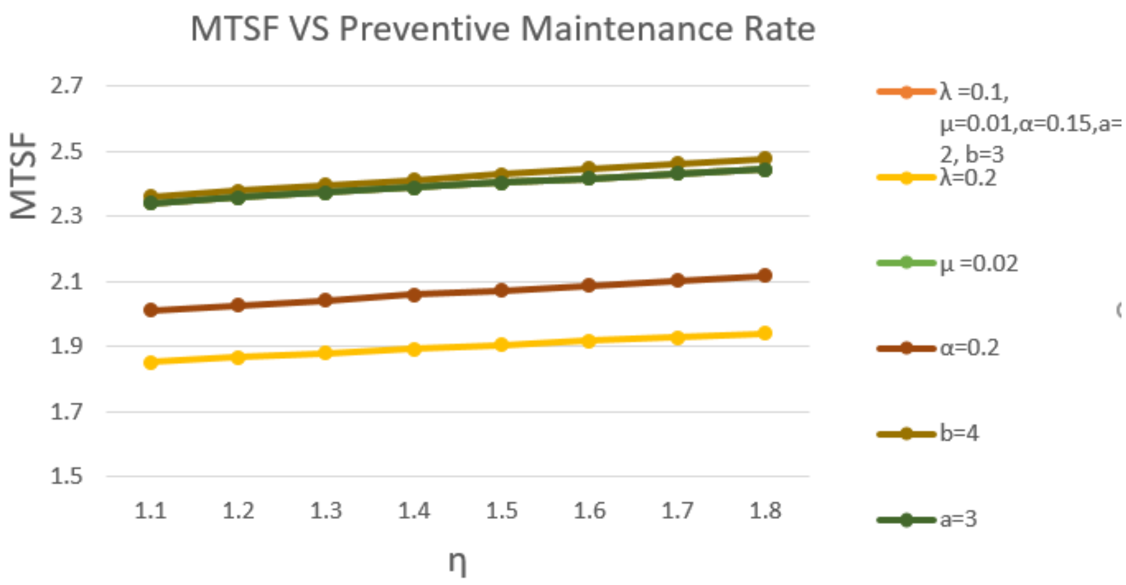

Figure 2: *MTSF vs Preventive Maintenance Rate*

From fig.2 it is quite evident that the MTSF increases with the increase in preventive maintenance rate, repair rate(b) of unit and server(a) as well. However, if the failure rate of the units is increasing (λ) from 0.1 to 0.2, MTSF decreases.

6.2) Availability vs Preventive maintenance Rate

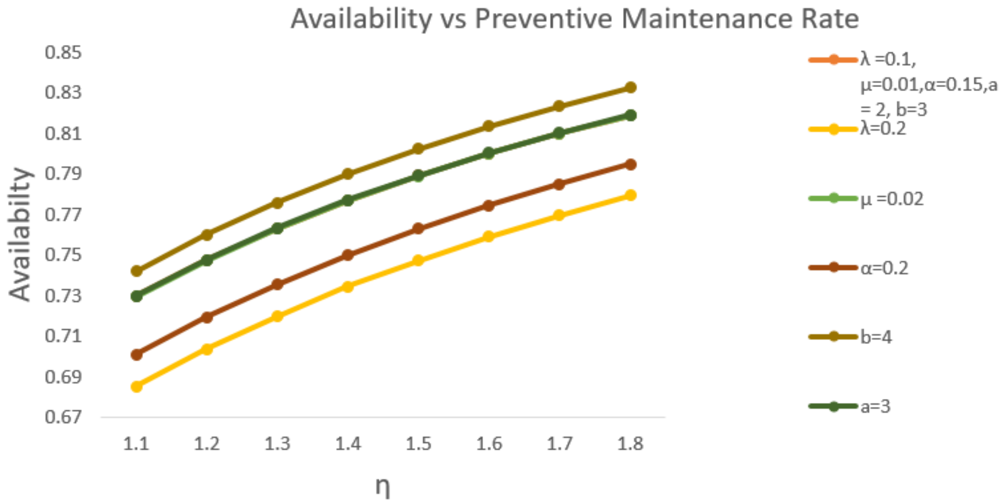

Figure 3: *Availability vs Preventive Maintenance Rate*

Fig.3 shows that availability of the system keeps on increasing if preventive maintenance rate (η) of the units is increased (from 1.1 to 1.8) and decreases with the increase in failure rate of units.

6.3) Profit vs Preventive Maintenance rate (η)

Figure 4: *Profit vs Preventive Maintenance Rate*

Fig.4 shows profit increases steadily with the increase in the preventive maintenance rate (1.1 to 1.8), repair rate (increased from 3 to 4) of the units however it sharply decreases with the increase in failure rates of the units and server.

7. Conclusion

A 2-out-of-3 system has been developed using the ideas of priority for preventive maintenance and conditional failure of the service facility. The MTSF, availability and profit function of this system have been obtained for particular values of the parameters. The study reveals that there is a decline in these measures with the increase of the rate by which unit undergoes for preventive maintenance, failure rates of the units and service facility. However, the values of MTSF, availability and profit function keep on increasing with the increase of treatment rate, repair rate of the unit and preventive maintenance completion rate.

The profit increases if the rate with which a unit completes its preventive maintenance. Hence, implementing the preventive maintenance repair policy for a 2-out-of-3 system is beneficial as it increases the availability and hence the profit of the system.

8.Application of the Proposed Model

Burning of toxic waste, especially waste produced in chemical factories, medical laboratories produce a lot of harmful and lethal gases. To neutralize the toxicity of these kind of gases huge amount of fire is required. Toxic waste incinerator is one such kind of combustion technique which helps in neutralising these harmful gases. It has a circuit named as flame detection circuit which works as a 2 -out-of-3system. The principal behind the working of the incinerator: As long as a sufficient amount of heat is maintained in the incinerator, it is safe to put the waste inside the chamber to neutralize the toxic gases produced due to combustion of waste. If the flame is not sufficient enough to neutralize the toxic gases produced, it would not be safe to keep inserting the waste inside the chambers because the gases produced would exit without being neutralized and may cause a severe health issue to anyone nearby. Therefore, our main focus is on designing the system in such a way that the system detects the sufficient amount of flame and permits waste insertion only when there is sufficient flame to neutralize the exhaust. Due to high risk involved in passing out the waste un neutralized it would be beneficial to make the flame detection system redundant. So that if one sensor fails to detect the flame, other may sense and cover up the risk involved. Hence it is very much necessary to design a system which opens the waste valve only if there is enough flame signalled by the sensors. The best designed system for this kind of incinerator is a 2-Out-of-3 system. There are three sensors to detect the flame which are identical in nature and the valve for injection of waste will open only if two out of three will signal that there is sufficient amount of flame inside the chamber to neutralize the toxic gases produced. If any of the two sensors fails to detect the flame it would lead to hazardous condition therefore it is highly recommended to keep an eye on the working condition of the sensors. Preventive maintenance is one such precautionary measure which ensures the proper functioning of the sensors hence it is kept as priority over repair while developing the system model.

A pictorial presentation of Toxic Waste Incinerator:

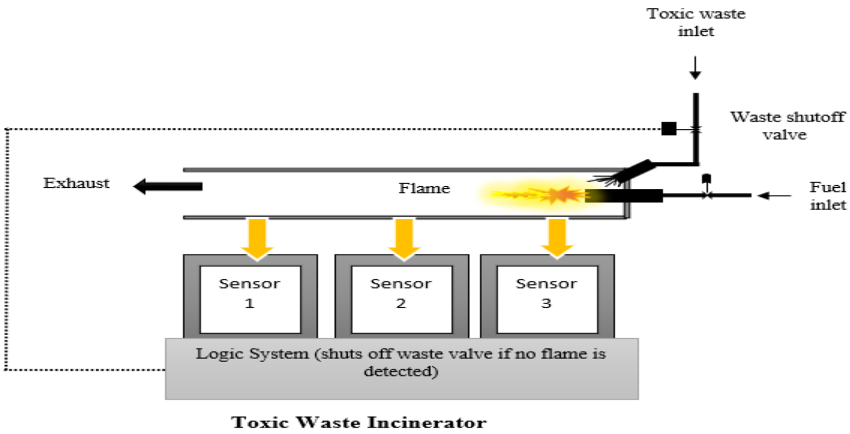

Figure 5: *Toxic Waste Incinerator*

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