A Novel Extended Version of the Ailamujia Inverted Weibull Distribution

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Abstract

Statististical distributions with support on the set of non-negative real numbers are important in modelling and describing the behaviour of lifetime data. Ailamujia distribution is one of the non-negative continuous distribution that has an application in lifetime data. In this paper, a new three-parameter non-negative continuous distribution which is an extension of the Ailamujia Inverted Weibull distribution is introduced. This extended distribution is labeled as the Cubic Transmuted Ailamujia Inverted Weibull distribution. The proposed distribution is derived from the cubic transmuted family of distributions by specifying Ailamujia Inverted Weibull distribution as a baseline distribution. The probability density function of the proposed distribution is derived and some of its plots are presented. It can be observed that the proposed distribution can model the data which are exponentially and skewed unimodal right tailed data. In addition, survival and hazard functions of the proposed distribution are derived. It reveals that the hazard function of the proposed distribution can model both monotonic and non-monotonic decreasing failure rate behaviour of the data. Some properties of the proposed distribution such as its moments, moment generating function, mean, variance are derived. The Maximum Likelihood approach is used to estimate the proposed distribution parameters. Furthermore, parameter estimates as well as the performance of the proposed distribution is investigated by utilizing two sets of lifetime data. For point of comparison, this paper uses the following criteria: Akaike Information Criterion, Bayesian Information Criterion, Kolmogorov - Smirnov statistics, Anderson-Darling and Cramer-von Mises. Results show that for both sets of data, the proposed distribution produce better estimate as compared to the Quasi Suja and the Weibull-Lindley distributions. So, the proposed distribution consider as the best model for modelling the given two real datasets.

Keywords: Ailamujia distribution, Ailamujia Inverted Weibull distribution, Cubic transmuted family of distributions

1. Introduction

Ailamujia distribution is one of the newly lifetime distributions that has many application in engineering [5] . Due to its flexibility in modelling lifetime data then it becomes an area of interest for some researchers.

Different extensions were considered in literature. Uzma [11] introduced the weighted analogue of Ailamujia distribution and derived some of its properties. Other identified extensions such as the area biased distribution [3] , the size biased Ailamujia distribution [8] and the inverse analogue of Ailamujia distribution [1].

Moreover, Smadi [10] proposed an extended version of the Ailamujia distribution called as Ailamujia inverted Weibull distribution (AIWD) and various properties such as reliability function, hazard function, moments, moment generating function, order statistics, mean residual function and Shannon's entropy were studied.

In this paper, a novel extension of the Ailamujia Inverted Weibull distribution called Cubic Transmuted Ailamujia Inverted Weibull distribution (CTAIWD) is derived. Some of its properties such as moments, moment generating function, mean and variance are studied. Maximum Likelihood approach is also discussed to estimate the proposed distribution parameters. Two real data sets are analyzed to evaluate the performance of the proposed distribution.

The rest of paper is organized as follows: The Cubic Transmuted Ailamujia Inverted Weibull (CTAIW) distribution is introduced in section 2. In section 3, some statistical properties of CTAIW distribution are presented. Maximum Likelihood Estimation of the proposed distribution is discussed in section 4. In section 5, the application of proposed distribution is illustrated. Some concluding remarks is presented in section 6.

2. Cubic Transmuted Ailamujia Inverted Weibull distribution

This section presents the derivation of Cubic Transmuted Ailamujia Inverted Weibull (CTAIW) distribution. The cumulative distribution function of the Ailamujia Inverted Weibull distribution is given by

$$F(x,\theta,\alpha) = (1+2\theta x^{-\alpha})e^{-2\theta x^{-\alpha}}, x>0, \alpha>0, \theta>0 \tag{1}$$

with corresponding probability density function

$$f(x,\theta,\alpha) = 4\alpha\theta^2 x^{-2\alpha-1}e^{-2\theta x^{-\alpha}}.$$

Rahman [7] proposed a cubic transmuted family of distributions for extending any continuous distribution. The cumulative distribution function of the cubic transmuted family of distributions is given by

$$F(x) = (1-\lambda)G(x) + 3\lambda G^2(x) - 2\lambda G^3(x), x \in R \tag{2}$$

where $\lambda \in [-1,1]$. The cumulative distribution function of the Cubic Transmuted Ailamujia Inverted Weibull distribution is obtain by inserting (1) into (2) then, we have

$$F(x,\theta,\alpha,\lambda) = (1-\lambda)ze^{-2\theta x^{-\alpha}} + 3\lambda z^2 e^{-4\theta x^{-\alpha}} - 2\lambda z^3 e^{-6\theta x^{-\alpha}}, \tag{3}$$

where $z = 1+2\theta x^{-\alpha}$. The corresponding probability density function is

$$f(x,\theta,\alpha,\lambda) = 4\alpha\theta^2 x^{-2\alpha-1}e^{-2\theta x^{-\alpha}}(1-\lambda+6\lambda z e^{-2\theta x^{-\alpha}} - 6\lambda z^2 e^{-4\theta x^{-\alpha}}). \tag{4}$$

Note that if $\lambda = 0$ then Cubic Transmuted Ailamujia Inverted Weibull distribution reduces to Ailamujia Inverted Weibull distribution.

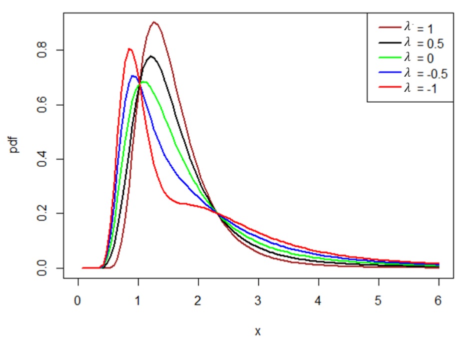

Figure 1: *pdf plots of CTAIW distribution for $\alpha = 1.5$, $\theta = 1.5$ and for varying values of λ*

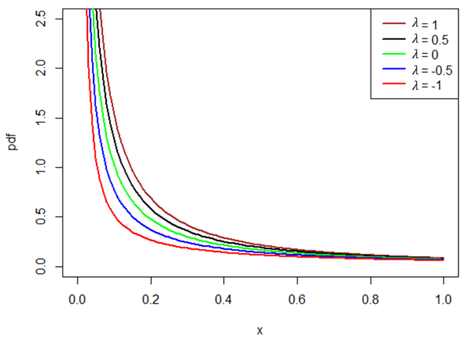

Figure 2: *pdf plots of CTAIW distribution for $\alpha = 0.2$, $\theta = 0.5$ and for varying values of λ*

Figures 1 and 2 present some possible shapes of the pdf of CTAIW distribution. It can be observed that the pdf of proposed distribution can generate a skewed unimodal and exponential shapes which are important in modelling lifetime data.

3. Statistical Properties

In this section, some statistical properties of the Cubic Transmuted Ailamujia Inverted Weibull distribution (CTAIW) such as moments, moment generating function, mean, variance, survival function and hazard function are derived.

3.1. Moments

Theorem 1. The *rth* moment of CTAIW distribution with density (4) is

$$\mu_r' = \lambda(2\theta)^{\frac{r}{\alpha}}[(1-2^{\frac{r}{\alpha}-1}3+3^{\frac{r}{\alpha}-1}2-\frac{1}{\lambda})\Gamma(-\frac{r}{\alpha}+2)-(2^{\frac{r}{\alpha}-2}3-3^{\frac{r}{\alpha}-2}4)\Gamma(\frac{r}{\alpha}+3)+3^{\frac{r}{\alpha}-3}2\Gamma(\frac{r}{\alpha}+4)] \tag{5}$$

The mean and variance are respectively, given by

$$\mu = \lambda(2\theta)^{\frac{1}{\alpha}}[(1-2^{\frac{1}{\alpha}3-1}+3^{\frac{1}{\alpha}2-1}-\tfrac{1}{\lambda})\Gamma(-\tfrac{1}{\alpha}+2)-(2^{\frac{1}{\alpha}-2}3-3^{\frac{1}{\alpha}-2}4)\Gamma(\tfrac{1}{\alpha}+3)+3^{\frac{1}{\alpha}2-3}\Gamma(\tfrac{1}{\alpha}+4)]$$

$$\begin{aligned}\sigma^2 =& \lambda(2\theta)^{\frac{2}{\alpha}}[(1-2^{\frac{2}{\alpha}-1}3+3^{\frac{2}{\alpha}-1}2-\frac{1}{\lambda})\Gamma(-\frac{2}{\alpha}+2)-(2^{\frac{2}{\alpha}-2}3-3^{\frac{2}{\alpha}-2}4)\Gamma(\frac{2}{\alpha}+3)+3^{\frac{2}{\alpha}-3}2\Gamma(\frac{2}{\alpha}+4)]\\ &-(\lambda(2\theta)^{\frac{1}{\alpha}}[(1-2^{\frac{1}{\alpha}-1}3+3^{\frac{1}{\alpha}-1}2-\frac{1}{\lambda})\Gamma(-\frac{1}{\alpha}+2)-(2^{\frac{1}{\alpha}-2}3-3^{\frac{1}{\alpha}-2}4)\Gamma(\frac{1}{\alpha}+3)\\ &+3^{\frac{1}{\alpha}-3}2\Gamma(\frac{1}{\alpha}+4)])^2.\end{aligned}$$

Proof. The rth moment is defined by

$$\begin{aligned}\mu_r' &= E[X^r]\\ &= \int_{-\infty}^{\infty} x^r f(x)dx\\ &= \int_0^{\infty} x^r 4\alpha\theta^2 x^{-2\alpha-1}e^{-2\theta x^{-\alpha}}(1-\lambda+6\lambda(1+2\theta x^{-\alpha})e^{-2\theta x^{-\alpha}}-6\lambda(1+2\theta x^{-\alpha})^2 e^{-4\theta x^{-\alpha}})dx\end{aligned}$$

$$= 4(1-\lambda)\theta^2\alpha \int_0^\infty x^r x^{-2\alpha-1} e^{-2\theta x^{-\alpha}} dx + 24\lambda\theta^2\alpha \int_0^\infty x^r x^{-2\alpha-1}(1+2\theta x^{-\alpha}) e^{-4\theta x^{-\alpha}} dx$$
$$- 24\lambda\theta^2\alpha \int_0^\infty x^r x^{-2\alpha-1}(1+2\theta x^{-\alpha})^2 e^{-6\theta x^{-\alpha}} dx$$
$$= (1-\lambda)[-(2\theta)^{\frac{r}{\alpha}}\Gamma(-\frac{r}{\alpha}+2)] - 6\lambda[\frac{(4\theta)^{\frac{r}{\alpha}}}{4}(\Gamma(-\frac{r}{\alpha}+2)+\frac{1}{2}\Gamma(-\frac{r}{\alpha}+3))]$$
$$+ 6\lambda[-\frac{1}{9}(6\theta)^{\frac{r}{\alpha}}(\Gamma(-\frac{r}{\alpha}+2)+\frac{2}{3}\Gamma(-\frac{r}{\alpha}+3)+\frac{1}{9}\Gamma(-\frac{r}{\alpha}+4))]$$
$$= \lambda(2\theta)^{\frac{1}{\alpha}}[(1-2^{\frac{1}{\alpha}3-1}+3^{\frac{1}{\alpha}2-1}-\frac{1}{\lambda})\Gamma(-\frac{1}{\alpha}+2)-(2^{\frac{1}{\alpha}-2}3-3^{\frac{1}{\alpha}-2}4)\Gamma(\frac{1}{\alpha}+3)+3^{\frac{1}{\alpha}2-3}\Gamma(\frac{1}{\alpha}+4)]$$

The mean of CTAIW distribution is obtained by using $r=1$ in (5) and is

$$\mu = \lambda(2\theta)^{\frac{1}{\alpha}}[(1-2^{\frac{1}{\alpha}3-1}+3^{\frac{1}{\alpha}2-1}-\tfrac{1}{\lambda})\Gamma(-\tfrac{1}{\alpha}+2)-(2^{\frac{1}{\alpha}-2}3-3^{\frac{1}{\alpha}-2}4)\Gamma(\tfrac{1}{\alpha}+3)+3^{\frac{1}{\alpha}2-3}\Gamma(\tfrac{1}{\alpha}+4)]$$

The $2nd$ raw moment of CTAIW distribution is obtained by using $r=2$ in (5) and is

$$\mu_2' = \lambda(2\theta)^{\frac{2}{\alpha}}[(1-2^{\frac{2}{\alpha}-1}3+3^{\frac{2}{\alpha}-1}2-\tfrac{1}{\lambda})\Gamma(-\tfrac{2}{\alpha}+2)-(2^{\frac{2}{\alpha}-2}3-3^{\frac{2}{\alpha}-2}4)\Gamma(\tfrac{2}{\alpha}+3)+3^{\frac{2}{\alpha}-3}2\Gamma(\tfrac{2}{\alpha}+4)]$$

The variance σ^2 of CTAIW distribution obtained as

$$\sigma^2 = \mu_2' - (\mu_1')^2$$
$$= \lambda(2\theta)^{\frac{2}{\alpha}}[(1-2^{\frac{2}{\alpha}-1}3+3^{\frac{2}{\alpha}-1}2-\frac{1}{\lambda})\Gamma(-\frac{2}{\alpha}+2)-(2^{\frac{2}{\alpha}-2}3-3^{\frac{2}{\alpha}-2}4)\Gamma(\frac{2}{\alpha}+3)+3^{\frac{2}{\alpha}-3}2\Gamma(\frac{2}{\alpha}+4)]$$
$$-(\lambda(2\theta)^{\frac{1}{\alpha}}[(1-2^{\frac{1}{\alpha}-1}3+3^{\frac{1}{\alpha}-1}2-\frac{1}{\lambda})\Gamma(-\frac{1}{\alpha}+2)-(2^{\frac{1}{\alpha}-2}3-3^{\frac{1}{\alpha}-2}4)\Gamma(\frac{1}{\alpha}+3)$$
$$+3^{\frac{1}{\alpha}-3}2\Gamma(\frac{1}{\alpha}+4)])^2.$$

■

3.2. Moment Generating Function

Theorem 2. Let X follows the CTAIW distribution then the moment generating function $M_X(t)$ is given by

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r \lambda (2\theta)^{\frac{r}{\alpha}}}{r!}[a\Gamma(-\frac{r}{\alpha}+2)-b\Gamma(\frac{r}{\alpha}+3)+c\Gamma(\frac{r}{\alpha}+4)]$$

where $a = 1-2^{\frac{r}{\alpha}-1}3+3^{\frac{r}{\alpha}-1}2-\frac{1}{\lambda}$, $b = 2^{\frac{r}{\alpha}-2}3-3^{\frac{r}{\alpha}-2}4$, $c = 3^{\frac{r}{\alpha}-3}2$ and $t \in R$.

Proof. By definition of moment generating function and equation (5), we have

$$M_X(t) = \mathbb{E}(e^{tX}) = \int_0^\infty e^{tx} f(x,\theta,\alpha,\lambda)dx.$$

Recall that $e^{tx} = \sum_{r=0}^\infty \frac{t^r}{r!}x^r$ then we have

$$M_X(t) = \int_0^\infty \sum_{r=0}^\infty \frac{t^r}{r!}x^r f(x,\theta,\alpha,\lambda)dx = \sum_{r=0}^\infty \frac{t^r}{r!}x^r \int_0^\infty f(x,\theta,\alpha,\lambda)dx = \sum_{r=0}^\infty \frac{t^r}{r!}\mu_r'.$$

Thus,

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r \lambda (2\theta)^{\frac{r}{\alpha}}}{r!}[a\Gamma(-\frac{r}{\alpha}+2)-b\Gamma(\frac{r}{\alpha}+3)+c\Gamma(\frac{r}{\alpha}+4)]$$

where $a = 1-2^{\frac{r}{\alpha}-1}3+3^{\frac{r}{\alpha}-1}2-\frac{1}{\lambda}$, $b = 2^{\frac{r}{\alpha}-2}3-3^{\frac{r}{\alpha}-2}4$ and $c = 3^{\frac{r}{\alpha}-3}2$. ■

3.3. Reliability Analysis

Let X be a random variable with cdf (3) and pdf (4) then the survival $S(x)$ and hazard $h(x)$ functions of CTAIW distribution are respectively, given as follows:

$$S(x,\theta,\alpha,\lambda) = 1-(1-\lambda)ze^{-2\theta x^{-\alpha}} - 3\lambda z^2 e^{-4\theta x^{-\alpha}} + 2\lambda z^3 e^{-6\theta x^{-\alpha}};$$

$$h(x,\theta,\alpha,\lambda) = \frac{4\alpha\theta^2 x^{-2\alpha-1} e^{-2\theta x^{-\alpha}} (1-\lambda+6\lambda z e^{-2\theta x^{-\alpha}} - 6\lambda z^2 e^{-4\theta x^{-\alpha}})}{1-(1-\lambda)ze^{-2\theta x^{-\alpha}} - 3\lambda z^2 e^{-4\theta x^{-\alpha}} + 2\lambda z^3 e^{-6\theta x^{-\alpha}}},$$

where $z = 1+2\theta x^{-\alpha}$, $x > 0$,$\alpha,\theta > 0$ and $\lambda \in [-1,1]$.

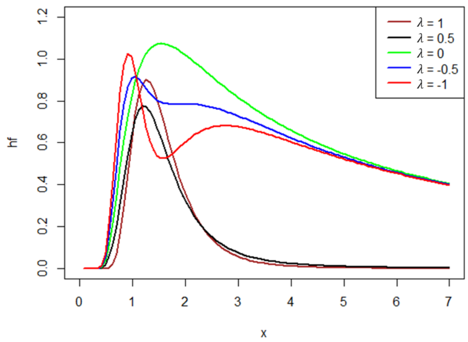

Figure 3: *hf plots of CTAIW distribution for* $\alpha = 1.5$, $\theta = 1.5$ *and for varying values of* λ

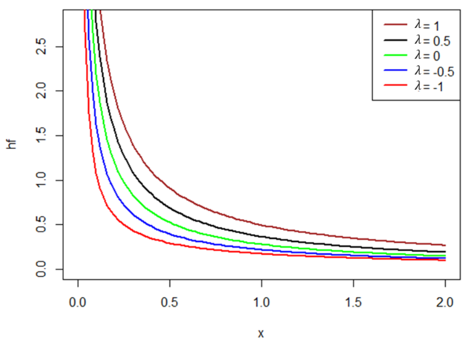

Figure 4: *hf plots of CTAIW distribution for* $\alpha = 0.2$, $\theta = 0.5$ *and for varying values of* λ

Figures 3 and 4 present some possible shapes of hazard rate function of CTAIW distribution. It reveals that the hazard rate function of the proposed distribution can model both monotonic and non-monotonic decreasing failure rate behavior of the data which are common in the reliability data.

4. Maximum Likelihood Estimation

In this section, the maximum likelihood approach is used to estimate the CTAIW distribution parameters.

Let X_1, X_2,..., X_n be a random sample of size n from CTAIW distribution, then the likelihood function is

$$\mathbf{L} = \prod_{i=1}^{n} [4\alpha\theta^2 x_i^{-2\alpha-1} e^{-2\theta x_i^{-\alpha}} (1-\lambda+6\lambda z e^{-2\theta x_i^{-\alpha}} - 6\lambda z^2 e^{-4\theta x_i^{-\alpha}})]$$

where $z = 1 + 2\theta x_i^{-\alpha}$. The log-likelihood function is

$$l = \sum_{i=1}^{n} log[4\alpha\theta^2 x_i^{-2\alpha-1} e^{-2\theta x_i^{-\alpha}} (1 - \lambda + 6\lambda z e^{-2\theta x_i^{-\alpha}} - 6\lambda z^2 e^{-4\theta x_i^{-\alpha}})]. \quad (6)$$

Taking the deivative of (6) with respect to the parameters α, θ and λ then, we have the following:

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - 2\sum_{i=1}^{n} log(x_i) + 2\theta \sum_{i=1}^{n} x_i^{-\alpha} log(x_i) + 24\lambda\theta^2 \sum_{i=1}^{n} \frac{log(x_i) x^{-2\alpha} e^{-2\theta x_i^{-\alpha}} (1 - 2z e^{-2\theta x_i^{-\alpha}})}{1 - \lambda - 6\lambda z e^{-2\theta x_i^{-\alpha}} - 6\lambda z^2 e^{-4\theta x_i^{-\alpha}}} \quad (7)$$

$$\frac{\partial l}{\partial \theta} = \frac{2n}{\theta} - 2\sum_{i=1}^{n} x_i^{-\alpha} - 24\lambda\theta \sum_{i=1}^{n} \frac{x_i^{-2\alpha} e^{-2\theta x_i^{-\alpha}} (1 - 2z e^{-2\theta x_i^{-\alpha}})}{1 - \lambda - 6\lambda z e^{-2\theta x_i^{-\alpha}} - 6\lambda z^2 e^{-4\theta x_i^{-\alpha}}} \quad (8)$$

$$\frac{\partial l}{\partial \lambda} = -\sum_{i=1}^{n} \frac{1 - 6z e^{-2\theta x_i^{-\alpha}} + 6z^2 e^{-4\theta x_i^{-\alpha}}}{1 - \lambda - 6\lambda z e^{-2\theta x_i^{-\alpha}} - 6\lambda z^2 e^{-4\theta x_i^{-\alpha}}} \quad (9)$$

Equating equations (7), (8) and (9) to 0, respectively, one can get numerical maximum likelihood estimates of the CTAIW distribution parameters.

5. Application

In this section, the proposed distribution is applied to two real data sets and compared with the following distributions:

- Quasi Suja (QS) distribution [9]

$$f(x, \alpha, \theta) = \frac{\theta^4}{\alpha\theta^3 + 24}(\alpha + \theta x^4)e^{-\theta x}, x > 0, \theta > 0, \alpha > 0.$$

- Weibull-Lindley (WL) distribution [2]

$$f(x, \alpha, \beta) = \alpha(\beta x^{\beta-1}(1 + x)e^{x^\beta})e^{-\alpha((1+x)e^{x^\beta}-1)}, x > 0, \alpha, \beta > 0.$$

A data analysis is performed using a package "fitdistrplus" in R. Moreover, AKaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Kolmogorov-Smirnov (K-S), Anderson-Darling (A) and Cramer-von Mises (W*) statistics are used for comparison in this analysis. In addition, the two sets of data used in the analysis are given as follows:

Data Set 1. This data set is from Murthy [6]. It is a set of failure times of 50 electronic components. The data set is given as follows: 0.036, 0.058, 0.061, 0.074, 0.078, 0.086, 0.102, 1.103, 0.114, 0.116, 0.148, 0.183, 0.192, 0.254, 0.262, 0.379, 0.381, 0.538, 0.570, 0.574, 0.590, 0.618, 0.645, 0.961, 1.228, 1.600, 2.006, 2.054, 2.804, 3.058, 3.076, 3.147, 3.625, 3.704, 3.931, 4.073, 4.393, 4.534, 4.893, 6.274, 6.816, 7.896, 7.904, 8.022, 9.337, 10.940, 11.020, 13.880, 14.730 and 15.080.

Data Set 2. This dataset is from Khan [4] and it is an ICU data set which assess the intensive care unit (ICU) patients agitation-sedation (A-S) status. The data set is given as follows: 9, 3, 27, 8, 4, 3, 4, 3, 23, 3, 3, 4, 28, 18, 19, 6, 3, 26, 3, 12, 6, 9, 43, 4, 4, 3, 5, 12, 4, 36, 6, 8, 6, 5, 3, 3 and 33.

Tables 1 and 3 list the MLEs of the CTAIW, QS and WL distributions fitted to first and second sets of data. Tables 2 and 4 indicate that the proposed distribution has a lower values of the AIC, BIC, K-S, A and W* compared to the QS and WL distributions for both sets of data. So, the propsed distribution fit well the said datasets. Furthermore, the estimated pdf of the fitted models to both sets of data are presented in figure 5 and 6, respectively.

Table 1: *MLEs of the fitted models for first set of data.*

Distribution	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$
CTAIW	0.5116727		0.6782907	−0.9999998
WL	0.07258343	0.12084431		
QS	211.8090902		0.6493223	

Table 2: *Numerical values of AIC, BIC, K-S, A and W* of the fitted models for first set of data.*

Distribution	AIC	BIC	$K-S$	A	$W*$
CTAIW	209.0186	214.7547	0.1531139	1.3078260	0.2494361
WL	228.7096	232.5337	0.2500551	3.3786131	0.5503281
QS	213.2069	217.031	0.2179699	3.6483026	0.4827523

Table 3: *MLEs of the fitted models for second set of data.*

Distribution	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$
CTAIW	1.1988356		8.5119091	−0.9999997
WL	0.02157018	0.11712539		
QS	20023.91		0.1678021	

Table 4: *Numerical values of AIC, BIC, K-S, A and W* of the fitted models for second set of data.*

Distribution	AIC	BIC	$K-S$	A	$W*$
CTAIW	232.7614	237.5941	0.1724946	1.4724752	0.2293943
WL	256.3175	259.5393	0.2192403	2.3094837	0.4026443
QS	253.2686	256.4904	0.3155350	2.5730059	0.4309077

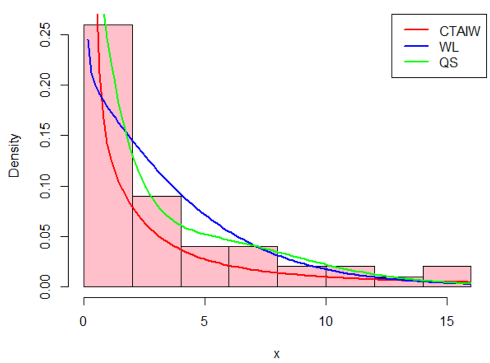

Figure 5: *Estimated pdf of the fitted models for the first set of data.*

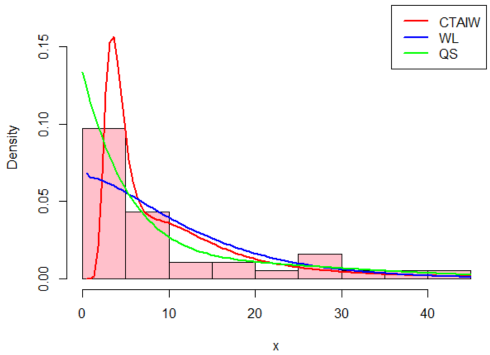

Figure 6: *Estimated pdf of the fitted models for the second set of data.*

6. Concluding Remarks

In this study, an extended version of the Ailamujia Inverted Weibull distribution called Cubic Transmuted Ailamujia Inverted Weibull distribution has been introduced. Some properties of the proposed distribution such as moment, moment generating function, mean and variance were derived. The maximum likelihood approach was used to estimate the proposed distribution parameters. Two real datasets were used to examine the flexibility of proposed distribution and compared to Quasi Suja and Weibull - Lindley distributions. It was found that the proposed distribution fit well the given data sets compared to said distributions.

References

[1] Aijaz, A., Ahmad, A. and Tripathi, R. (2020). Inverse analogue of Ailamujia distribution with statistical properties and applications. *Asian Research Journal of Mathematics*, 16:36–46.

[2] Chacko, V. M., S, D. K, Thomas, B. and C, R. (2018) Weibull-Lindley Distribution: A Bathtub Shaped Failure Rate Model. *Reliability: Theory and Application*, 13:9–20.

[3] Jayakumar B. and Elangovan, R. A. (2019). New generalization of Ailamujia distribution with applications in bladder cancer data. *IJSRMSS*, 6:61–68.

[4] Khan, M. S., King, R. and Hudson, I. L. (2017). Transmuted generalized exponential distribution: A generalization of the exponential distribution with applications to survival data. *Communication Statistics : Simulation and Computation*, 46:4377–4398.

[5] Kotz, S. L. and Pensky, M. The stress strength model and its generalizations, Theory and Applications, World Scientific, Singapore, 2003.

[6] Murthy, D. N. P., Xie, M., Jiang, R. Weibull Models, John Wiley and sons Inc, Hoboken, 2004.

[7] Rahman, M. M., Al-Zahrani, B. and Shahbaz, S. H. (2019). Cubic Transmuted Uniform Distribution: An Alternative to Beta and Kumaraswamy Distributions. *European Journal of Pure and Applied Mathematics*, 12:1106–1121.

[8] Rather, A., Subramanian, C., Shafi, S., Malik, K. A., Ahmad, P. J., Para, B. A. and Jan, T. A. (2018). New size biased distribution with Applications in Engineering and Medical Science. *IJSRMSS*, 5:75–85.

[9] Shanker, R., Upadhyay, R. and Shukla, K. K. (2022). A Quasi Suja Distribution. *Reliablilty: Theory and Application*,17:162–178.

[10] Smadi, M. M. and Ansari, S. I. (2022). Ailamujia Inverted Weibull Distribution with Application to Lifetime Data. *Pakistan Journal of Statistics*, 38:341–358.

[11] Uzma, J., Kawsar, F. and Ahmad, S. P. (2017). On weighted Ailamujia distribution and its applications to lifetime data. *Journal of Statistics Applications and Probability an International Journal*, 6:619–633.