MARKOV APPROACH FOR RELIABILITY AND AVAILABILITY ANALYSIS OF A FOUR UNIT REPAIRABLE SYSTEM

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Abstract

Efforts have been made to analyze reliability and availability of a repairable system using Markov approach. The system has four non-identical units which work simultaneously. The system is assumed as completely non-functional at the failure of all the units. The failure and repair times as usual follow negative exponential distribution. The reliability measures of the system have been obtained by solving the Chapman-Kolmogorov equations using Laplace transform technique. The values of availability, reliability and mean time to system failure have been evaluated for particular values of the parameters considering all the units identical in nature. The effect of failure rate, repair rate and operating time on reliability, MTSF and availability has been studied. The application of the work has also been discussed with a real life example.

Keywords: Repairable System, Non-Identical Units, Markov Approach, Reliability Measures, Chapman-Kolmogorov Equations, Laplace Transform Technique

I. Introduction

In the era of fast-growing technology, everyone is interested to buy such a system which has a smaller number of design defectives and works as per the expectations. Therefore, the prime responsibility of the manufacturers is to produce the reliable products in order to stay for long in the competitive market. Today, the purpose of the system designers and reliability engineers is not only to develop a reliable system but also to identify the techniques that can be used to improve the system reliability. Over the years, several reliability improvement techniques have been evolved including provision of spare units, proper structure of the components, appropriate repair-maintenance policies and use of high-quality components. As a result of which researchers have also been succeeded in pointing out the guidelines for enhancing availability of the systems. It has been revealed that the availability of systems can be improved using the concept of redundancy in cold standby or in parallel mode.

There exist many systems in which functioning of the components (or units) are required in parallel mode not only to share the working stress but also minimize the failure risk of the systems. It has been observed that the use of some important methodologies including semi-Markov process and regenerative point technique has been made extensively by the researchers to analyze the well known reliability measures such as reliability, MTSF and availability of the repairable and non- repairable systems. The use of Markov approach has also been made by the researchers to assess reliability of systems with different structural designs of components. However, not much attention has been given by the researchers for analyzing reliability and availability of repairable systems using Markov approach. Chao et al. [1] carried out the reliability of large series system using Markov structure. El-Damcese et al. [2] analyzed a parallel repairable system with different failure modes. Umamaheshwari et al. [9] discussed a Markov model with human error and common cause. Li [4] obtained the reliability of a redundant system. Kalaiarasi et al. [3] analyzed system reliability using Markov Technique. Nandal and Bhardwaj [5] analyzed the profit of a parallel cold standby system using Lindley distribution. Saritha et al. [8] considered the reliability and availability for non-repairable & repairable systems using Markov modeling. Wang et al. [10] evaluated the reliability for multi state Markov repairable system. Reni et al. [7] studied the reliability of Markov models. Rathi et al. [6] discussed the reliability characteristics of a parallel system with priority concept. In that paper, authors considered that at least two, three and four modules must operate for the successful operation of the system. But they have not considered the case of repairable system and also when at least one module must work as we know that parallel system will work until it's all units fail.

Here we describe reliability and availability of a repairable system using Markov approach. The system has four non-identical units which work simultaneously. The system is assumed as completely non-functional at the failure of all the units. The failure and repair times as usual follow negative exponential distribution. The reliability measures of the system have been obtained by solving the Chapman-Kolmogorov equations using Laplace transform technique. The values of availability, reliability and mean time to system failure have been evaluated for particular values of the parameters considering all the units identical in nature. The effect of failure rate, repair rate and operating time on reliability, MTSF and availability has been studied. The application of the work has also been discussed with a real life example.

II. Assumptions and State Descriptions

1. The transition rates of the units follow negative exponential distribution
2. The repair of the failed unit is done immediately at the availability of the repair facility.
3. The system is declared failed at the failure of all units.
4. All the units are operative at time 't' in state '0' of the system.
5. The system is in state '1' at time 't' upon the failure of one unit, the repair is made immediately and other units are still in operation.
6. The system is in state '2' at time 't' upon the failure of two units and one of the failed units went into repair immediately and third unit is still operating.
7. The system is in state '3' at time 't' upon the failure of three units and one of the failed units went into repair immediately and fourth unit is still operating
8. All the units are failed at time 't' in state '4' of the system.

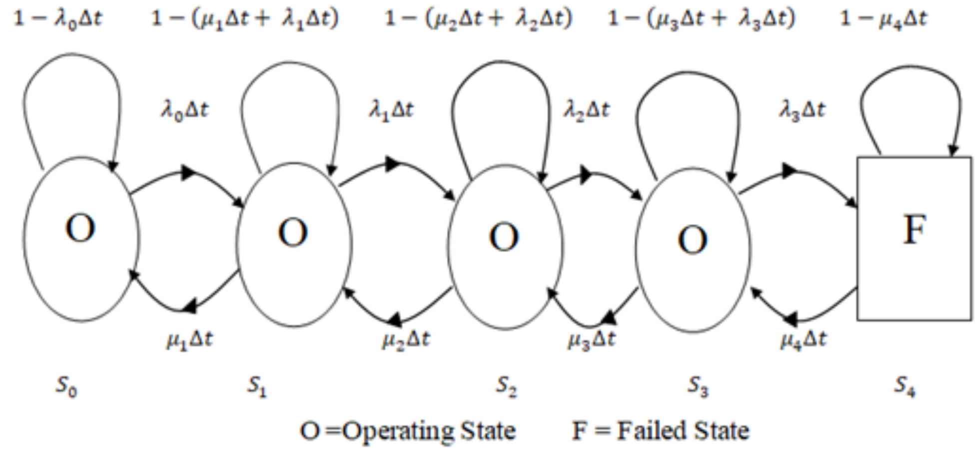

Figure 1: *State Transition Diagram*

a) Notations and Abbreviations

$\boldsymbol{\lambda_x}$	Failure rate of the system at state where x units have failed (x=0,1,2,3)
$\boldsymbol{\mu_y}$	Repair rate of the system at state where y units have failed (y=1,2,3,4)
$\boldsymbol{S_x}$	States (x=0,1,2,3,4)
t	Time
$\boldsymbol{P_x(t)}$	Probability that the system is in state x at time t (x=0,1,2,3,4)
$\boldsymbol{A}(\infty)$	Steady State Availability of the system
R(t)	Reliability of the system
MTSF	Mean Time to System Failure

III. Reliability Measures of the System

a) Reliability

In passing from state i at time s to state j at time t (s<t), we must pass through some intermediate state k at some intermediate time u. When the continuous-time Markov chain is homogeneous, the Chapman-Kolmogorov equation may be written as:

$$P_{ij}(t+\Delta t)=\sum_{all\ k} P_{ik}(t)P_{kj}(\Delta t) \qquad for\ t,t\geq 0$$

$$=\sum_{k\neq j} P_{ik}(t)P_{kj}(\Delta t)+P_{ik}(t)P_{kj}(\Delta t) \tag{1}$$

The Chapman- Kolmogorov equations of the system can also be obtained from expression (1) as

$$P_0(t+\Delta t)=P_0(t)(1-\lambda_0\Delta t)+P_1(t)\mu_1\Delta t$$
$$P_1(t+\Delta t)=P_0(t)\lambda_0\Delta t+P_1(t)\big(1-(\mu_1\Delta t+\lambda_1\Delta t)\big)+P_2(t)\mu_2\Delta t$$
$$P_2(t+\Delta t)=P_1(t)\lambda_1\Delta t+P_2(t)\big(1-(\mu_2\Delta t+\lambda_2\Delta t)\big)+P_2(t)\mu_3\Delta t$$
$$P_3(t+\Delta t)=P_2(t)\lambda_2\Delta t+P_3(t)\big(1-(\mu_3\Delta t+\lambda_3\Delta t)\big)$$
$$P_4(t+\Delta t)=P_3(t)\lambda_3\Delta t+P_4(t) \tag{2-6}$$

These Markov equations are being developed by taking the probability of each state at time $t+\Delta t$. Above equations (2-6) can be rewritten as

$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t}=P_0(t)(-\lambda_0)+P_1(t)\mu_1$$
$$\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t}=P_0(t)\lambda_0+P_1(t)\big(-(\mu_1+\lambda_1)\big)+P_2(t)\mu_2$$
$$\frac{P_2(t+\Delta t)-P_2(t)}{\Delta t}=P_1(t)\lambda_1+P_2(t)\big(-(\mu_2+\lambda_2)\big)+P_2(t)\mu_3$$

$$\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t} = P_2(t)\lambda_2 + P_3(t)\big(-(\mu_3+\lambda_3)\big)$$
$$\frac{P_4(t+\Delta t)-P_4(t)}{\Delta t} = P_3(t)\lambda_3 \qquad (7\text{-}11)$$

Converting these equations (7-11) to a differential equation and taking Lim $\Delta t \to 0$, we get

$$P_0'(\mathrm{t}) = P_0(t)(-\lambda_0) + P_1(t)\mu_1$$
$$P_1'(\mathrm{t}) = P_0(t)\lambda_0 + P_1(t)\big(-(\mu_1+\lambda_1)\big) + P_2(t)\mu_2$$
$$P_2'(\mathrm{t}) = P_1(t)\lambda_1 + P_2(t)\big(-(\mu_2+\lambda_2)\big) + P_3(t)\mu_3$$
$$P_3'(\mathrm{t}) = P_2(t)\lambda_2 + P_3(t)\big(-(\mu_3+\lambda_3)\big)$$
$$P_4'(\mathrm{t}) = P_3(t)\lambda_3 \qquad (12\text{-}16)$$

Above equations (12-16) can be solved by using LT method.

$$sp_0(\mathrm{s}) - P_0(0) = p_0(s)(-\lambda_0) + p_1(s)\mu_1$$
$$sp_1(\mathrm{s}) - P_1(0) = p_0(s)\lambda_0 + p_1(s)\big(-(\mu_1+\lambda_1)\big) + p_2(s)\mu_2$$
$$sp_2(\mathrm{s}) - P_2(0) = p_1(s)\lambda_1 + p_2(s)\big(-(\mu_2+\lambda_2)\big) + p_3(s)\mu_3$$
$$sp_3(\mathrm{s}) - P_3(0) = p_2(s)\lambda_2 + p_3(s)\big(-(\mu_3+\lambda_3)\big)$$
$$sp_4(\mathrm{s}) - P_4(0) = p_3(s)\lambda_3 \qquad (17\text{-}21)$$

Boundary conditions are

$$P_0(0) = 1, P_1(0) = 0, P_2(0) = 0, P_3(0) = 0, P_4(0) = 0$$

So, the system of equations will be

$$(s+\lambda_0)p_0(\mathrm{s}) - p_1(s)\mu_1 = 1$$
$$-\lambda_0 p_0(s) + (s+\mu_1+\lambda_1)p_1(\mathrm{s}) - \mu_2 p_2(s) = 0$$
$$-\lambda_1 p_1(s) + (s+\mu_2+\lambda_2)p_2(\mathrm{s}) - \mu_3 p_3(s) = 0$$
$$-\lambda_2 p_2(s) + (s+\mu_3+\lambda_3)p_3(\mathrm{s}) = 0$$
$$-\lambda_3 p_3(\mathrm{s}) + (s)p_4(\mathrm{s}) = 0$$

It can be written as

$$\begin{bmatrix} s+\lambda_0 & -\mu_1 & 0 & 0 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & 0 \\ 0 & 0 & 0 & -\lambda_3 & s \end{bmatrix} \begin{bmatrix} p_0(\mathrm{s}) \\ p_1(\mathrm{s}) \\ p_2(\mathrm{s}) \\ p_3(\mathrm{s}) \\ p_4(\mathrm{s}) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, solving for $p_0(\mathrm{s}), p_1(\mathrm{s}), p_2(\mathrm{s}), p_3(\mathrm{s})$ *and* $p_4(\mathrm{s})$ using Cramer's Rule, we have

$$\Delta_R = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 0 & 0 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & 0 \\ 0 & 0 & 0 & -\lambda_3 & s \end{vmatrix}$$

$$= s(s^4 + s^3(\lambda_0+\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3) + s^2(\lambda_0\lambda_1+\lambda_0\lambda_2+\lambda_0\lambda_3+\lambda_0\mu_2+\lambda_0\mu_3+\lambda_1\lambda_2+ \lambda_1\lambda_3+\lambda_1\mu_3+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_3\mu_1+\lambda_3\mu_2+\mu_1\mu_2+\mu_1\mu_3+\mu_2\mu_3) + s\big(\lambda_0\lambda_1\lambda_2+\lambda_0\lambda_1\lambda_3+ \lambda_0\lambda_2\lambda_3+\lambda_0\lambda_3\mu_2+\lambda_0\lambda_1\mu_3+\lambda_0\mu_2\mu_3+\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_1+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3\big)+\lambda_0\lambda_1\lambda_2\lambda_3)$$
$$= s(s-m)(s-n)(s-o)(s-p)$$

Here m, n ,o and p are roots of Δ_R.

$$\Delta_{R_1} = \begin{vmatrix} 1 & -\mu_1 & 0 & 0 & 0 \\ 0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & 0 \\ 0 & 0 & 0 & -\lambda_3 & s \end{vmatrix}$$

$$\Delta_{R_1} = s(s^4 + s^3(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3) + s^2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\mu_3+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_3\mu_1+ \lambda_3\mu_2+\mu_1\mu_2+\mu_1\mu_3+\mu_2\mu_3) + s\big(\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_1+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3\big))$$

$$\Delta_{R_2} = \begin{vmatrix} s+\lambda_0 & 1 & 0 & 0 & 0 \\ -\lambda_0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & 0 \\ 0 & 0 & 0 & -\lambda_3 & s \end{vmatrix}$$

$$\Delta_{R_2} = \lambda_0 s^3 + \lambda_0 s^2(\lambda_2 + \lambda_3 + \mu_2 + \mu_3) + \lambda_0 s(\lambda_2\lambda_3 + \lambda_3\mu_2 + \mu_2\mu_3)$$

$$\Delta_{R_3} = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 1 & 0 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & -\mu_3 & 0 \\ 0 & 0 & 0 & s+\lambda_3+\mu_3 & 0 \\ 0 & 0 & 0 & -\lambda_3 & s \end{vmatrix}$$

$$\Delta_{R_3} = \lambda_0\lambda_1 s^2 + s(\lambda_0\lambda_1\lambda_3 + \lambda_0\lambda_1\mu_3)$$

$$\Delta_{R_4} = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 0 & 1 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & 0 & 0 \\ 0 & 0 & -\lambda_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & s \end{vmatrix}$$

$$\Delta_{R_4} = \lambda_0\lambda_1\lambda_2 s$$

$$\Delta_{R_5} = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 0 & 0 & 1 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & 0 \\ 0 & 0 & 0 & -\lambda_3 & 0 \end{vmatrix}$$

$$\Delta_{R_5} = \lambda_0\lambda_1\lambda_2\lambda_3$$

Now,

$$p_0(\mathrm{s}) = \frac{\Delta_{R_1}}{\Delta_R}$$

$$= \frac{\begin{array}{c}[s(s^4 + s^3(\lambda_1 + \lambda_2 + \lambda_3 + \mu_1 + \mu_2 + \mu_3) \\ + s^2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\mu_3 + \lambda_2\lambda_3 + \lambda_2\mu_1 + \lambda_3\mu_1 + \lambda_3\mu_2 + \mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3) \\ + s\left(\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\mu_1 + \lambda_3\mu_1\mu_2 + \mu_1\mu_2\mu_3\right))]\end{array}}{\Delta_R}$$

$$p_1(\mathrm{s}) = \frac{\Delta_{R_2}}{\Delta_R}$$

$$= \frac{\lambda_0 s^3 + \lambda_0 s^2(\lambda_2 + \lambda_3 + \mu_2 + \mu_3) + \lambda_0 s(\lambda_2\lambda_3 + \lambda_3\mu_2 + \mu_2\mu_3)}{\Delta_R}$$

$$p_2(\mathrm{s}) = \frac{\Delta_{R_3}}{\Delta_R}$$

$$= \frac{\lambda_0 s^3 + \lambda_0 s^2(\lambda_2 + \lambda_3 + \mu_2 + \mu_3) + \lambda_0 s(\lambda_2\lambda_3 + \lambda_3\mu_3 + \mu_2\mu_3)}{\Delta_R}$$

$$p_3(\mathrm{s}) = \frac{\Delta_{R_4}}{\Delta_R}$$

$$= \frac{\lambda_0\lambda_1 s^2 + s(\lambda_0\lambda_1\lambda_3 + \lambda_0\lambda_1\mu_3)}{\Delta_R}$$

$$p_4(\mathrm{s}) = \frac{\Delta_{R_5}}{\Delta_R}$$

$$= \frac{\lambda_0\lambda_1\lambda_2\lambda_3}{\Delta_R}$$

Taking Laplace Inverse of $p_0(\mathrm{s}), p_1(\mathrm{s})$, $p_2(\mathrm{s})$ and $p_3(\mathrm{s})$, we get

$$p_0(\mathrm{t}) = e^{mt}\left[\frac{\begin{array}{c}\mathrm{m}^3 + \mathrm{m}^2(\lambda_1 + \lambda_2 + \lambda_3 + \mu_1 + \mu_2 + \mu_3) \\ + \mathrm{m}(\mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3 + \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\mu_3 + \lambda_2\lambda_3 + \lambda_2\mu_1 + \lambda_3\mu_1 + \lambda_3\mu_2) \\ + \left(\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\mu_1 + \lambda_3\mu_1\mu_2 + \mu_1\mu_2\mu_3\right)\end{array}}{(m-n)(m-o)(m-p)}\right]$$

$$+e^{nt}\left[\frac{\begin{gathered}n^3+n^2(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3)\\+n(\mu_1\mu_2+\mu_1\mu_3+\mu_2\mu_3+\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\mu_3+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_3\mu_1+\lambda_3\mu_2)\\+\left(\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_1+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3\right)\end{gathered}}{(n-m)(n-o)(n-p)}\right]$$

$$+e^{ot}\left[\frac{\begin{gathered}o^3+o^2(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3)\\+o(\mu_1\mu_2+\mu_1\mu_3+\mu_2\mu_3+\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\mu_3+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_3\mu_1+\lambda_3\mu_2)\\+(\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_1+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3)\end{gathered}}{(o-m)(o-n)(o-p)}\right]$$

$$+e^{pt}\left[\frac{\begin{gathered}p^3+p^2(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3)\\+p(\mu_1\mu_2+\mu_1\mu_3+\mu_2\mu_3+\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\mu_3+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_3\mu_1+\lambda_3\mu_2)\\+(\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\mu_1+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3)\end{gathered}}{(p-m)(p-n)(p-o)}\right]$$

$$p_1(\mathrm{t})=e^{mt}[\frac{\lambda_0m^2+\lambda_0m^1(\lambda_2+\lambda_3+\mu_2+\mu_3)+\lambda_0(\lambda_2\lambda_3+\lambda_3\mu_3+\mu_2\mu_3)}{(m-n)(m-o)(m-p)}$$
$$+e^{nt}\frac{\lambda_0n^2+\lambda_0n^1(\lambda_2+\lambda_3+\mu_2+\mu_3)+\lambda_0(\lambda_2\lambda_3+\lambda_3\mu_3+\mu_2\mu_3)}{(n-m)(n-o)(n-p)}$$
$$+e^{ot}\frac{\lambda_0o^2+\lambda_0o^1(\lambda_2+\lambda_3+\mu_2+\mu_3)+\lambda_0(\lambda_2\lambda_3+\lambda_3\mu_3+\mu_2\mu_3)}{(o-m)(o-n)(o-p)}$$
$$+e^{pt}\frac{\lambda_0p^3+\lambda_0p^2(\lambda_2+\lambda_3+\mu_2+\mu_3)+\lambda_0(\lambda_2\lambda_3+\lambda_3\mu_3+\mu_2\mu_3)}{(p-m)(p-n)(p-o)}]$$

$$p_2(\mathrm{t})=\lambda_0\lambda_1[\frac{e^{mt}\,(\mu_3+\lambda_3+m)}{(m-n)(m-o)(m-p)}+\frac{e^{nt}(\mu_3+\lambda_3+n)}{(n-m)(n-o)(n-p)}+\frac{e^{ot}(\mu_3+\lambda_3+o)}{(o-m)(o-n)(o-p)}$$
$$+\frac{e^{nt}(\mu_3+\lambda_3+p)}{(p-m)(p-n)(p-o)}]$$

$$p_3(\mathrm{t})=\lambda_0\lambda_1\lambda_2[\frac{e^{mt}}{(m-n)(m-o)(m-p)}+\frac{e^{nt}}{(n-m)(n-o)(n-p)}$$
$$+\frac{e^{ot}}{(o-m)(o-n)(o-p)}+\frac{e^{nt}}{(p-m)(p-n)(p-o)}]$$

$$p_4(\mathrm{t})=\lambda_0\lambda_1\lambda_2\lambda_3[\frac{e^{mt}}{(m-n)(m-o)(m-p)}+\frac{e^{nt}}{(n-m)(n-o)(n-p)}$$
$$+\frac{e^{ot}}{(o-m)(o-n)(o-p)}+\frac{e^{nt}}{(p-m)(p-n)(p-o)}+\frac{1}{mnop}]$$

Thus, Reliability of the system is

$$\mathrm{R(t)}=p_0(t)+p_1(t)+p_2(t)+p_3(t)=1-p_4(t)$$
$$=1-\lambda_0\lambda_1\lambda_2\lambda_3[\frac{e^{mt}}{(m-n)(m-o)(m-p)}+\frac{e^{nt}}{(n-m)(n-o)(n-p)}+\frac{e^{ot}}{(o-m)(o-n)(o-p)}+\frac{e^{nt}}{(p-m)(p-n)(p-o)}+\frac{1}{mnop}] \quad (22)$$

b) Mean Time to System Failure (MTSF)

On integrating equation (22), we get the expression for MTSF as

$$MTSF = \int_0^\infty R(t)dt$$

$$MTSF = \frac{\lambda_1\lambda_2\lambda_3 + \mu_1(\lambda_3(\lambda_2+\mu_2)+\mu_2\mu_3) + \lambda_0(\lambda_3(\lambda_2+\mu_2)+\mu_2\mu_3+\lambda_1(\lambda_2+\lambda_3+\mu_3))}{mnop}$$

c) Availability

Now, using equation (1), we get the Chapman- Kolmogorov equations for the system to derive the expression for availability as

$$\begin{gathered}
P_0(t+\Delta t) = P_0(t)(1-\lambda_0\Delta t) + P_1(t)\mu_1\Delta t \\
P_1(t+\Delta t) = P_0(t)\lambda_0\Delta t + P_1(t)\big(1-(\mu_1\Delta t + \lambda_1\Delta t)\big) + P_2(t)\mu_2\Delta t \\
P_2(t+\Delta t) = P_1(t)\lambda_1\Delta t + P_2(t)\big(1-(\mu_2\Delta t + \lambda_2\Delta t)\big) + P_2(t)\mu_3\Delta t \\
P_3(t+\Delta t) = P_2(t)\lambda_2\Delta t + P_3(t)\big(1-(\mu_3\Delta t + \lambda_3\Delta t)\big) + P_3(t)\mu_4\Delta t \\
P_4(t+\Delta t) = P_3(t)\lambda_3\Delta t + P_4(t)(1-\mu_4\Delta t) \qquad (23\text{-}27)
\end{gathered}$$

These Markov equations are being developed by taking the probability of each state at time $t + \Delta t$. Above equations (23-27) can be rewritten as

$$\begin{gathered}
\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = P_0(t)(-\lambda_0) + P_1(t)\mu_1 \\
\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t} = P_0(t)\lambda_0 + P_1(t)\big(-(\mu_1+\lambda_1)\big) + P_2(t)\mu_2 \\
\frac{P_2(t+\Delta t)-P_2(t)}{\Delta t} = P_1(t)\lambda_1 + P_2(t)\big(-(\mu_2+\lambda_2)\big) + P_2(t)\mu_3 \\
\frac{P_3(t+\Delta t)-P_3(t)}{\Delta t} = P_2(t)\lambda_2 + P_3(t)\big(-(\mu_3+\lambda_3)\big) + P_4(t)\mu_4 \\
\frac{P_4(t+\Delta t)-P_4(t)}{\Delta t} = P_3(t)\lambda_3 + P_4(t)(-\mu_4) \qquad (28\text{-}32)
\end{gathered}$$

Converting equations (28-32) to a differential equation and taking Lim $\Delta t \rightarrow 0$, we get

$$\begin{gathered}
P_0'(\mathrm{t}) = P_0(t)(-\lambda_0) + P_1(t)\mu_1 \\
P_1'(\mathrm{t}) = P_0(t)\lambda_0 + P_1(t)\big(-(\mu_1+\lambda_1)\big) + P_2(t)\mu_2 \\
P_2'(\mathrm{t}) = P_1(t)\lambda_1 + P_2(t)\big(-(\mu_2+\lambda_2)\big) + P_3(t)\mu_3 \\
P_3'(\mathrm{t}) = P_2(t)\lambda_2 + P_3(t)\big(-(\mu_3+\lambda_3)\big) + P_4(t)\mu_4 \\
P_4'(\mathrm{t}) = P_3(t)\lambda_3 + P_4(t)(-\mu_4) \qquad (33\text{-}37)
\end{gathered}$$

Above equations (33-37) can be solved by using LT method.

$$\begin{gathered}
sp_0(\mathrm{s}) - P_0(0) = p_0(s)(-\lambda_0) + p_1(s)\mu_1 \\
sp_1(\mathrm{s}) - P_1(0) = p_0(s)\lambda_0 + p_1(s)\big(-(\mu_1+\lambda_1)\big) + p_2(s)\mu_2 \\
sp_2(\mathrm{s}) - P_2(0) = p_1(s)\lambda_1 + p_2(s)\big(-(\mu_2+\lambda_2)\big) + p_3(s)\mu_3 \\
sp_3(\mathrm{s}) - P_3(0) = p_2(s)\lambda_2 + p_3(s)\big(-(\mu_3+\lambda_3)\big) + p_4(s)\mu_4 \\
sp_4(\mathrm{s}) - P_4(0) = p_3(s)\lambda_3 + p_4(s)(-\mu_4) \qquad (38\text{-}42)
\end{gathered}$$

Boundary conditions are

$$P_0(0) = 1, P_1(0) = 0, P_2(0) = 0, P_3(0) = 0, P_4(0) = 0$$

So, the system of equations will be

$$\begin{gathered}
(s+\lambda_0)p_0(\mathrm{s}) - p_1(s)\mu_1 = 1 \\
-\lambda_0 p_0(s) + (s+\mu_1+\lambda_1)p_1(\mathrm{s}) - \mu_2 p_2(s) = 0 \\
-\lambda_1 p_1(s) + (s+\mu_2+\lambda_2)p_2(\mathrm{s}) - \mu_3 p_3(s) = 0 \\
-\lambda_2 p_2(s) + (s+\mu_3+\lambda_3)p_3(\mathrm{s}) - \mu_4 p_4(s) = 0 \\
-\lambda_3 p_3(s) + (s+\mu_4)p_4(\mathrm{s}) = 0
\end{gathered}$$

It can be written as

$$\begin{bmatrix} s+\lambda_0 & -\mu_1 & 0 & 0 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & -\mu_4 \\ 0 & 0 & 0 & -\lambda_3 & s+\mu_4 \end{bmatrix} \begin{bmatrix} p_0(\mathrm{s}) \\ p_1(\mathrm{s}) \\ p_2(\mathrm{s}) \\ p_3(\mathrm{s}) \\ p_4(\mathrm{s}) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, solving for $p_0(\mathrm{s}), p_1(\mathrm{s}), p_2(\mathrm{s})$ *and* $p_3(\mathrm{s})$using Cramer's Rule, we have

$$\Delta_A = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 0 & 0 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & -\mu_4 \\ 0 & 0 & 0 & -\lambda_3 & s+\mu_4 \end{vmatrix}$$

$$\Delta_A = s(s^4 + s^3(\lambda_0+\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3+\mu_4) + s^2(\lambda_0\lambda_1+\lambda_0\lambda_2+\lambda_0\mu_3+\lambda_0\mu_3+\lambda_0\mu_4+\lambda_1\lambda_3+\lambda_1\lambda_2+\lambda_1\mu_3+\lambda_1\mu_4+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_2\mu_4+\lambda_3\mu_1+\lambda_3\mu_2+\mu_1\mu_2+\mu_1\mu_3+\mu_1\mu_4+\mu_2\mu_3+\mu_2\mu_4+\mu_3\mu_4) + s\left(\lambda_0\lambda_1\lambda_2+\lambda_0\lambda_1\lambda_3+\lambda_0\lambda_2\lambda_3+\lambda_0\lambda_3\mu_2+\lambda_0\lambda_1\mu_3+\lambda_0\mu_2\mu_3+\lambda_0\lambda_1\mu_4+\lambda_0\lambda_2\mu_4+\lambda_0\mu_2\mu_4+\lambda_0\mu_3\mu_4+\lambda_1\lambda_2\mu_3+\lambda_1\lambda_2\mu_4+\lambda_1\mu_3\mu_4+\lambda_2\lambda_3\mu_1+\lambda_2\mu_1\mu_4+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3+\mu_1\mu_2\mu_4+\mu_2\mu_3\mu_4\right) + (\mu_1\mu_2\mu_3\mu_4+\lambda_0\mu_2\mu_3\mu_4+\lambda_0\lambda_1\mu_3\mu_4+\lambda_0\lambda_1\lambda_2\mu_4+\lambda_0\lambda_1\lambda_2\lambda_3))$$

$$= s(s-a)(s-b)(s-c)(s-d)$$

Here, a ,b ,c and d are the roots of Δ_A.

$$\Delta_{A_1} = \begin{vmatrix} 1 & -\mu_1 & 0 & 0 & 0 \\ 0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & -\mu_4 \\ 0 & 0 & 0 & -\lambda_3 & s+\mu_4 \end{vmatrix}$$

$$\Delta_{A_1} = s^4 + s^3(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3+\mu_4) + s^2(\lambda_1\lambda_3+\lambda_1\lambda_2+\lambda_1\mu_3+\lambda_1\mu_4+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_2\mu_4+\lambda_3\mu_1+\lambda_3\mu_2+\mu_1\mu_2+\mu_1\mu_3+\mu_1\mu_4+\mu_2\mu_3+\mu_2\mu_4+\mu_3\mu_4) + s\left(\lambda_1\lambda_2\mu_3+\lambda_1\lambda_2\mu_4+\lambda_1\mu_3\mu_4+\lambda_2\lambda_3\mu_1+\lambda_2\mu_1\mu_4+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3+\mu_1\mu_2\mu_4+\mu_2\mu_3\mu_4\right) + \mu_1\mu_2\mu_3\mu_4$$

$$\Delta_{A_2} = \begin{vmatrix} s+\lambda_0 & 1 & 0 & 0 & 0 \\ -\lambda_0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & -\mu_4 \\ 0 & 0 & 0 & -\lambda_3 & s+\mu_4 \end{vmatrix}$$

$$\Delta_{A_2} = as^3 + as^2(\lambda_2+\lambda_3+\mu_2+\mu_3+\mu_4) + as(\lambda_2\lambda_3+\lambda_2\mu_4+\lambda_3\mu_2+\mu_2\mu_3+\mu_2\mu_4+\mu_3\mu_4) + \lambda_0\mu_2\mu_3\mu_4$$

$$\Delta_{A_3} = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 1 & 0 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & -\mu_3 & 0 \\ 0 & 0 & 0 & s+\lambda_3+\mu_3 & -\mu_4 \\ 0 & 0 & 0 & -\lambda_3 & s+\mu_4 \end{vmatrix}$$

$$\Delta_{A_3} = \lambda_0\lambda_1 s^2 + s(\lambda_0\lambda_1\lambda_3+\lambda_0\lambda_1\mu_3+\lambda_0\lambda_1\mu_4) + \lambda_0\lambda_1\mu_3\mu_4$$

$$\Delta_{A_4} = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 0 & 1 & 0 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & 0 & 0 \\ 0 & 0 & -\lambda_2 & 0 & -\mu_4 \\ 0 & 0 & 0 & 0 & s+\mu_4 \end{vmatrix}$$

$$\Delta_{A_4} = \lambda_0\lambda_1\lambda_2\mu_4 + \lambda_0\lambda_1\lambda_2 s$$

$$\Delta_{A_5} = \begin{vmatrix} s+\lambda_0 & -\mu_1 & 0 & 0 & 1 \\ -\lambda_0 & s+\lambda_1+\mu_1 & -\mu_2 & 0 & 0 \\ 0 & -\lambda_1 & s+\lambda_2+\mu_2 & -\mu_3 & 0 \\ 0 & 0 & -\lambda_2 & s+\lambda_3+\mu_3 & 0 \\ 0 & 0 & 0 & -\lambda_3 & 0 \end{vmatrix}$$

$$\Delta_{A_5} = \lambda_0\lambda_1\lambda_2\lambda_3$$

Now,

$$p_0(\mathrm{s}) = \frac{\Delta_{A_1}}{\Delta_A}$$

$$= [s^4 + s^3(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3+\mu_4) + s^2(\lambda_1\lambda_3+\lambda_1\lambda_2+\lambda_1\mu_3+\lambda_1\mu_4+\lambda_2\lambda_3+\lambda_2\mu_1+\lambda_2\mu_4+\lambda_3\mu_1+\lambda_3\mu_2+\mu_1\mu_2+\mu_1\mu_3+\mu_1\mu_4+\mu_2\mu_3+\mu_2\mu_4+\mu_3\mu_4) + s(\lambda_1\lambda_2\mu_3+\lambda_1\lambda_2\mu_4+\lambda_1\mu_3\mu_4+\lambda_2\lambda_3\mu_1+\lambda_2\mu_1\mu_4+\lambda_3\mu_1\mu_2+\mu_1\mu_2\mu_3+\mu_1\mu_2\mu_4+\mu_2\mu_3\mu_4) + \mu_1\mu_2\mu_3\mu_4]/\Delta_A$$

$$p_1(\mathrm{s}) = \frac{\Delta_{A_2}}{\Delta_A}$$

$$= \frac{[\lambda_0 s^3 + \lambda_0 s^2(\lambda_2 + \lambda_3 + \mu_2 + \mu_3 + \mu_4) + \lambda_0 s(\lambda_2\lambda_3 + \lambda_2\mu_4 + \lambda_3\mu_2 + \mu_2\mu_3 + \mu_2\mu_4 + \mu_3\mu_4) + \lambda_0\mu_2\mu_3\mu_4]}{\Delta_A}$$

$$p_2(\mathrm{s}) = \frac{\Delta_{A_3}}{\Delta_A} = \frac{[\lambda_0\lambda_1 s^2 + s(\lambda_0\lambda_1\lambda_3 + \lambda_0\lambda_1\mu_3 + \lambda_0\lambda_1\mu_4) + \lambda_0\lambda_1\mu_3\mu_4]}{\Delta_A}$$

$$p_3(\mathrm{s}) = \frac{\Delta_{A_4}}{\Delta_A} = \frac{[\lambda_0\lambda_1\lambda_2\mu_4 + \lambda_0\lambda_1\lambda_2 s]}{\Delta_A}$$

$$p_4(\mathrm{s}) = \frac{\Delta_{A_5}}{\Delta_A} = \frac{\lambda_0\lambda_1\lambda_2\lambda_3}{\Delta_A}$$

Taking Laplace Inverse of $p_0(\mathrm{s}), p_1(\mathrm{s})$, $p_2(\mathrm{s}), p_3(\mathrm{s})$ and $p_4(\mathrm{s})$, we get

$$p_0(\mathrm{t}) = \frac{e^{at}(a\lambda_1(a\lambda_3 + (a+\mu_3)(a+\mu_4) + \lambda_2(a+\lambda_3+\mu_4)) + (a+\mu_1)(a\lambda_2(a+\lambda_3+\mu_4)) + (a+\mu_2)(a\lambda_3 + (a+\mu_3)(a+\mu_4)))}{a(a-b)(a-c)(a-d)}$$
$$+ \frac{e^{bt}(b\lambda_1(b\lambda_3 + (b+\mu_3)(b+\mu_4) + \lambda_2(b+\lambda_3+\mu_4)) + (b+\mu_1)(b\lambda_2(b+\lambda_3+\mu_4) + (b+\mu_2)(b\lambda_3 + (b+\mu_3)(b+\mu_4))))}{b(b-a)(b-c)(b-d)}$$
$$+ \frac{e^{ct}(c\lambda_1(c\lambda_3 + (c+\mu_3)(c+\mu_4) + \lambda_2(c+\lambda_3+\mu_4)) + (c+\mu_1)(c\lambda_2(c+\lambda_3+\mu_4) + (c+\mu_2)(c\lambda_3 + (c+\mu_3)(c+\mu_4))))}{c(c-a)(c-b)(c-d)}$$
$$+ \frac{e^{dt}(d\lambda_1(d\lambda_3 + (d+\mu_3)(d+\mu_4) + \lambda_2(d+\lambda_3+\mu_4)) + (d+\mu_1)(d\lambda_2(d+\lambda_3+\mu_4) + (d+\mu_2)(d\lambda_3 + (d+\mu_3)(d+\mu_4))))}{d(d-a)(d-b)(d-c)}$$
$$+ \frac{\mu_1\mu_2\mu_3\mu_4}{abcd}$$

$$p_1(\mathrm{t}) = \lambda_0\Big(\frac{\mu_2\mu_3\mu_4}{abcd} + \frac{e^{at}(a\lambda_2(a+\lambda_3+\mu_4) + (a+\mu_2)(a\lambda_3 + (a+\mu_3)(a+\mu_4)))}{a(a-b)(a-c)(a-d)}$$
$$+ \frac{e^{bt}(b\lambda_2(b+\lambda_3+\mu_4) + (b+\mu_2)(b\lambda_3 + (b+\mu_3)(b+\mu_4)))}{b(b-a)(b-c)(b-d)}$$
$$+ \frac{e^{ct}(c\lambda_2(c+\lambda_3+\mu_4) + (c+\mu_2)(c\lambda_3 + (c+\mu_3)(c+\mu_4)))}{c(c-a)(c-b)(c-d)}$$
$$+ \frac{e^{dt}(d\lambda_2(d+\lambda_3+\mu_4) + (d+\mu_2)(d\lambda_3 + (d+\mu_3)(d+\mu_4)))}{d(d-a)(d-b)(d-c)}\Big)$$

$$p_2(\mathrm{t}) = \lambda_0\lambda_1\Big(\frac{\mu_3\mu_4}{abcd} + \frac{e^{at}(a\lambda_3 + (a+\mu_3)(a+\mu_4))}{a(a-b)(a-c)(a-d)} + \frac{e^{bt}(b\lambda_3 + (b+\mu_3)(b+\mu_4))}{b(b-a)(b-c)(b-d)}$$
$$+ \frac{e^{ct}(c\lambda_3 + (c+\mu_3)(c+\mu_4))}{c(c-a)(c-b)(c-d)} + \frac{e^{dt}(d\lambda_3 + (d+\mu_3)(d+\mu_4))}{d(d-a)(d-b)(d-c)}\Big)$$

$$p_3(\mathrm{t}) = \lambda_0\lambda_1\lambda_2\Big(\frac{\mu_4}{abcd} + \frac{e^{at}(a+\mu_4)}{a(a-b)(a-c)(a-d)} - \frac{e^{bt}(b+\mu_4)}{b(b-a)(b-c)(b-d)}$$
$$- \frac{e^{ct}(c+\mu_4)}{c(c-a)(c-b)(c-d)} - \frac{e^{dt}(d+\mu_4)}{d(d-a)(d-b)(d-c)}\Big)$$

$$p_4(\mathrm{t}) = \lambda_0\lambda_1\lambda_2\lambda_3\Big(\frac{1}{abcd} + \frac{e^{at}}{a(a-b)(a-c)(a-d)} + \frac{e^{bt}}{b(b-a)(b-c)(b-d)}$$
$$+ \frac{e^{ct}}{c(c-a)(c-b)(c-d)} + \frac{e^{dt}}{d(d-a)(d-b)(d-c)}\Big)$$

The availability is given by

$$A(t) = p_0(\mathrm{t}) + p_1(\mathrm{t}) + p_2(\mathrm{t}) + p_3(\mathrm{t}) = 1 - p_4(t)$$
$$= 1 - \lambda_0\lambda_1\lambda_2\lambda_3\Big(\frac{1}{abcd} + \frac{e^{at}}{a(a-b)(a-c)(a-d)} + \frac{e^{bt}}{b(b-a)(b-c)(b-d)}$$
$$+ \frac{e^{ct}}{c(c-a)(c-b)(c-d)} + \frac{e^{dt}}{d(d-a)(d-b)(d-c)}\Big)$$

The steady state availability is given by

$$A(\infty) = \lim_{s\to 0} s(p_0(s) + p_1(s) + p_2(s) + p_3(s))$$

$$= \lim_{s\to 0} [\, s[s^4 + s^3(\lambda_1 + \lambda_2 + \lambda_3 + \mu_1 + \mu_2 + \mu_3 + \mu_4) + s^2(\lambda_1\lambda_3 + \lambda_1\lambda_2 + \lambda_1\mu_3 + \lambda_1\mu_4 + \lambda_2\lambda_3 + \lambda_2\mu_1 + \lambda_2\mu_4 + \lambda_3\mu_1 + \lambda_3\mu_2 + \mu_1\mu_2 + \mu_1\mu_3 + \mu_1\mu_4 + \mu_2\mu_3 + \mu_2\mu_4 + \mu_3\mu_4) + s(\lambda_1\lambda_2\mu_3 + \lambda_1\lambda_2\mu_4 + \lambda_1\mu_3\mu_4 + \lambda_2\lambda_3\mu_1 + \lambda_2\mu_1\mu_4 + \lambda_3\mu_1\mu_2 + \mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_4 + \mu_2\mu_3\mu_4) + \mu_1\mu_2\mu_3\mu_4] [\lambda_0 s^3 + \lambda_0 s^2(\lambda_2 + \lambda_3 + \mu_2 + \mu_3 + \mu_4) + \lambda_0 s(\lambda_2\lambda_3 + \lambda_2\mu_4 + \lambda_3\mu_2 + \mu_2\mu_3 + \mu_2\mu_4 + \mu_3\mu_4) + \lambda_0\mu_2\mu_3\mu_4] + [\lambda_0\lambda_1 s^2 + s(\lambda_0\lambda_1\lambda_3 + \lambda_0\lambda_1\mu_3 + \lambda_0\lambda_1\mu_4) + \lambda_0\lambda_1\mu_3\mu_4] + \lambda_0\lambda_1\lambda_2\mu_4 + \lambda_0\lambda_1\lambda_2 s]/\Delta_A$$

$$= \frac{\mu_1\mu_2\mu_3\mu_4 + \lambda_0(\mu_2\mu_3\mu_4 + \lambda_1(\lambda_2\mu_4 + \mu_3\mu_4))}{abcd}$$

Or it can also be expressed in the form of

$$= \frac{\mu_1\mu_2\mu_3\mu_4 + \lambda_0\mu_2\mu_3\mu_4 + \lambda_0\lambda_1\mu_3\mu_4 + \lambda_0\lambda_1\lambda_2\mu_4}{\mu_1\mu_2\mu_3\mu_4 + \lambda_0\mu_2\mu_3\mu_4 + \lambda_0\lambda_1\mu_3\mu_4 + \lambda_0\lambda_1\lambda_2\mu_4 + \lambda_0\lambda_1\lambda_2\lambda_3}$$

b) Particular Case

Now, if all the components are taken as identical units
For Availability we have,

$$\lambda_0 = 4\lambda, \lambda_1 = 3\lambda, \lambda_2 = 2\lambda, \lambda_3 = \lambda, \mu_4 = 4\mu, \mu_3 = 3\mu, \mu_2 = 2\mu \text{ and } \mu_1 = \mu$$

$$A(t) = 1 - 24\lambda^4\Big(\frac{1}{abcd} + \frac{e^{at}}{a(a-b)(a-c)(a-d)} + \frac{e^{bt}}{b(b-a)(b-c)(b-d)} + \frac{e^{ct}}{c(c-a)(c-b)(c-d)} + \frac{e^{dt}}{d(d-a)(d-b)(d-c)}\Big)$$

$$A(\infty) = \frac{48\lambda^3\mu + 144\lambda^2\mu^2 + 96\lambda\mu^3 + 24\mu^4}{12\lambda^4 + 48\lambda^3\mu + 144\lambda^2\mu^2 + 96\lambda\mu^3 + 24\mu^4}$$

For Reliability we have

$$\lambda_0 = 4\lambda, \lambda_1 = 3\lambda, \lambda_2 = 2\lambda, \lambda_2 = \lambda, \mu_4 = 0, \mu_3 = 3\mu, \mu_2 = 2\mu \text{ and } \mu_1 = \mu$$

$$R(t) = 1 - 24\lambda^4\Big[\frac{e^{mt}}{(m-n)(m-o)(m-p)} + \frac{e^{nt}}{(n-m)(n-o)(n-p)} + \frac{e^{ot}}{(o-m)(o-n)(o-p)} + \frac{e^{nt}}{(p-m)(p-n)(p-o)} + \frac{1}{mnop}\Big]$$

For MTSF we have

$$MTSF = \frac{6\lambda^3 + \mu(6\mu^2 + \lambda(2\lambda + 2\mu)) + 4\lambda(6\mu^2 + \lambda(2\lambda + 2\mu) + 3\lambda(3\lambda + 3\mu))}{mnop}$$

VI. Numerical and Graphical Presentation

Here, we evaluate the reliability, availability and MTSF for the arbitrary values of repair rate (μ) and failure rate (λ) with operating time (t) of the components. The numerical and graphical representation of the results is given below:

From Figure 2, it is observed that the reliability of the system declines with the increase of failure rate and operating time. Also, the reliability of the system is increases with an increase in repair rate of the units.

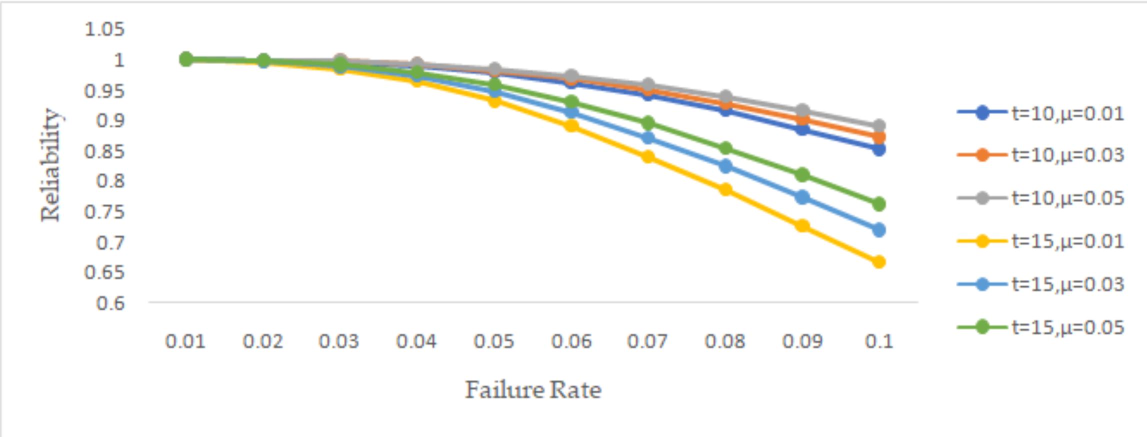

Figure 2: *Reliability V/s Repair Rate (μ) and Failure Rate(λ) with Operating Time(t)*

From Figure 3, it is observed that the MTSF of the system declines with the increase of failure rate. Also, the MTSF of the system is increases with an increase in repair rate of the units.

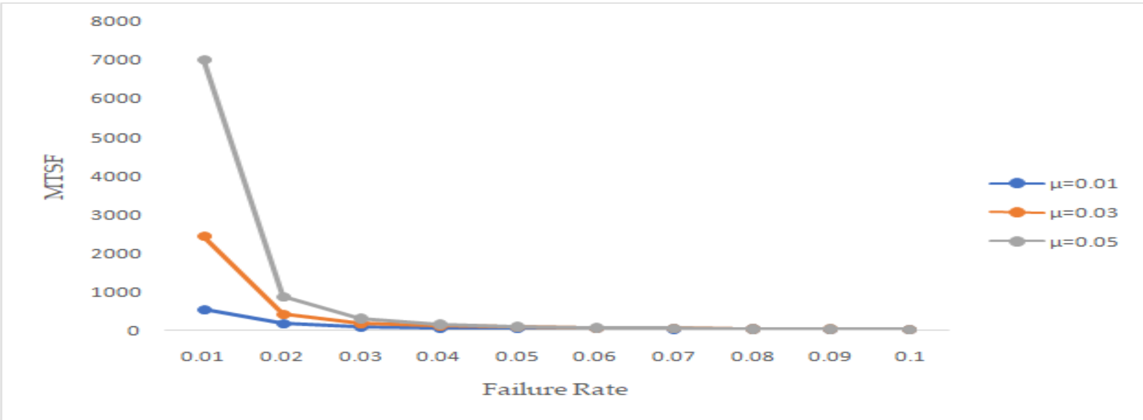

Figure 3: *MTSF V/s Repair Rate (μ) and Failure Rate(λ)*

From Figure 4, it is observed that the availability (A(t)) of the system declines with the increase of failure rate and operating time. Also, the availability of the system is increases with an increase in repair rate of the units.

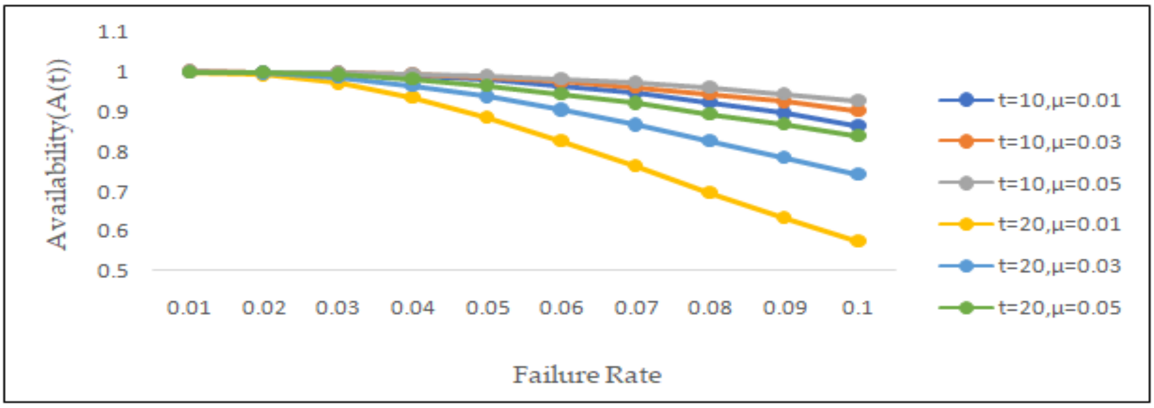

Figure 4: *Availability V/s Repair Rate (μ) and Failure Rate(λ) with Operating Time(t)*

From Figure 5, it is observed that the availability (A(∞)) of the system declines with the increase of failure rate. Also, the availability of the system is increases with an increase in repair rate of the units.

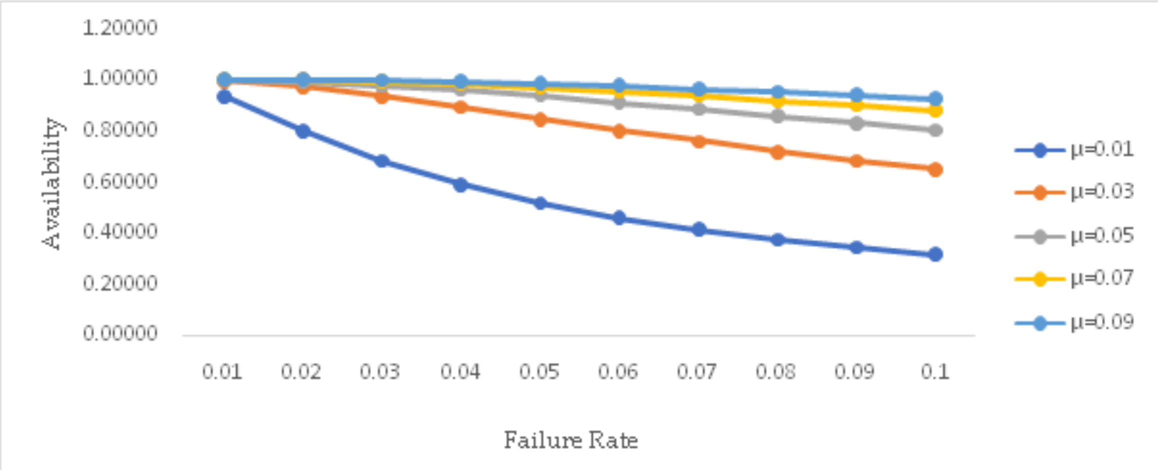

Figure 5: *Availability V/s Repair Rate (μ) and Failure Rate (λ)*

VII. Application

The real-life application of this study can be visualized at toll plaza. On a toll plaza, vehicles enter and exit the mainline roadway from certain locations along the highway. All vehicles must stop and pay the toll at any of the tollbooths that work simultaneously to reduce the gathering of traffic vehicles. The drivers prefer a tollbooth that has a shorter lane rather than a longer lane to minimize their own travel time. .

It is a common fact that delivery of services at the tollbooths plays an important role at the toll plaza. Nowadays, at some toll plazas traffic delay has become a common problem because of failure of the timely services at the tollbooths. And, therefore to improve the working efficiency of the toll plaza it becomes necessary to use more than one tollbooth at a time simultaneously. Here, we have considered a system in which four tollbooths are connected in parallel which works simultaneously as shown in the figure 6.

Figure 6: *Toll Plaza*

VIII. Conclusion

Reliability, MTSF, availability and steady state availability have been evaluated for a four-unit repairable parallel redundant system by using Markov approach. The values for these measures have been obtained for particular values of repair rate, failure rate and operating time. The study reveals that the availability and mean time to system failure keeps on increasing with the increase of the repair rate while they decline sharply when failure rate increases. The reliability and availability of the system is declines with the increase of failure rate and operating time. On the other hand, it is also analyzed that these measures keep on increasing with the increase of repair rate. Hence, it is observed that system is more reliable and available to use by increasing the repair rate of the units.

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