TRUNCATED PRANAV DISTRIBUTION: PROPERTIES AND APPLICATIONS

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Abstract

In this paper, truncated Pranav distribution has been proposed. The behavior of truncated Pranav distribution has been presented graphically. Moment based measures including coefficient of variation, skewness, kurtosis, and Index of dispersion have been derived and presented graphically. Nature of survival and hazard rate functions are presented graphically. Maximum likelihood method has been used to estimate the parameter of proposed model. Simulation based study of proposed distribution has also been discussed. It has been applied on two data sets and its superiority has been compare and checked using goodness of fit (AIC and K. S. test) over other truncated distributions as well as one parameter distribution, such as exponential, Lindley, Pranav, Ishita, truncated Akash, truncated Lindley, and truncated Akash distribution. It was found good fit over above-mentioned distributions. It can be considered as good lifetime distribution especially for non-skewed data.

Keywords: Akash distribution, Lindley distribution, Moments, Right Truncated, Left Truncated

1. Introduction

In the recent past decades, lifetime modeling has been becoming popular in distribution theory, where many statisticians are involved in introducing new models. Some of the life time models are very popular and applied in biological, engineering and agricultural areas, such as Lindley distribution of Lindley [1], weighted Lindley distribution introduced by [2], Akash distribution suggested by Shanker[3], Ishita distribution proposed by Shanker and Shukla [4], Pranav distribution introduced by Shukla [5], are some among others and extension of above mentioned distribution has also been becoming popular in different areas of statistics.

Shukla [5] proposed Pranav distribution convex combination of exponential and gamma distributions which is defined by its pdf and cdf

$$f_1(y;\theta) = \frac{\theta^4}{\theta^4+6}(\theta + y^3)e^{-\theta y}; y > 0, \theta > 0 \tag{1}$$

$$F_2(y;\theta) = 1 - \left[1 + \frac{\theta y(\theta^2 y^2 + 3\theta y + 6)}{\theta^4 + 6}\right] e^{-\theta y}; y > 0, \theta > 0 \quad (2)$$

The rth moment about origin $\mu_r{}'$ of Pranav distribution given as

$$\mu_r{}' = \frac{r!\{\theta^4 + (r+1)(r+2)(r+3\}}{\theta^r(\theta^4+6)}; r = 1,2,3,\ldots \quad (3)$$

Shukla [5] has discussed in detailed about its mathematical and statistical properties, estimation of parameters and applications to model lifetime data from engineering and biomedical engineering. Truncated type of distribution is more effective for modeling lifetime data because its limits used as bound either upper or lower or both according to the given data. Truncated normal distribution is proposed by Johnson et al. [6]. It has wide application in economics and statistics. Many researchers have been proposed truncated type of distribution and applied in different areas of statistics, especially in censor data such as truncated Weibull distribution of Zange and Xie [7], truncated Lomax distribution of Aryuyuen and Bodhisuwan [8], truncated Pareto distribution of Janinetti and Ferraro [9], truncated Lindley distribution of Singh et al. [10]. Some researchers have proposed distribution such as Sindhu and Hussai [11] proposed a Mixture of two generalized inverted exponential distributions with censored sample model and applied on cancered data, and Shukla [12] proposed Inverse Ishita distribution and applied on using the data sets of bladder cancer patient and failure times of the air conditioning system. Recently truncated version of Akash distribution introduced by Shukla and Shanker [13] and truncated version of two parameter Pranav distribution has been proposed by Shukla [14] and its superiority has been shown in their paper over other truncated distribution. Some distributions and their introducer names have been listed in the table1.

Table1: *Pdf of some selected distribution and their introducer's name*

Distribution name	Pdf (probability density function)	Introducer' name
Truncated Lindley (TLD)	$f(x;\theta) = \dfrac{\theta^2(x+1)e^{-\theta x}}{(a\theta+1)e^{-\theta a} - (b\theta+1)e^{-\theta b} + (\theta+1)(e^{-\theta a} - e^{-\theta b})}$	Singh et al. [10]
Truncated Akash (TAD)	$f(x;\theta) = \dfrac{\theta^3(x^2+1)e^{-\theta x}}{a\theta(a\theta+2)e^{-\theta a} - b\theta(b\theta+2)e^{-\theta b} + (\theta^2+2)(e^{-\theta a} - e^{-\theta b})}$	Shukla & Shanker [13]
Akash	$f(x;\theta) = \dfrac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta}; x > 0, \theta > 0$	Shanker [3]
Ishita	$f(x;\theta) = \dfrac{\theta^3}{\theta^3+2}(\theta+x^2)e^{-\theta}; y > 0, \theta > 0$	Shanker& Shukla [4]
Lindley	$f(x;\theta) = \dfrac{\theta^2}{\theta+1}(1+x)e^{-\theta x}; x > 0, \theta > 0$	Lindley [1]
Exponential	$f(x;\theta) = \theta e^{-\theta x}; x > 0, \theta > 0$	

Truncated version of a continuous distribution can be defined as:

Definition1. Let X be a random variable distributed according to some pdf $g(x;\theta)$ and cdf $G(x;\theta)$, where θ is a parameter vector of X. Let X lies within the interval $[a,b]$, where $-\infty < a \le x \le b < \infty$, then X, conditional on $a \le x \le b$ is distributed as truncated distribution. The pdf of truncated distribution as reported by Singh et al (2014) defined by:

$$f(x;\theta) = g(x/a \le x \le b;\theta) = \frac{g(x;\theta)}{G(b;\theta) - G(a;\theta)} \quad (4)$$

where $f(x;\theta) = g(x;\theta)$ for all $a \le x \le b$ and $f(x;\theta) = 0$ elsewhere. Note that $f(x;\theta)$ in fact is a pdf of X on interval $[a,b]$. We have

$$f(x;\theta) = \int_a^b f(x;\theta)\,dx = \frac{1}{G(b;\theta)-G(a;\theta)}\int_a^b g(x;\theta)\,dx$$

$$= \frac{1}{G(b;\theta)-G(a;\theta)} G(b;\theta) - G(a;\theta) = 1 \tag{5}$$

The cdf of truncated distribution is given by

$$F(x;\theta) = \int_a^x f(x;\theta)dx = \frac{G(x;\theta)-G(a;\theta)}{G(b;\theta)-G(a;\theta)} \tag{6}$$

The main objective of this paper is to propose new truncated distribution using Pranav distribution which is called as truncated Pranav distribution, and to know the behavior and properties of proposed distribution over other truncated as well as parent distributions. Present paper has been divided into eight sections. Introduction about the paper is described in the first section. In the second section, truncated Pranav distribution has been derived. Behavior of hazard rate has been presented in third section Statistical properties including its moment have been discussed in the fourth section. Estimation of parameters of the proposed distribution has been discussed in the fifth section. Simulation study of proposed distribution has been discussed in the sixth section. Its application and comparative study with one parameter lifetime distribution have been illustrated in the section seven. Finally, the conclusion of the paper has been given in the eighth section.

2. Truncated Pranav Distribution

In this section, pdf and cdf of new truncated distribution is proposed and named Truncated Pranav distribution, using (5) & (6) of definition1 and from (1) & (2) , which is defined as :

Definition 2: Let X be random variable which is distributed as Truncated Pranav distribution (TPD) with scale parameter θ and location parameters a &b, and denoted by TPD(a, b, θ). The pdf and cdf of X are derived respectively as:

$$f(x;\theta) = g(x/a \le x \le b;\theta) = \frac{g(x;\theta)}{G(b;\theta)-G(a;\theta)}$$

$$F(x;\theta) = \frac{G(x;\theta)-G(a;\theta)}{G(b;\theta)-G(a;\theta)}$$

Where $g(x;\theta) = \frac{\theta^4}{\theta^4+6}(\theta + x^3)e^{-\theta x}$

$$G(b;\theta) = 1 - \left[1 + \frac{\theta b(\theta^2 b^2 + 3\theta b + 6)}{\theta^4 + 6}\right]e^{-\theta b}$$

$$G(a;\theta) = 1 - \left[1 + \frac{\theta a(\theta^2 a^2 + 3\theta a + 6)}{\theta^4 + 6}\right]e^{-\theta a}$$

$$f(x;\theta) = \frac{\theta^4(x^3+\theta)e^{-\theta x}}{(a^3\theta^3+3a^2\theta^2+\theta^4+6a\theta+6)e^{-\theta a}-(b^3\theta^3+3b^2\theta^2+\theta^4+6b\theta+6)e^{-\theta b}} \tag{7}$$

$$F(x;\theta) = \frac{(a^3\theta^3+3a^2\theta^2+\theta^4+6a\theta+6)e^{-\theta a}-(x^3\theta^3+3x^2\theta^2+\theta^4+6x\theta+6)e^{-\theta x}}{(a^3\theta^3+3a^2\theta^2+\theta^4+6a\theta+6)e^{-\theta a}-(b^3\theta^3+3b^2\theta^2+\theta^4+6b\theta+6)e^{-\theta b}} \tag{8}$$

where $-\infty < a \le x \le b < \infty$, and $\theta > 0$

Three cases can be considered of proposed doubly truncated distribution as:

a) When $a = 0$and $b = \infty$, it reduced to parent model (Pranav distribution).,
b) When $a = 0$, it is known as right (upper) truncated distribution of the parental model (Right truncated Pranav distribution)
c) When $b = \infty$, it is known as left (lower) truncated distribution of the parent model (left truncated Pranav distribution.

Properties of TPD are explained as follows:

i. TPD has three parameters, where two parameters a and b were considered lowest and largest value from the data.

ii. Parameter θ is considered as scale parameter, for the fixed value of a and b, TPD is observed decreasing and increasing as increased value of θ for $\theta < 1$ and $\theta > 1$ respectively.

Performance of pdf of TPD for varying values of parameters has been illustrated in the figure 1.

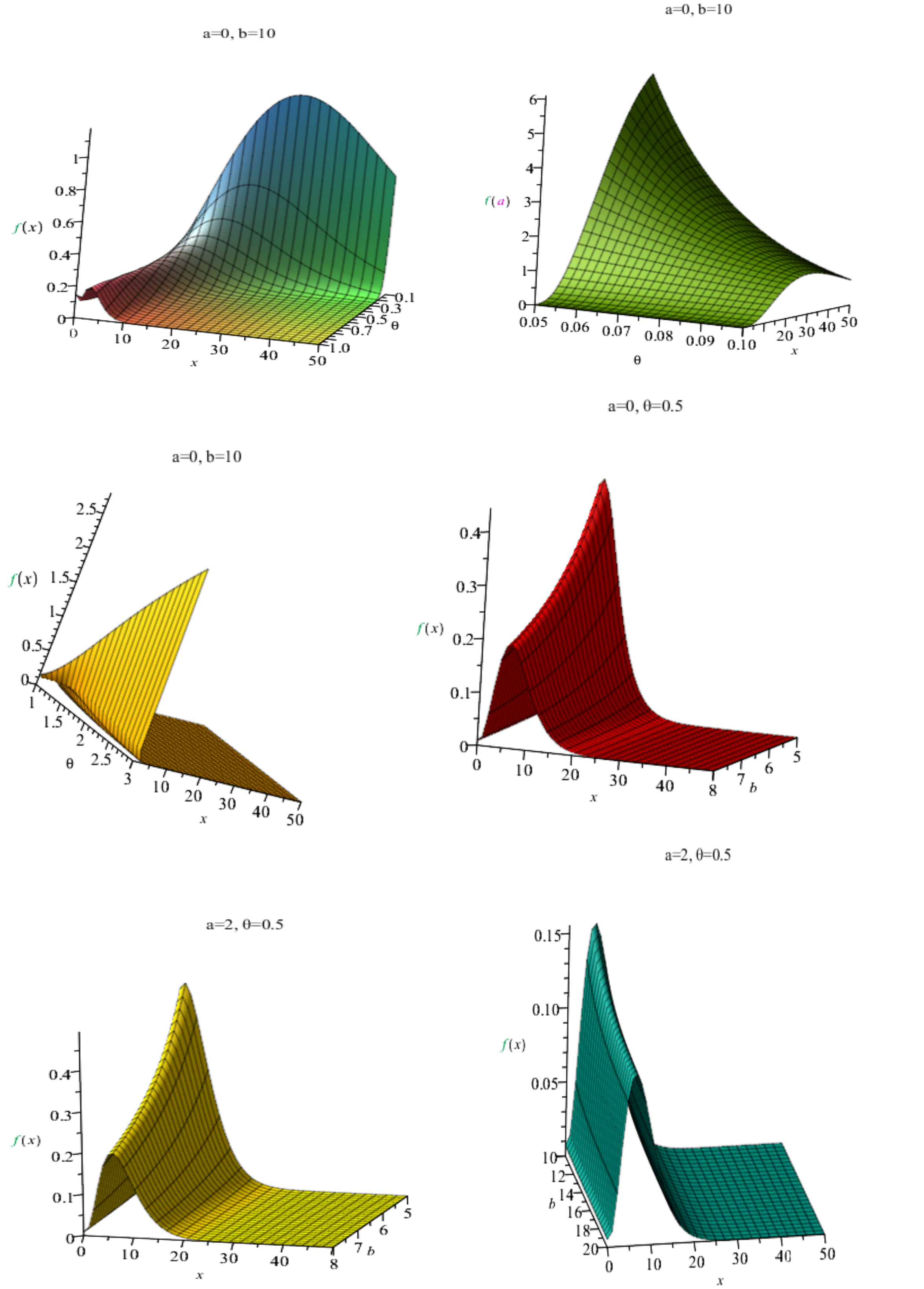

Figure 1: *pdf plots of TPD for varying values of parameters*

3. Survival and Hazard function

The survival function $S(x)$ and the hazard function $h(x)$ of TPD are defined as

$$S(x) = 1 - F(x) = \frac{(x^3\theta^3 + 3x^2\theta^2 + \theta^4 + 6x\theta + 6)e^{-\theta x} - (b^3\theta^3 + 3b^2\theta^2 + \theta^4 + 6b\theta + 6)e^{-\theta b}}{(a^3\theta^3 + 3a^2\theta^2 + \theta^4 + 6a\theta + 6)e^{-\theta a} - (b^3\theta^3 + 3b^2\theta^2 + \theta^4 + 6b\theta + 6)e^{-\theta b}}$$

$$h(x) = \frac{f(x)}{S(x)} = \frac{\theta^4(x^3 + \theta)e^{-\theta x}}{(x^3\theta^3 + 3x^2\theta^2 + \theta^4 + 6x\theta + 6)e^{-\theta x} - (b^3\theta^3 + 3b^2\theta^2 + \theta^4 + 6b\theta + 6)e^{-\theta b}}$$

It is obvious that $h(x)$ is independent from parameter a. Behavior of Survival and hazard function of TPD for varying values of parameter are presented in figures 2&3. It was observed from the figure 3 that value of hazard rate is increasing as increased value of parameter of θ when value of rest of the parameters (a, b)fixed.

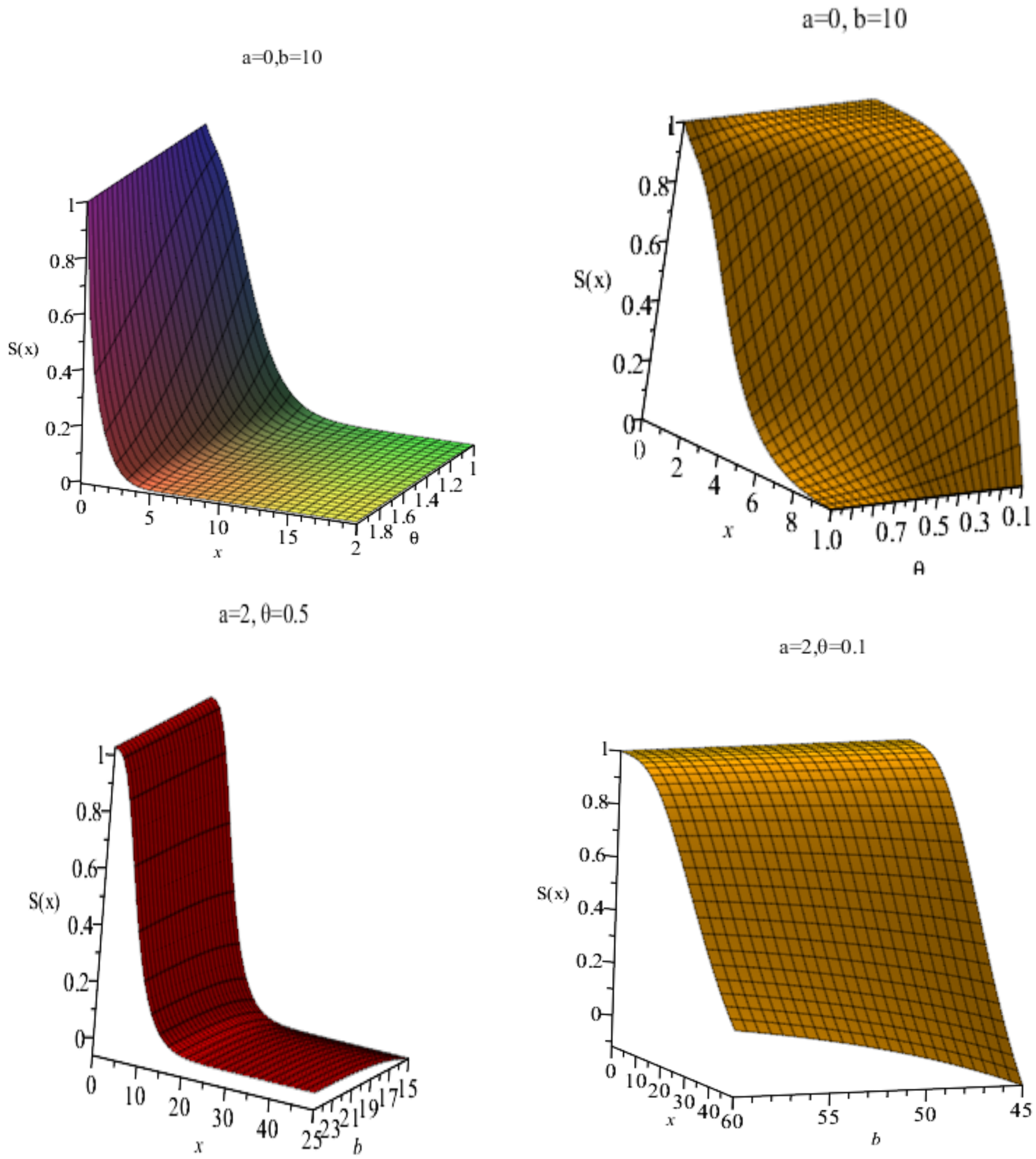

Figure 2: *S(x) plots of TPD for varying values of parameter*

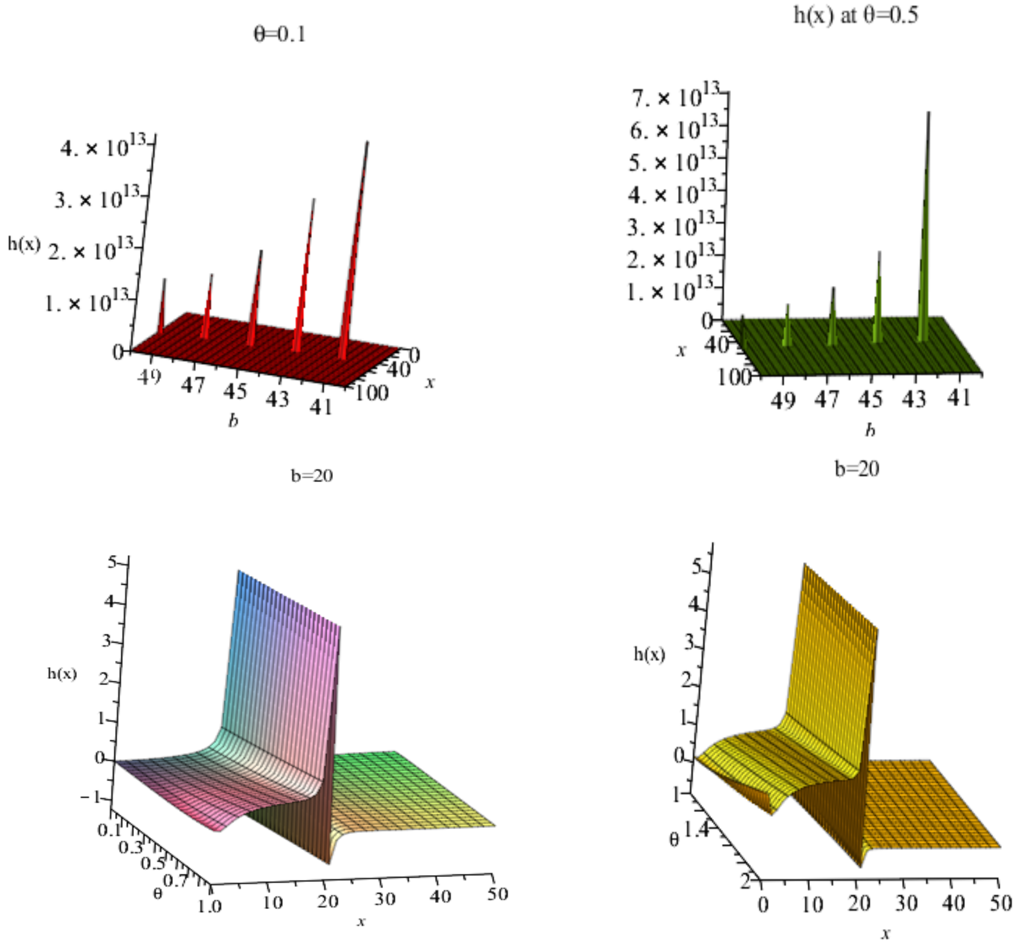

Figure3: *h(x) plots of TPD for varying values of parameter*

4. Moments and mathematical properties

Moments of a distribution are used to study the most important characteristics of the distribution including mean, variance, skewness, kurtosis, etc. The rth moment of origin$\mu_r{}'$ of TPD can be expressed in explicit expression in terms of incomplete gamma functions.

Theorem1: Suppose $X_i(a > 0, b\sim\infty)$ follows left (lower) truncated TPD (θ, a, ∞). Then the rth moment about origin $\mu_r{}'$ of TPD is

$$\mu_r{}' = \frac{\theta^4\gamma(r+1,\theta a)+\gamma(r+4,\theta a)}{\theta^r[(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}-(\theta^4+6)]}; r = 1,2,3,\ldots$$

Proof: Considering $K = \{(a^3\theta^3 + 3a^2\theta^2 + 6a\theta + \theta^4 + 6)e^{-\theta a} - (\theta^4 + 6)\}$ in (7), we have

$$\mu_r{}' = \frac{\theta^4}{K}\int_a^\infty x^r(\theta + x^3)\,e^{-\theta x}dx$$

$$= \frac{\theta^4}{K}\left[\int_a^\infty \theta e^{-\theta x}x^r dx + \int_a^\infty e^{-\theta x}x^{r+3}dx\right]$$

Taking $u = \theta x, x = \frac{u}{\theta}$

$$= \frac{\theta^4}{K}\left[\frac{\theta}{\theta^{r+1}}\left\{\int_0^{\theta a} e^{-u}x^r du\right\} + \frac{1}{\theta^{r+4}}\left\{\int_0^{\theta a} e^{-u}x^{r+3}du\right\}\right]$$

Where $\gamma(\alpha, z) = \int_z^\infty e^{-x}x^{\alpha-1}dx\,, \alpha > 0, x > 0$ is the upper incomplete gamma function

$$= \frac{\theta^4}{K}\left[\frac{\gamma(r+1,\theta a)}{\theta^r} + \frac{\gamma(r+4,\theta a)}{\theta^{r+4}}\right]$$

$$=\frac{1}{K}\left[\frac{\theta^4\gamma(r+1,\theta a)+\gamma(r+4,\theta a)}{\theta^r}\right]$$

$$\mu_r{}' = \frac{\theta^4\gamma(r+1,\theta a)+\gamma(r+4,\theta a)}{\theta^r\left[(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}-(\theta^4+6)\right]}$$

Now taking $r = 1,2$, mean and variance can be obtained as

$$\mu_r{}' = \frac{\theta^4\gamma(2,\theta a)+\gamma(5,\theta a)}{\theta\left[(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}-(\theta^4+6)\right]}$$

$$\mu_r{}' = \frac{\theta^4\gamma(3,\theta a)+\gamma(6,\theta a)}{\theta^2\left[(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}-(\theta^4+6)\right]}$$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

Theorem2. Suppose $X_i(b > 0, a \sim 0)$ follows upper (right) truncated TPD$(\theta, 0, b)$. Then the rth moment about origin$\mu_r{}'$ of TPD is

$$\mu_r{}' = \frac{\theta^4\{\gamma(r+1,\theta b)\}+\gamma(r+4,\theta b)}{\theta^r\left[(\theta^4+6)-(b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\right]}; r = 1,2,3, \ldots$$

Proof: Considering $K = \{(\theta^4+6)-(b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\}$ in (7), we have

$$\mu_r{}' = \frac{\theta^4}{K}\int_a^b x^r(\theta+x^3)\, e^{-\theta x}dx$$

$$= \frac{\theta^4}{K}\left[\int_0^b \theta e^{-\theta x}x^r dx + \int_0^b e^{-\theta x}x^{r+3}dx\right]$$

Taking $u = \theta x, x = \frac{u}{\theta}$

$$= \frac{\theta^4}{K}\left[\frac{\theta}{\theta^{r+1}}\left\{\int_0^{\theta b} e^{-u}x^r du\right\} + \frac{1}{\theta^{r+4}}\left\{\int_0^{\theta b} e^{-u}u^{r+3}du\right\}\right]$$

Where $\gamma(\alpha, z) = \int_0^z e^{-x}x^{\alpha-1}dx, \alpha > 0, x > 0$ is the lower incomplete gamma function

$$=\frac{\theta^4}{K}\left[\frac{\gamma(r+1,\theta b)}{\theta^r} + \frac{\gamma(r+4,\theta b)}{\theta^{r+4}}\right]$$

$$=\frac{1}{K}\left[\frac{\theta^4\{\gamma(r+1,\theta b)\}+\gamma(r+4,\theta b)}{\theta^r}\right]$$

$$=\mu_r{}' = \frac{\theta^4\{\gamma(r+1,\theta b)\}+\gamma(r+4,\theta b)}{\theta^r\left[(\theta^4+6)-(b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\right]}$$

Now taking$r = 1,2$, mean and variance can be obtained as

$$\mu_1' = \frac{\theta^4\gamma(2,\theta b)+\gamma(5,\theta b)}{\theta\left[(\theta^4+6)-(b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\right]}$$

$$\mu_2' = \frac{\theta^4\gamma(3,\theta b)+\gamma(6,\theta b)}{\theta^2\left[(\theta^4+6)-(b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\right]}$$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

Theorem3: Suppose X follows doubly TPD (θ, a, b). Then the rth moment about origin $\mu_r{}'$ of TPD is

$$\mu_r{}' = \frac{\theta^4\{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)\}+\{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)\}}{\theta^r\begin{pmatrix}(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}- \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\end{pmatrix}}; r = 1,2,3, \ldots$$

Proof: Considering $K = \begin{Bmatrix}(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a} - \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\end{Bmatrix}$

in (7), we have

$$\mu_r{}' = \frac{\theta^4}{K}\int_a^b x^r(\theta+x^3)\, e^{-\theta x}dx$$

$$= \frac{\theta^4}{K}\left[\int_a^b \theta e^{-\theta x}x^r dx + \int_a^b e^{-\theta x}x^{r+3}dx\right]$$

Taking $u = \theta x, x = \frac{u}{\theta}$

$$= \frac{\theta^4}{K}\left[\begin{array}{l}\frac{\theta}{\theta^{r+1}}\left\{\int_0^{\theta b} e^{-u}x^r du - \int_0^{\theta a} e^{-u}x^r du\right\} + \\ \frac{1}{\theta^{r+4}}\left\{\int_0^{\theta b} e^{-u}u^{r+3} du - \int_0^{\theta a} e^{-u}x^{r+3} du\right\}\end{array}\right]$$

Where $\gamma(\alpha, z) = \int_0^z e^{-x}x^{\alpha-1}dx, \alpha > 0, x > 0$ is the lower incomplete gamma function

$$=\frac{\theta^4}{K}\left[\frac{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)}{\theta^r} + \frac{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)}{\theta^{r+4}}\right]$$

$$=\frac{1}{K}\left[\frac{\theta^4\{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)\}+\{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)\}}{\theta^r}\right]$$

$$=\frac{\theta^4\{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)\}+\{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)\}}{\theta^r\begin{pmatrix}(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}- \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\end{pmatrix}} \quad (9)$$

Now taking$r = 1,2$, mean and variance can be obtained as

$$\mu_1' = \frac{\theta^4\{\gamma(2,\theta b)-\gamma(2,\theta a)\}+\{\gamma(5,\theta b)-\gamma(5,\theta a)\}}{\theta\begin{pmatrix}(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}- \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\end{pmatrix}}$$

$$\mu_2' = \frac{\theta^4\{\gamma(3,\theta b)-\gamma(3,\theta a)\}+\{\gamma(6,\theta b)-\gamma(6,\theta a)\}}{\theta^2\begin{pmatrix}(a^3\theta^3+3a^2\theta^2+6a\theta+\theta^4+6)e^{-\theta a}- \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\theta^4+6)e^{-\theta b}\end{pmatrix}}$$

Variance $\quad \mu_2 = \mu_2' - (\mu_1')^2$

Similarly rest two moments of origin as well as coefficient of variation, coefficient of skewness, coefficient of kurtosis and Index of dispersion can be obtained, substituting $r = 3,4$ in the equation (9), which are as follows:

$$\mu_3' = \frac{\theta^4\{\gamma(4,\theta b) - \gamma(4,\theta a)\} + \{\gamma(7,\theta b) - \gamma(7,\theta a)\}}{\theta^3\begin{pmatrix}(a^3\theta^3 + 3a^2\theta^2 + 6a\theta + \theta^4 + 6)e^{-\theta a} - \\ (b^3\theta^3 + 3b^2\theta^2 + 6b\theta + \theta^4 + 6)e^{-\theta b}\end{pmatrix}}$$

$$\mu_4' = \frac{\theta^4\{\gamma(5,\theta b) - \gamma(5,\theta a)\} + \{\gamma(8,\theta b) - \gamma(8,\theta a)\}}{\theta^4\begin{pmatrix}(a^3\theta^3 + 3a^2\theta^2 + 6a\theta + \theta^4 + 6)e^{-\theta a} - \\ (b^3\theta^3 + 3b^2\theta^2 + 6b\theta + \theta^4 + 6)e^{-\theta b}\end{pmatrix}}$$

Coefficient of Variation=$\frac{\left(\mu_2'-(\mu_1')^2\right)^{1/2}}{\mu_1'}$, Coefficient of Skweness=$\frac{\left(\mu_3'+3\mu_2'\mu_1'-(\mu_1')^2\right)}{(\mu_2'-(\mu_1')^2)^{3/2}}$, Coefficient of Kurtosis= $\frac{\left(\mu_4'-4\mu_3'\mu_1'+6\mu_2'(\mu_1')^2-3(\mu_1')^4\right)}{(\mu_2'-(\mu_1')^2)^2}$,

Index of dispersion= $\frac{\left(\mu_2'-(\mu_1')^2\right)}{\mu_1'}$, graph of above measures is presented in figures 4 to 9. From the figure 4 &5, it was observed that mean and variance are decreasing with increased value of θ and slightly increasing with *b* while parameter *a* is kept constant.

Coefficient of variation of TPD was found deceasing with increased value of θ and *b*.

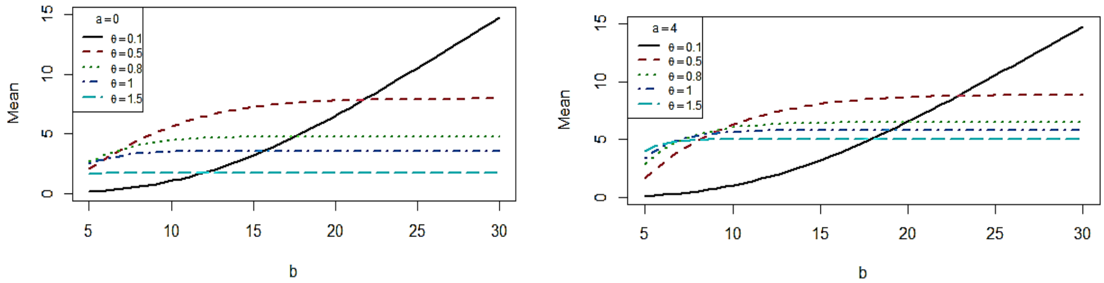

Figure 4: *Mean of TPD (doubly truncated) for varying value of parameter*

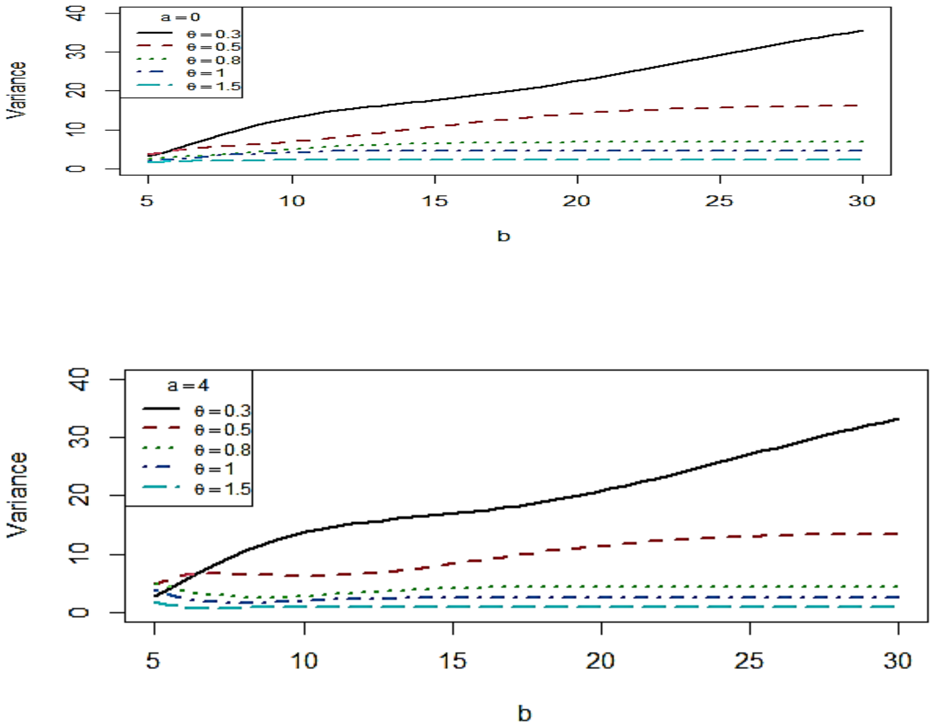

Figure5: *Variance of TPD for varying value of parameter*

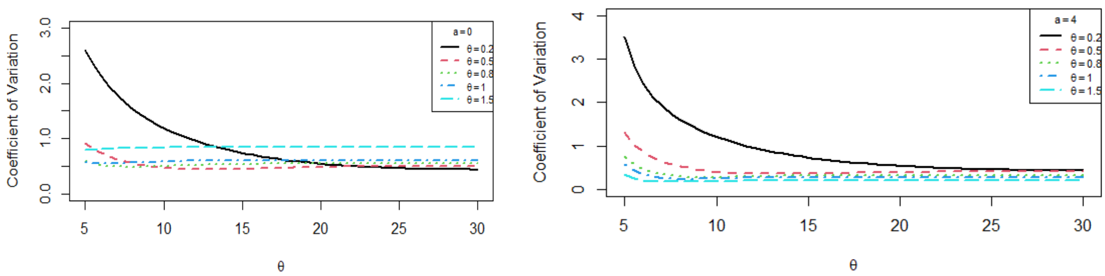

Figure 6*: Coefficient of variation of TPD (Doubly truncated) for varying value of parameter*

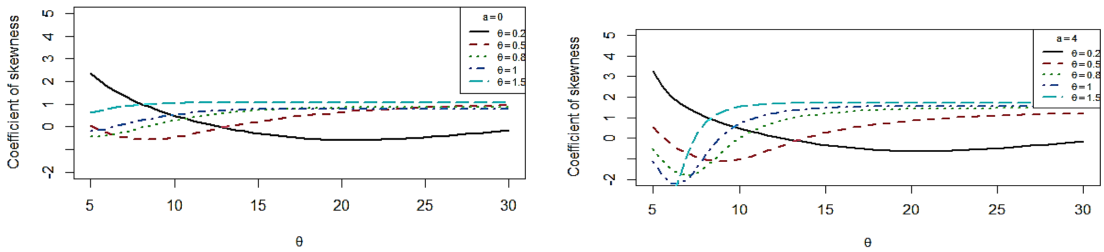

Figure 7: *Coefficient of skewness of TPD (Doubly truncated) for varying value of parameter*

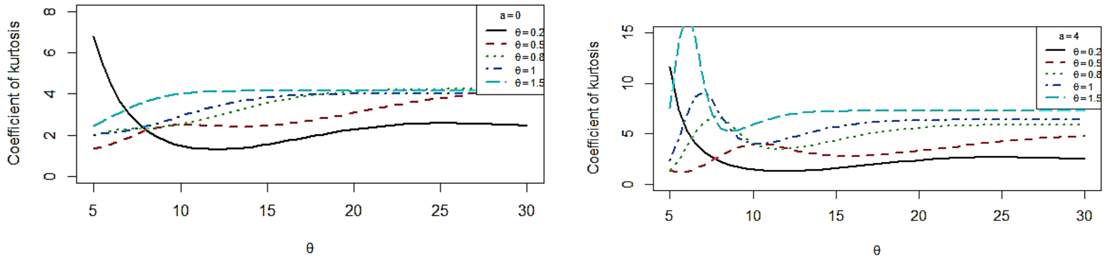

Figure 8: *Coefficient of kurtosis of TPD (Doubly truncated) for varying value of parameter*

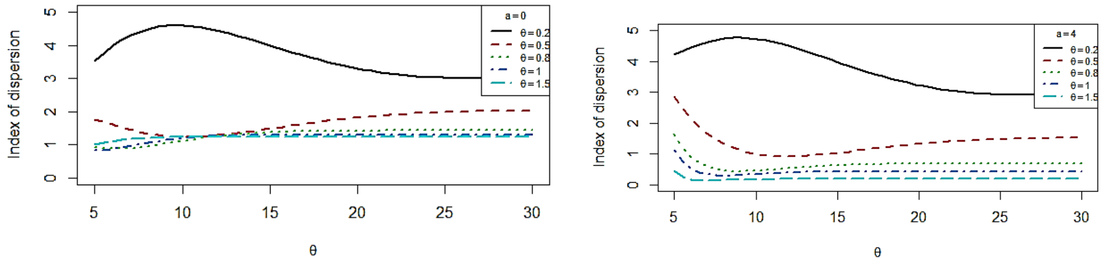

Figure9: *Index of dispersion of TPD (doubly truncated) for varying value of parameter*

5. Maximum likelihood Method Estimation

Let $(x_1, x_2, x_3, \ldots, x_n)$ be a random sample of size n from (7).
The likelihood function, L of TPD is given by

$$L = \left(\frac{\theta^4}{(a^3\theta^3+3a^2\theta^2+\theta^4+6a\theta+6)e^{-\theta a}-(b^3\theta^3+3b^2\theta^2+\theta^4+6b\theta+6)e^{-\theta b}}\right)^n \prod_{i=1}^{n}(\theta + {x_i}^3)\, e^{-n\theta\bar{x}}$$

The log likelihood function is thus obtained as

$$\ln L = n \ln\left(\frac{\theta^4}{(a^3\theta^3+3a^2\theta^2+\theta^4+6a\theta+6)e^{-\theta a}-(b^3\theta^3+3b^2\theta^2+\theta^4+6b\theta+6)e^{-\theta b}}\right) + \sum_{i=1}^{n} \ln(\theta + {x_i}^3) - n\theta\bar{x}$$

Taking $\hat{a} = min(x_1, x_2, x_3, \ldots, x_n)$, $\hat{b} = max(x_1, x_2, x_3, \ldots, x_n)$, the maximum likelihood estimate $\hat{\theta}$of parameter θ is the solution of the log-likelihood equation $\frac{\partial \log L}{\partial \theta} = 0$.
It is obvious that $\frac{\partial \log L}{\partial \theta} = 0$ will not be in closed form and hence some numerical optimization technique can be used e the equation forθ. In this paper the nonlinear method available in R software has been used to find the MLE of the parameterθ.

6. Simulation study

In this section, simulation study has been carried out using R-software. Acceptance and rejection method is used to generate random number, where sample size,$n = 20,40,60,80,100$, value of $\theta = 0.1,0.5,1.0,1.5$& $(a = 10, b = 100)$ have been used for calculating Bias error and MSE (Mean square error) of parameter θ, which is presented in table2.

General algorithm for generating the data which was given by Robert and Casella [15] are as follows: The constraints we impose on this candidate density f_Y are that:

(i) Y be simulate-able (the data simulation from Y be actually possible).

(ii) There is a constant c with $\frac{f_X(x)}{f_Y(x)} \leq c$ for all $x \in S_x = \{x: f_X(x) > 0\}$

(iii) $f_X(x)$ and $f_Y(x)$ have compatible supports (i.e., $S_X \subseteq S_Y$).

In this case, X can be simulated as follows by Accept-Reject method. First, we generate y from $Y \sim f_Y$ and, independently, we generate u from $U \sim U(0,1)$.

If $u \leq \frac{f_X(y)}{c f_y(y)}$ then we set $x = y$. If the inequality is not satisfied, we then discard/reject y and u and start again.

Table2: *Average Bias (AB) and Average MSE (AM) of the simulated MLEs of θ at fixed value of* $a = 10, b = 100$

Sample	θ	A B	AM	AB	AM
20	0.1	0.0606620	0.07359774	0.04834145	0.04673793
	0.5	0.0509281	0.05187350	0.05092813	0.00518735
	1.0	0.02592813	0.01344536	0.02592813	0.01344536
	1.5	0.00092813	0.000017228	0.00092813	0.00001722
40	0.1	0.02391497	0.02287705	0.02420517	0.02343561
	0.5	0.02546406	0.02593675	0.02546406	0.0259367
	1.0	0.01296406	0.006722681	0.01296406	0.00672268
	1.5	0.00046406	0.0000086143	0.00046406	0.00008614
60	0.1	0.01535922	0.01415435	0.01619160	0.01573008
	0.5	0.016976044	0.01729117	0.01697604	0.00172911
	1.0	0.008642711	0.004481787	0.00864271	0.00044817
	1.5	0.000309377	0.0000057428	0.00030937	0.00005742
80	0.1	0.010465770	0.008762596	0.01131883	0.01024928
	0.5	0.012732033	0.01296837	0.01273203	0.01296837
	1.0	0.006482033	0.003361341	0.00648203	0.00336134
	1.5	0.00023203	0.000004.307	0.00023204	0.00000430
100	0.1	0.00882808	0.007793502	0.00890854	0.07936211
	0.5	0.010185626	0.01037470	0.01018562	0.01037470
	1.0	0.005185626	0.002689072	0.00518562	0.00268907
	1.5	0.000185626	0.0000034457	0.00018562	0.00000344

From the above table, it is observed that Bias and Mean square error are decreasing as increased value of sample size.

7. Applications on lifetime data

In this section, TPD has been applied to two datasets using maximum likelihood estimates. Parameter θ estimated whereas another parameters a, and b are considered as lowest and highest values of data, i.e.

$\hat{a} = min(x_1, x_2, x_3, \ldots, x_n)$& $\hat{b} = max(x_1, x_2, x_3, \ldots, x_n)$. Goodness of fit has been decided using Akaike information criteria (AIC), Bayesian Information criteria (BIC) and Kolmogorov Simonov test (KS) values respectively, which are calculated for each distribution and compared with p-value. As we know that best goodness of fit of the distribution can be decided based on minimum value of KS, AIC and BIC.

Data Set 1: The data is given by Birnbaum and Saunders [16] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second.

Data Set 2: This data set is the strength data of glass of the aircraft window reported by Fuller et al. [17]:

18.83 20.8 21.657 23.03 23.23 24.05 24.321 25.5 25.52 25.8 26.69 26.77 26.78 27.05 27.67 29.9 31.11 33.2 33.73 33.76 33.89 34.76 35.75 35.91 36.98 37.08 37.09 39.58 44.045 45.29 45.381

Table 3: *MLE's, Standard Errors, - 2ln L, AIC, K-S and p-values of the fitted distributions for data set-1*

Distributions	ML Estimates	Standard Errors	$-2\ln L$	AIC	BIC	K-S	p-value
TPD	$\boldsymbol{\hat{\theta} = 0.055278}$	**0.00330**	**927.37**	**929.37**	**928.76**	**0.136**	**0.048**
TAD	$\hat{\theta} = 0.03917$	0.00303	939.13	941.13	942.05	0.153	**0.017**
TLD	$\hat{\theta} = 0.02199$	0.00273	958.88	960.88	962.31	0.186	**0.001**
Pranav	$\hat{\theta} = 0.04387$	0.00253	950.97	952.97	954.40	0.194	**0.001**
Ishita	$\hat{\theta} = 0.04390$	0.002533	950.92	9952.92	954.35	0.194	**0.001**
Lindley	$\hat{\theta} = 0.02886$	0.002038	983.10	985.10	986.54	0.252	**0.000**
Exponential	$\boldsymbol{\hat{\theta} = 0.01463}$	**0.001457**	**1044.87**	**1046.87**	**1048.30**	**0.336**	**0.000**

Table 4: *MLE's, Standard Errors, - 2ln L, AIC, K-S and p-values of the fitted distributions for data set-2*

Distributions	ML Estimates	Standard Errors	$-2\ln L$	AIC	BIC	K-S	p-value
TPD	**0.12067**	**0.02455**	**201.80**	**203.80**	**203.19**	**0.107**	**0.829**
TAD	$\hat{\theta} = 0.08776$	0.024241	201.96	203.96	205.58	0.112	**0.786**
TLD	$\hat{\theta} = 0.05392$	0.023917	202.18	204.18	205.61	0.117	**0.738**
Pranav	$\hat{\theta} = 0.09706$	0.01004	240.68	242.68	242.67	0.298	**0.005**
Ishita	$\hat{\theta} = 0.097328$	0.01008	240.48	242.48	243.48	0.297	**0.006**
Lindley	$\hat{\theta} = 0.06299$	0.00800	253.98	255.98	256.98	0.365	**0.000**
Exponential	$\boldsymbol{\hat{\theta} = 0.032452}$	**0.00582**	**274.52**	**276.52**	**277.52**	**0.458**	**0.000**

From the above tables 3&4 , it was observed that TPD gives good fit over other selected distributions. Fitted plots of the considered distributions are presented in figure 10 and 11 respectively.

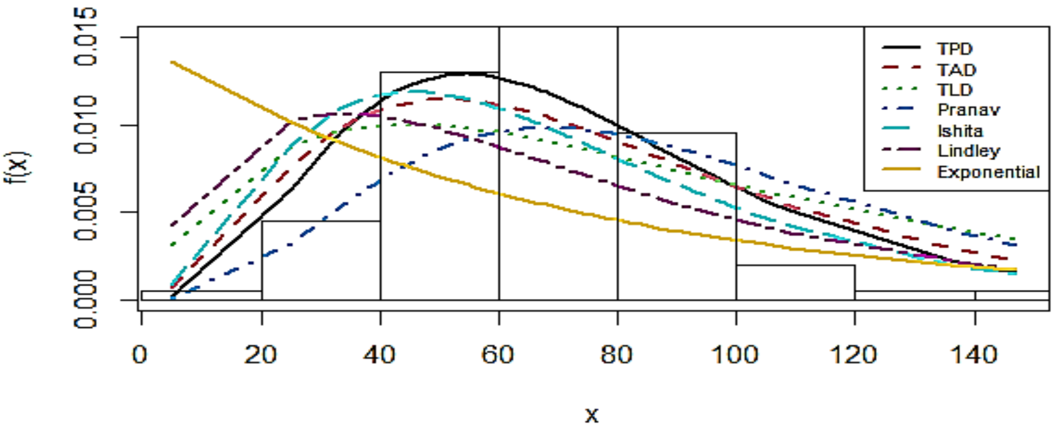

Figure10: *Fitted plots of distributions for the dataset 1*

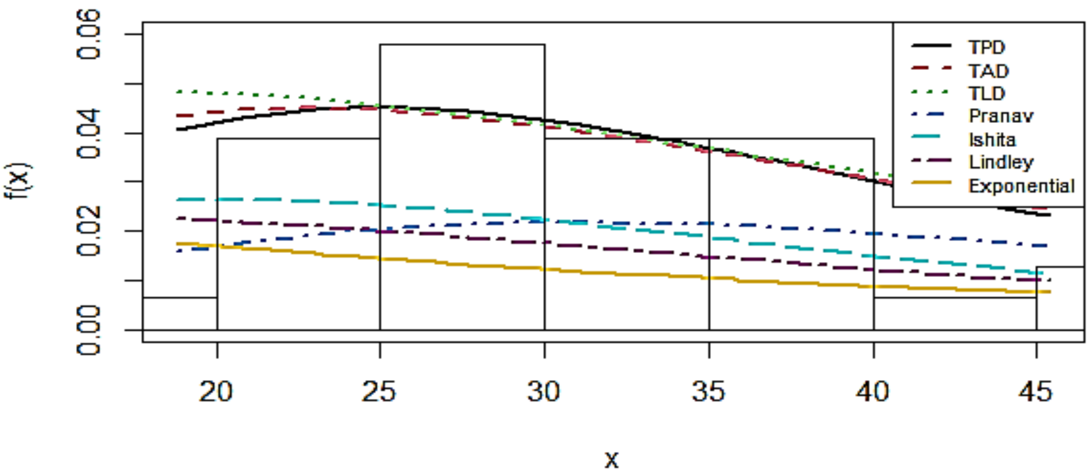

Figure11: *Fitted plot of distributions for dataset 2*

7. Conclusions

In this paper, truncated Pranav distribution (TPD) has been proposed. Its statistical and mathematical properties have been discussed. Maximum likelihood method has been used for estimation of its parameter. Simulation study has also been conducted to know behavior of proposed distribution. Goodness of fit of TPD has been discussed with two lifetime datasets and superiority has been checked with truncated Akash, truncated Lindley, Pranav, Ishita, Lindley, and exponential distributions. It has been observed that TPD gives good fit on both the data sets. In the first data set, value of AIC 929.37 and KS value- 0.136, p-value =0.048 <0.01, and for the second data set, value of AIC-203.80 and KS value- 0.107 (p-value =0.829 >0.05) were observed which were compared over two parameter TAD (truncated Akash Distribution), TLD (truncated Lindley Distribution) and one parameter Pranav, Ishita, Lindley and exponential distribution. Therefore, it may be considered good distribution for the lifetime data specially on fixed values (lower limit, upper limit).

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