Study of reliability of the on-tether subsystem of a tethered high-altitude unmanned telecommunication platform

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Abstract

High-altitude platform (HAP) are stations on an object at an altitude of around 15-50 km at a specified nominal fixed point relative to Earth. Tethered high-altitude platform (tHAP) are unmanned aerial vehicle that are connected to the ground via a tether with a lift height of 100 - 150 meters, and a multi-copter as high altitude mode. The reliability of the tHAP can be assessed with a focus on the tether that connects it to the ground. This article proposes a Markov model which obtain the reliability of the tHAP. The tether is considered to be made up of multiple wires in such a way that the tether still operates for a given number of functioning wires. The failure rates of the wires are dependent on the number of failed wires. Through the reliability analysis of the proposed Markov model, the key performance measures such as reliability of the system, mean time between failures and the probability of the system being reliable are computed. The optimal number of wires is also obtained via the numerical computation of the performance measures.

Keywords: Markov chain, reliability, tethered high-altitude platform, unmanned aerial vehicle

1. INTRODUCTION

Autonomous unmanned aerial vehicles (UAVs) are currently in widespread use. The biggest disadvantage of UAVs is their limited operational period due to a limited energy resource of batteries or fuel-carrying capacity. High-altitude platform (HAP)s are stations on an object at an altitude of around 15-50 km at a specified nominal fixed point relative to Earth. The long-term operation can be provided by a tethered high-altitude platform (tHAP), with a lift height of 100-150 meters, in which power supply of engines and payload equipment are provided from ground-based power sources via copper cables [1].

tHAPs have several advantages when compared to terrestrial stations or satellite stations ([2] [3]). tHAPs have an intermediate geographical range between that of terrestrial stations and a satellite stations. tHAPs can be deployed in a matter of hours. They can be operated for long duration and can return to ground easily. Furthermore they have a secure and efficient back-haul, since tHAPs are connected to the core network via wire. tHAPs are also extensively used in the case of disaster situation in the concerned region due to the easy deployment.

Components of a tHAP are shown in Fig. 1. tHAPs consists of a flying platform connected to the Ground Control Station (GCS) via a tether. The GCS houses a system for diagnostics and control of the tHAP from the ground. The tether also includes power transmission cable and data transmission cable. The navigation and stabilization system comprises of on-board sensors,

on-ground anchor points, etc., that measures various parameters for navigation, and the circuits that stabilize the platform based on these parameters. The main power source is located on the ground. The intelligent wench is responsible for controlling the length of the tether.



Figure 2: UAV Failure Causes

Fig. 2 shows the contribution of various factors in the failure of an ordinary UAV. Most of the failures can be attributed to the power system. These failures should be less frequent in the case of tHAP, since the power source is stationed on the ground and more reliable. The failures of navigation system should also be fewer since the tHAP is mostly stationary. The failure rate of GCS, electric system, mainframe and payload should not differ much. This data does not include the failures of the components in the tether [5]. The tether will be around 15-50km long. It will also have a total mass around a few hundred kg. There is a need to focus on improving the reliability of the tether that connects the tHAP to the ground as well as the power and communication cables. Vishnevsky et al. [7] studied the reliability of the functioning of a

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flight module of a tethered high-altitude telecommunication platform, utilizing the k-out-of-n : F model. The authors of [9] proposed an analytical model of the k-out-of-n:G system under two system failure scenarios. Ivanova [10] presented a hot standby repairable k-out-of-n system.

This study proposes a Markov model setup for the prediction of reliability of the tether. It has been assumed that the tether is constructed of multiple wires. The functioning of the tether depends on the number of working wires. If the number is below some level, the tether will stop working. To this end, numerical examples have been carried out. The obtained results have been found to be very close to the existing results of [6] which uses a different criterion - minimizing the weight of the cable.

The paper is divided into four parts. Section 2 provides a general overview and a broad description of the approach taken. Section 3 comprises of the analysis of the given approach using Markov model. Section 4 shows the various numerical results obtained based on the analysis. Section 5 provides the conclusions and future work.



2. MODEL DESCRIPTION

Figure 3: Diagram of the proposed model

Consider the cable be made of multiple parallel wires, such that the cable still operates provided a given number of these wires are still functioning. As seen in Fig. 3, the cable consists of n wires running parallel to each other. The wires have current flowing through them with different phases. Their phases will be symmetrically distributed such that the total current is zero. Whenever a wire has a fault, the phases of the currents in the remaining wires will be adjusted such that the total current is still zero.

Assume that a cable made up of *n* wires will still function reasonably provided *k* wires are still working. Whenever a wire fails there is, in general, a higher probability for the next wire to fail. In this work, take this into consideration by assuming that the failure rates of the wires are dependent on the number of wires already failed. Furthermore, assume that the failure rates of the remaining wires are the same. Let λ_i be failure rate of each wire after *i* wires have failed.



Figure 4: State transition diagram for the proposed Markov model

The system can be assessed using a Markov model [8]. The Markov model of the system after applying state aggregation is shown in Fig. 4. Here S_i ($0 \le i \le n - k$) is the state in which exactly *i* wires failed and n - i wires are working. S_{n-k+1} is the state in which more than *k* wires have faulted, i.e., the whole system fails.

3. Reliability Analysis

Let $P_n(t)$ be the probability that the system in the state S_n at time t. The governing equations are given by

$$\frac{dP_0(t)}{dt} = -n\lambda_0 P_0(t) \tag{1}$$

$$\frac{dP_i(t)}{dt} = (n-i+1)\lambda_{i-1}P_{i-1}(t) - (n-i)\lambda_i P_i(t), \quad 1 \le i \le n-k$$
(2)

$$\frac{dP_{n-k+1}(t)}{dt} = k\lambda_{n-k}P_{n-k}(t).$$
(3)

Initial condition is given by $P_0(0) = 1$, $P_i(0) = 0$, $1 \le i \le n - k + 1$. Solving the above Equations (1-3) with the initial condition, we get,

$$P_0(t) = e^{-n\lambda_0 t} \tag{4}$$

$$P_i(t) = \sum_{j=0}^{i} A_{ij} e^{-(n-j)\lambda_j t}, 0 < i \le n-k$$
(5)

$$P_{n-k+1}(t) = 1 - \sum_{i=0}^{n-k} P_i(t)$$
(6)

where

$$A_{ij} = \frac{\prod_{h=0}^{i-1} (n-h)\lambda_h}{\prod_{h=0, h\neq j}^i (n-h)\lambda_h - (n-j)\lambda_j}.$$

3.1. Measures

1. Reliability of the system is computed by using Equations (4) - (5) and is given by,

$$R_{n}(t) = \sum_{i=0}^{n-k} P_{i}(t)$$

= $\sum_{j=0}^{n-k} \frac{\prod_{h=0}^{n-k} (n-h)\lambda_{h}}{\prod_{h=0,h\neq j}^{n-k} (n-h)\lambda_{h} - (n-j)\lambda_{j}} e^{-(n-j)\lambda_{j}t}.$ (7)

2. Mean time between failures $(MTBF_n)$ is computed using Equation (7) and is given by,

$$MTBF_n = \int_0^\infty R_n(t)dt.$$

3. Let T_{conf} be the largest value of t for which probability of system being reliable for time t is greater than or equal to p_{conf} .

$$T_{conf} = max(t|R_n(t) \ge p_{conf}).$$

3.2. Special Case

Suppose λ_i are equal to λ , Equations (4) - (6) become

$$P_{i}(t) = {}^{n}C_{i}e^{-n\lambda t}(e^{\lambda t}+1)^{i}$$

$$P_{n-k+1}(t) = 1 - \sum_{i=0}^{n-k}P_{i}(t).$$

$$R_{n}(t) = \sum_{i=0}^{n-k}\sum_{j=0}^{i}{}^{n}C_{i}{}^{i}C_{j}e^{-(n-j)\lambda t}.$$

Further,

$$MTBF_n = \frac{1}{\lambda} \sum_{i=0}^{n-k} \sum_{j=0}^{i} \frac{{}^nC_i{}^iC_j}{n-j}.$$

4. NUMERICAL RESULTS

In this section, numerical illustration is presented for the proposed model. In order to plot the graphs, the parameters' values are considered depending on the stability conditions of the proposed model. The graphs have been plotted considering $\lambda = 0.1 year^{-1}$ for various values of ρ .

Fig. 6 shows the plot of the reliability function for various n taking $\lambda = 0.1 year^{-1}$ for the special case $\lambda_i = \lambda$. It can be observed from the figure that the value of $R_n(t)$ decreases with respect to the time as there will be more chances for the system to fail due to the circumstances. As the value of n increases, the reliability can be seen to increase. After n = 6, the reliability starts to decrease which provides the optimal value of n.

The similar behaviour can be observed from Fig. 5(b) for $\rho = 1.5$. Here, it can be observed that $R_n(t)$ is exhibiting the decreasing behaviour with respect to t. Also, $R_n(t)$ starts decreasing after n = 6 which shows that n = 6 is the optimal value. On the similar track, Fig. 5(a) demonstrates the similar behaviour of $R_n(t)$ with respect to t for $\rho = 0.8$. Here also the obtained optimal value of n is 6. Similarly, Table 1 gives the values of $MTBF_n$ for various n. The values of T_{conf} for various values of n and p_{conf} taking $\lambda = 0.1 year^{-1}$ is shown in Table 2.



Figure 5: Reliability function for various n.

n ρ n	1	1.5	2	2.5			
4	5.8333	4.7222	4.1667	3.8333			
5	6.7193	5.0724	5.3225	4.4667			
6	6.9698	5.4921	5.8811	4.5921			
7	4.4421	3.7880	3.5552	3.2986			
8	1.3850	1.4006	1.3080	1.7920			
9	1.1108	1.3276	1.2102	1.3677			

Table 1: *MTBF_n* vs n taking $\lambda = 0.1$ year⁻¹, for various ρ



Figure 6: *Reliability function for various n for* $\lambda_i = \lambda$ *.*

<i>p</i> _{conf} n	0.9	0.95	0.99	0.995	0.999
4	0.334	0.221	0.189	0.096	0.024
5	1.221	0.967	0.746	0.544	0.100
6	1.975	1.464	1.002	0.866	0.377
7	1.556	1.115	0.988	0.545	0.323
8	1.215	0.966	0.766	0.432	0.307
9	0.977	0.851	0.676	0.402	0.211

Table 2: *Values of* T_{conf} (years) for $\rho = 1.5$ and k = 3 for various n and p_{conf} .

5. Conclusions and Future Work

Based on the reliability graphs, Table 1 and Table 2, the reliability improves as increase the number of wires n until n = 6, after which the reliability decreases. Thus, a 6- core wire will be the best configuration for the chosen parameters. Vishnevsky et al. [6] obtained in their study of tether HAP system, by a different criterion, i.e., minimizing the weight of the cable. The obtained results are very close to the results obtained by [6].

In this study, all distributions in the proposed model are considered as independent exponential distributions. Hence, the underlying stochastic process is a Markov process. By extension of the above consideration, for future work, this research work can be considered with non Markov processes. As a future work, the non Markov model can be simulated to test whether Markov process is a good approximation for more complex non Markov processes.

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