New Cosine-Generator With an Example of Weibull Distribution: Simulation and Application Related to Banking Sector

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Abstract

In this work, we propose a novel trigonometric-based generator entitled the "New Cosine-Generator" to acquire elevated distribution adaptability. This generator is formed without the insertion of extra parameters. Adopting the Weibull distribution as the baseline distribution, and this distribution is referred to as the New Cosine-Weibull Distribution. Several statistical features of the investigated distribution were studied, including moments, moment generating functions, order statistics, and reliability measures. For different parameter values, a graphical representation of the probability density function (pdf) and the cumulative distribution function (cdf) is provided. The distribution's parameters are determined using the well-known maximum likelihood estimation approach. Finally, simulation analysis and an application is used to evaluate the effectiveness of the distribution.

Keywords: Cosine function, moments, maximum likelihood estimation, reliability indicators, simulation.

1. Introduction

In applied sciences such as biological sciences, medical sciences, environmental sciences, engineering, finance, and actuarial science, among others, statistical evaluation of lifetime data is unpredictable, and statistical modelling is the finest and most effective technique to examine the ambiguity of any occurrence. Because of the complex nature and distinctive characteristics of data, life time data serves a critical function in sectors such as insurance and finance. Thus, there is an apparent need for expansion and modification of current traditional statistical distributions. Indeed, various initiatives have been made to develop additional classes of lifetime distributions in order to extend various families of distributions and offer more adaptability to the novel model. Numerous investigators have added new classes of life time distributions throughout the last few years, which are now available in the statistical literature. The implementation of trigonometric functions to construct new statistical distributions is becoming a prevalent approach, and employing these trigonometric distributions for data interpretation demonstrates greater versatility. Looking back over the literature, we can see that numerous authors employed many generators or transformations. For instance, Eugene et al. [16], the gamma-G family by Zagrofos and Balakrishana [20], the transformedtransformer(T-X) by Alzaatrah et al [1], the Weibull-G by Bourguignon et al. [6], Morad Alizadeh et al. [17] constructed the Gompertz-G distribution family, Brito et al. [7], formulated the Topp-Leone odd log-Logistic family of distributions and Aijaz et al. [4] a noval aproach for constructing distributions with an example of Rayleigh distribution, SS-transformation based on trigonometric functions is proposed by kumar et al. [2], Chesneau et al.[10], Mahmood. Z and Chesneau. C [18], Souza.l et al.[13], Jammal.F et al. [11], M.A.Lone et al.[19],I.H. Dar et al [5] and Aijaz Ahmad et al. [3]. This work aims to present the cosine-generator distributions, a novel family of trigonometric function-based generator. The benefit of this generator is that flexibility is achieved without the insertion of further parameters.

Let us suppose $F(x;\zeta)$ be cdf of a random variable X, then the cumulative distribution function of new cosine-generator family of distributions is described as.

$$\begin{aligned} F(x;\zeta) &= -\int_0^{\frac{\pi(2-2^{\bar{G}(x;\zeta)})}{2}} sinxdx \\ &= 1 - cos\left(\frac{\pi(2-2^{\bar{G}(x;\zeta)})}{2}\right) \quad ; \quad x \in \mathbb{R}, \zeta > 0 \end{aligned} \tag{1}$$

The related probability density function of equation (1) is stated as

$$f(x;\zeta) = \frac{\pi}{2} log(2) 2^{\bar{G}(x;\zeta)} g(x;\zeta) sin\left(\frac{\pi(2-2^{\bar{G}(x;\zeta)})}{2}\right); \quad x \in \mathbb{R}, \zeta > 0 \tag{2}$$

Where $\bar{G}(x;\zeta)) = 1 - G(x;\zeta)$ and $\frac{dG(x;\zeta)}{dx} = g(x;\zeta)$.
Futhermore, the reliability function represented as $R(x;\zeta)$, hazard rate function represented as $H(x;\zeta)$ and reverse hazard rate function represented as $h(x;\zeta)$ are respectively stated in general form by

$$\begin{aligned} R(x;\zeta) &= 1 - F(x;\zeta) = cos\left(\frac{\pi(2-2^{\bar{G}(x;\zeta)})}{2}\right) \\ H(x;\zeta) &= \frac{f(x;\zeta)}{R(x;\zeta)} = \frac{\pi}{2} log(2) 2^{\bar{G}(x;\zeta)} g(x;\zeta) tan\left(\frac{\pi(2-2^{\bar{G}(x;\zeta)})}{2}\right) \\ h(x;\zeta) &= \frac{f(x;\zeta)}{F(x;\zeta)} = \frac{\frac{\pi}{2} log(2) 2^{\bar{G}(x;\zeta)} g(x;\zeta) sin\left(\frac{\pi(2-2^{\bar{G}(x;\zeta)})}{2}\right)}{1 - cos\left(\frac{\pi(2-2^{\bar{G}(x;\zeta)})}{2}\right)} \end{aligned}$$

The Weibull distribution has been employed in a wide range of domains and applications. The hazard function of the Weibull distribution can only be monotone. As a consequence, it cannot be used to replicate lifespan data with a bathtub-shaped hazard function. We adopt the Weibull distribution as the baseline distribution for the newly formed generator and exhibit its numerous characteristics.
Suppose X denotes a random variable that follows the Weibull distribution, then its cumulative distribution function is stated as

$$G(x;\alpha,\beta) = 1 - e^{-\alpha x^{\beta}}; \quad x > 0, \alpha, \beta > 0 \tag{3}$$

The related probability density function is given as

$$g(x;\alpha,\beta) = \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}; \quad x>0,\alpha,\beta>0 \tag{4}$$

1.1. Usefull expansion

We apply Taylor's series of $sinx = \sum_{p=0}^{\infty}(-1)^p\frac{x^{2p+1}}{(2p+1)!}$ in equation (2) to get its mixture form

$$f(x;\zeta) = \sum_{p=0}^{\infty}\frac{(-1)^p}{(2p+1)!}\frac{\pi}{2}log(2)g(x;\zeta)2^{\bar{G}(x;\zeta)}2^{2p+1}\left(1-2^{-G(x;\zeta)}\right)^{2p+1} \tag{5}$$

Now, we apply $(1-u)^a = \sum_{q=0}^{\infty}(-1)^q\binom{a}{q}u^q$ in equation (5), we obtain

$$f(x;\zeta) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-1)^{p+q}}{(2p+1)!}log(2)\binom{2p+1}{q}\pi^{2p+2}g(x;\zeta)2^{-(q+1)G(x;\zeta)} \tag{6}$$

Again we apply Taylor's series of $a^y = \sum_{r=0}^{\infty}\frac{(log(a))^r}{r!}y^r$in equation (6), we have

$$f(x;\zeta) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\frac{(-1)^{p+q+r}}{(2p+1)!}\frac{(log(2))^{r+1}}{r!}\binom{2p+1}{q}\pi^{2(p+1)}(q+1)^r g(x;\zeta)\left(G(x;\zeta)\right)^r \tag{7}$$

Equations (3) and (4) in equation (7) enable us to construct the baseline model's probability density function in mixture form, which has been employed as an illustration for the established generator.

$$f(x;\alpha,\beta) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\alpha\beta x^{\beta-1}e^{-\alpha(r+1)x^{\beta}} \tag{8}$$

$$\Psi_{pqrs} = \frac{(-1)^{p+q+r+s}}{(2p+1)!}\frac{(log(2))^{r+1}}{r!}\binom{2p+1}{q}\binom{r}{s}\pi^{2(p+1)}(q+1)^r$$

2. New Cosine-Weibull Distribution and its Mathematical properties

By applying the new cosine-generator, we exhibit the probability density function (pdf) and cumulative distribution function (cdf) of a newly formed distribution called new Cosine-Weibull distribution (NCWD) in this part and strengthen certain of its mathematical features. Using equation (3) in equation(4), we obtain the cdf of desired distribution as

$$F(x;\alpha,\beta) = 1 - cos\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right) \quad ; \quad x>0,\alpha,\beta>0 \tag{9}$$

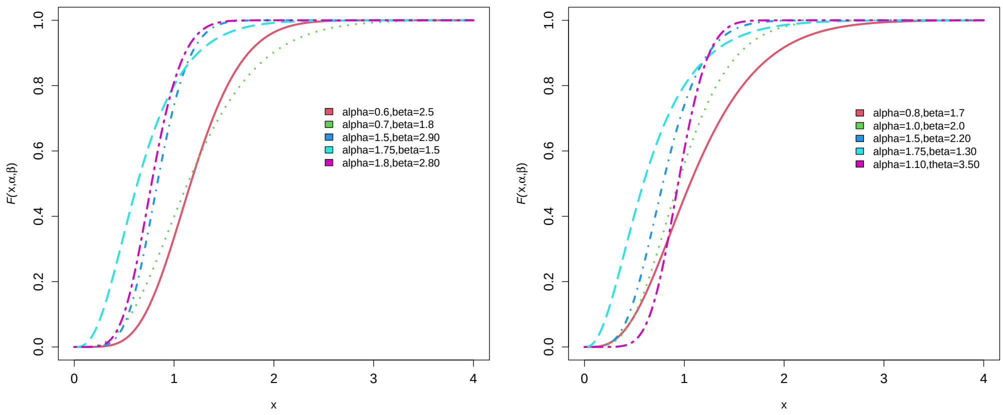

Figure 1: *cdf plot of NCWD for different choise of parameters*

The related probability density function is stated as

$$f(x;\alpha,\beta) = \frac{\pi log(2)}{2}\alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}} sin\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right) \quad ; \quad x>0,\alpha,\beta>0 \tag{10}$$

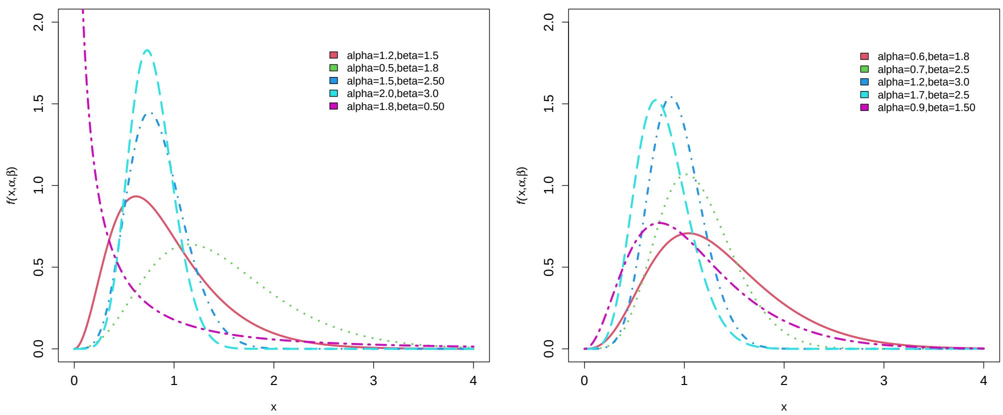

Figure 2: *pdf plot of NCWD for different choise of parameters*

2.1. Moments of new cosine-Weibull distribution

Let x denotes a random variable, then the k^{th} moment of NCWD is denoted as $\mu_k^{'}$ and is given by

$$\mu_k^{'} = E(x^k) = \int_0^{\infty} x^k f(x;\alpha,\beta)dx \tag{11}$$

Using equation (8) in equation (11), it yields

$$\mu_k^{'} = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\alpha\beta\int_0^{\infty} x^{k+\beta-1}e^{-\alpha(r+1)x^{\beta}}dx$$

Making substitution $\alpha(r+1)x^{\beta} = z$, sothat $0 < z < \infty$, we have

$$\mu_k^{'} = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\frac{\alpha}{(\alpha(r+1))^{\frac{k+\beta}{\beta}}}\int_0^{\infty} z^{\frac{k}{\beta}}e^{-z}dz$$

After solving the integral, we obtain

$$\mu_k^{'} = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\frac{\alpha}{(\alpha(r+1))^{\frac{k+\beta}{\beta}}}\Gamma\left(\frac{k+\beta}{\beta}\right)$$

Where $\Gamma(.)$ denotes the gamma function.

2.2. Moment generating function of new cosine-Weibull distribution

Suppose x denotes a random variable follows NCWD. Then the moment generating function of the distribution denoted by $M_X(t)$is given

$$\begin{aligned}
M_X(t) = E(e^{tx}) &= \int_0^{\infty} e^{tx}f(x;\alpha,\beta)dx \\
&= \int_0^{\infty}\left(1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + ...\right)f(x;\alpha,\beta)dx \\
&= \sum_{k=0}^{\infty}\frac{t^k}{k!}\int_0^{\infty} x^k f(x;\alpha,\beta)dx \\
&= \sum_{k=0}^{\infty}\frac{t^k}{k!}E(x^k) \\
&= \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\sum_{k=0}^{\infty}\Psi_{pqrs}\frac{\alpha t^k}{k!\,(\alpha(r+1))^{\frac{k+\beta}{\beta}}}\Gamma\left(\frac{k+\beta}{\beta}\right)
\end{aligned}$$

2.3. Incomplete moments of new cosine-Weibull distribution

The v^{th} incomplete moment for density function in general is stated as

$$I(v) = \int_0^{v} x^k f(x;\alpha\beta)dx$$

Using the equation (8), we have

$$I(v) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\alpha\beta\int_0^{v} x^{k+\beta-1}e^{-\alpha(r+1)x^{\beta}}dx$$

Making substitution $\alpha(r+1)x^{\beta} = z$, sothat $0 < z < \alpha(r+1)v^{\beta}$, we have

$$I(v) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\frac{\alpha}{(\alpha(r+1))^{\frac{k+\beta}{\beta}}}\int_0^{\alpha(r+1)v^{\beta}} z^{\frac{k}{\beta}}e^{-z}dz$$

After solving the integral, we obtain

$$I(v) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\frac{\alpha}{(\alpha(r+1))^{\frac{k+\beta}{\beta}}}\gamma\left(\frac{k+\beta}{\beta},\alpha(r+1)v^{\beta}\right)$$

Where $\gamma(a,x) = \int_0^{x} u^{a-1}e^{-u}du$ denotes lower incomplete gamma function

2.4. Quantile function of new cosine-Weibull distribution

The quantile function of any distribution may be described as follows:

$$Q(u) = X_q = F^{-1}(u)$$

Where $Q(u)$ denotes the quantile function of $F(x)$ for $u \in (0, 1)$.
Let us suppose

$$F(x) = 1 - cos\left(\frac{\pi(2 - 2^{e^{-\alpha x^{\beta}}})}{2}\right) = u \tag{12}$$

After simplifying equation (12), we obtain quantile function of NCWD distribution as

$$Q(u) = X_q = \left\{\frac{-1}{\alpha} log\left[\frac{1}{log(2)} log\left(\frac{2\pi - 2\arccos(1-u)}{\pi}\right)\right]\right\}^{\frac{1}{\beta}}$$

3. Reliability Measures of new cosine-Weibull distribution

This section is focused on researching and developing distinct ageing indicators for the formulated distribution.

3.1. Survival function

Suppose X be a continuous random variable with cdf $F(x)$.Then its Survival function which is also called reliability function is defined as

$$S(x) = p_r(X > x) = \int_x^{\infty} f(x)dy = 1 - F(x)$$

Therefore, the survival function for NCWD is given as

$$\begin{aligned} S(x; \alpha, \beta) =& 1 - F(x; \alpha, \beta) \\ =& cos\left(\frac{\pi(2 - 2^{e^{-\alpha x^{\beta}}})}{2}\right) \end{aligned} \tag{13}$$

3.2. Hazard rate function

The hazard rate function of a random variable x is denoted as

$$H(x; \alpha, \beta) = \frac{f(x; \alpha, \beta)}{S(x; \alpha, \beta)} \tag{14}$$

Using equation (10) and (13) in equation (14), then the hazard rate function of NCWD is given as

$$H(x; \alpha, \beta) = \frac{\pi log(2)}{2} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} 2^{e^{-\alpha x^{\beta}}} tan\left(\frac{\pi}{2}\left(2 - 2^{e^{-\alpha x^{\beta}}}\right)\right)$$

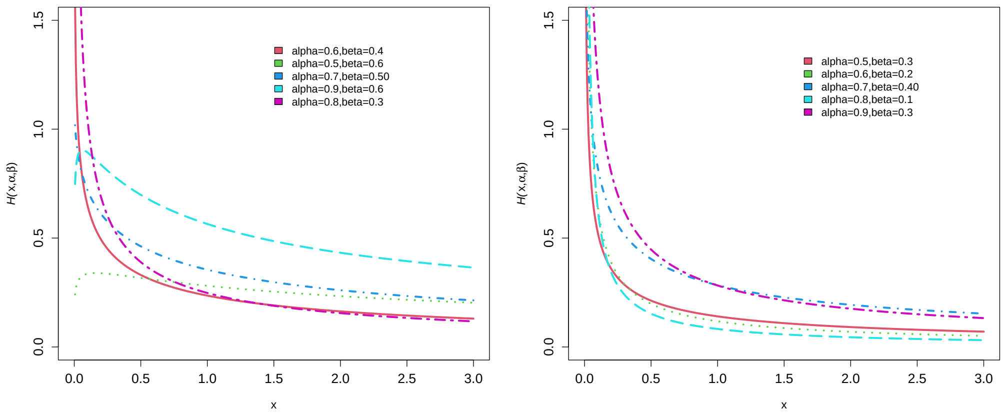

Figure 3: *HRF plot of NCWD for different choise of parameters*

3.3. Reverse hazard rate function

The reverse hazard rate function of a random variable x is given as

$$h(x;\alpha,\beta) = \frac{f(x;\alpha\beta)}{F(x;\alpha\beta)} \tag{15}$$

using equations (9) and (10) in equation (15), then we obtain reverse hazard rate function as

$$h(x;\alpha,\beta) = \frac{\frac{\pi log(2)}{2}\alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}} sin\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)}{1 - cos\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)}$$

3.4. Mean residual function

The mean residual lifetime is the predicted residual life or the average completion period of the constituent after it has exceeded a certain duration x. It is extremely significant in reliability investigations.

Mean residual function of random x variable can be obtained as

$$\begin{aligned} m(x;\alpha,\beta) &= \frac{1}{S(x;\alpha,\beta)}\int_x^{\infty} tf(t,\alpha,\beta)dt - x \\ &= \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\Psi_{pqrs}\alpha\beta \sec\left((\pi/2)(2-2^{e^{-\alpha x^{\beta}}})\right)\int_x^{\infty} t^{\beta}e^{-\alpha(r+1)t^{\beta}}dt - x \end{aligned}$$

Making substitution $\alpha(r+1)t^{\beta} = z$, sothat $\alpha(r+1)x^{\beta} < z < \infty$, we have

$$m(x;\alpha,\beta) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{\Psi_{pqrs}\alpha \sec\left((\pi/2)(2-2^{e^{-\alpha x^{\beta}}})\right)}{(\alpha(r+1))^{\frac{1+\beta}{\beta}}}\int_{\alpha(r+1)x^{\beta}}^{\infty} z^{\frac{1}{\beta}}e^{-z}dz - x$$

After solving the integral, we get

$$m(x;\alpha,\beta) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{\Psi_{pqrs}\alpha \sec\left((\pi/2)(2-2^{e^{-\alpha x^{\beta}}})\right)}{(\alpha(r+1))^{\frac{1+\beta}{\beta}}}\Gamma\left(\frac{\beta+1}{\beta},\alpha(r+1)x^{\beta}\right) - x$$

Where $\Gamma(a,x) = \int_x^{\infty} u^{a-1}e^{-u}du$ denotes the upper incomplete gamma function

4. Order Statistics and Maximum likelihood Estimation of New cosine-Weibull distribution

Let us suppose $X_1, X_2, ..., X_n$ be random samples of size n from NCWD with pdf $f(x)$ and cdf F(x). Then the probability density function of the k^{th} order statistics is given as

$$f_X(k) = \frac{n!}{(k-1)!(n-1)!} f(x) \left[F(x)\right]^{k-1} \left[1-F(x)\right]^{n-1} \tag{16}$$

Using equation (3) and (4) in equation (10), we have

$$f_X(k) = \frac{n!}{(k-1)!(n-1)!} \frac{\pi log(2)}{2} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} sin\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\left[cos\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\right]^{k-1}$$
$$\left[1-cos\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\right]^{n-1}$$

The pdf of the first order X_1 and nth order X_n statistics of new cosine-Weibull distribution are respectively given as

$$f_X(1) = \frac{n\pi log(2)}{2} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} sin\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\left[1-cos\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\right]^{n-1}$$

And

$$f_X(n) = \frac{n\pi log(2)}{2} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} sin\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\left[cos\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\right]^{n-1}$$

Let the random samples $x_1, x_2, x_3, ..., x_n$ are drawn from new cosine-Weibull distribution. The likelihood function of n observations is given as

$$L = \prod_{i=1}^{n} \frac{\pi log(2)}{2} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} sin\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)$$

The log-likelihood function is given as

$$l = nlog(log(2)) + nlog(\alpha) + nlog(\beta) - \alpha\sum_{i=1}^{n} x_i^{\beta} + (\beta-1)\sum_{i=1}^{n} log(x_i) + log(2)\sum_{i=1}^{n} e^{-\alpha x_i^{\beta}}$$
$$+\sum_{i=1}^{n} log\left(sin\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right)\right) \tag{17}$$

The partial derivatives of the log-likelihood function with respect to α and β are given as

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} x_i^{\beta} - log(2)\sum_{i=1}^{n} x_i^{\beta} e^{-\alpha x_i^{\beta}} + (\frac{\pi}{2}) log(2)\sum_{i=1}^{n} x_i^{\beta} 2^{e^{-\alpha x_i^{\beta}}} cot\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right) \tag{18}$$

$$\frac{\partial l}{\partial \beta} = \frac{n}{\beta} - \alpha\beta\sum_{i=1}^{n} x_i^{\beta-1} + \sum_{i=1}^{n} log(x_i) + log(2) - \alpha\sum_{i=1}^{n} x_i^{\beta} log(x_i) e^{-\alpha x^{\beta}}$$
$$+ (\frac{\alpha\pi}{2}) log(2)\sum_{i=1}^{n} x_i^{\beta} 2^{e^{-\alpha x_i^{\beta}}} e^{-\alpha x_i^{\beta}} cot\left(\frac{\pi(2-2^{e^{-\alpha x^{\beta}}})}{2}\right) log(x_i) \tag{19}$$

For interval estimation and hypothesis tests on the model parameters, an information matrix is required. The 2 by 2 observed matrix is

$$I(\zeta) = \frac{-1}{n}\begin{bmatrix} E\left(\frac{\partial^2 logl}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 logl}{\partial\alpha\partial\beta}\right) \\ E\left(\frac{\partial^2 logl}{\partial\beta\partial\alpha}\right) & E\left(\frac{\partial^2 logl}{\partial\beta^2}\right) \end{bmatrix}$$

The elements of above information matrix can be obtain by differentiating equations (18) and (19) again partially. Under standard regularity conditions when $n \to \infty$ the distribution of $\hat{\zeta}$ can be approximated by a multivariate normal $N(0, I(\hat{\zeta})^{-1})$ distribution to construct approximate confidence interval for the parameters. Hence the approximate $100(1-\psi)\%$ confidence interval for α and β are respectively given by

$$\hat{\alpha} \pm Z_{\frac{\psi}{2}}\sqrt{I^{-1}_{\alpha\alpha}(\hat{\zeta})} \quad and \quad \hat{\beta} \pm Z_{\frac{\psi}{2}}\sqrt{I^{-1}_{\beta\beta}(\hat{\zeta})}$$

5. Simulation Analysis

The bias, variance and MSE were all addressed to simulation analysis. From NCWD taking N=500 with samples of size n=50,150,150,250,350,450 and 500. The following expression has been used to produce random numbers.

$$X = \left\{\frac{-1}{\alpha} log\left[\frac{1}{log(2)} log\left(\frac{2\pi - 2\arccos(1-u)}{\pi}\right)\right]\right\}^{\frac{1}{\beta}}$$

Where u is uniform random numbers with $u \in (0,1)$. For various parameter combinations, simulation results have been achieved. The bias, variance and MSE values are calculated and presented in table 1 and 2. As the sample size increases, this becomes apparent that these estimates are relatively consistent and approximate the actual values of parameters. Interestingly, with all parameter combinations, the bias and MSE reduce as the sample size increases.

Table 1: *Bias, variance and their corresponding MSE's for different parameter values* $\alpha = 1.2, \beta = 0.5$

Sample size	Parameters	Bias	Variance	MSE
50	α	0.01450	0.02467	0.02488
	β	0.01371	0.00268	0.00287
150	α	0.00415	0.00614	0.00616
	β	0.00213	0.00091	0.00091
250	α	-0.00031	0.00411	0.00411
	β	0.00406	0.00049	0.00051
350	α	-0.00411	0.00276	0.00277
	β	0.00248	0.00037	0.00038
450	α	0.00223	0.00218	0.00219
	β	0.00255	0.00025	0.00026
500	α	0.00388	0.00214	0.00216
	β	0.00096	0.00024	0.00024

6. Data Aanalysis

This subsection evaluates a real-world data set to demonstrate the new cosine-Weibull distribution's applicability and effectiveness. The new cosine-Weibull distribution (NCWD) adaptability

Table 2: *Bias, variance and their corresponding MSE's for different parameter values* $\alpha = 1.5, \beta = 1.2$

Sample size	Parameters	Bias	Variance	MSE
50	α	0.01450	0.03773	0.03794
	β	0.03003	0.01942	0.02032
150	α	0.01324	0.01125	0.01143
	β	0.01132	0.00508	0.00521
250	α	0.00982	0.00698	0.00708
	β	0.00446	0.00324	0.00326
350	α	0.00286	0.00517	0.00518
	β	0.01186	0.00243	0.00257
450	α	0.00248	0.00370	0.00371
	β	0.00319	0.00165	0.00166
500	α	0.00160	0.00311	0.00311
	β	0.00073	0.00140	0.00140

is determined by comparing its efficacy to that of other analogous distributions such as Weibull distribution (BD), Frechet distribution (FD), Inverse Burr distribution (IBD), Nadrajah Haghigi distribution (NHD),Rayleigh distribution (RD) and Exponential distribution (ED).
To compare the versatility of the explored distribution, we consider the criteria like AIC (Akaike information criterion), CAIC (Consistent Akaike information criterion), BIC (Bayesian information criterion), HQIC (Hannan-Quinn information criterion), Kolmogorov-Smirnov tes (K.S), the Cramer-Van Mises criteria (W*) and the Anderson-Darling test (A*). Distribution having lesser AIC, CAIC, BIC,HQIC, K.S, W* and A* values is considered better.
Data set: The data set given below represents the waiting times (in minutes) before service of 100 bank customers, information provided by Ghitany et al.[10].
0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2,4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9,5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2,6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9,12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5
The ML estimates with corresponding standard errors in parenthesis of the unknown parameters are presented in Table 3 and the comparison statistics, AIC, BIC, CAIC, HQIC and the goodness-of-fit statistic for the data set are displayed in Table 4.

Table 3: *Descriptive statistics for data set*

Min.	Max.	Ist Qu.	Med.	Mean	3rd Qu.	kurt.	Skew.
0.800	38.500	4.675	8.100	9.877	13.025	5.5402	1.4727

Table 4: *The ML Estimates (standard error in parenthesis) for data set*

Model	$\hat{\alpha}$	$\hat{\beta}$
NCWD	0.0803	1.1474
	(0.0174)	(0.0830)
WD	0.0304	1.4585
	(0.0094)	(0.1087)
FD	6.535	1.1629
	(0.8918)	(0.0799)
IBD	8.8661	1.2900
	(1.2087)	(0.0827)
NHD	0.0212	3.3292
	(0.0138)	(1.8244)
ED	0.1012	....
	(0.0101)	....
RD	0.0066	....
	(0.00065)	....

Table 5: *Comparison criterion and goodness-of-fit statistics for data set*

Model	-l	AIC	CAIC	BIC	HQIC	K.S statistic	W*	A*	p-value
NCWD	316.98	637.96	638.09	643.17	640.07	0.0353	0.0168	0.1243	0.9996
WD	318.73	641.46	641.58	646.67	643.57	0.0577	0.0629	0.3962	0.8922
FD	334.38	672.76	672.88	677.97	674.87	0.1167	0.3832	2.505	0.1312
IBD	330.42	664.85	664.97	670.06	666.96	0.1026	0.2922	1.9478	0.2425
NHD	323.44	650.89	651.02	656.10	653.00	0.1076	0.1111	0.6958	0.197
ED	329.02	660.04	660.08	662.64	661.09	0.1730	0.0270	0.1790	0.0050
RD	329.24	660.48	660.52	663.08	661.53	0.1734	0.1268	0.7877	0.0048

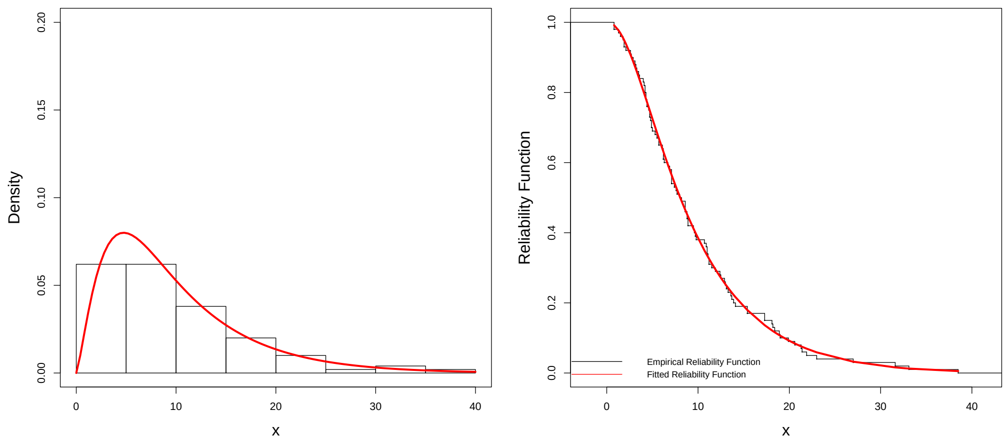

Figure 4: *Estimated pdf of the fitted model and Empirical versus fitted reliability function for data set.*

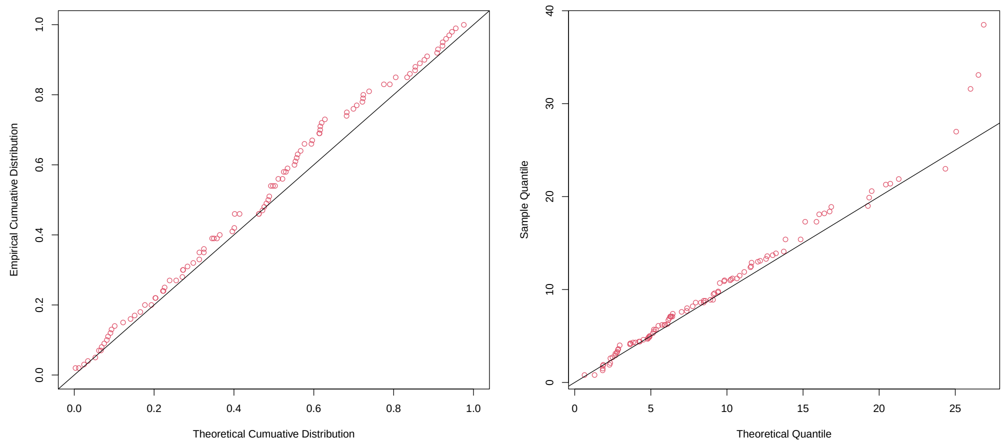

Figure 5: *PP and QQ plot for NCWD.*

It is observed from table 4 that NCWD provides best fit than other competative models based on the measures of statistics, AIC, BIC, CAIC, HQIC, K-S statistic W* and A*. Along with p-values of each model.

7. Conclusion

There is a growing concern among both statisticians and applied researchers in constructing versatile lifetime models to enhance the modelling of survival data. In this research, we established a two-parameter new cosine-Weibull distribution, which is created by employing the weibull distribution as the baseline. Several structural properties of the proposed distribution including moments, moment generating function, order statistics and reliability measures has been discussed. The parameters of the distribution are estimated by famous method of maximum likelihood estimation. Finally the efficiency of the distribution is examined through an application.

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