WEIBULL COMPARISON BASED ON RELIABILITY, AVAILABILITY, MAINTAINABILITY, AND DEPENDABILITY (RAMD) ANALYSIS

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Abstract

As a continuous probability distribution, the Weibull distribution is widely used in the study of reliability, availability and other life data. In this research, we propose the RAMD analysis to estimate the three-parameter Weibull distribution. The estimation of the distribution parameters is an important problem that has received a lot of attention from researchers because of their effects in several measurements. The real data results indicate that our proposed estimation method is significantly consistent in estimation compared to the RAMD analysis method. The numerical values of filtration system reliability and availability were calculated using Maple software. The system of first-order differential equations is formulated using a mnemonic approach and solved recursively. Several scenarios were examined to determine the impact of the models under consideration. The calculations were done with Maple 13 software. Other reliability measures such as mean time to failure (MTTF), mean time to repair (MTTR), and dependability ratio was estimated. The comparative analysis was conducted using a reverse osmosis (RO) filtration system.

Keywords: Availability, life data, Weibull-distribution, Maple, Reliability, Reverse Osmosis

1. Introduction

A reliability study's main goal is to provide information that can be used to make decisions. Performance Analysis of K out of N Reverse Osmosis (RO) system in the treatment of wastewater using RAMD analysis was carried out by Ibrahim et al. [19]. Based on the analyses, Jun et al.[9] Concluded The key to the life model is the estimating parameters; it can accurately predict the life of a product in terms of reliability. Because obtaining the estimated parameters is difficult, the process of estimating 3-parameter Weibull distribution is important. Iterative methods are required for the estimation process using the maximum likelihood function, which takes a long time and effort. RAMD analysis is the proposed method for the estimation of the 3- parameter weibull distribution. In comparison to the maximum likelihood method, the real data results show that our proposed estimation method is significantly more consistent in estimation. Maihulla A. S. et al. [10] investigated the Reliability, Availability, Maintainability, and Dependability Analysis of a Complex Reverse Osmosis Machine System in Water Purification. The swedish physicist Weibull proposed the Weibull distribution (Walooddi weibull, 1939). He used it to determine the tensile strength of various materials. Since then, it's been widely used in reliability and life testing problems, such as determining a component's time to failure or life length, which is measured from a specified time until it fails as stated by [14].

Jukić and Marković, [8] and Walpole et al., [18] studied the Reliability Estimation of Three Parameters Weibull Distribution based on Particle Swarm Optimization. Basheer et al. [5] demonstrated how to treat the pollutants found in olive mill wastewater (total organic carbon (TOC), dissolved organic carbon (DOC), total phosphorus (TP), total nitrogen (TN), and total polyphenols), a sequential Direct Contact Membrane Distillation (DCMD) and a Reverse Osmosis (RO) hybrid membrane system were used. Similar study was also conducted by Teresa [3] but with recoveries of some economical merits. The influence of permeate flow and pressure on pollutant parameter removals was also included in his study. One of the biggest challenges that humanity must address in the twenty-first century is the scarcity of freshwater. That is why Biniaz et al. [17] investigated potential development of an environmentally friendly, economical, and energy-efficient membrane distillation process. Industrial and household wastes can lessen pollution than to the membrane distillation (MD) method. The Gumbel–Hougaard family copula was used to model the reliability and performance of a solar photovoltaic system was analyzed by Maihulla A. S. et al. [12]. Failure of the purification filter is a serious issue that the water purification industries are facing. Modern technical challenges include removing human-caused contaminants from drinking water. After a brief discussion of the treatment stages that municipal water through before it reaches your tap, you'll learn about the detectable contamination of drinking water caused by anthropogenic (human-made) toxins that is still present. These were all examined by Maihulla et al. [11]. The issue surrounding the comparison of the RAMD analysis and the 3-parameter Weibull was not covered in any of the aforementioned publications.

Our motivation for using a reverse osmosis filtration system as a test bed for the two methods was to see how well they worked together (3-parameter Weibull and RAMD analysis) stems from a serious problem that the water purification industry is facing due to purification filter failure. As a result, there has been a slow advancement in technological advancement in water purification, as well as its importance in the lives of people all over the world. The filtration industry is working hard to keep up with the increasing complexity of the systems. RAMD analysis was used to examine the filtration system's strength, efficiency, and performance improvement, according to the findings of the paper. If the strength, efficiency, and performance of the filtration system are evaluated, users will be able to save money on medical care due to contaminated water. Protect yourself from pollution in the water. The study is divided into five parts, one of which is the current introduction. Modeling of filtration is discussed in the section. Materials and methods are

Covered in Section 2. Section 3 discussed the proposed RAMD analysis method, which was compared to the 3-parameter Weibull, as well as an analytical analysis of the system. Section 4 presented the result. Section 5 included a discussion and explanation of the findings, as well as a conclusion.

1.1. Filtration

One of the most common methods for removing these materials is gravity filtration. This procedure involves passing water containing solid impurities (for example, precipitates after water softening) through a porous material, usually sand and gravel layers. The force of gravity pushes the water through the medium. The gaps between the sand and gravel grains allow small water molecules to flow through. Precipitation-derived solids, on the other hand, become trapped in the pores and thus remain in the porous medium. The solid contaminants have been removed from the water that passes through the bottom of the filter.

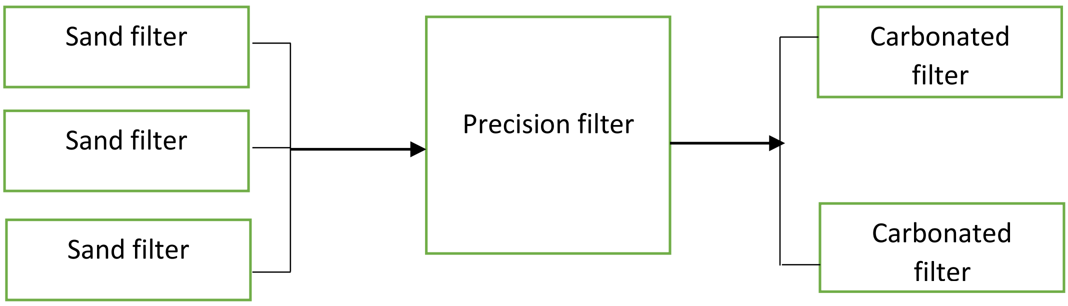


Figure 1: RO Filtration System's Block diagram

1.2. RAMD indices for subsystem 1 (sand filter)

Sand Filter: George et al. [6] stated that, Sand filters are widely used in water purification and work on a completely different mechanism to remove suspended matter. Rather than passing through small orifices through which particles cannot pass, the water passes through a 750 mm deep bed of filter medium, typically 0.75 mm sand.

1.3. RAMD indices for subsystem 2. (Precision filter)

The cylinder shell is usually made of stainless steel, and the internal filter elements are made of PP melt-blown, wire burning, folding, titanium filter, activated carbon filter, and other tubular filter elements, which are chosen based on the different filter media and design process to meet the effluent quality requirements.

1.4. RAMD indices for subsystem 3 (activated carbon filter)

Activated carbon filters are used to purify water without leaving any harmful chemicals behind according to Y. K. Siong et al. [21]. For water treatment, a prototype is being created using activated carbon and an ultraviolet radiation system. Analysis of surface area and porosity. For comparison of surface morphology, scanning electron microscopy (SEM) was used to obtain magnified images of GAC-A and GAC-B.

2. Methods

With the probability density function, let T be a random variable that represents time to failure. f(t), where f(t) is the 3-parameter Weibull distribution's probability density function, which can be written in (1) below, as analyzed by: Jukić and Marković, [8] and Jun et al., [9].

$$f(t;\alpha,\beta,\Upsilon) = \begin{cases} \frac{\alpha}{\beta}\left(\frac{t-\Upsilon}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{t-\Upsilon}{\beta}\right)^{\alpha}\right] & t \geq 0 \\ 0 & t \leq 0 \end{cases} \tag{1}$$

Where $\alpha > 0$ denotes the shape parameter, $\beta > 0$ denotes the scale parameter, and $\Upsilon \leq t$ denotes the location parameter.
Integrating the probability density function yields the cumulative distribution function for the 3-parameter Weibull distribution in (2).

$$F(t) = P(T \leq t) = \int_0^t f(t;\alpha,\beta,\Upsilon)dt = 1 - e^{-\left(\frac{t-\Upsilon}{\beta}\right)^{\alpha}} \tag{2}$$

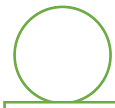
The three-parameter Weibull distribution's mean and variance are (3) and (4) below, as in (Muraleedharan, [14])

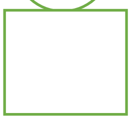
$$\boldsymbol{\mu} = \Upsilon + \beta\Gamma\left(\frac{\alpha+1}{\alpha}\right) \tag{3}$$

Where $\boldsymbol{\mu} = E(t) = MTTF$ (Mean time to failure)

$$\sigma^2 = \beta^2\left[\Gamma\left(\frac{\alpha+2}{\alpha}\right) - \Gamma^2\left(\frac{\alpha+1}{\alpha}\right)\right] \tag{4}$$

2.1. List of notations and definitions


: Represent the system in working state


: Represent the system in failed state

P_0 : Represent the initial state of the system working in full capacity state.
P_1 : Represent the state in which one parallel unit is failed
P_2 : Represent the state in which two parallel unit is failed
P_3 : Represent the state in which three parallel unit is failed
α_i $_{i=i,2,3}$ Represent the failure rates subsystems
β_j $_{j=1,2,3}$ Represent the repair rates subsystems
$P_x(t)$ Probability to remain at xth state at time t
$\frac{d}{dt}P_x(t)$, $x = 0,1,2,3$. Represent the derivative with respect to time t

2.2. Reliability

The chance that a device will run without failure for a particular period of time is referred to as reliability under the operational conditions indicated.

$$R(t) = \int_t^{\infty} f(x)dx \qquad \text{Reliability function} \qquad (5)$$

2.3. MTBF

Mean Time between Failures (MTBF): The mean time between failures refers to the average period of good system functioning. The MTBF is the reciprocal of the constant failure rate or the ratio of the test time to the number of failures when the failure rate is reasonably consistent over the operating period. It is given by (6), as in Ashish K. and Monika S. [13].

$$\text{MTBF} = \int_0^{\infty} R(t)dt = \int_0^{\infty} e^{-\theta t} = \frac{1}{\theta} \qquad \text{Mean Time between Failure} \qquad (6)$$

2.4. MTTR

Mean Time between repairs (MTTR): is the reciprocal of the system repair rate, as in (7) below:

$$\text{MTTR} = \frac{1}{\boldsymbol{\beta}} \qquad \text{Mean Time to Repair} \qquad (7)$$

2.5. Availability

Availability: For repairable systems, availability is a performance criterion that considers both the system's dependability and maintainability. It's defined as the probability that the system will work correctly when it's needed. JG Wohl [20]. That is the ratio of life time to total time, as in (8) below.

$$\text{Availability} = \frac{Life\ time}{Total\ time} = \frac{Life\ time}{Life\ time + Repair\ time} = \frac{MTTF}{MTTF+MTTR} \qquad (8)$$

2.6. Maintainability

Maintainability: When maintenance is necessary, is a design, installation, and operation feature that is often stated as the possibility that a machine can be kept in, or returned to, a particular operational condition within a specified time frame, according to Wohl, JG [20]. It is given by (9) below.

$$M(t) = 1 - e^{(-\frac{-t}{MTTR})} \qquad \text{Maintainability function} \qquad (9)$$

2.7. Dependability A. Ebeling [4] defined dependability as a measure of a system's availability, reliability, maintainability. The advantage of dependability is that it allows for cost, reliability, and maintainability comparisons. For random variables with exponential distribution, the dependability ratio is as follows:

$\beta = Repair\ rate$, α = Failure rate

$d = \frac{\alpha}{\beta} = \frac{MTBF}{MTTR}$

The importance of maintenance is reflected in the high value of the dependability ratio. The dependability value increases if availability is greater than 0.9 and decreases if availability is less than 0.1, according to C. Li, S. Besarati, and colleagues [2]. The following formula in (10) calculates the minimum value of dependability:

$$D_{min} = 1 - (\frac{1}{d-1})(e^{-\frac{lnd}{d-1}} - e^{-\frac{d\ln d}{d-1}}) \quad (10)$$

2.8. Functions of reliability and failure rate

$R(t)$ Is the reliability function (also known as the survival function). The probability that the device will operate without failure for a given period of time under the specified operating conditions is known as reliability. This included in the study carried out by Haldar and Mahadevan, [7] and

Walpole et al., [18]. The R(t) in (11) below is in terms of the failure and repair rates of the three parameter weibull distribution, unlike that of (5).

$$R(t) = P(T \leq t) = \int_0^t f(t;\alpha,\beta,\Upsilon)dt = 1 - F(t) = e^{-\left(\frac{t-\Upsilon}{\beta}\right)^{\alpha}} \quad (11)$$

$h(t)$ Represents the failure rate (also known as the hazard rate). The failure rate is expressed in terms of failures per unit time. It is computed as the ratio of number of failures of the items undergoing the test time. From $T = t$ to $T = t + \Delta t$, given that it survived to time t, this was investigated by Nachlas, [15] and Walpole et al.,[18].

$$P(t < T < t + \Delta t | T > t) = \frac{P(t<T<t+\Delta t)}{P(T>t)} = \frac{F(t+\Delta t)-F(t)}{R(t)} = \frac{\Delta F(t)}{R(t)} \quad (12)$$

We get the failure rate by dividing this ratio by Δt and finding the limit as $\Delta t \rightarrow 0$: in the three parameter weibull distribution, as below in (13). The computation of availability, MTTF, MTTR, MTBF as well as dependability analyses for RAMD against three parameter weibull distribution were presented in table 2.

$$h(t) = \lim_{\Delta t \rightarrow 0} \frac{P(t < T < t + \Delta t | T > t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta F(t)}{\Delta R(t)} = \frac{F(t)}{R(t)} \quad (13)$$

And for each correspondent subsystem: by substituting h(t) as failure rate, we got (14) below:

$$MTTF = \frac{1}{Failure\ rate} = \frac{1}{h(t)} \quad \text{(Mean time between failures)} \quad (14)$$

And

$$MTTR = \frac{1}{Repair\ rate} \quad \text{(Mean time to repair)} \quad (15)$$

And therefore the Availability of the system can be calculated using the relation below as in Monika et al. [16]. Availability, also using three parameter weibull distribution can be computed as (16) below:

$$\text{Availability} = \frac{MTBF}{MTBF+MTTR} \quad (16)$$

3. The proposed RAMD Analysis method

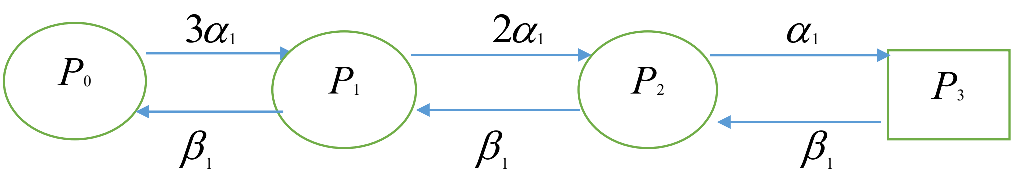


Figure 2: *Transition diagram for subsystem 1*

$$\frac{d}{dt}P_0(t) = -3\alpha_1 P_0 + \beta_1 P_1 \tag{17}$$
$$\frac{d}{dt}P_1(t) = -(2\alpha_1 + \beta_1)\,P_1 + 3\alpha_1 P_0 + \beta_1 P_2 \tag{18}$$
$$\frac{d}{dt}P_2(t) = -(\alpha_1 + \beta_1)\,P_2 + 2\alpha_1 P_1 + \beta_1 P_3 \tag{19}$$
$$\frac{d}{dt}P_2(t) = -\beta_1 P_2 + \alpha_1 P_1 \tag{20}$$

The steady-state probabilities of the system are obtained by imposing the following restrictions: $\frac{d}{dt} \to 0$, as $t \to \infty$. see Arora and Kumar [1].

Under steady state, equation (17) - (20) reduces to

$$P_1 = \frac{\alpha_1}{\beta_1} P_0 \tag{21}$$

Substituting (21) into (18)

$$P_2 = \frac{\alpha_1^2}{\beta_1^2} P_0 \tag{22}$$

Substituting (22) into (19)

$$P_3 = \frac{\alpha_1^3}{\beta_1^3} P_0 \tag{23}$$

Using normalization condition

$$P_0 + P_1 + P_2 + P_3 = 1 \tag{24}$$

Substituting (21) and (22) and (23) into (24) we have:

$$P_0 + = \frac{\alpha_1}{\beta_1} P_0 + \frac{\alpha_1^2}{\beta_1^2} P_0 + \frac{\alpha_1^3}{\beta_1^3} P_0 = 1$$
$$P_0 = \frac{\beta_1^3}{\beta_1^3 + 6\alpha_1^3 + 3\beta_1^2\alpha_1 + 6\,\beta_1\alpha_1^2} \tag{25}$$

Table 2 contains important device output metrics that have been extracted

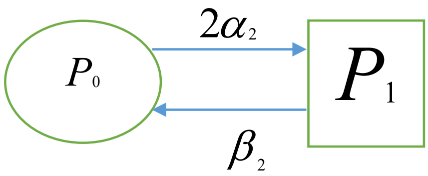


Figure 3: *Transition diagram for subsystem 2*

$$\frac{d}{dt}P_0(t) = -2\alpha_4 P_0 + \beta_4 P_1 \quad (26)$$
$$\frac{d}{dt}P_1(t) = 2\alpha_4 P_0 - \beta_4 P_1 \quad (27)$$

Under steady state, equation (20) and (21) reduces to:

$$-2\alpha_4 P_0 + \beta_4 P_1 = 2\alpha_4 P_0 - \beta_4 P_1$$

And

$$P_1 = \frac{2\alpha_4}{\beta_4} P_0 \quad (28)$$

Using normalization condition

$$P_0 + P_1 = 1 \quad (29)$$

Substituting (22) into (23) we have:

$$P_0 + \frac{2\alpha_4}{\beta_4} P_0 = 1$$
$$P_0 = \frac{\beta_4}{\beta_4 + 2\alpha_4} \quad (30)$$

Table 2 contains important device output metrics that have been extracted.

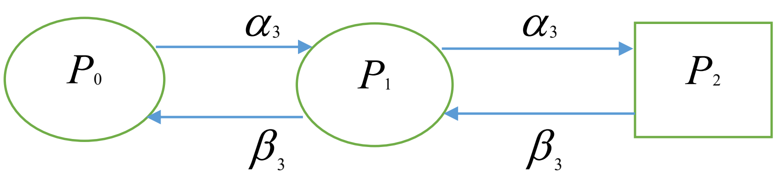


Figure 4: *Transition diagram for subsystem 3*

$$\frac{d}{dt}P_0(t) = -\alpha_3 P_0 + \beta_3 P_1 \quad (31)$$
$$\frac{d}{dt}P_1(t) = -(\beta_3 + \alpha_3)P_1 + \alpha_3 P_0 + \beta_3 P_2 \quad (32)$$
$$\frac{d}{dt}P_2(t) = -\beta_3 P_2 + \alpha_3 P_1 \quad (33)$$

Under steady state, equation (25)- (27) yield

$$P_1 = \frac{\alpha_3}{\beta_3} P_0 \quad (34)$$

Substituting (28) into (26) under steady state we have:

$$P_2 = \frac{\alpha_3^2}{\beta_3^2} P_0 \tag{35}$$

Using normalization condition

$$P_0 + P_1 + P_2 = 1 \tag{36}$$

It follows that:

$$P_0 = \frac{\beta_3^2}{\beta_3^2 + \alpha_3^2 + \beta_3 \alpha_3} \tag{37}$$

3.1. System Reliability

$$R_{sys}(t) = R_{s1}(t) \times R_{s2}(t) \times R_{s3}(t) \tag{38}$$

$$R_{sys}(t) = e^{-\alpha_1(t)} \times e^{-\alpha_2(t)} \times e^{-\alpha_3(t)} \tag{39}$$

$$R_{sys}(t) = e^{-(\alpha_1 + \alpha_2 + \alpha_3)t} \tag{40}$$

3.2. System Availability

Arranged in series, failure of one cause the complete failure of the system.

$$A_{sys} = A_{s1} \times A_{s2} \times A_{s3} \tag{41}$$

$$A_{sys} = \left(\frac{\beta_1^3}{\beta_1^3 + 6\alpha_1^3 + 3\beta_1^2 \alpha_1 + 6\, \beta_1 \alpha_1^2}\right) \times \left(\frac{\beta_4}{\beta_4 + 2\alpha_4}\right) \times \left(\frac{\beta_3^2}{\beta_3^2 + \alpha_3^2 + \beta_3 \alpha_3}\right) \tag{42}$$

$$A_{sys} = \left(\frac{0.07}{0.08005}\right) \times \left(\frac{0.09}{0.11}\right) \times \left(\frac{0.0121}{0.01585}\right) \tag{43}$$

$$A_{sys} = 0.87445 \times 0.81818 \times 0.76341 \tag{44}$$

$$A_{sys} = 0.18249 \tag{45}$$

Table 2 contains important device output metrics that have been extracted.

4. Results

The numerical outcomes from RAMD and Weibull analysis performed using Maple software are presented in this section. The discussion of the comparative analysis in this section is found in the part that follows. The tables' contents are translated into graphs. Table 1 below shows the failure and repair rates for the corresponding subsystems. The comparison findings for the two aforementioned methodologies were shown in Tables 2, 3, and 4.

Table 1: *Failure and repair rates for subsystems*

Subsystem	Failure Rate (α)	Repair Rate (β)
S_1	α_1= 0.004	β_1 = 0.54
S_2	α_2= 0.006	β_2 = 0.67
S_3	α_3= 0.008	β_3 = 0.90

Table 2: *RAMD/Weibull indices for the RO filtration system*

RAMD indices of Subsystems	Subsystem S_1	Subsystem S_2	Subsystem S_3	System
Reliability	$e^{-0.009t}$	$e^{-0.005t}$	$e^{-0.014t}$	$e^{-0.028t}$
Availability (Weibull)	0.96755	0.77853	0.89706	0.67573
Availability (RAMD)	0.84556	0.69453	0.77453	0.45486
MTBF (Weibull)	104.34	198.92	88.06	391.32
MTBF(RAMD)	83.33 21.01	166.67 32.25	62.50 25.56	312.50 78.82
MTTR (Weibull)	4.77	3.21	1.96	9.94
MTTR(RAMD)	1.85	1.49	1.11	4.45
Dependability (Weibull)	0.99302	0.99746	0.99753	0.98805
Dependability (RAMD)	0.97497	0.94301	0.92891	0.85405
Dependability ratio (Weibull)	21.87	61.97	44.93	
Dependability ratio (RAMD)	45.04	111.86	56.31	

Table 3: *Availability of a system*

Subsystems	3-Parameter weibull distribution	RAMD (Exponential distribution)
1	0.96755	0.84556
2	0.77853	0.69453
3	0.89706	0.77453

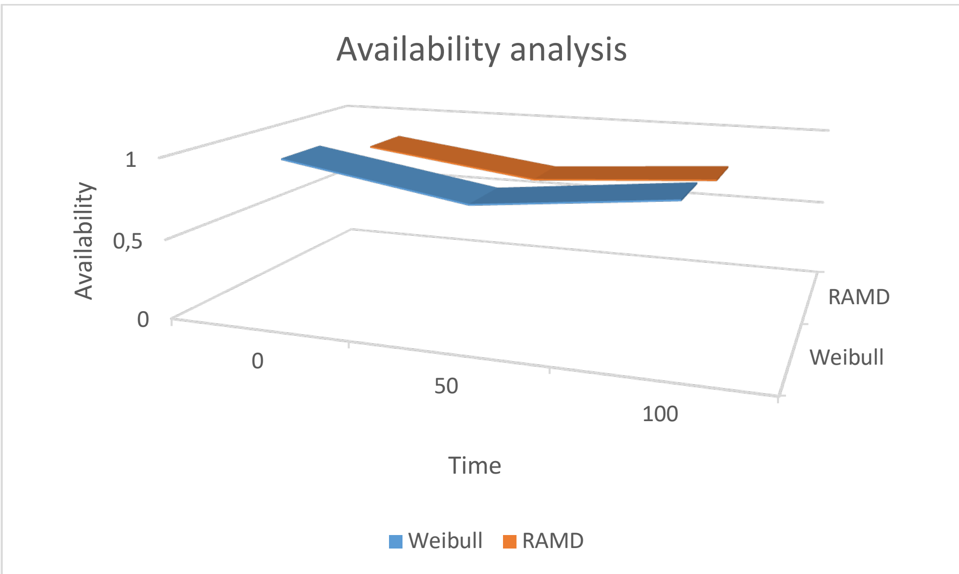


Figure 5: *Variation of Availability with time*

Table 4: *Reliability of a system*

Time(in days)	3-Parameter weibull distribution	RAMD (Exponential distribution)
0	1.00000	0.99999
10	0.79918	0.79384
20	0.71540	0.70030
30	0.66808	0.64285
40	0.63288	0.59827
50	0.60240	0.55945
60	0.57436	0.52410
70	0.54801	0.49140
80	0.52308	0.46100
90	0.49942	0.43268
100	0.47695	0.40628

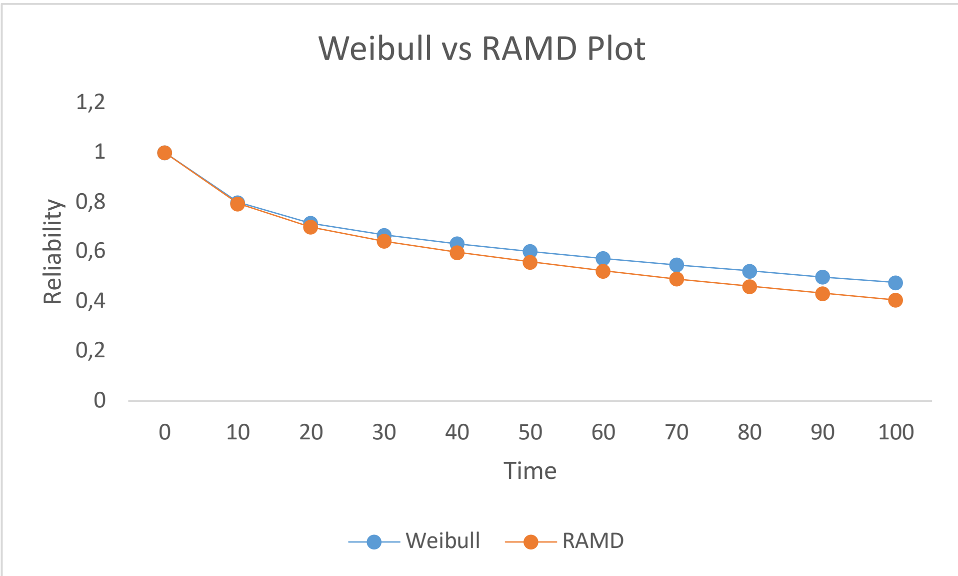


Figure 6: *Variation of Reliability with time*

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5. Discussion

Figures 2, 3, and 4 show the transition diagrams for the three reverse osmosis filtering units/subsystems, namely the sand filters, precision filters, and carbonated filters. A comparison of the efficiency of two methods (RAMD analysis and 3-parameter Weibull) was conducted. Table 2 lists all of the additional RAMD-Weibull metrics. It demonstrated how effective the Weibull is when compared to the RAMD. The variation in the system's availability is 0.22087, which is due to the fact that subsystem 3 has a larger variation than subsystems 1 and 2. When the MTBF of the entire system is considered, it is found that the weibull is more efficient in terms of the average operating time from the time a failed device is restored to the time it becomes failed again than RAMD analysis, with a variation of nearly 78.82. When compared to 1 and 3, the same subsystem 2, contributed significantly. Despite the fact that it had fewer units than 1 and 3, it was still a good choice. Using the RAMD analysis, the system's reliability after 50 days of operation is only 0.55945. The reliability of the same system using a 3-parameter Weibull distribution after the same period is 0.60240.

Figure 5 depicted the availability of the two methods as a function of time (3-parameter Weibull and RAMD analysis). The efficiency of 3-parameter Weibull over RAMD analysis is clearly observed. According to the numerical analysis in Table 4 and the corresponding values in Figure 6.

6. Conclusion

For desalinating saltwater to produce drinking water, reverse osmosis (RO) filtration system is an important technology. The performance of a desalination system is determined by the failure behavior of its components. Because the RO filtration system was designed to be low-power, the subsystems' dependability must be maintained through proper design and material selection in order for the plant to operate continuously. Furthermore, the estimation of the system's strength in terms of reliability characteristics using the 3-parameter Weibull distribution is more efficient than the RAMD analysis method. As a result, the study investigated the system's availability and reliability, as well as other features such as MTBF, MTTR, and dependability analysis. To address the issue of precision filter redundancy, more research can be done.

7. Acknowledgement

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