A COMPREHENSIVE CASE STUDY ON INTEGRATED REDUNDANT RELIABILITY MODEL USING k-out-of-n CONFIGURATION

¹Srinivasa Rao Velampudi

Department of Sciences and Humanities, Raghu Institute of Technology, Visakhapatnam, Andhra Pradesh, India, vsr.rit@gmail.com

²Sridhar Akiri *

Department of Mathematics, GITAM School of Science, GITAM (Deemed to be University), Visakhapatnam, Andhra Pradesh, India, sakiri@gitam.edu

³Pavan Kumar Subbara

Department of Mathematics, GITAM School of Science, GITAM (Deemed to be University), Bangalore Campus, Karnataka, India, psubbar@gitam.edu

⁴Yadavalli V S S

Department of Industrial & Systems Engineering, Pretoria University, 0002 Pretoria, South Africa, sarma.yadavalli@up.ac.za

Abstract

Designers may introduce a system with multiple technologies in series to improve system efficiency. The configuration can be applied to k out of n systems if each technology contains k out of n factors. The k out of n configuration method is successful until every component of the system is successful. The efficiency of the entire system is more in amount than that of a single system factor in a k out of n shape. An Integrated Reliability Model (IRM) for the k out of n, here, an additional system is suggested to account for both the efficiencies of the factors and the number of factors in every phase and the different constraints to optimize the efficiency of the system. To enhance system efficiency, the authors employed the numerous methods of Lagrangean approach to determine the numbers and efficiency of the factors as well as the reliabilities of the phase under different parameters namely load, size, and cost. The dynamic programming approach and simulation method have been adapted to attain an integer result as well as to see the values real.

Keywords: Reliability Theory, IRM, Lagrangean Approach, Stage Efficiency, D P Approach, System Efficiency

1. Introduction

The structure's reliability [1] can be improved by either placing superfluous units, applying the element of greater reliability or by adopting the two methods at a time and both of them use extra resources. Optimizing structure reliability, and conditions to resource availability viz. size, value, load, are examined. In general, reliability is tested as an element of value; But, when tested with

real-world problems, the invisible effect of other restraints such as load, size [4], etc. has a special effect on improving structural reliability. The specific functionality of the over-reliability model with several limitations to optimize the recommended setup was examined to maximize the recommended setup. The problem examines the unknowns that is, various elements (X_{ej}), the element reliability (r_{ej}), and the stage reliability (R_{sj}) at a specific point for disposing of multiple restraints to magnify the structure reliability that is described as a [14] United Reliability Model (URM). In literature, United Reliability Models [8] are enhanced by applying value restraints where there is a fixed association between value and reliability. A unique pattern of planned work is a deliberation of the load and size as supplementary restraints along with value to form and improve the superfluous reliability system for [15] k out of n structure composition [6, 7].

2. Methods

2.1. Assumptions and Notations:

- Each stage's elements are believed to be identical, i.e., all elements have the same level of reliability.
- All elements are supposed to be statistically independent, meaning that their failure has no bearing on the performance of other elements in the structure.

$R_{SR} =$	Structure Efficiency
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R_{si} = Efficiency of phase 'sj', $0 < R_s$	_i <1
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- r_{ej} = Efficiency of each component in phase 'ej'; $0 < r_{ej} < 1$
- X_{ej} = Number of components in phase 'ej'
- C_{ei} = Worth coefficient of each component in phase 'ej'
- L_{ej} = Load coefficient of each component in phase 'ej'
- S_{ei} = Size coefficient of each component in phase 'ej'
- C_{e0} = Greatest allowable structure Value
- L_{e0} = Greatest allowable structure Load
- S_{e0} = Greatest allowable structure Size
- LMM Lagrangean Multiplier Method
- DPA Dynamic Programming Approach
- IRRM Integrated Redundant Reliability Model

 c_i , d_i , i_i , k_i , m_i , n_i are Constants.

2.2 Mathematical Analysis:

The efficiency of the system to the provided worth function

$$R_{SR} = \sum_{i=1}^{n} B(m, i) p^{i} (1 - p)^{m - i}$$
(1)

The following relationship between worth and efficiency is used to calculate the worth coefficient of each unit in phase 'ej'.

$$r_{ej} = \sinh^{-1} \left[\frac{c_{ej}}{f_j} \right]^{\frac{1}{d_j}}$$
(2)

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Therefore	$C_{ej} = f_j \sinh \left[r_{ej} \right]^{d_j}$	(2a)
Similarly,	$L_{ej} = p_j \sinh [r_{ej}]^{k_j}$	(2b)
	$S_{ej} = q_j \sinh [r_{ej}]^{n_j}$	(2c)

Since value-components are linear in *ej*,

$$\sum_{j=1}^{n} c_{ej}. \ X_{ej} \le C_{e0}$$
(3a)

Similarly load-components and size-components are also linear in *ej*,

$$\sum_{j=1}^{n} l_{ej} \cdot X_{ej} \leq L_{e0} \tag{3b}$$

$$\sum_{j=1}^{n} S_{ej} \cdot X_{ej} \leq S_{e0} \tag{3c}$$

Substituting (2) in (3)

$$\sum_{j=1}^{n} f_j \sinh[r_{ej}]^{d_j} \cdot X_{ej} - C_{e0} \le 0$$
(4a)

$$\sum_{j=1}^{n} p_j \sinh[r_{ej}]^{k_j} \cdot X_{ej} - L_{e0} \le 0$$
(4b)

$$\sum_{j=1}^{n} q_j \sinh[r_{ej}]^{n_j} \cdot X_{ej} - S_{e0} \le 0$$
(4c)

The transformed equation through the relation $X_{ej} = \frac{\ln R_{SJ}}{\ln r_{ej}}$ (5)

Where

$$R_{sj} = \sum_{k=2}^{n} B(ej,k) (r_{ej})^k (1 - r_{ej})^{ej-k}$$
(6)

Subject to the constraints

$$\sum_{j=1}^{n} f_j \sinh[r_{ej}]^{d_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - C_{e0} \le 0$$
(7a)

$$\sum_{j=1}^{n} p_j \sinh [r_{ej}]^{k_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - L_{e0} \le 0$$
(7b)

$$\sum_{j=1}^{n} q_j \sinh[r_{ej}]^{n_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - S_{e0} \le 0$$
(7c)

Positivity restrictions $e_j \ge 0$

A Lagrangean function is defined as

$$L_{F} = R_{SR} + \delta_{1} \left[\sum_{j=1}^{n} f_{j} \sinh [r_{ej}]^{d_{j}} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - C_{e0} \right] + \delta_{2} \left[\sum_{j=1}^{n} p_{j} \sinh [r_{ej}]^{k_{j}} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - L_{e0} \right] + \delta_{3} \left[\sum_{j=1}^{n} q_{j} \sinh [r_{ej}]^{n_{j}} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - S_{e0} \right]$$
(8)

The Lagrangean function can be used to find the ideal point and separating it by R_{sj} , r_{ej} , δ_1 , δ_2 and δ_3 .

$$\frac{\partial L_F}{\partial R_{SR}} = 1 + \delta_1 \left[\sum_{j=1}^n f_j \sinh\left[r_{ej}\right]^{d_j} \cdot \frac{1}{\ln r_{ej}} \frac{1}{R_{sj}} \right] + \delta_2 \left[\sum_{j=1}^n p_j \sinh\left[r_{ej}\right]^{k_j} \cdot \frac{1}{\ln r_{ej}} \frac{1}{R_{sj}} \right] + \delta_3 \left[\sum_{j=1}^n q_j \sinh\left[r_{ej}\right]^{n_j} \cdot \frac{1}{\ln r_{ej}} \frac{1}{R_{sj}} \right]$$
(9)

$$\frac{\partial L_F}{\partial r_{ej}} = \delta_1 \left[\sum_{j=1}^n f_j \sinh\left[r_{ej}\right]^{d_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} \right] \left[d_j \cdot \coth r_{ej} - \frac{1}{r_{ej} \cdot \ln r_{ej}} \right] + \\ \delta_2 \left[\sum_{j=1}^n p_j \sinh\left[r_{ej}\right]^{k_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} \right] \left[d_j \cdot \coth r_{ej} - \frac{1}{r_{ej} \cdot \ln r_{ej}} \right] + \\ \delta_3 \left[\sum_{j=1}^n q_j \sinh\left[r_{ej}\right]^{n_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} \right] \left[d_j \cdot \coth r_{ej} - \frac{1}{r_{ej} \cdot \ln r_{ej}} \right]$$
(10)

$$\frac{\partial L_F}{\partial \delta_1} = \sum_{j=1}^n f_j \sinh \left[r_{ej} \right]^{d_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - C_{e0}$$
(11)

$$\frac{\partial L_F}{\partial \delta_2} = \sum_{j=1}^n p_j \sinh \left[r_{ej} \right]^{k_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - L_{e0}$$
(12)

 $\frac{\partial L_F}{\partial \delta_3} = \sum_{j=1}^n q_j \sinh \left[r_{ej} \right]^{n_j} \cdot \frac{\ln R_{sj}}{\ln r_{ej}} - S_{e0}$ (13)

Where δ_1 , δ_2 and δ_3 are Lagrangean multipliers.

The number of elements in each phase (X_{ej}) , the best element reliability (r_{ej}) , the phase reliability (R_{sj}) and the structure reliability (R_{SR}) are derived by using the Lagrangean method [12]. This method provides a real (valued) solution concerning worth, load, and size.

2.3 Case Problem

To derive the multiple parameters of a given mechanical system using optimization techniques [9], where all the assumptions like value, weight, and volume are directly proportional to system reliability has been considered in this research work. The same logic may not be true in the case of electronic systems. Hence, the optimal element accuracy (r_{ej}) , phase reliability (R_{sj}) , Number of elements in each phase (X_{ej}) , and structure accuracy (R_{SR}) can be evaluated in any given mechanical system [10]. In this work, an attempt has been made to evaluate the Structure accuracy [13] of a special purpose machine that is used for single phase industrial power generators assembly.

The machine is used for the assembly of 3 or 4 components on the base of the power generator. The machine's approximate worth was \$3000, which is considered a structure value, the load of the machine is 120 pounds which is the load of the structure, and the space occupied by the machine is 100 cm^3 , which is the volume or size of the structure. To attract the authors from different cross sections, the authors attempted to use hypothetical numbers, which can be changed according to the environment.

2.4 Constants

The data required for the constants for the case problem are provided in Table 1.

Phase	Worth	Constants	Loa	d Constants	Size Constants		
rnase	f_j	d_{j}	p_j	k_{j}	q_j	n_j	
1	2200	0.85	100	0.92	100	0.94	
2	2400	0.88	80	0.88	90	0.89	
3	2600	0.91	60	0.91	80	0.86	

Table1: Worth, Load and Size Pre-fixed Constant Values

The efficiency of each factor, phase, and number of factors in each stage, as well as the structural efficiency [2, 3], are shown in the tables below.

2.4.1 The Details of Component-Worth Constraint by using Lagrangean Multiplier Method without Rounding-Off

The value-related efficiency design is described in the Table 2.

Table2: Worth Constraint Analysis by using Lagrangean Multiplier Method

Phase	f_j	d_j	r _{ej}	Log r _{ej}	R_{sj}	$Log R_{sj}$	X _{ej}	C _{ej}	$C_{ej} . X_{ej}$	
01	2200	0.85	0.8741	-0.0584	0.6777	-0.1690	2.89	2233	6456	
02	2400	0.88	0.8445	-0.0734	0.6487	-0.1880	2.56	2334	5977	
03	2600	0.91	0.8456	-0.0728	0.5461	-0.2627	3.61	2516	9077	
	Final Worth									

2.4.2 The Details of Component-Load Constraint by using Lagrangean Multiplier Method without Rounding-Off

The equivalent results for the load are shown in the Table 3.

Phase	p_j	k_j	r _{ej}	Log r _{ej}	R _{sj}	$Log R_{sj}$	X _{ej}	L _{ej}	L_{ej} . X_{ej}
01	100	0.92	0.8741	-0.0584	0.6777	-0.1690	2.89	100	290
02	80	0.88	0.8445	-0.0734	0.6487	-0.1880	2.56	78	199
03	60	0.91	0.8456	-0.0728	0.5461	-0.2627	3.61	58	209
				Final Loa	ıd				698

Table3: Load Constraint Analysis by using Lagrangean Multiplier Method

2.4.3 The Details of Component-Size Constraint by using Lagrangean Multiplier Method without Rounding-Off

Equivalent results for size are described in the Table 4.

Phase	q_j	n_j	r _{ej}	Log r _{ej}	R_{sj}	$Log R_{sj}$	X_{ej}	S_{ej}	$S_{ej} . X_{ej}$
01	100	0.94	0.8741	-0.0584	0.6777	-0.1690	2.89	100	289
02	90	0.89	0.8445	-0.0734	0.6487	-0.1880	2.56	87	224
03	80	0.86	0.8456	-0.0728	0.5461	-0.2627	3.61	78	282
				Final Siz	e				795

 Table4: Size Constraint Analysis by using Lagrangean Multiplier Method

3. Efficiency Design by using Lagrangean Multiplier Method

The efficiency design [11] summarizes the e_j values as integers (rounding the value of e_j to the nearest integer), and the acceptable outcomes for the worth, load, and size are listed in the tables. Calculate variance due to worth, load, size, and construction capacity (before and after rounding off e_j to the nearest integer) to obtain information.

3.1 Efficiency Design by using Lagrangean Multiplier Method Concerning Worth, Load and Size with Rounding-Off

Table5: Efficiency design relating to Worth, Load and Size Constraint Analysis by using Lagrangean

Phase	r _{ej}	R_{sj}	X _{ej}	C _{ej}	$C_{ej} . X_{ej}$	L _{ej}	$L_{ej} . X_{ej}$	$S_{ej}S_{ej}$	S_{ej} . X_{ej}
01	0.8741	0.6777	3	2233	6699	100	300	100	300
02	0.8445	0.6487	3	2334	7002	78	234	87	261
03	0.8456	0.5461	4	2516	10066	58	232	78	312
Total V	North, Lo	oad and	Size	2	3767		766	873	
		Struct	ure Ef	$y(R_{SR})$			0.9	9987	

Multiplier Method with Rounding Off is shown in the following table

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Variation in Worth	= Total Worth with rounding off-Total Worth without rounding off Total Value without rounding off	= 10.49%
Variation in Load	= Total Load with rounding off-Total Load without rounding off Total Load without rounding off	= 09.72%
Variation in Size	= Total Size with rounding off-Total Size without rounding off Total Size without rounding off	= 05.00%
Variation in Efficiency	= Effciency with rounding off-Efficiency without rounding off Structure Efficiency without rounding off	= 10.06%.

4. Dynamic Programming Approach

Using the Lagrangean technique [5], which has a number of drawbacks, such as having to provide the amount of components needed at each stage ($X_{ej}('ej')$) in real numbers, which may be difficult to apply. The generally used approach of rounding down the value of results in changes in worth, load, and size, affecting system reliability and having a significant impact on the model's efficiency design. This flaw could be considered, for which the author suggests a substitute empirical implementation that uses the dynamic programming method to obtain an integer solution by using the solutions produced from the Lagrangean approach as parameters for the proposed dynamic programming method.

Table6: *Phase I of the Dynamic Programming*

Phase–I('ej')	Phase - I - Reliability (\mathbf{R}_{sj})
01	0.5789
02	0.7183
03	0.8577
04	0.9265
05	0.9605
06	<u>0.9945</u>

Phase – II (' ej ')		Phase - II - Reliability (\mathbf{R}_{sj})							
04	0.4370	0.4490	0.4540						
05	0.4666	0.4916	0.4978	0.5238					
06	0.4962	0.5342	0.5416	0.6125	0.3991				
07	0.5258	0.5768	0.5854	0.7012	0.5496	0.4285			
08	0.5406	0.5981	0.6073	0.7899	0.7571	0.6614	0.6734	0.4523	
09	0.5554	0.6194	0.6292	0.8786	0.8936	0.7258	0.7254	0.4835	
10	0.5628	0.6407	0.6511	0.9229	0.9187	0.9021	0.8852	0.7264	

Table7: Phase II of the Dynamic Programming

Table8: Phase III of the Dynamic Programming

Phase – III (' ej ')		Phase - III - Reliability (\mathbf{R}_{sj})							
09	0.4932	0.6235	0.6461	0.5938	0.5814	0.4235	0.2824	-	
10	0.5383	0.7045	0.7044	0.6451	0.6287	0.4516	0.3125	0.1818	
11	0.5824	0.7436	0.7654	0.6821	0.7345	0.5216	0.4514	0.2345	
12	0.6265	0.7874	0.7952	0.7514	0.7841	0.6834	0.5387	0.3678	
13	0.6706	0.7951	0.8164	0.8354	0.8547	0.7893	0.6868	0.4864	
14	0.7147	0.8356	0.8573	0.9125	0.9571	0.8514	0.7256	0.6454	

5. Results

The application of the Lagrangean multiplier technique yielded a real-valued solution for the suggested Integrated Redundant Reliability Systems for the k-out-of-n configuration mathematical models under investigation, as well as the much-required integer solution. The author obtained new phase reliability (\mathbf{R}_{sj}) by using a dynamic programming approach. The new values for stage reliability (\mathbf{R}_{sj}) are (0.9445, 0.9229, and 0.9571). The Dynamic Programming Approach is utilised, and the results for the given mathematical function are outlined in tables 9, 10, and 11 that follow in order to derive the required conclusions.

5.1 The Details of Component-Worth Constraint by using Dynamic Programming Approach

Phase	f_j	d_j	r _{ej}	Log r _{ej}	R_{sj}	Log R _{sj}	X _{ej}	C _{ej}	$C_{ej} . X_{ej}$
01	2200	0.85	0.9982	-0.0008	0.9945	-0.0024	3	2580	7740
02	2400	0.88	0.9736	-0.0116	0.9229	-0.0348	3	2735	8205
03	2600	0.91	0.9891	-0.0048	0.9571	-0.0190	4	3016	12064
Final Worth									28009

The value-related efficiency design is described in the Table 9.

Table9: The Details of Component-Worth constraint by using Dynamic Programming Approa	ıch
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Mutation in Worth -Component= 30.21%Mutation in Structure Efficiecy= 01.23%

5.2 The Details of Component-Load Constraint by using Dynamic Programming Approach

The equivalent results for the load are shown in the Table 10.

Table10: The Details of Component-Load constraint by using Dynamic Programming Approach

Phase	p_j	k _j	r _{ej}	Log r _{ej}	R_{sj}	$Log R_{sj}$	X _{ej}	L _{ej}	$L_{ej} . X_{ej}$
01	100	0.92	0.9982	-0.0008	0.9945	-0.0024	3	117	351
02	80	0.88	0.9736	-0.0116	0.9229	-0.0348	3	91	273
03	60	0.91	0.9891	-0.0048	0.9571	-0.0190	4	70	280
Final Load									904

Mutation in Load-Component= 29.32%Mutation in Structure Efficiecy= 01.23%

5.3 The Details of Component-Size Constrain by using Dynamic Programming Approach

The equivalent results for size are described in the Table 11.

Phase	q_j	n_j	r _{ej}	Log r _{ej}	R_{sj}	Log R _{sj}	X_{ej}	S_{ej}	$S_{ej} . X_{ej}$
01	100	0.94	0.9982	-0.0008	0.9945	-0.0024	3	117	351
02	90	0.89	0.9736	-0.0116	0.9229	-0.0348	3	103	309
03	80	0.86	0.9891	-0.0048	0.9571	-0.0190	4	93	372
Final Size									1032
Structure Efficiency (R_{SR})									0.9864

Table11: The Details of Component-Size constraint by using Dynamic Programming Approach

Mutation in Size-Component= 29.81%Mutation in Structure Efficiecy= 01.23%

5.4 Comparison of Optimization of Integrated Redundant Reliability k out of n systems – LMM with rounding-off and Dynamic Programming approach for Worth

Table12: Results Correlated LMM with rounding off approach and Dynamic programming approach for Worth

			Dynamic Programming						
Phase	X _{ej}	r _{ej}	R_{sj}	C _{ej}	$C_{ej} . X_{ej}$	r _{ej}	R _{sj}	C _{ej}	$C_{ej} . X_{ej}$
01	3	0.8741	0.6777	2233	6699	0.9982	0.9945	2580	7740
02	3	0.8445	0.6487	2334	7002	0.9736	0.9229	2735	8205
03	4	0.8456	0.5461	2516	10066	0.9891	0.9571	3016	12064
Total Worth 23767				28009					
Structure Efficiency		Using Wi	th LMM Ap (R_{SR})	0.9987	Using DP Approach (R_{SR})			0.9999	

5.5 Comparison of Optimization of Integrated Redundant Reliability k out of n systems – LMM with rounding-off and Dynamic Programming approach for Load

Table13: Results Correlated wi	h LMM rounding off a	proach and Dynamic pro	gramming approach for Load
	0 11 1	1	8 11 9

		With Rounding Off				Dynamic Programming				
Phase	X _{ej}	r _{ej}	R_{sj}	L _{ej}	$L_{ej} . X_{ej}$	r _{ej}	R_{sj}	L _{ej}	$L_{ej} . X_{ej}$	
01	3	0.8741	0.6777	100	300	0.9982	0.9945	117	352	
02	3	0.8445	0.6487	78	234	0.9736	0.9229	91	273	
03	4	0.8456	0.5461	58	232	0.9891	0.9571	70	278	
Total	Load	767 904								
Structure		Using With LMM			0.9987	Using DP Approach (R_{SR})			0.9999	
Effic	iency	Ар	proach (R	(_{SR})						

5.6 Comparison of Optimization of Integrated Redundant Reliability k out of n systems – LMM with rounding-off and Dynamic Programming approach for Size

With Rounding Off				Dynamic Programming					
Phase	X _{ej}	r _{ej}	R _{sj}	S _{ej}	$S_{ej} . X_{ej}$	r _{ej}	R _{sj}	S _{ej}	S_{ej} . X_{ej}
01	3	0.8741	0.6777	100	300	0.9982	0.9945	117	352
02	3	0.8445	0.6487	87	261	0.9736	0.9229	103	307
03	4	0.8456	0.5461	78	312	0.9891	0.9571	93	372
Total	l Size	873				1031			
Structure		Using With LMM			0.9987	Using DP Approach (R_{SR}) 0.9999			
Effici	iency	Ap	proach (R	(_{SR})					

Table14: Results Correlated LMM with rounding off approach and Dynamic programming approach for Size

6. Discussion

This work proposes an integrated reliability model for a k out of n configuration system with many efficiency criteria. When the data are discovered to be in reals, the Lagrangean multiplier approach is used to compute the number of components (X_{ej}), component efficiencies (r_{ej}), phase efficiencies (R_{sj}), and system efficiency (R_{SR}). To obtain practical applicability, the dynamic way of programming approach is employed to construct an integer solution using the inputs from the Lagrangean method.

This work proposes an integrated reliability model for a k out of n configuration system with many efficiency criteria, when the data are discovered to be in real solution. The Lagrangean multiplier approach is used to compute the number of components (X_{ej}) and the respective component efficiencies (r_{ej}) are 0.8741, 0.8445 & 0.8456, stage reliabilities (R_{sj}) are 0.6777, 0.6487 & 0.5461, and structure reliability (R_{SR}) is 0.9987. To obtain practical applicability, Dynamic programming approach is employed to construct an integer solution whereas component reliabilities (r_{ej}) are 0.9982, 0.9736 & 0.9891, stage reliabilities (R_{SJ}) are 0.9945, 0.9229 & 0.9571, and the system reliability (R_{SR}) is 0.9999, Using the inputs from the Lagrangean method. Finally, we observed that the worth, load and size components changed slightly, but compare with stage reliability, resulting in increased system reliability.

The IRM generated in this manner is quite valuable, particularly in real-world settings when a k from n configuration IRM with reliability engineer redundancy is required. In circumstances where the system value is low, the proposed model is especially valuable for the dependability design engineer to build high-quality and efficient materials.

In future study, the authors recommend utilizing a unique approach that limits the minimum and maximum component reliability values while maximizing system dependability using any of the current heuristic processes to build similar IRMs with redundancy.

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