PROBABILISTIC ANALYSIS OF A TWO UNIT COLD STANDBY SYSTEM WITH REPAIR AND REPLACEMENT POLICIES

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Abstract

The present paper deals with two identical units, one is operative and the other of which other is kept on cold standby. If the operative unit fails, it goes under repair and after repair, it is not considered as good as new. If the unit fails after the first repair, it is replaced with a new unit. A single repairman is always available with the system to repair a failed unit. Failure time, repair time and replacement time distributions are taken as exponential to reduce the complexity of the system model. By using the regenerative point technique, the various important measures of system effectiveness have been obtained and are shown with the help of graph.

Keywords: Cold standby, replacement policy, MTSF, regenerative point, availability, transition probability and mean sojourn time.

1. Introduction

Various researchers have widely used the redundancy technique in systems of identical units to improve system reliability. Since the demand for improving system reliability is increasing, the field of research in reliability theory is becoming more advanced. Many researchers in the field of reliability theory, including [2, 5], have analyzed the model of a two-unit cold standby system. The performance measure of the model of a two-unit system with repair and replacement policies has been studied by various authors [1, 3, 4, 6, 7].

The current paper deals with the study of a probabilistic analysis of a two-unit cold standby system with repair and replacement policies. One unit is operational, while the other is kept on cold standby. Whenever the operative unit fails, it goes into repair. A repaired unit is not considered as good as new. If the repaired unit fails after the first repair, a new unit is installed to replace it. For the purpose of repairing the failed units, a single repairman is always ready with the system. To reduce the complexity of the system, the exponential form of failure time, repair time, and replacement time is taken. After using the regenerative point technique, the following measures of system effectiveness are obtained:

- Transition probabilities and mean sojourn times in various states of the system.

- Reliability and mean time to system failure (MTSF).
- Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
- The expected busy period of repairman during time interval (0, t).
- Net expected profit earned by the system in time interval (0, t) and in steady-state.

2. Model description and assumptions

- The system consists of two identical units, one operational (o) and the other in standby (s).
- A single repairman is always available with the system to repair and replace a failed unit.
- After repair, it is not considered as good as new.
- If the unit fails after the first repair, replace it with a new one.
- The failures of the units are independent and the failure time, repair time and replacement time distributions of the units are as taken exponentials.

3. Notations and states of the system

3.1. Notations

E : Set of regenerative states $=\{S_0 \text{ to } S_3\}$.

θ : Constant failure rate of operative unit.

β : Constant failure rate after one time repair of a unit.

α : Repair rate of a unit.

η : Replacement rate of failed unit.

$q_{ij}(\bullet)$: p.d.f. of transition time from regenerative state S_i to S_j.

$q_{ij}^{(k)}(\bullet)$: p.d.f. of transition time from regenerative state S_i to S_j via non-regenerative state S_k.

$\sim$: Symbol for Laplace-stieltjes transforms i.e.

$$\tilde{A}(s)=\int e^{-st}dA(t)$$

$*$: Symbol for Laplace-transform i.e.

$$A^*(s)=\int e^{-st}A(t)dt$$

© : Symbol for ordinary convolution i.e.

$$A(t)©B(t)=\int_0^t A(u)B(t-u)du$$

Here, $\beta>\theta$, Also the limits of the integration are not mentioned whenever they are 0 to ∞.

3.2. Symbols for the states of the systems

N_o, N_s : Unit-1 and Unit-2 in operative and in standby mode.

$\overline{N}s, N_O$: Unit-1 in standby mode and Unit-2 is waiting for repair.

$\overline{N}o, F_r$: Unit-1 and Unit-2 in operative and standby mode.

F_w, F_{wr} : Unit-1 in repair mode and Unit-2 is waiting for repair.

F_R, F_{wr} : Replaced unit-1 after repair and unit-2 waiting for repair.

Considering the above symbols and keeping in view the assumptions stated in section-2, the possible states of the system model are shown in transition diagram (Figure.1).

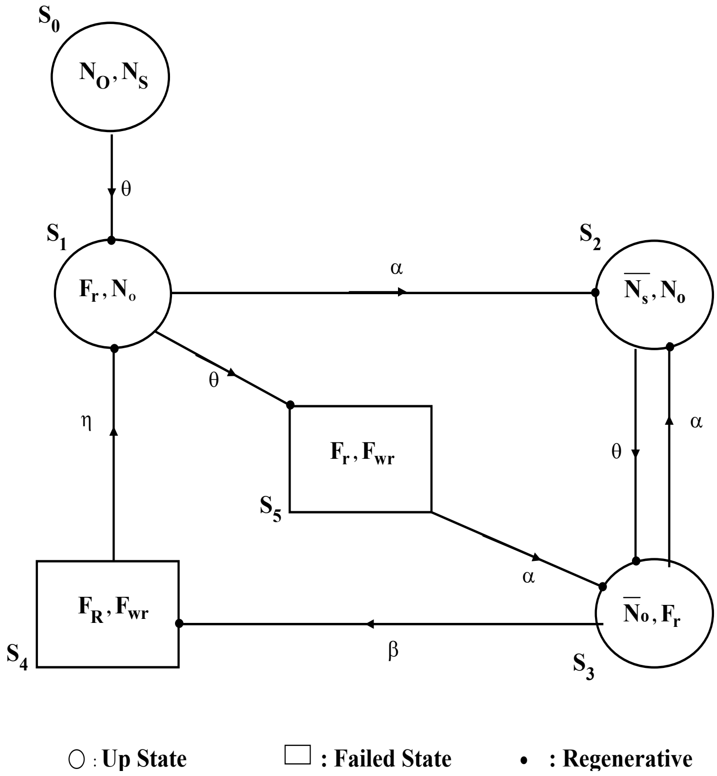

Figure.1: *Transition diagram*

4. Transition probabilities

The direct or one-step steady state transition probabilities can be obtained as follows:

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t)$$

Similarly,

$$p_{01} = \int \theta e^{-\theta t}\, dt, \qquad p_{12} = \int \alpha e^{-(\alpha+\theta)t}\, dt = \frac{\alpha}{\alpha+\theta}$$

$$p_{15} = \int \theta e^{-(\alpha+\theta)t}\, dt = \frac{\theta}{\alpha+\theta}, \qquad p_{01} = \int \theta e^{-\theta t}\, dt = 1$$

$$p_{32} = \int \alpha e^{-(\alpha+\beta)t}\, dt = \frac{\alpha}{\alpha+\beta}, \qquad p_{34} = \int \beta e^{-(\alpha+\beta)t}\, dt = \frac{\beta}{\alpha+\beta}$$

$$p_{41} = \int \eta e^{-\eta t}\, dt = 1, \qquad p_{53} = \int \alpha e^{-\alpha t}\, dt = 1$$

It can be easily verified that,

$$p_{01} = p_{23} = p_{41} = p_{53} = 1, \qquad p_{12} + p_{15} = 1, \qquad p_{32} + p_{34} = 1 \tag{1-3}$$

5. Mean sojourn times

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before making the transition into any other state. If T_i denotes the sojourn time in state S_i, then mean sojourn time in state S_i is;

$$\psi_i = \int P\left(T_i > t\right)dt \tag{4}$$

Therefore,

$$\psi_0 = \int e^{-\theta t}dt = \frac{1}{\theta} = \psi_2$$
$$\psi_1 = \int e^{-(\alpha+\theta)t}dt = \frac{1}{\alpha+\theta}$$
$$\psi_3 = \int e^{-(\alpha+\beta)t}dt = \frac{1}{\alpha+\beta}$$
$$\psi_4 = \int e^{-\eta t}dt = \frac{1}{\eta}$$
$$\psi_5 = \int e^{-\alpha t}dt = \frac{1}{\alpha} \tag{5-9}$$

6. Analysis of characteristics

6.1 Reliability of the system and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0, t) given that at t = 0, it starts from state $S_i \in E$..To obtain it, we assume the failed states S_4 and S_5 as absorbing. Now using simple probabilistic arguments we have the following recurrence relations in $R_i(t)$; i =0, 1,2,3.

$$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t)$$
$$R_1(t) = Z_1(t) + q_{12}(t)©R_2(t)$$
$$R_2(t) = Z_2(t) + q_{23}(t)©R_3(t)$$
$$R_3(t) = Z_3(t) + q_{32}(t)©R_2(t) \tag{10-13}$$

Where, $Z_0 = e^{-\theta t},\ Z_1 = e^{-(\alpha+\theta)t},\ Z_2 = e^{-\theta t},\ Z_3 = e^{-(\alpha+\beta)t}$

Taking Laplace transforms of relations (10-13) and simplifying the resulting set of algebraic equations for $R_0^*(s)$, we get;

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

Where,

$$N_1(s) = (1 - q_{23}^* q_{32}^*)Z_0^* + q_{01}^*(1 - q_{23}^* q_{32}^*)Z_1^* + q_{01}^* q_{12}^* Z_2^* + q_{01}^* q_{12}^* q_{23}^* Z_3^*$$
$$D_1(s) = 1 - q_{23}^* q_{32}^* \tag{14}$$

Here, also for brevity we have omitted the argument's' from $q_{ij}^*(s), Z_0^*(s)$.

The expression of mean time to system failure is given by,

$$E(T_0) = \lim_{s \to 0} R_0^*(s)$$

Observing $q_{ij}^*(0) = p_{ij}, Z_0^*(0) = \psi_0$,

We have,

$$E[T_0] = \frac{p_{34}(\psi_0 + \psi_1) + p_{12}(\psi_2 + \psi_3)}{p_{34}} \tag{15}$$

6.2. Availability analysis

Let $A_i(t)$ be the probability that the system is up (operative) at epoch t, when system initially starts from state $S_i \in E$. Using the basic probabilistic concepts in regenerative point technique as in case of reliability, one can obtain the recurrence relations for $A_i(t)$; i=0, 1, ---, 5. Taking the Laplace Transformations and solving the resulting set of algebraic equations for $A_0^*(s)$, we get,

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \tag{16}$$

Where,

$$\begin{aligned} N_2(s) = {} & (1 - q_{23}^* q_{32}^* - q_{41}^* q_{12}^* q_{23}^* q_{34}^* - q_{41}^* q_{15}^* q_{53}^* q_{34}^*) Z_0^* + (q_{01}^* - q_{01}^* q_{32}^* q_{23}^*) Z_1^* \\ & + (q_{01}^* q_{12}^* + q_{01}^* q_{15}^* q_{53}^* q_{32}^*) Z_2^* + (q_{01}^* q_{12}^* q_{23}^* + q_{01}^* q_{15}^* q_{53}^*) Z_3^* \end{aligned}$$

and

$$D_2(s) = 1 - q_{23}^* q_{32}^* - q_{12}^* q_{23}^* q_{34}^* q_{41}^* - q_{15}^* q_{53}^* q_{34}^* q_{41}^*$$

The steady state availability of the system is given by,

$$A_0 = \lim_{s \to 0} s A_0^*(s) = \lim_{s \to 0} s \frac{N_2(s)}{D_2(s)}$$

As $D_2(0) = 0$, so by using L. Hospitals rule, we get

$$A_0 = \frac{N_2(0)}{D_2'(0)} \tag{17}$$

Where,

$$N_2(0) = p_{34}\psi_1 + (p_{12} + p_{15}p_{32})\psi_2 + \psi_3$$

and

$$D_2'(0) = p_{34}(\psi_1 + \psi_4) + (p_{12} + p_{15}p_{32})\psi_2 + \psi_3 + p_{15}p_{34}\psi_5 \tag{18}$$

The expected up time of the system in interval (0, t) are given by;

$$\mu_{up}(t) = \int_0^t A_0(u)du$$

So that

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s} \tag{19}$$

6.3. Busy period analysis

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the unit is under repair and under replacement at time t due to the failure unit, when the system initially starts from regenerative states $S_i \in E$. Using the probabilistic arguments as in case of availability, we develop the recurrence relations for $B_i^1(t)$ and $B_i^2(t)$; i=0,1,2,3,4,5. Then, taking the Laplace Transforms of these recurrence relations and solving the resulting algebraic equations for $B_0^{1*}(s)$ and $B_0^{2*}(s)$ we get;

$$B_0^{1*}(s) = \frac{N_3(s)}{D_2(s)} \text{ and } B_0^{2*}(s) = \frac{N_4(s)}{D_2(s)} \tag{20-21}$$

Where,

$$N_3(s) = (q_{01}^* - q_{01}^* q_{32}^* q_{23}^*)Z_1^* + (q_{01}^* q_{12}^* q_{23}^* + q_{01}^* q_{53}^* q_{15}^*)Z_3^* \tag{22}$$

$$N_4(s) = (q_{01}^* q_{12}^* q_{23}^* q_{34}^* + q_{01}^* q_{53}^* q_{15}^* q_{34}^*)Z_4^* \tag{23}$$

In the long run, the probabilities that the repairman will be busy are respectively given by:-

$$B_0^1 = \frac{N_3(0)}{D_2'(0)} \text{ and } B_0^2 = \frac{N_4(0)}{D_2'(0)} \tag{24}$$

Where,

$$N_3(0) = p_{34}\psi_1 + \psi_3$$
$$N_4(0) = p_{34}\psi_1$$

The value of $D_2'(0)$ is same as given by expression (18).

The expected busy period of skilled repairman and regular repairman during (0,t) are given by,

$$\mu_b^1(t) = \int_0^t B_0^1(u)du \text{ and } \mu_b^2(t) = \int_0^t B_0^2(u)du$$

So that

$$\mu_b^{1*}(s) = \frac{B_0^{1*}(s)}{s} \text{ and } \mu_b^{2}(s) = \frac{B_0^{2*}(s)}{s} \tag{25-26}$$

7. Profit function analysis

We are now in the position to obtain the net expected profit incurred during time (0, t) by considering the characteristics obtained in earlier sections.
Let us consider,

K_0 = revenue per unit time by the system when it is operative.

K_1 = cost per unit time when repairman is busy in repair of failed unit.

K_2 = cost per unit time when repairman is busy in replacement of failed unit.

Then, the net expected profit incurred during time (0, t),

$$\begin{aligned} P(t) &= \text{Expected total revenue in } (0,t) - \text{Expected amount spent on repair in } (0,t) \\ &= K_0\mu_{up}(t) - K_1\mu_b^1(t) - K_2\mu_b^2(t) \end{aligned}$$

The expected profit per-unit time in steady state is given by,

$$P = K_0A_0 - K_1B_0^1 - K_2B_0^2$$

8. Graphical study of system behaviour

The Behavioral characteristics of MTSF and Profit function for different values of Failure rate θ shown in figures 2 and 3 respectively.

In figure 2, the graphical analysis of MTSF in relation to failure rate θ for three different values of repair rate α i.e. 0.09, 0.35 and 0.85 and for two fixed values of β at 0.05 and 0.09 has been plotted.

The graphical analysis of Profit function in relation to failure rate θ for different values of repair rate α i.e. 0.35, 0.55 and 0.95 and for fixed values of β at 0.32 and 0.45 has been shown in figure 3.

In the analysis at figure 2 and 3, the MTSF and Profit function shows a decline as the value of failure rate increases and when the value of repair rate increases, the graph shows the increase in the value of MTSF and Profit function.

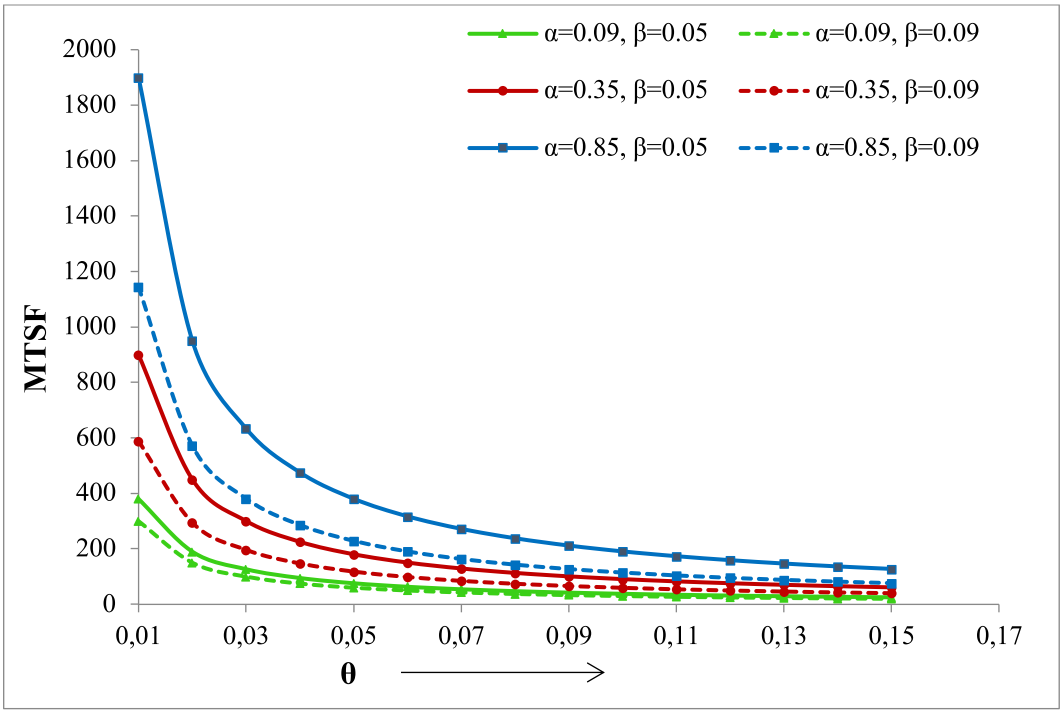

Figure.2: *Behavior of MTSF with respect to* α , β *and* θ

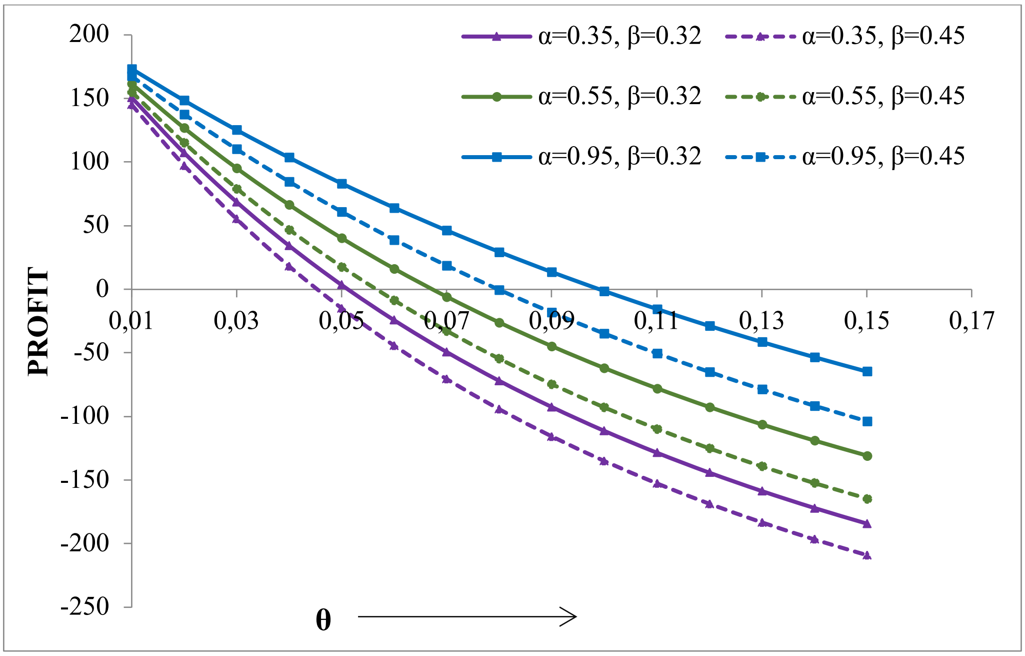

Figure.3: *Behavior of Profit with respect to* α , β *and* θ

According to the dotted curves in the MTSF graph in relation to failure rate, in order to achieve MTSF for at least 300 units, the failure rate θ must be less than 0.01, 0.02, and 0.04 for values of α 0.09, 0.35, and 0.85 when β is fixed at 0.09.

Similarly, for smooth curves the upper bounds for failure rate (θ) 0.012, 0.031 and 0.081 corresponding to α i.e., 0.09, 0.35and 0.85 when β is fixed at 0.05.

Figure 3 shows that the system is profitable only when the failure rate θ is less than 0.045, 0.058, and 0.078, respectively, for values of α 0.35, 0.55, and 0.95 when β fixed at 0.45.

Similarly, we conclude from smooth curves that the system is profitable only when failure rate (θ) is less than 0.05, 0.067 and 0.098 respectively for values of repair rate α i.e., 0.35, 0.55and 0.95 when β is fixed at 0.032.

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