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Assessment And Prediction Of Reliability Of An Automobile Component Using Warranty Claims Data

Tahsina Aziz, M. Rezaul Karim

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Department of Statistics, University of Rajshahi, Rajshahi-6205, Bangladesh. Email: taziz_stat@yahoo.com
Department of Statistics, University of Rajshahi, Rajshahi-6205, Bangladesh. Email: mrezakarim@yahoo.com; mrkarim@ru.ac.bd

Abstract

This paper presents an analysis of warranty claims data of a component of an automobile. The objectives of the analysis are to assess and predict the reliability of the component. To do these the paper present nonparametric and parametric analyses for the lifetime variable, age in month, based on warranty claims data. It also investigates on the variation of reliability of the component with respect to month of production and dominant failure modes. The paper will be useful to the manufacturer for assessing and predicting reliability and warranty costs and for assuring customer satisfaction and product reputation.

Keywords. Automobile component; Failure mode; Maximum likelihood estimate; Reliability; Warranty claims data; Warranty claim rate.

1. Introduction

The complexity of products has been increasing with technological advances. Over the last few decades there has been a heightened interest in improving quality, productivity and reliability of manufactured products. Rapid advances in technology and constantly increasing demands of customers for sophisticated products have put new pressure on manufacturers to produce highly reliable products. As a result, a product must be viewed as a system consisting of many elements and capable of decomposition into a hierarchy of levels, with the system at the top level and parts at the lowest level. Blischke, Karim and Murthy (2011) mentioned that there are many ways of describing this hierarchy. The modern automobile is a complex system consisting of over 15,000 components (Blischke et al., 2011). In this paper the warranty claims data of a component of automobile which belongs to the electrical sub-system, manufactured and sold in Asia, is considered. We analyze the warranty claims data of the component to investigate questions of interest to the manufacturers regarding reliability assessment and prediction.

As there are many aspects to warranty, a number of procedures have been developed for analyzing product warranty data, and the literature on this topic is very large. Detailed discussion on various aspects of warranty and reviews of subsequent recent literature on warranty analysis can be found in Thomas and Rao (1999), Murthy and Djamaludin (2002), Karim and Suzuki (2005), Blischke et al. (2011), Wu (2012) and Wang and Xie (2017). Many factors contribute to product failures that result in warranty claims. One of the most important factors is the age of the product. Age is calculated by the service time measured in terms of calendar time since the product was sold or entered service. The age-based (or age-specific) analysis of product failure data has engendered considerable interest in the literature (Kalbfleisch et al., 1991; Kalbfleisch and Lawless, 1996; Lawless, 1998; Karim et al., 2001; Suzuki et al., 2001) and a number of approaches have been developed with regard to addressing age-based analysis of warranty claims data.

Recently, Blischke et al. (2011) discussed the age-based analysis of an automobile component failure data in a case study. Here first we find the non-parametric estimates of cumulative density function $F(t)$ and reliability function $R(t)$ of lifetime random variable T measured by age in month. Next we apply the parametric approach to select the suitable lifetime models for the component and of different failure modes assuming that the number of failures at age t depends on the age of the product and is independent on other factors. The age-based warranty claim rates for different month of production are estimated for checking the quality variation problems with respect to production period. We also determine the dominant failure modes for the component and investigate how the reliability improves by successively removing the dominant failure modes. We consider a month as the unit for age without loss of generality. If necessary, the unit 'month' can be easily substituted with 'week', 'day' and so on.

The outline of the paper is as follows. Section 2 describes the warranty claims data set. Section 3 discusses the nonparametric approach of data analysis. Section 4 presents the parametric approach to analysis the warranty claims data. Finally, Section 5 concludes the paper.

2. Description of Data

This paper analyses a set of failure data of an automobile component manufactured and sold in Asia. The failure data are the warranty claims data of the component produced over 12 month period of a year and sold over a 26 month period. For reasons of commercial sensitivity we cannot disclose the names of the component and manufacturing company and call simply the "component". The component is non-repairable and the automobiles on which it is used are sold with a non-renewing free-replacement warranty (FRW) with 18 months (age limit) warranty period.[2] The data are collected during 26 months observation period. There are total 4746 failed observations and 64567 censored observations. For each claim, the available data relating to component consisted of the following:

- Serial number of claim
- Month of production
- Date of sale
- Date of failure
- Age of the component

[2] Generally in case of automobile components, the offered warranty is two-dimensional, where the warranty is characterized by a region in a two-dimensional plane, usually with one axis representing age and the other representing usage, whichever occurs first. However, the warranty of this component is one-dimensional, which is characterized by a single variable, age.

- Odometer readings (mileage in kilometers)
- Failure modes
- Used region

The manufacturer has identified 8 different failure modes for the component denoted by FM01, FM02, FM03, FM04, FM05, FM06, FM07 and FM08. Additional these, the database consists of the supplementary data: Production amount (monthly, for 12 months) and Sales amount (monthly, for 26 months).

3. Nonparametric Analysis

The nonparametric approach allows the user to analyze data without assuming an underlying distribution, that is, it does not require that the form of the sampled population be known. Blischke et al. (2011) recommended that any set of warranty data first be subjected to a nonparametric analysis before moving on to parametric analyses assuming a specific underlying probability distribution. Here we look at the nonparametric approach to inferences regarding quantities such as the cumulative density function (cdf) $F(t)$, reliability function $R(t)$, as well as warranty claim rates (WCR) of the component.

Kaplan and Meier (1958) derived the nonparametric estimator of the survival function for censored data which is known as the *product-limit* (PL) *estimator*. This estimator is also widely known as the *Kaplan-Meier* (KM) *estimator* of the survival function. We find the age-based Kaplan-Meier estimator of the survival function $S(t)$ or reliability function $R(t)$. Suppose that there are observation on n individuals and that there are k ($k \leq n$) distinct times (say, age in month) $t_1 < t_2 < \ldots < t_k$ at which failures occur. Let d_i denote the number of units that failed at t_i and r_i represent the number of units that are right-censored at t_i, $i = 1, 2, \ldots, k$. Then the size of the *risk set* (number of units that are alive) at the beginning of time t_i is

$$n_i = n - \sum_{j=0}^{i-1} d_j - \sum_{j=0}^{i-1} r_j, \quad i = 1, 2, \ldots, k \tag{1}$$

where $d_0=0$ and $r_0=0$. Then, the estimator of the conditional probability that a unit fails in the time interval from $t_i - \delta t$ to t for small δt, given that the unit enters this interval, is the sampling proportion failing $\hat{p}_i = d_i / n_i, i = 1, 2, \ldots, k$ and the estimator of the corresponding survival probability is $1 - \hat{p}_i = (n_i - d_i)/n_i, i = 1, 2, \ldots, k$. Under this condition, the Kaplan-Meier estimator of the survival function $S(t)$ is given by

$$S(t) = P(T > t) = \prod_{j:t_j < t} (1 - \hat{p}_j) = \prod_{j:t_j < t} \frac{n_j - d_j}{n_j}, t \geq 0. \tag{2}$$

The nonparametric estimator of $F(t)$ is obtained using the Kaplan-Meier estimator as

$$\hat{F}(t) = 1 - \hat{S}(t), t \geq 0 \tag{3}$$

Meeker and Escobar (1998) discussed estimation methods for the variance and point-wise normal-approximation confidence intervals for $F(t)$. By using the logit transformation, they showed that two-sided approximate 100(1-α)% confidence intervals for $F(t)$ can be calculated as

$$\left[\frac{\hat{F}(t)}{\hat{F}(t)+(1-\hat{F}(t))\times w}, \frac{\hat{F}(t)}{\hat{F}(t)+(1-\hat{F}(t))/w}\right] \tag{4}$$

where $w = \exp\left\{z_{(1-\alpha/2)} s\hat{e}_{\hat{F}(t)} / [\hat{F}(t)(1-\hat{F}(t))]\right\}$ and $s\hat{e}_{\hat{F}(t)} = \sqrt{V(\hat{F}(t))} = \hat{S}(t)\sqrt{\sum_{j=1}^{t} \frac{\hat{p}_j}{n_j(1-\hat{p}_j)}}$.

The nonparametric estimates of reliability function $R(t)$ and cumulative density function $F(t)$ with their 95% confidence intervals are plotted in Figure 1. Minitab software and the R-function survfit(Surv()) under the library survival can be applied to estimate these functions. Figure 1 indicates that about 96% of the component is estimated to survive until 12 months. The value of the cdf at age 18 months is $F(t=18)=0.068$, indicating a claims rate of 6.8% within the warranty period. We are 95% confident that the probability of failing of the component within the warranty period of 18 months is between 6.6% and 7.0%.

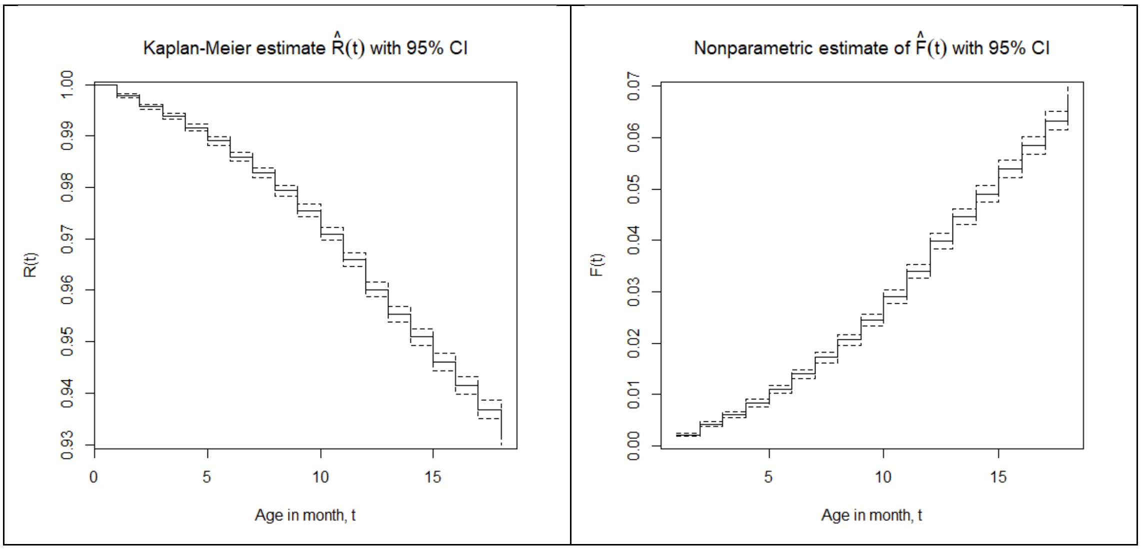

Figure 1: Nonparametric estimates of $R(t)$ (left side) and $F(t)$ (right side) with 95% confidence intervals

The Pareto chart of different failure modes, given in Figure 2, indicates that the three failure modes FM02, FM01, and FM03 account for 83.5% of the total claims. Failure modes from FM04 to FM08 have considerably lower frequencies. Based on Figure 2, we may conclude that efforts should be concentrated to eliminate or reduce the risks associated with the failure modes FM02, FM01 and FM03 in order to improve the reliability of the product and thereby decrease warranty claims and costs.

The summary statistics for the variables Age and Usage for important failure modes are given in Table 1. These summary statistics are the conditional estimates in the sense that they are estimated based on the items that failed during the warranty period and led to claims. This means the summary statistics for the variables Age and Usage given that the Age is less than or equal to 18 months.

Table 1 indicates that the conditional average and median lifetimes with respect to Age and Usage are smaller for failure mode FM02 among the three failure modes. In case of

Usage variable for three failure modes, the mean exceeds the median, indicating skewness to the right. On the other hand, Age variable shows negative skewness.

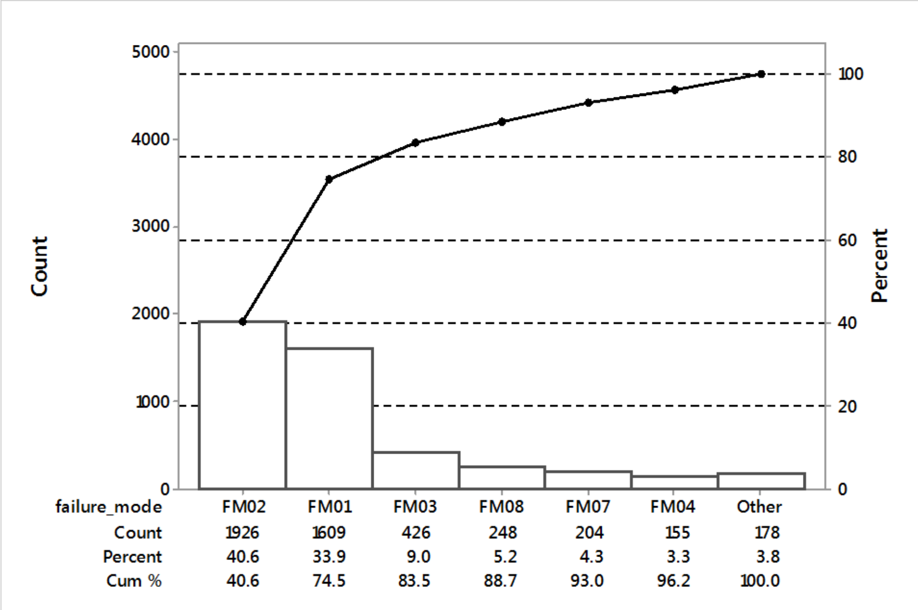

Figure 2: Pareto chart of failure modes

Table 1: Summary statistics for the variables Age and Usage for important failure modes

Failure Mode	Age (in month)							
	Count	Mean	StDev	Q1	Median	Q3	Skewness	Kurtosis
FM01	1609	11.6650	4.4155	9	12	15	-0.5080	-0.5107
FM02	1926	10.1277	4.7288	7	10	14	-0.1509	-0.9037
FM03	426	13.0493	4.4547	11	14	17	-0.9541	0.1410
Failure Mode	Usage (in km)							
	Count	Mean	StDev	Q1	Median	Q3	Skewness	Kurtosis
FM01	1609	27221.76	15053.99	16067.50	25819.00	36479.00	0.6809	0.7755
FM02	1926	24785.99	15905.73	12735.25	22593.50	34533.50	0.7978	0.6639
FM03	426	30018.70	16246.41	18825.25	26984.00	40408.50	0.8214	0.7578

Figure 3 shows the interval plots of Usage (km) for eight failure modes (FM01, FM02, …, FM08) under four used regions (geographic areas of a country with different environments, denoted by R1, R2, R3 & R4). Interval plot can be used to assess and compare both a measure of central tendency and variability of the data. The confidence intervals allow to assess the differences between group means in relation to within-group variance.

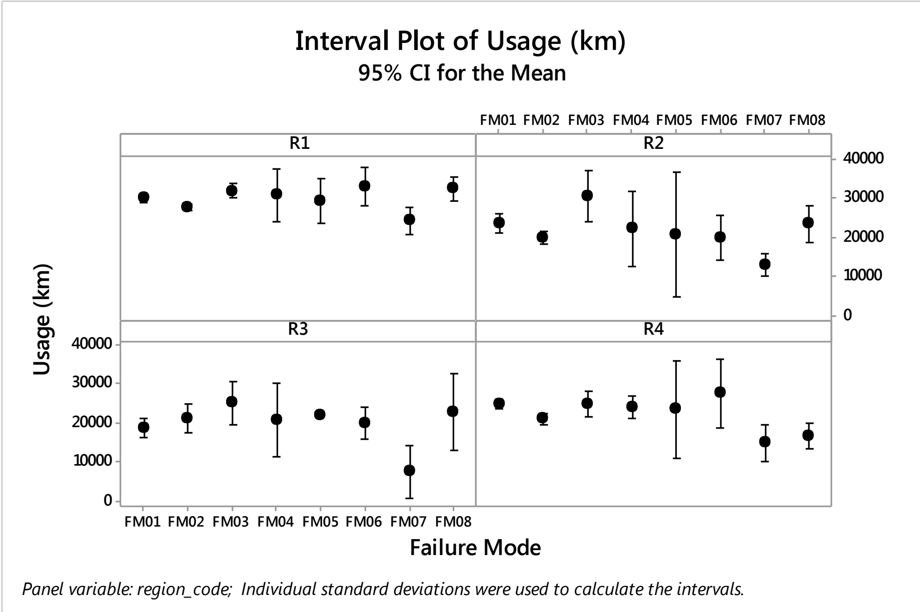

Figure 3: Interval plot of Usage (km) for various failure modes under four used regions

The interval plots show the means of usage for failure mode FM07 are shorter in all used regions. The means of usage for almost all failure modes are longer for used region R1 than other regions. The intervals of means for various failure modes are not all overlap, this indicates that some of the means are different. This indicates the variation of average lifetimes for different failure modes with respect to used regions.

The component considered here was produced during one year from month January to month December. The information on the month of production for the failed items are given in the database. The following Figures 4 and 5 can be used to investigate whether there is any variation in quality with respect to the month of production. Figures 4 and 5 indicate that the items produced in month September have smallest mean lifetimes for both Age and Usage. The intervals (except the production month September) all overlap, so we cannot conclude that any of the means (except September) are different. This preliminary graphical investigation indicates that there might be some quality related problems for the items produced in month September.

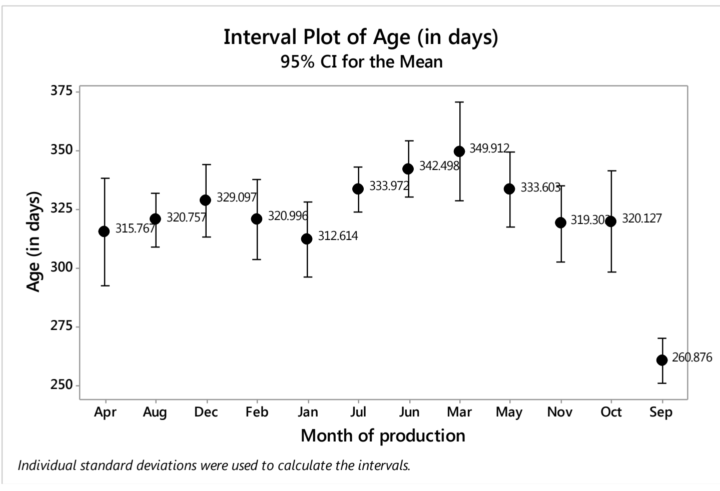

Figure 4: Interval plot of Age (in days) based on month of production

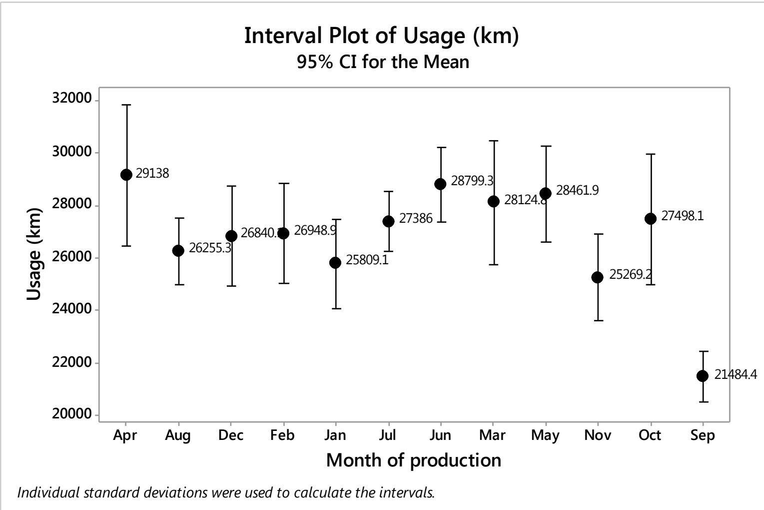

Figure 5: Interval plot of Usage (km) based on month of production

Figure 6 makes a comparison of nonparametric estimates of reliability functions for the main three failure modes FM01, FM02 and FM03. The figure indicates that $R(t)$ for FM02 less than the $R(t)$ for FM01 less than the $R(t)$ for FM03, for all t = 1,2,…, 18. For example, $R_{FM02}(t{=}12){=}0.9814 < R_{FM01}(t{=}12){=}0.9875 < R_{FM03}(t{=}12){=}0.9976$. Therefore, to improve the overall reliability of the component, efforts should be concentrated to eliminate or reduce the failure modes FM02 first, then FM01 and then FM03.

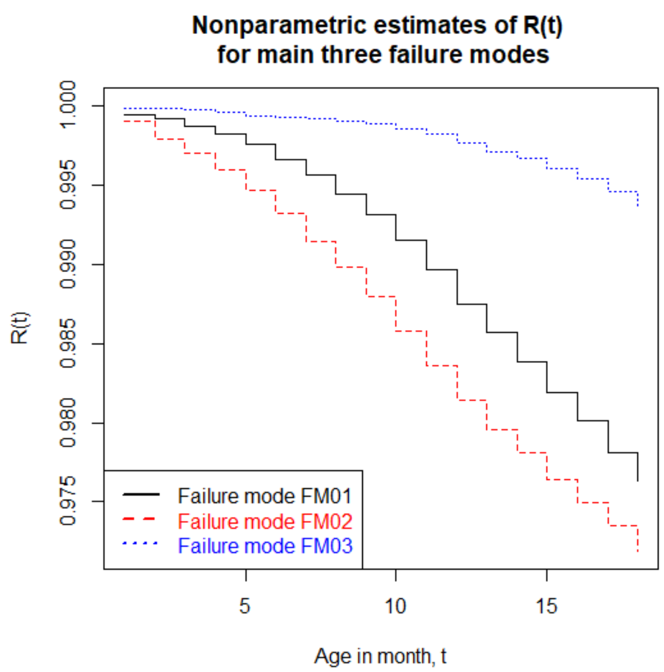

Figure 6: Comparison of nonparametric estimates of $R(t)$
for main three failure modes

The warranty claims data of the component considered here were manufactured over a 12 month period (Jan., Feb., .., Dec.) of a particular year. Monthly production amounts are given as supplementary data. The production month-wise monthly failure counts can be estimated from the warranty database. Due to variations in materials and/or production, the quality of components can vary from batch to batch or month of production (MOP) to MOP. We estimate the age-based warranty claim rates (WCR) for various MOP to provide a basis for checking quality variation problems with respect to production period. We define the WCR for MOP=i and Age=t as follows

$$WCR(i,t) = \frac{r_{it}}{M_i}, i = 1, 2, ..., 12; t = 1, 2, ..., 18 \tag{5}$$

where r_{it} represents the count of claims at age t occurred from month of production i and M_i is the total number of items produced in month i, i=1, 2, …, 12; t=1,2,…, 18. More detail on the estimation of WCR can be found in Blischke et al. (2011). The estimates of WCR (i, t) are shown in Figure 7.

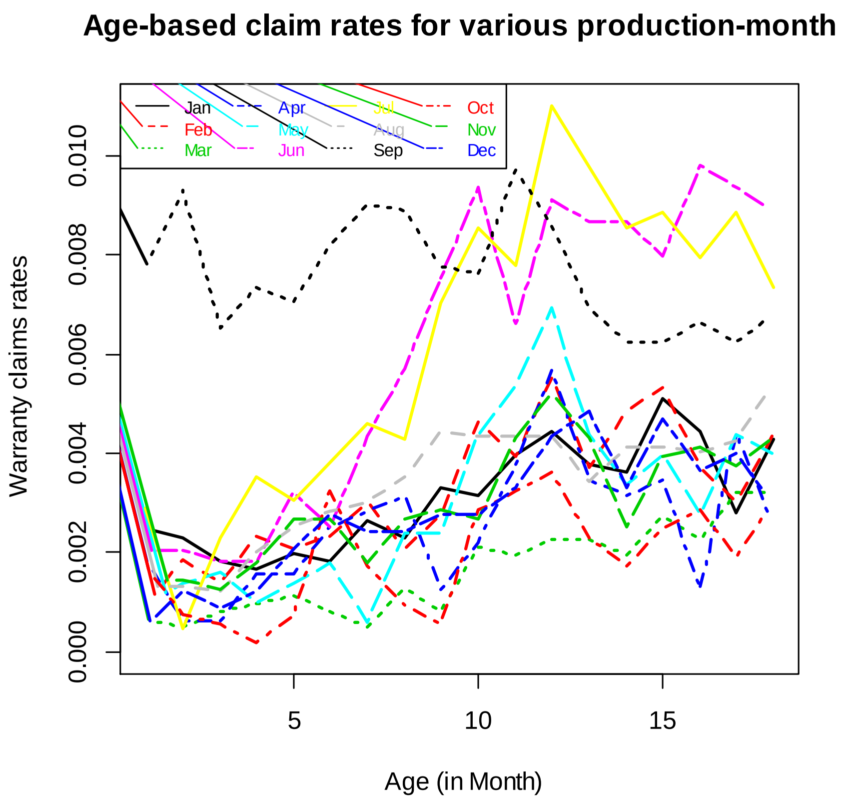

Figure 7: Age-based warranty claim rates for different months of production (Jan. to Dec.)

Figure 7 indicates that the warranty claim rates are very high for the three months of production September, July and June compared with other months of production. For the MOP September, the WCRs are approximately constant with respect to age. The WCRs for the MOP July and June are seems to be increasing with age. The quality of the items produced in the remaining MOP (January to May, August and October to December) is the best in the sense that the claim rates are low and age-wise approximately similar. This suggests that there were some problems in materials and/or production process in the MOP September, July and June.

To make the differences among WCRs clear in Figure 7, we separate the MOP in two groups: Group 1 contains three MOP September, July and June and Group 2 contains the remaining nine MOP (January to May, August and October to December). Then we estimate the age-based average WCRs for Group 1 and Group 2. For example, for Group 1, the average WCR at age t equals to $\{WCR(9,t)+WCR(7,t)+WCR(6,t)\}/3$, t=1,2,…, 18. Similarly, it can be estimated for Group 2 by averaging on nine MOP. The age-based average warranty claim rates for two groups are shown in Figure 8 which clearly indicates that the average warranty claim rates for Group 1 are higher than that of Group 2.

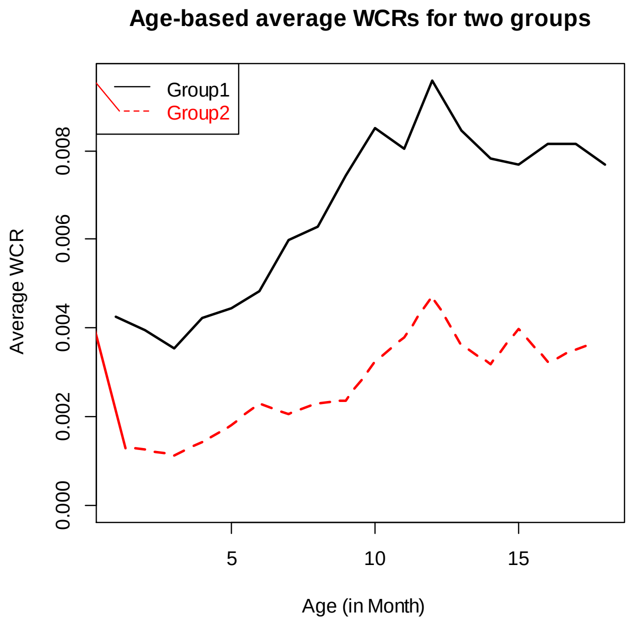

Figure 8: Age-based average warranty claim rates for two groups

4. Parametric Analysis

This section presents the parametric approach to analysis the warranty claims data set discussed in Section 2. The parametric approach to data analysis is concerned with the construction, estimation, and interpretation of mathematical models as applied to empirical data. This involves the tasks model selection, estimation of model parameters and validation of the model. Once these tasks are completed, the model may be used for prediction and other inferences.

To apply the parametric approach, we arrange the data in a concentrated form. Let t_i be the observed failure/censored lifetimes for the random variable T measured in month, m_i denote the number of units (frequency) that failed/censored at t_i and δ_i represent the failure-censoring indicator for t_i (taking on value 1 for failed items and 0 for censored), i = 1, 2, …, k (for the data set k=18). We assume a parametric model $f(t;\theta)$, with corresponding survival or reliability function $R(t;\theta)$, for the failure time variable T, where θ is a vector of model parameters. Under this scenario of data, the likelihood function can be written as

$$L(\theta) = \prod_{i=1}^{k} f(t_i;\theta)^{\delta_i m_i} R(t_i;\theta)^{(1-\delta_i)m_i} \tag{6}$$

The log likelihood becomes

$$\log L(\theta) = \sum_{i=1}^{k} \left[\delta_i m_i \log\{f(t_i;\theta)\} + (1-\delta_i) m_i \log\{R(t_i;\theta)\} \right] \tag{7}$$

We assume the eleven popular distributions, given in Appendix A (Table A.1), in the likelihood function (6) or log-likelihood function (7) and obtain the maximum likelihood estimator of θ by maximizing any of these likelihood functions. The log-likelihood function (7) is evaluated for the variable Age in month, T, and maximize to obtain the MLEs of the parameters assuming eleven distributions: (i) Smallest extreme value, (ii) Two-parameter Weibull, (iii) One-parameter exponential, (iv) Two-parameter exponential, (v) Normal, (vi) Two-parameter lognormal, (vii) Logistic, (viii) Loglogistic, (ix) Three-parameter Weibull, (x) Three-parameter lognormal and (xi) Three-parameter Loglogistic. We use the Minitab software to do this task.[3] The adjusted Anderson-Darling (AD) test statistic is used to select the best fitted distribution among the eleven distributions.[4] Figure 9 shows the Minitab output of distribution ID plots for the four distributions (Weibull, lognormal, loglogistic and 3-parameters lognormal) which give the smaller AD values among eleven distributions.

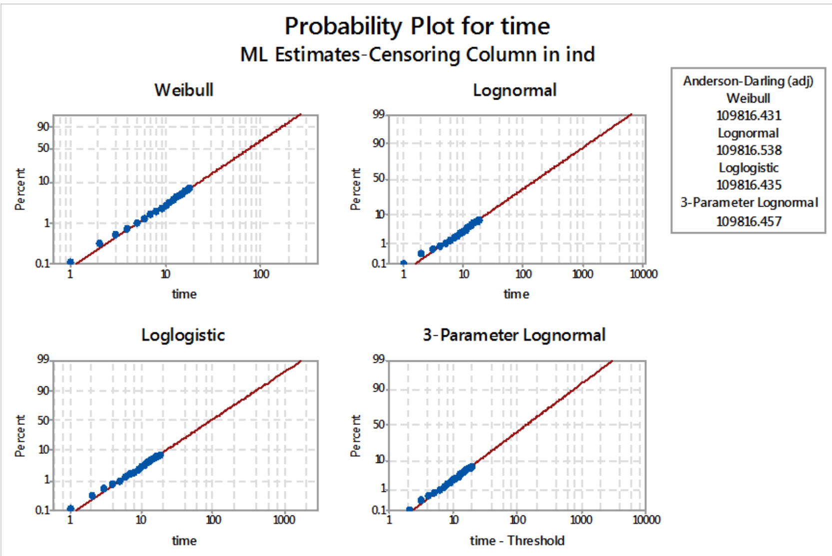

Figure 9: Four distributions probability plots of Age in month

In Figure 9, the overall appearance of the plots are not much changed, and the values of the AD statistic are approximately equal. However, the Weibull distribution shows the smallest AD statistic and so this distribution can be considered as the best distribution for the data among eleven distributions.

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[3] The R functions mle(), optimize(), optim() or nlm() can also be used to do this task.

[4] Minitab (version 17) software creates probability plots and estimates adjusted Anderson-Darling (AD) test statistic for these eleven distributions.

The Weibull distribution overview plot shown in Figure 10, where the maximum likelihood estimates of the parameters are scale parameter $\hat{\eta} = 99.0176$ and shape parameter $\hat{\beta} = 1.5553$. The maximum likelihood estimates of mean age and median age are respectively 89.0239 months and 78.2292 months. As the estimate of the shape parameter of Weibull distribution is greater than one, the hazard function in Figure 10 indicates an increasing failure rate (IFR) with respect to age.

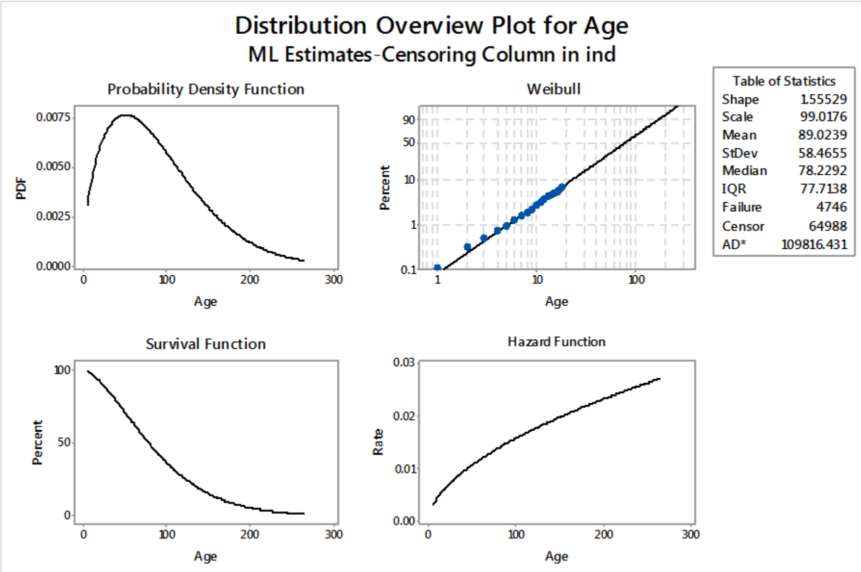

Figure 10: Weibull distribution overview plot for Age in month

The fitted Weibull cumulative density function, $\hat{F}(t;\eta,\beta)$, can be utilized to predict warranty cost of the component for a given warranty period. Let c_s denotes the average warranty cost (the cost incurred by the seller for servicing a claim which can be estimated from the warranty-service database) under a one-dimensional warranty with only first failure coverage. Then an estimate of the expected average cost per unit to the manufacturer for servicing a warranty up to t_w, denoted by $\hat{C}(t_w)$, is c_s times the proportion of units expected to fail within t_w (Karim and Suzuki, 2008), that is, $\hat{C}(t_w) = c_s\hat{F}(t_w;\eta,\beta),\ t_w > 0$.

4.1. Analysis by Individual Failure Mode

If the manufacturer wants to improve the overall reliability of the component, it is important to find the suitable parametric distributions for each failure modes separately. Comparing the reliability functions of each failure modes, the manufacturer can redesign the component, if necessary, to optimize the overall reliability. This can be done by analyzing the competing risk models. In the competing risk setup, when we look at a single failure mode, all of the remaining items, including those that failed by another mode and common censored items, are right-censored. The distributions for individual failure modes are selected based on the minimum adjusted AD values and probability plots from a set of 11 distributions. It is found that the 3-parameter lognormal distribution can be selected as the best distribution for each failure modes. The maximum likelihood estimates of parameters of 3-parameter lognormal distribution for different failure modes are summarized in Table 2.

Figure 11 plots the individual reliability functions for eight different failure modes. It indicates that the reliability of failure modes FM02 and FM01 are very low compared with other failure modes.

Table 2: MLEs of the parameters of 3-parameter lognormal distribution

Failure Modes	Maximum likelihood estimates (MLEs)		
	Location ($\hat{\mu}$)	Scale ($\hat{\sigma}$)	Threshold ($\hat{\tau}$)
FM01	5.3630	1.1930	-2.0349
FM02	6.7479	2.0279	0.2808
FM03	4.7649	0.2886	-39.1793
FM04	6.6998	1.2714	-4.1681
FM05	7.9435	1.5531	-2.1135
FM06	7.8058	1.6704	-1.6149
FM07	22.1312	7.0101	0.9993
FM08	9.2202	2.3635	0.1461

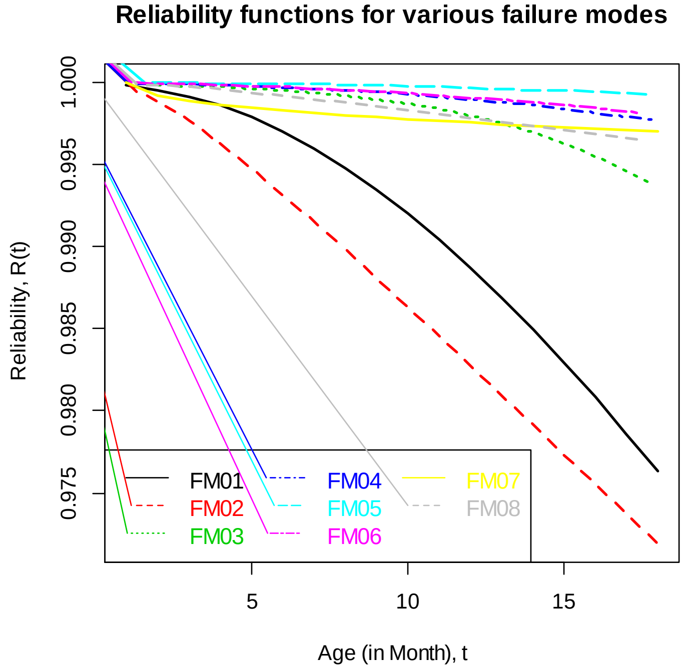

Figure 11: Reliability functions for different failure modes

Therefore, to increase the overall reliability of the component, effort should be concentrated on failure modes FM02 and FM01. Elimination of these or reducing the risks associated with them would significantly increase reliability and decrease warranty claims and costs. This investigation is important not only for assessing reliability and warranty costs, but also for assuring customer satisfaction and product reputation.

4.2. Elimination of Dominant Failure Mode

In this section, we look at modeling through elimination of the main failure modes one at a time. This enables us to investigate how the reliability of the component improves by successively removing failure modes. If $\hat{R}_{\mathrm{FM}k}(t)$ be the estimated reliability function associated with the k^{th} failure mode, under competing risk setup, the estimate of overall reliability of the component at age t, $\hat{R}(t)$, can be expressed as

$$\hat{R}(t) = \prod_{k=01}^{K} \hat{R}_{\mathrm{FM}k}(t),\ t = 1, 2, \ldots, 18 \qquad (8)$$

where K is the number of failure modes and here K=08. FM01 eliminated means the first term of the right side of (8) equals 1, and so on for other failure modes. For example, the reliability of the component after eliminating failure mode FM*z*, let us denote by $\hat{R}_{[\text{-FMz}]}(t)$, z=01, 02, …, 08, can be estimated as

$$\hat{R}_{[-\mathrm{FMz}]}(t) = \prod_{k=01}^{K} \hat{R}_{\mathrm{FM}k}(t) / \hat{R}_{\mathrm{FMz}}(t),\ t = 1, 2, \ldots, 18 \qquad (9)$$

Figure 12 shows a comparison of reliability functions after eliminating failure modes FM01 or FM02. In this figure, "2-parameter-Weibull" means the estimated reliability function based on 2-parameter Weibull distribution fitted in Section 5, "Comp-risk All FM included" means the estimated reliability function based on competing risk model (8), "FM01 Eliminated" and "FM02 Eliminated" mean the estimated reliability functions by eliminating failure modes respectively FM01 and FM02 by (9).

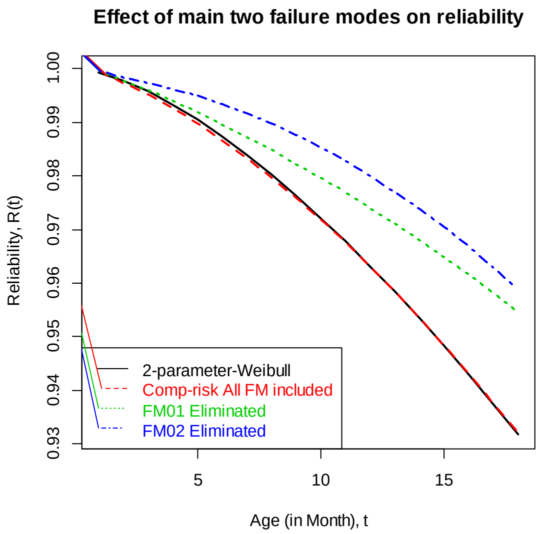

Figure 12: Comparison of reliability functions after eliminating failure modes FM01 or FM02

The overall reliability of the component estimated by 2-parameter Weibull distribution and by competing risk model are almost equal. The reliability of the component improves vastly after eliminating failure modes FM02 or FM01. For example, at age 18 months, the component reliability is 0.9319. This reliability improves to 0.9546 if failure mode FM01 eliminated and to 0.9589 if failure mode FM02 eliminated. The analysis suggests that if we design out failure mode FM02 and/or FM01, the reliability of the component improves vastly. This investigation is important in effective maintenance management (Murthy, et al., 2015) and managerial implications for cost-benefit analysis, including improvement in reliability, reduction in warranty cost, and forecasting claims rates and costs.

5. Conclusions

In this paper, we have attempted to analyze warranty claims data on a component of an automobile. Nonparametric and parametric analyses were employed for analyzing the warranty claims data. Some findings and recommendations are as follows:

- For this component, the warranty claim rates are significantly very high for the three months of production June, July and September (called Group1) compared with other months of production (called Group2). The claim rates for Group1 is approximately 2.5 times higher than that of Group2.
- The component has two dominating failure modes (denoted by FM02 and FM01) which vastly contribute in decreasing the reliability of the component. The overall 18-month component reliability is 0.9319. That is, 93.19% of the components survive past 18 months. If the failure modes FM01 or FM02 can be eliminated, 95.46% or 95.89% of the component will survive at the age of 18 months. To improve the overall reliability, we may need to improve both the failure modes FM01 and FM02. This analysis would be useful to the manufacturer if they decide to eliminate the dominant failure modes and to address the problem whether it is due to manufacturing or design.
- The paper presents age-based analysis of warranty claims data. The limitation of the paper is that it does not considered usage-based analysis. Future research on applications of usage-based modeling (e.g., Rai and Singh, 2005; Jiang and Jardine, 2006; Manna et al., 2007; Dai et al., 2017; He et al., 2018) and bivariate modeling (e.g., Moskowitz and Chun, 1994; Murthy et al., 1995; Blischke and Murthy, 1996; Kim and Rao, 2000; Pal and Murthy, 2003; Baik et al., 2004; Manna et al., 2008; Gupta et al., 2017) would enrich the analysis of the data.

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