Two A. N. Kolmogorov's Early Letters About Mathematics Education

B. V. Gnedenko
Publication and notes by D. B. Gnedenko

Abstract

It is well known that A.N. Kolmogorov made a great contribution to the development of Russian mathematics pedagogy. Mr. Kolmogorov became increasingly interested in pedagogical issues in early 1962, when he came up with many new ideas that encouraged teachers and mathematicians to use their creativity in search of ways to solve these issues. Those ideas were set forth in the letters I received from Mr. Kolmogorov in 1962. My son came across two of them while sorting out books and papers, and both of them deserve publication. We will present the part of the letters devoted to the topic of our discussion in full. The publication takes the reader to the early 1960s and introduces him or her to Andrey Nikolaevich Kolmogorov's vision of mathematics education at schools and universities (particularly at the faculties of mathematics, physics, chemistry, and at physics and technology institutes, as well as at engineering and physics institutes).

§ 1. Issues of secondary education

It was long ago that Mr. Kolmogorov showed interest to mathematics education. The first Moscow School Mathematics Competition was held in 1935. Since then mathematics competitions became an integral part of this country's cultural life. Mr. Kolmogorov actively participated in the first competition and all those that followed it: he took part in analyzing problems, gave lectures to schoolchildren, was a Chairman of the Organizing Committee of the Moscow Mathematics Competition (in 1937, 1963 and 1975). But in the late 50s - early 60s he took interest in a broad range of aspects of this issue. He noted that the country that prioritized the scientific development sought to improve mathematics education. At that time he took great interest in the progress of physics, mathematics, and technology, and noticed that mathematics school education did not meet new requirements.

I remember a meeting that was devoted to probability theory and took place in Uzhgorod in August 1959. When the meeting was over all the participants who lived in Moscow and Kiev, including Mr. Kolmogorov, went back by train. They talked about mathematics school education.

Since autumn 1959 special mathematics courses were introduced in some schools. Postgraduates and young teachers who graduated from Moscow State University did lecturing in Moscow, Moscow region and in Ivanovo. Mr. Kolmogorov engaged his students into these activities.

In the letters, which were mentioned in the beginning, Mr. Kolmogorov set forth interesting ideas that explained the need to renounce a unified approach to teaching mathematics at different schools.

We will begin with a letter that is devoted exclusively to school and higher mathematics education[1].

"1. The transition to universal secondary education is one of the fundamental measures needed to overcome the opposition between physical and mental labor. It will undoubtedly lead

[1] Boris Vladimirovich did not mention in the text of the letter when he wrote it, and I could not find the original letter so far. (note D. B.)

to a profound change in the structure of our secondary school, that is gradually taking shape. However narrower special needs of this country should be factored in while changing the methods of upbringing and education of tens of millions of our youth.

2. Accelerated development of scientific research in physics, mathematics and chemistry, and modern technology based on them is one of the urgent needs of this country. Today expenditures on scientific and technical researches account for a considerable share of about 10-15% of the total national revenue. Thus, it is clear that research activities have become an integral part of production. There are practically no boundaries between research institutes and industrial enterprises in many branches of modern technology.

3. There may be many different ways of how to engage in research activities, however successful development of physics and mathematics and modern technology is certainly impossible without involving a lot of young people aged 17 to 19 in a close study of physics and mathematics and in the atmosphere of scientific research activities.

4. Faculties of physics and mathematics, mechanics and mathematics, physics, and chemistry in universities and technical universities, such as physics and technology institutes and engineering and physics institutes should be open to young people of various background. Young people, whose life journey began with simple physical labour, should get access to universities of this type in an organized manner. However, in order to achieve scientific and technological development it is necessary to recruit the majority of youth, rather than its significant part, aged 17 to 19 studying at such universities. It is essential to find ways to provide this part of youth with good vocational raining before university entrance, as well as during their studies.

5. First-year students should gain profound knowledge of the basics of physics and mathematics at the aforementioned universities. Extra-mural forms of studies and on-the-job training may be regarded only as an additional form of work, rather than training of highly qualified workers. It will be appropriate only for undergraduates studying at such universities to have a closer connection with the production sector, and work at enterprises and scientific institutes while studying.

6. The aforementioned universities need tens of thousand of new students annually. Technical universities which are not part of this group and faculties of physics and mathematics and chemistry in pedagogical institutes should also enroll a number of talented young people aged 17 to 19 who have good knowledge of physics and mathematics. Naturally, young people aged 17 to 19 who are to be enrolled in the universities right after they finish secondary school should satisfy high demands with regard to their capacities and knowledge of mathematics, physics and chemistry, and their serious approach to labour. Underestimating such career management scheme applied to mathematics and physics, modern technical sciences could have led to the most deplorable consequences.

7. Thus, the demand for personnel consisting of young people well trained in physics and mathematics who enter in universities at the age of 17-19 amounts to 50,000 if not 100,000.

8. It appears inappropriate to establish secondary schools with a focused specialization for outstandingly gifted young people (separate classes for would-be mathematicians, physicists, chemists) collaborating with universities and at universities that young people think they will definitely enter (Academy of pedagogy project). Indeed a group of students who are good at and interested in mathematics, physics and chemistry, enjoy making research, constructing apparatuses is formed when they reach 15, i.e. by the end of 8-year studies at school. A narrower specialization is usually defined later.

9. The most serious part of the aforementioned group of high-school students of a 8-year school can be prepared for studying at universities and technical institutes in two or three years.

Less serious ones can find application for their propensities and interests after undergoing the necessary training and work as laboratory assistants at scientific and industrial laboratories, as qualified workers in such industrial sectors as radio-technology, as computists and operators at computation centres and enterprises, as draughtsman at design engineering bureaus and the like.

Tens of thousands of such employees are needed now.

10. From this one can obviously conclude that it is appropriate for 8-year schools of various types to have special physics and mathematics classes or physics and technology classes which will produce approximately 100,000-150,000 students a year.

11. If principles of vocational training are to be introduced in schools in a reasonable manner and one bears in mind that since these schools are situated primarily in cities they should be available to talented 8-year school graduates from villages and towns, it is sensible to let them study for three years (from 15 to 18). It will let enroll young people who finished 2-year training courses in secondary schools of different type for the second year. This is what we adopt from the Academy of pedagogy project.

12. Young people should enter in physics and mathematics schools by competition, and to a large extent (but not exclusively) upon the recommendation obtained from 8-year schools. The majority of students should receive scholarships and places in dormitories.

§ 2. Higher mathematics education

We will address the issues related to higher mathematics education which were outlined in this letter.

"13. Teachers of mathematics, physics, chemistry who work at physics and mathematics schools should study mainly at universities. As far as mathematics is concerned, there should be about 3,000-5,000 teachers, thus universities training 300-500 teachers per year will not be overstretched. On the contrary, universities will benefit from such forms of interaction with secondary school.

14. About half of physics and mathematics school graduates should apply for universities right after finishing schools. Still they should not regard admission to universities as something self-evident and guaranteed. Naturally, they will gain access by a competition.

15. Despite such broad horizons physics and mathematics schools will not look like closed schools, "cultivating" talents. Their name itself should not suggest any "specially gifted" students (which certainly does not prevent such students from applying for them sometimes).

16. As it was noted in paragraph 4, physics and mathematics schools should not be the only normal way to get access to universities we are interested in (physics and mathematics faculties of universities and technical universities of a new format). If it turns out that young people studying at physics and mathematics schools for three years are better trained than at other secondary schools, it is likely to be more reasonable to let first-year students learn a wider variety of subjects (for example, without separating mathematicians, mechanics and physicists at universities) and to let physics and mathematics school graduates apply for the second year of studies (the third year of studies at physics and mathematics schools will probably include big courses of analytical geometry and elements of analysis).

Mr. Kolmogorov always took great interest in university education. He systematically monitored the content of the courses and formulated wishes about changes in curricula and making sure that both mandatory and special classes have a practical aspect. Mr. Kolmogorov was a true think tank of mechanics and mathematics faculty of Moscow State University. He was well aware of both advantages and disadvantages of courses lectured to students. This allowed him to suggest new ideas every year and carry them out into practice. During the war he suggested that mathematical analysis should be lectured for third-year students. This part of the course was based on functional analysis. Parts of contemporary mathematics was presented coherently, rather than separately to students. Moreover, he proposed to introduce a workshop on mathematical analysis, which implied fundamental problems to be solved on the basis of all the material that has been studied. Many students found the tasks difficult, but their solution facilitated rapid development of mathematical skills. No wonder those years brought him many talented students. We would

like to note today the faculty mainly comprises Mr. Kolmogorov's students, who stepped into the scientific field with the help of a broad programme that made them get in-depth knowledge of its parts and constituents.

§ 3. The contents of the letter dated from 28 September 1962

In the letter dated from 28 September 1962 Andrey elaborates on the ideas he has just outlined.

"It is common knowledge that people should manage their own lives rather go with the tide. Now you're engaging me in all sorts of new long-term undertakings, which means that if I take a serious approach to their implementation, I will spend on them the rest of my active life. I will not start with these undertakings, but with your plans of work, then I will turn to mine, after that I will touch upon different institutes, faculties, laboratories to be established.

About you

When you moved to Moscow, you intended to give your work a new, interesting and productive direction. I highly value you as a mathematician who at the same time educates young people and can organize scientific work.

a) You will PROBABLY bring something new to mathematics when you are between 50 and 60 years old that will be as good as the series of your works I was honoured to write about in the article on the occasion of your fiftieth birthday (I have deliberately stressed what I consider essential) or may be something more valuable? But I'm afraid that for the past few years you haven't even had a chance to take any visible effort to learn other people's ideas that are SIGNIFICANTLY NEW to you, or at least to place understanding and presentation (in lectures, books) of a sphere that you used to have a profound knowledge of about twenty years ago (then were aware of the most recent data!) to a significantly new level to .

b) Hence the result: it is still unclear what will encourage young people in Moscow who might one day turn into someone similar to your Korolyuk[2] or Skorokhodov living in Kiev to attend your classes[3]. And you would deserve[4] that success. I cannot see from what you have said

[2] Vladimir Semenovich Korolyuk (born 1925) – Candidate of Physical and Mathematical Sciences (1954), Doctor of Physical and Mathematical Sciences (1964), Professor (1965), Corresponding Member of the Academy of Sciences of the Ukrainian SSR (Academy of Sciences of Ukraine since 1991, the National Academy of Sciences of Ukraine 1994) (1967), Member of the Academy of Sciences of the Ukrainian SSR (1976). (D. B.)

[3] Anatoliy Vladimirovich Skorokhod (1930-2011) – Candidate of Physical and Mathematical Sciences (1957), Doctor of Physical and Mathematical Sciences (1962), Professor (1964), Corresponding Member of the Academy of Sciences of the Ukrainian SSR (1967), Member of the Academy of Sciences of the Ukrainian SSR (1985). (D. B.)

[4] At that time I got such a student as I. N. Kovalenko who made a major contribution to the development of the theory of waiting lines.

/Boris Vladimirovich was wrong. Igor Nikolaevich Kovalenko (born 1935) embarked on postgraduate studies to B.V. in 1957 ("...to study at postgraduate classes <...> it was only I. N. Kovalenko who I took, he was one of the most capable of my students," B. V. writes in his memoirs "My Life in Mathematics and Mathematics in My Life"), became Candidate of Physical and Mathematical Sciences in 1960, Doctor of Engineering in 1964, Doctor of Physical and Mathematical Sciences in 1970, Corresponding Member of the Academy of Sciences of the Ukrainian SSR in 1972, Member of the Academy of Sciences of the Ukrainian SSR in 1978.

In 1962 Boris Vladimirovich had such postgraduate students as Bronyus Igno Grigelionis (1935-2014) (would-be Corresponding Member of the Academy of Sciences of the Lithuanian SSR (Academy of Sciences of Lithuania since 1990) (1972), member of the Academy of Sciences of the

that you have students for whom you could have similar hopes. It is natural that you do not have them yet, what is important though is that there should be conditions for them to appear.

c) You have enthusiastically spoken about Keldysh[5] worrying about the fact that we have no serious practical statistics. The well-known message you try to deliver through "reliability" classes is evidently just as good: it is a desire to introduce in this country something that others have already had. It is in balance with works demanding great "creative" efforts I have already mentioned in paragraph a). These undertakings are good and they do not necessarily prevent that deeper scientific work (that is inevitably more individual). But, if one does not create (in statistics or reliability theory) something radically new from the international point of view, one can take on a serious and interesting task of dramatically improving the work culture in this country. And that starts with big critical "sanitary" work that aims at exposing any false pseudoscience, introducing serious practical training for young people (my last-year workshop for the staff was certainly just a tiny haphazard attack, but still I had that goal in mind). Our young people who are as old as Mr. Meshalkin[6] or Mr. Belyaev[7] also need primarily the examples of work on the spot that will be more substantive than what they can do themselves. Unfortunately, the number of any more superficial undertakings devoted to the reliability theory that you are now dealing with has a priori exceeded the one that is needed for such an approach.

d) However, I do not deny that it may be exciting to be engaged in a broader organizational work, which implies that a mentor does not show others a good example but takes advantage of his ability to select people. Our "laboratory" is certainly too small for such interesting work. You are a perfect director of an institution of any size devoted to the development of statistics without additional training (in terms of traditional significance tests, experiment design with the least required number of tests etc.), who is aware of everything.

As far as cybernetics is concerned, in the broader sense, at the moment you would lack understanding of overall development prospects. This new science requires a mentor who has a keen scent for the prospects of new emerging areas. You would possibly get such skills in the future but under the current circumstances in case the authorities wanted to see someone as a director of a future institute you would have the advantage of an honest decent man who is not obsessed with any far-fetched ideas (it is the combination of honesty and the absence of too narrow personal interests that is rare in this case).

About me

I intended to limit my activities to paragraphs a) and b), and a little bit of c)[8]. I have succeeded so far only in the organizational work that implied my authority should be based on the ability to show a personal example. Between 1933 and 1939 I took care of practically all the post-graduate students of mathematics institute, selecting mentors for them, giving advice to them together with their mentors, replacing a mentor in case of a failure, in any case, I got to the core of their post-graduate programmes. On the contrary, my activities in the capacity of the secretary of Physics, Mathematics Department, or Dean was not HIGHLY successful: here the ability to assess people and assign functions to them that a mentor is unable to carry out is at the forefront. These are, in fact, administrative talents. I have no intention to belittle them by characterizing.

Lithuanian SSR (1987)) and Tadeush Pavlovich Marjanovich (1932-2014) (would-be Corresponding Member of the National Academy of Sciences of Ukraine (1992))/. (D. B.)

[5] Mstislav Vsevolodovich Keldysh (1911-1978) – member of the Academy of Sciences of the USSR (1946), President of the USSR Academy of Sciences (1961-1975). (D. B.)

[6] Lev Dmitrievich Meshalkin (1934-2000) – A. N. Kolmogorov's student, Candidate of Physical and Mathematical Sciences (1979), Professor (1991). (D. B.)

[7] Yuriy Konstantinovich Belyaev (born in 1932) – A. N. Kolmogorov's student, Candidate of Physical and Mathematical Sciences (1960), Doctor of Physical and Mathematical Sciences (1970). (D. B.)

[8] See in the text below. (D. B.)

It was that idea that stipulated my lengthy evasion from all proposals to be a director of an institution.

However, certainly, self-consistency has never been one of my traits of character. And I immediately become enthusiastic if there is a chance to facilitate the creation of active and interesting groups of young mathematicians. Especially due to the fact that over the past years, despite some very talented students, I still lack the usual environment I used to enjoy back then when I was surrounded by a group of young colleagues who I found nice (a friendly generation of my students, or just colleagues in charge of mathematics workshop at the faculty, or even a group of students at Humboldt University, etc.).

However, I needed some staff positions, at most, that I have and will get without taking an administrative position to realize my personal research plans, or to write books (the activity I should resume). In addition to that I am interested in an opportunity to have more freedom when it comes to spending a semester in Leningrad, Novosibirsk, Calcutta, Paris, or even Vilnius, or Dresden, where I could be not only a guest of honor, but rather take part in productive teamwork.

Indeed, I even dreamed about giving our laboratory and chair to you, for example.

Due to various circumstances I am not in a very good shape now and I am still trying to pave the way for such a specifically organized productive work beyond the limits that are too restraining.

3. Now, about a new mathematics institute and faculty

Today these ideas have suddenly taken the shape of a "small Novosibirsk" in Fryazino that still looks more like a dream. Now the talks about this may be more interesting for you to understand the nature of ideas I am passionate about which have changed little in the course of time.

a) This year I am once again in charge of another Moscow competition[9], but I am not contented with that either. It means that I like my closest assisstants (Sasha Kirillov[10] and Andrey Yegorov[11] and Kolya Vasiliev[12] who have just entered a PhD programme), but I do not enjoy the style of "the club members"[13] created by Mr.Kronrod, and it got deeply rooted. There is a good secondary school in Fryazino (which produces sometimes Moscow Competition winners, although they are not among the first ones) and there are definitely many children whose parents, engineers and qualified workers, are extremely interested making secondary education more physics-and-

[9] Andrey Nikolaevich is referring to the current (1962-1963) academic year (Moscow mathematics competitions are held in the spring). (D. B.)

[10] Alexander Aleksandrovich Kirillov (born 1936) – student of I. M. Gelfand, Doctor of Physical and Mathematical Sciences (1962), Professor (1965). (D. B.)

[11]Andrey Aleksandrovich Yegorov – Candidate of Physical and Mathematical Sciences, since 1963he has been working at Moscow Physics and Mathematics Boarding School №18 named after A. N. Kolmogorov at Moscow State University, Senior Professor of the Mathematics Chair of AESC MSU. Member of the organizing committee and the jury of all-Russian and all-Union competitions between1961 and 1979. (D. B.)

[12] Nikolai Borisovich Vasiliev (1940-1998) – graduated from MSU Mechanics and Mathematics Faculty in 1962 and ambarked on post-graduate studies in this faculty. After finishing post-graduate studies, he spent the rest of his life working at MSU Interfaculty Laboratory of Mathematical Methods in Biology. He spent many years dealing with Moscow, all-Russian and then all-Union mathematics competitions for schoolchildren. (D. B.)

[13] Alexander Semenovich Kronrod (1921-1986) – Doctor of Physical and Mathematical Sciences (1949). In the 1950s he was the head of the laboratory at the Institute for Theoretical and Experimental Physics, the main purpose of which was to solve issues related to the development of nuclear weapons. Stalin prize recipient. He started to work in School №7 in 1961. Professor (1966). He worked at the Patent Information Institute, since 1974 he worked in the Central Geophysical Expedition of the USSR Ministry of Oil Industry. (D. B.)

mathematics-oriented, just as directors of institutes and enterprises. It will be natural to launch the corresponging work parallel to School № 7 headed by Landis[14] and Kronrod and situated at the end of Leninsky Prospekt in the view of moving a score of our faculty graduates and post-graduates to Fryazino.

b) The establishment of a faculty (say, physics and mathematics faculty) that is cybernetics-oriented and has the chairs of 1) analysis and function theory, 2) differential equations and vibration theory, 3) equations of mathematical physics and functional analysis, 4) geometry and algebra complemented by scientific work on the theory of continuous groups, bundle spaces, etc., 5) theoretical physics, 6) electronics, 7) logic and discrete automatons, etc. (It is not a very big contribution, though the names reflect a trend, to forming[15] a robust basis for recruiting not only those who specialize in cybernetics, probability theory, but in topology, number theory who are capable of giving lectures on new "combinatorics" who are interested in automatons (like Tolya Karatsuba) to work as members of the faculty and Cybernetics Institute that is subordinate to the Academy of Sciences[16]).

c) It would be natural to find a dean who specializes in physics and radio electronics, or the like, and I could be a Director of the Mathematics and Cybernetics Institute for the first five years, in case the dean (who is a party member) is very young, I could assume the functions of Academic Council Chairman. Anyway, these are just possible examples, it is not a plan.

d) If you have no intention to move to Fryazino, then you will, presumably, take over my Chair in Moscow and will be invited as an advisor – hopefully, the salary will adequate reckoning with the novelty of the work you will do.

Yours Andrey

If we take it more seriously: such a plan suggests at least three people involved who are Members or Corresponding Members of the Academy of Sciences mathematician (there is one), physicist and radio engineer, ready to take up the initiative.

But seriously, could you, please, learn who is the head of research institutes that already exist in Fryazino.

Meantime, the details are fantastic: the faculty that can accept 200-300 people a year and specializes in pure mathematics, computational mathematics, cybernetics, vibration theory, radioelectronics, automation, cybernetics, and theoretical physics. Thus, these are physical professions, that require only advanced mathematical training! General training courses are provided for all first-year students".

[14] Evgenii Mikhailovich Landis (1921-1997) – Candidate of Physical and Mathematical Sciences (1953), Doctor of Physical and Mathematical Sciences (1957), Professor (1961). (D. B.)

[15] It is what is written in the original letter. (D. B.)

[16] Anatoly Alekseevich Karatsuba (1937-2008) – Candidate of Physical and Mathematical Sciences (1962), Doctor of Physical and Mathematical Sciences (1966). He was a head of the Number Theory Department at the Steklov Institute of Mathematics and a Professor of Number Theory Chair (1970), Mathematical Analysis Chair (since 1980), Mechanics and Mathematics Faculty of MSU. (D. B.)