THIS RANDOM, RANDOM, RANDOM WORLD...

TALES ABOUT SCIENTIFIC INSIGHTS

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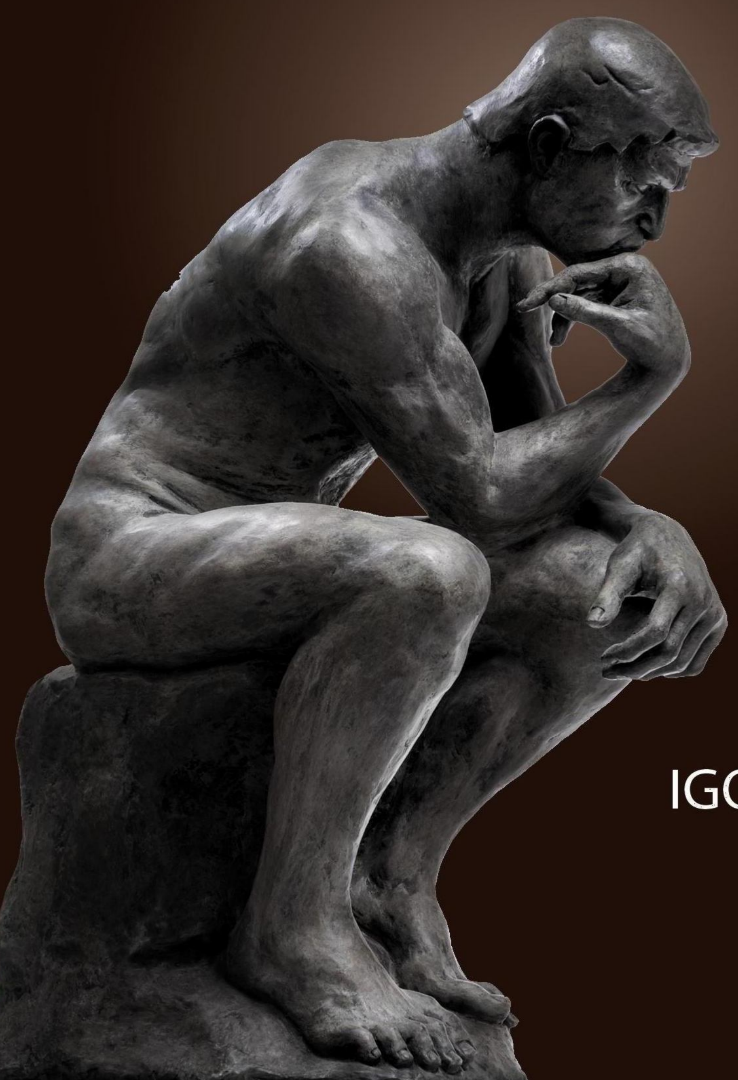

IGOR USHAKOV

Tales and Legends about Mathematical Insights

Igor Ushakov

This Random,
Random,
Random World…

San Diego
2012

Series

"Tales and Legends about Mathematical Insights"

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These books help teachers make their classes more interesting and help pupils to know even more than their teachers!

Cover design: Igor A. Ushakov.

To my beloved granddaughters
Anastasia & Alisa

Preface

The subject of mathematics is so serious that nobody should miss an opportunity to make it a little bit entertaining.

Blaise Pascal*[1]*.

What is this book about? For whom is it written? Why is it written in this manner, not in another? Discussion about geometry, algebra and similar topics definitely hint that this is about mathematics. On the other hand, you cannot find within a proof of any statement or strong chronology of facts. Thus, this is not a tutorial book on mathematics or on of mathematics. This is just a collection of interesting and sometimes exciting stories and legends about human discoveries in one or another way connected to mathematics…

Our book is open for everybody who likes to enrich their intelligence with the stories of genius insights and great mistakes (mistakes also can be great!), and with biographies of creators of mathematical thinking and mathematical approaches in the study of the World.

Who are the readers of the proposed book? We believe that there is no special audience in the sense of education or age. The books could be interesting to schoolteachers and university professors (not necessarily mathematicians!) who would like to make their lectures more vivid and intriguing. At the same time, students of different educational levels as well as their parents may find here many interesting facts and ideas. We can imagine that the book could be interesting even for state leaders whose educational level is enough to read something beyond speeches prepared for them by their advisors.

Trust us: we tried to write the book clearly! Actually, it is non-mathematical book around mathematics.

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[1] **Blaise Pascal** (1623 –1662), was a French mathematician, physicist, inventor, writer and philosopher.

This book is not intended to convert you to a "mathematical religion". Indeed, there is no need to do this: imagine how boring life would be if everybody were a mathematician? Mathematics is the world of ideas, however any idea needs to be realized: integrals cannot appease your hunger, differential equations cannot fill gas tank of your car....

However, to be honest, we pursued the objective: we tried to convince you, the reader, that without mathematics *homo erectus* would never transform into *homo sapiens*.

Now, let us travel into the very interesting place: Terra Mathematica. We'll try to make this your trip interesting and exciting.

Igor Ushakov

Contents

1. Random Regularity or Regular Randomness?

The older one gets the more convinced one becomes that his Majesty King Chance does three-quarters of the business of this miserable universe..

Frederick the Great[2]

1.1. Determinism or randomness?

Even deterministic things, as a rule, happen in random.

(Unknown author)

The thought about causality began long ago. As far back as Democritus[3] told that he "would prefer to find a reasonable explanation of a problem than to possess a crown". Afterwards Aristotle[4] developed a principle of four types of reasons, including the

[2] **Frederick the Great** (1712 -1786), King of Prussia, a prominent conqueror of his time..

[3] **Democritus of Abdera** (460 - 370 BC), ancient Greek philosopher who believed that all matter is made up of various imperishable, indivisible elements which he called "atoma" (singular atomon) or "indivisible units"..

[4] **Aristotle** (384 BC – 322 BC), ancient Greek philosopher, a student of Plato and teacher of Alexander the Great. He wrote on diverse subjects, including physics, metaphysics, poetry (including theater), logic, rhetoric, politics, government, ethics, biology and.

idea of the goal as a universal reason (that was laid in the foundation of theology[5]).

What is determinism? This is a philosophical teaching about reasons, causality, interaction and conditionality of events and processes occurring in the World. So, on the first glance, it seems that determinism is a real science as soon as randomness is… Indeed, what does it mean: randomness? Maybe it is just still has not been recognized? Or even non-cognizable?

Great French thinker Voltaire[6] told, for instance: "Chance is a word void of sense; nothing can exist without a cause. There is no action without a cause; there is no existence without a cause to exist".

Great French scientist Pierre Laplace strongly believed in causal determinism, which is expressed in the following quote:

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes".

In 1814 Laplace introduced a hypothetical creature – "demon[7]" such that if it knew the precise location and momentum of every atom in the universe then it could use Newton's laws to reveal the entire course of cosmic events, past and future.

It is very logical! Really, a body dropped from a horizontally flying plane will fell to the Earth by parable… Stop! Stop! First of all, not a body but an immaterial point; then not from a plane in real atmosphere but in ideal environments without resistance; and let us remember that the

[5] Theology pursues providing reasoned discourse of religion, spirituality and God or the gods.

[6] **François-Marie Arouet**, better known by the pen name **Voltaire** (1694-1778), French writer, essayist, deist and philosopher known for his wit, philosophical sport, and defense of civil liberties, including freedom of religion and the right to a fair trial.

[7] Afterwards this hypothetical creature was named "Laplace's demon". By the way, "demon" in Greek means "god" and "genius" – only in Christian traditions it became an evil.

gravity is changing (though insignificantly)… And in addition a slight wind is blowing… Where is the parable?

And finally, don't forget spontaneous and unpredictable human behavior that intervenes almost in any "natural" process…

However, assume that we are considering ideal mechanical system of the Universe scale and it evolutes in accordance definite deterministic laws: let me know, please, what kind of a demon could collect instantaneously all current data? Where one could find a memory to put it there? Who determine the law of interaction of uncountable number of all particles? And, finally, where a demon could find computing capacity to do the calculations? (And how these results afterwards could be presented?)

It seems that problem of the "Laplace's demon" is not in possibility of deterministic prognosis but in principal possibility of realization of such prognosis. An opinion exists that Laplace invented his demon just to demonstrate that there were a necessity of statistical description of most of phenomena in the real World. Though majority consider Laplace was a convinced determinist because the "Laplace's determinism principles" are laid in the basis of classical mechanics. (Could you imagine that the man created the probability theory could be "inflexible" determinist?!)

Since "pure determinism" cannot answer all arose questions, randomness appears on the scene inevitably. And with developing thermodynamics and statistical physics scientists began to use "probabilistic determinism"! Though some remind that even Albert Einstein[8] told: "God does not play dice with the universe ". Everywhere one tried to find an intelligent substance...

Modern determinism theory is a synthesis of many scientific approaches in the world cognition. It finds its reflection in ideas of self-organization of the matter. Questions about essence of randomness and casualty and their interrelations are still the most principal in determinism.

So, where is the origin? A hen or an egg? Determinism or randomness? Random casualty or casual randomness?

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[8] **Albert Einstein** (1879 -1955), great physicist, one of founders of modern physics, creator of the Relativity Theory, Nobel Prize laureate. ***For more details see Chapter "Pantheon" in Book 1.***

Even not trying to answer these questions, let us consider – just in illustrative purposes – something "more ground", for instance, the modern theory of life development on the Earth.

Relation between casualty and randomness was investigated by number philosophers and biologists. From on hand, evolution is a very much regular process, though, from other hand, in the fundament of it lie random mutations. In other words, a series of random events generates something regular...

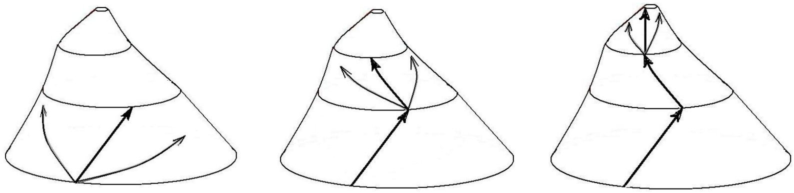
With discovery of general genetic laws, the Darwin's evolution theory was definitely revised in part of inheriting mechanism. The modern "synthetic evolution theory appears in 1930-s as the result of convolution of genetics (discrete mutations of individual genes) and the Darwin's theory where the natural selection plays the main role.

There are even two directions tychogenesis, or evolution determined by chance, and nomogenesis, or evolution determined by law. Frankly speaking, there is no principal contradiction between them: non-effective branches die and effective ones continue it's "regularly developing".

Let's consider an immune system: here everything happens in accordance with the method "trials and errors". As soon as unknown agent appears within the body, special proteins begin to mutate until such a type of antibody that is able to recognize the agent and start to fight with it. The process itself is stochastic, though if an enough attempts have been made, the positive result could be reached. Of course it should be mentioned that originally "switched" proteins are not chosen in random.

Thus, scientists agree that evolution is a globally regular process, though its details are random.

Probably this process more than anything is close to a mathematical model of random search: at each step of the process one makes several trials and chooses the one with the best indication of a goal function. Afterwards, one makes a step in the chosen direction, and the procedure is continuing. In optimization theory it is proved that that such procedure leads to the extreme of a unimodal function, i.e. a function with the only extreme.



Procedure of random search.

Most brightly one can see regularity of the evolution process on the so-called "parallelism" effect. It is the name of the phenomenon when different species habituating in the same environments possess similar properties. For instance, various mammals living in water have "fish-like" forms (shape of the body, flippers)

Notice that the human history, reflecting evolution development of the societies, represents a much tied interlacing of randomness and casualties. Doubtless, some regular historical laws exist, however we know examples when really insignificant events have changed people's destiny. And how many historical examples of "what-if" type we know!

Yes, if Persian Emperor Darius the Great[9], having token Scythians' gifts did not try to interpret its "prophetic" sense and did not turn his army back… (By legend, Scythians sent him symbolical gift of obedience: a frog, a bird, a mouse, and an arrow. Darius' court fortune-teller interpreted it in the following way: in Persians could not jump as frogs, could not fly as birds, could not hide as mice, then they all would die from arrows. Darius trusted his fortune-teller because Persians could not do anything of described!)

Yes, if Julius Caesar[10], who hesitated to cross the Rubicon or not, had casted another die, the fate of the entire Roman Empire could be quite different… (And we would have lost two so beloved expressions: "The lot has casted" and "To cross the Rubicon"!).

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[9] **Darius the Great** (550–485 BC), Persian Emperor, famous conqueror of the ancient times.

[10] **Gaius Julius Caesar** (100-44 BC), a Roman military and political leader. Played a critical role in the transformation of the Roman Republic into the Roman Empire.

Yes, if Napoleon[11] did not get a cold before the Waterloo battle, probably, the outcome could be different...

Of course, all these events happened by chance are considered by historians as regular and inevitable processes. And who can check it? The time is irreversible as our life: it is impossible to start the life from the beginning!

Well... After all these remarks, it's the time to turn to probability and statistics!

Aphorisms concerning randomness

- Whatever people think about greatness of their acts, they are often the product not of great ideas, but simply an accident. (*Rochefoucauld*[12])
- Winners do not believe in coincidence. (*Nietzsche*[13])
- Blind event changes everything. (*Vergilius*[14])
- "Case" is God's nickname when he does not want to sign his own name. (*Anatole France*[15])
- In life it is very important to know when to seize the opportunity, but it is equally important to know when not to do it. (*Disraeli*[16])
- In each great deal some part depends on chance. (*Napoleon*)
- Everything is influenced by randomness. (*Lucan*[17])
- Everything is ruled by chance. It would be perfect to know who ruled the chance. (*Lec*[18])
- Unusual cases often repeat. (*Chapek*[19])
- Unexpected happens in our life more frequent than expected. (*Plautus*[20])

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[11] **Napoleon I Bonaparte** (1769-1821), French Emperor, great conqueror.
[12] **François VI, duc de La Rochefoucauld** (1613-1680), noted French author of maxims and memoirs.
[13] **Friedrich Wilhelm Nietzsche** (1844-1900), German philosopher.
[14] **Publius Vergilius Maron** (1970-1919 BC), one of the most significant Roman poets.
[15] **Anatole France**, the pen name of **Jacques Anatole François Thibault** (1844-1924), French writer.
[16] **Benjamin Disraeli** (1804 -1881), British statesman and writer.
[17] **Marcus Annaeus Lucanus** (38-65), ancient Roman poet.
[18] **Stanislaw Jerzy Lec** (1909 1966), Polish satirist and poet.
[19] **Karel Chapek** (1890-1938), Check writer and playwright.
[20] **Titus Macchius Plautus .** (254-184 BC)**,** playwright of Ancient Rome.

- It is impossible to become a good man by chance. (*Plato*[21])
- Chance is one pole of dependency, and another one is necessity. (*Engels*[22])

1.2. Dice and probability theory

It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge.
Pierre Simon de Laplace[23]

Why people so like to play different hazardous games? Probably, because its outcome is random, i.e. is unknown a priori. Of course, there are many hazardous games where to win you should possess some special skill, though chance is always chance! And every gambler believed in his or her "special" fortune.

One of the most unpretentious games where a skill is insignificant enough is a game with dice. Here the role of a chance is the most. Probably, because of it namely a game with dice led to first thoughts about definition of randomness and probability, we will start with this game.

When did people begin to play with dice first?

Sofocl[24] told that dice were invented by Greek hero Palamed[25] during long and boring siege of Troy to enjoy tired soldiers. However, Herodotus[26] wrote that dice were invented by Lydians[27].

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[21] **Plato,** original name was **Aristocles** (428-347 B.C.), ancient Greek philosopher.
[22] **Friedrich Engels** (1820-1895), German social scientist and philosopher, one of founder of communist theory.
[23]**Pierre-Simon Marquis de Laplace (1749-1827),** great French mathematician, physician and astronomer. He was one of the founders of the Probability Theory, mathematical physics and celestial mechanics. ***For more details, see Chapter "Pantheon" I Book 1.***
[24] **Sofocl** (496-406 BC), Athenian playwright, considered one of the great tragic poets of classical antiquity.
[25] **Palamed** was a Greek hero, participant of the Trojan War. He is said to have invented not only game with dice as well as counting, currency, weights and measures and other important things.

However, multiple archeological diggings show that dice existed far before. In ancient times, predecessors of dice were ankle bones of animals, or *astragals*. The popularity of astragals is seen in the variety of cultures that employed its use. One believes that dice firs were used for fortune-telling rather than for hazard games.

By the way, do you know where the word “hazard” came from? Some tell us that in Old French game of dice was called hazard. Yes, it is true, however where did this word come from? As a matter of fact, Arabic “az-zahr” means dice or a kind of dice-game!
It is difficult even imagine where one could not find dice! Eskimos and Africans, Aztecs and Mayas, Polynesians – all of the played dice! And dice were of different shapes and colors; sometimes they were of wood or of clay.


A WALL DRAWING FROM POMPEII

In ancient Greece and Rome dice were mostly of precious elephant bones, though there were dice of bronze, amber or porcelain... Notice that dice dated about one millennium B.C. were found in archeological diggings in Egypt, China and India.

Now there are at least two most popular dice games: craps and poker. Craps is a game where one should guess the points before two dice are casted. Poker seems to us more interesting: one casts five dice and after looking on the result has a choice to cast again any number of dice to reach a needed result. This dice game is very similar to a card game with the same name.

[26] **Herodotus of Halicarnassus** (484-425 BC), Greek historian who is regarded as the "Father of History".
[27] **Lydia** was an ancient kingdom at the Western part of the Anatolian peninsula. By the way, the wealth of the last Lydian king Croesus was and is proverbial.

1.3. Poker on dice

Probability theory is laid on bones…
Fortunately, these bones are astragals
for playing dice!
(Unknown author)

Each player (usually, there are from two to five players) casts five dice. The objective of the game is to play successfully some a priori defined combination of points.

Each turn consists of 3 rolls of the dice. At the start of each turn, the player rolls all the dice. Prior to each subsequent roll, the player may choose to keep or release any combination of dice. After the third roll, the player must score the set of dice in one of the scoring boxes. If the player completes a combination before the third roll, he may take his score without rolling again. If the player gets a resulting combination that already filled in the table, he gets nothing and waits until the next turn. The game does not end until somebody fills all 13 cells of the table.

Below in the table all possible combinations are given.

Example	Type of combination	Score	Bonus
■-□-◘-●-▲	Chance	Sum of points	
1-1-1-■-□	Three or more ones	3	
2-2-2-■-□	Three or more twos	6	
3-3-3-■-□	Three or more threes	9	
4-4-4-■-□	Three or more fours	12	
5-5-5-■-□	Three or more fives	25	
6-6-6-■-□	Three or more sixes	36	35 if first 6 lines filled
■-■-▲-□-□	Two and Two	Sum of points	
■-■-■-▲-□	Three of a kind	Sum of points	
■-■-■-■-▲	Four of a kind	Sum of points	
■-■-□-□-□	Full House	Sum of points	25
1-2-3-4-5	Small Straight	Sum of points	30
2-3-4-5-6	Large Straight	Sum of points	40
■-■-■-■-■	Five of a kind	Sum of points	50
■,□,◘,●,▲ are conditional notation of different points			

Each turn consists of 3 rolls of the dice. At the start of each turn, the player rolls all the dice. Prior to each subsequent roll, the player may choose to keep or release any combination of dice. After the third roll, the player must score the set of dice in one of the scoring boxes. If the player completes a combination before the third roll, he may take his score without rolling again. If the player gets a resulting combination that already filled in the table, he gets nothing and waits until the next turn. The game does not end until somebody fills all 13 cells of the table.

Let us briefly demonstrate a tactics of the game. Assume that in the very beginning of the game you cast the following combination:

It seems to you that it is not a bad idea to cast again dice with "one", "three" and "four".

Assume that after casting these dice, you get

Obviously, you will try casting the last time dice "two" and "five". And …

You are so luck! You get the best possible combination with $6\times6+50=86$ points!

Though assume that it is not a beginning of the game and you have already filled combination poker on sixes. And imagine that you badly need Large Straight, having a situation similar to the previous one.

What you should do in this case? Of course, you should take one of sixes and try getting two. After first attempt you get, say,

But be persistent! Try to cast now die with "one" to get two. If you are lucky again, it will be at last

However, a chance is a chance: you cast the die with "one" and get "one" again,

or get a combination ▲-3-4-5-6, where ▲ is everything but "two"…

So, starting the game you should try to fill the most "difficult" cells of the table. You don't know what is less or more probable? Don't worry: those who prepared the table already put less probable combinations on the bottom!

And now, when you can play one of dice games, you are interested in the role of dice in creation of one of the most interesting branch of modern mathematics – the Probability Theory!

2. THE PROBABILITY THEORY ORIGINATING

Probability theory is nothing but
common sense reduced to calculation.
Pierre Simon de Laplace

2.1. First steps

In the basis of this section we took "The essay on the history of probability theory", written by one of outstanding modern statisticians Boris Gnedenko[28]. We illustrated some most interesting subjects keeping the historical track of origin source.

Scientists, dealing with science history, notice that first counting of possible outcomes with three dice was made in 960 by bishop named Wibold who ran a church in a tiny city Cambrai in the Northern France. He counted 56 different combinations. It is clear how he could do his calculations by systematic enumerating, though instead of methodical and dull explanations, let us construct the following set of outcomes:

1-1-1 1-1-2 1-1-3 1-1-4 1-1-5 1-1-6
1-2-2 1-2-3 1-2-4 1-2-5 1-2-6

[28] **Boris Vladimirovich Gnedenko** (1912-1995), one of the prominent specialists in the field of probability theory, mathematical statistics, stochastic processes and its applications. He was Kolmogorov's pupil. Member of Ukrainian Academy of Sciences, Chair of the Probability Theory Department of the Lomonosov Moscow State University.

1-3-3 1-3-4 1-3-5 1-3-6
1-4-4 1-4-5 1-4-6
1-5-5 1-5-6
1-6-6

2-2-2 2-2-3 2-2-4 2-2-5 2-2-6
2-3-3 2-3-4 2-3-5 2-3-6
2-4-4 2-4-5 2-4-6
2-5-5 2-5-6
2-6-6

3-3-3 3-3-4 3-3-5 3-3-6
3-4-4 3-4-5 3-4-6
3-5-5 3-5-6
3-6-6

4-4-4 4-4-5 4-4-6
4-5-5 4-5-6
4-6-6

5-5-5 5-5-6
5-6-6

6-6-6

Construction of such a set could be easily explained: one takes first fixed pair, say "one & one" and change "the tail" from "one" to "six". Then one chooses the next ordered two dice – "one & two" and continue "attaching the tail".

Probably, something like this has been done by the bishop, so let's call it "Wibold's table". Note that "Wibold's table" possesses an interesting "triangle ordering" structure: on the top it has six threes and then six lines with decreasing numbers of sixes. The next triangle has five threes at the upper line and so on. It is easy to calculate the total number of threes in the "Wibold's table":

$$N = \frac{6+1}{2} \times 6 + \frac{5+1}{2} \times 5 + \frac{4+1}{2} \times 4 + \frac{3+1}{2} \times 3 + \frac{2+1}{2} \times 2 + 1 = 56.$$

It is easy to notice that the bishop counted not ***all possible*** combinations but only combinations with different set of points on the upper sides of dice. However, his problem he solved correctly.

By the way, it is possible to make "imaginary" experiment with a die that has 5 sides (notice that there is no such a polyhedral in the nature!) Actually, you can "construct" such a "pentahedral" in standard Excel using five cells, each of which has a generator of random numbers from 1 to 5. In this case analogous "Wibold's table" will have the form:

1-1-1 1-1-2 1-1-3 1-1-4 1-1-5
1-2-2 1-2-3 1-2-4 1-2-5
1-3-3 1-3-4 1-3-5
1-4-4 1-4-5
1-5-5

2-2-2 2-2-3 2-2-4 2-2-5
2-3-3 2-3-4 2-3-5
2-4-4 2-4-5
2-5-5

3-3-3 3-3-4 3-3-5
3-4-4 3-4-5
3-5-5

4-4-4 4-4-5
4-5-5

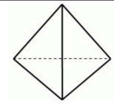
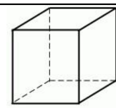
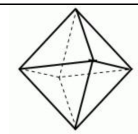
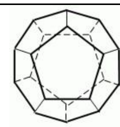
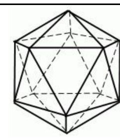
5-5-5

For this case the number of possible Для этого случая число различающихся комбинаций равно

$$N = \frac{5+1}{2}\times 5 + \frac{4+1}{2}\times 4 + \frac{3+1}{2}\times 3 + \frac{2+1}{2}\times 2 + 1 = 35.$$

Of course, one can increase dimension of a "hypothetical" die with no limit if you use computer with random number generator. However, let us use an opportunity to mention that there are only five symmetrical (or regular) polyhedrons that also called "Platonic[29] solids":

[29] These solids are called after Plato who first gave a detailed description of all of them. **Plato** (428-348 BC), Greek philosopher, Socrates' pupil and Aristotle's mentor. His actual name was Aristocles, and Plato was his nickname meaning "broad-shouldered".

Tetrahedron	4 sides	
Hexahedron, or Cube	6 sides	
Octahedron	8 sides	
Dodecahedron	12 sides	
Icosahedron	20 sides	

However, let's return back to Wibold. It is possible to assume that he was interested not only in different sets of points but also in frequencies of their appearance. And here he even never stood close to the solution… Probably it happened because he observes three dice already on the table. But if he would make dice casting one-by-one he could guess about something…

Consider the process of getting some particular outcome: for instance, there is only one possibility to have three "sixes": each of dice has to show "six". (Same with any "triplet".) However, outcome "pair of sixes" and "one" can be obtained by three ways:

1-6-6, or 6-1-6, or 6-6-1. With three different points there are even more possibilities. For instance, let we observe "four", "five" and "six". There are 3! = 6 possible outcomes:

- first die – 4, second die – 5, third die – 6;
- first die – 5, second die – 4, third die – 6;
- first die – 4, second die – 6, third die – 5;

- first die – 5, second die – 6, third die – 4;
- first die – 6, second die – 4, third die – 5;
- first die – 6, second die – 5, third die – 4;

We will not consider other situations: it is clear that chances of appearance of Wibold's virtues are quite different! For instance, the virtue, which corresponds to the set 6-6-6 is met rarer than the virtue 4-5-6!!

After Wibold, there were multiple efforts to count the number of different outcomes with casting three dice. No success!

Almost half of millennium later, such serious mathematicians like Girolamo Cardano[30] and Niccolo Tartaglia[31] submerged into this problem.

In a manuscript "*De Ludo alea*" dated 1526 and dedicated to dice gambling (the book was published only in 1563), Cardano solved a number of interesting problems concerning chances of various dice casting outcomes. Not everything was strong and correct, though he presented many fresh ideas. By the way, Cardano exactly defined the total number of different outcomes of three dice casting 216, though his method was not to clear and straightforward.

Cardano did not come to a concept of probability, though on intuitive level he understood even the Law of Large Numbers though on very qualitative level: "Long series of castings has small deviation, though it happens in a single game.

And at last genius of Galileo Galilei[32] came on the scene. In his work "Outcomes in dice gambling" that was published only 70 years after author's death, he analyzed three dice casting. He solved the problem with genial simplicity: if the first die might produce 6 outcomes, the second on

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[30] **Girolamo Cardano** (1501-1576), outstanding Italian mathematician and physicist.
[31] **Niccolo Fontana Tartaglia** (1500-1557), Italian mathematician and engineer. Published many books, including the first Italian translations of Archimedes and Euclid.
[32] **Galileo Galilei** (1564-1642), Italian physicist, mathematician, astronomer, and philosopher. For his achievements, he is often referred as "Father of modern astronomy", the "Father of modern physics", and even the "Father of science". ***For more details see Chapter "Pantheon" in Book 1***.

does the same as well as the third one, so the total number of outcomes equals $6^3=216$!

Galileo gave the methodology of counting number of outcomes for any fixed sum of points. However even Galileo did not formulate the concept of probability...

A real beginning of probability theory formation began later in a correspondence between two outstanding mathematicians – Pierre Fermat[33] and Blaise Pascal. And history around this correspondence is really intriguing!

2.2. Chevalier de Mere

The most important questions of life are, for the most part, really only problems of probability.
Pierre Simon de Laplace

Chevalier de Mere was a man of Paris high society. He was a bit writer, a bit mathematician and most of all… a gambler! He would leave this world as imperceptible as he entered it, though by fate chances his name was tied to the birth of one of the most important branch of modern mathematics – Probability Theory. As a matter of fact, de Mere (1610-1684) was inveterate and very hazardous dice gambler. As a man who was not stranger in mathematics, he tried to find rules, following which he could win for sure. So he formulated several problems, which he could not solve himself. And now we came very close to the beginning of the story.

It occurred that the chevalier was a closed friend of Blaise Pascal whom he asked to solve his problems.

The first Mere's problem.

This problem concerned a new game invented by him: he suggested casting a die four times in a row and betted that at least once

[33] **Pierre de Fermat** (1601-1665), French lawyer at the Parliament of Toulouse, France, who was a mathematician made a great input a lot in modern mathematics.

"six" would be observed. Multiple trials showed that chances of both sides were about equal, though de Mere asked his friend to make strong calculations.

The second Mere's problem was about fair haring the stake in the case when the game has been interrupted. Pascal was interested by both problems, since they seemed to him quite new. In 1654 Pascal wrote about the problems to Fermat whom he did not know in person. The problem of fair sharing the stake has been solved by them almost simultaneously.

That correspondence itself made Pascal and Fermat friends. Pascal wrote in his letter from Paris to Toulouse where Fermat lived: "Since this moment I would like to open you my soul because I am so glad that out thoughts were met. I see that the truth is the same in Paris and in Toulouse".

The year when that correspondence began might be considered as a moment of the probability theory birth. It is interesting a remark by Pascal from one of his letters "… in the World chances are ruling and, at the same time, there exists order and law, which are formed from these chances by the laws of randomness".

Let us come back to the Mere's problems.

In the first case arguments are relatively simple (though take into account that Pascal and Fermat did it first time!) There is one chance of six to observe "six", i.e. chance not to get "six" equals 5/6. After two castings we have 36 possible outcomes, which are presented in the figure below. It is obvious that 5×5=25 outcomes of total 36 possibilities do not contain "six", and 36 – 25 = 11 contain it.

points on the second die

points of the first die

	1	2	3	4	5	6
1	1;1	1;2	1;3	1;4	1;5	1;6
2	2;1	2;2	2;3	2;4	2;5	2;6
3	3;1	3;2	3;3	3;4	3;5	3;6
4	4;1	4;2	4;3	4;4	4;5	4;6
5	5;1	5;2	5;3	5;4	5;5	5;6
6	6;1	6;2	6;3	6;4	6;5	6;6

In other words, if each outcome is equally possible (i.e., dice are "fair" and – that is even more important – a gambler is not a crook) then chance to get no "six" equals 25/36.

Continuing the arguments, one finds that after three castings $5\times5\times5=125$ outcomes of total number of possible outcomes $6\times6\times6=216$ do not contain a "six", i.e. a chance to get at least one "six" equals $(216 - 125):216=91:216 \approx 0.422$. After four castings, the total number of possible outcomes equals $6\times6\times6\times6 = 1296$, and the number of outcomes with no "six" equals $5\times5\times5\times5 = 625$, i.e. chances to get at least one "six" is equal to $(1296 - 625):1296 = 671:1296 \approx 0.518$.

As you can see, experienced gambler chevalier de Mere was very close to a "fair game" though his intuition hinted him that his bet is a little bit more profitable on the average.

1;1	1;2	1;3	1;4	1;5	1;6
2;1	2;2	2;3	2;4	2;5	2;6
3;1	3;2	3;3	3;4	3;5	3;6
4;1	4;2	4;3	4;4	4;5	4;6
5;1	5;2	5;3	5;4	5;5	5;6
6;1	6;2	6;3	6;4	6;5	6;6

By the way, a table with outcomes for two dice game, which was presented above, is very convenient for some simple calculations. For instance, what is more probable: to get 7 point or to get a double? Do you try to make calculations? Don't do it, just take a look at the table: all outcomes with total 7 points locate on the NW-SE diagonal and all doubles locates on the NE-SW diagonal, so the both outcomes are equally probable: there are six chances of 36 possible ones.

You could find simple solutions of some other interesting problems, you might formulate for yourself.

<u>The second de Mere's problem</u>.

The next problem formulated by de Mere occurred much more difficult though verbal formulation of it was simple. Indeed, how to share a stake if both gamblers have different number of wins? It is clear for equal numbers of wins, they should share the stake in equal. However if they play until 10 wins and the first gambler won 3 games and the second one only 2? It seems that they are "almost equal". Though if they have 9 and 8 wins, are they "more equal" then in the previous case? And if the first one won 9 games and another one has 0 wins, does it mean that the first gambler should take the total stake? Or maybe, "almost all stake"? And if so, what does it mean "almost"?

Actually that problem had a long history: it is known that Luca Pacioli[34] in 1487 in his book "*Summa de arithmetica, geometrica, proportioni et proportionalita*" considered problems of the type. His solution was mistaken though he suggested a productive methodology.

Later Cardano in his book "*De Ludo alea*", which we already mention, came very close to the concept of a "fair game" (or "a game with zero sum" in modern terminology). In his next book "*Practica arithmetice et mensurandi singularis*" he criticized Pacioli's solution though gave his own solution incorrect in a general case.

In parallel, the same problem was considered in 1556 by Niccolo Tartaglia in his "*General tractate on measure and number*" where he also criticized Pacioli and, as Cardano did, he had some mistakes in his own solution.

Only Blaise Pascal had found an elegant solution of the problem of fair stake sharing. His arguments were as follows. Assume that two gamblers put 10 golden ducats on the table. They decided to play until three wins and a winner should keep total stake. However, the game has been interrupted when the first gambler has two wins and the second one has only one. What sharing of the total stake between the two should be considered as "fair"? (We don't take into account that concept of "fairness" depends on whose the decision is: wolf's or sheep's?)

If gamblers would continue the game then at the next step there are two equally probable outcomes: (a) the first gambler wins and (b) the second gambler wins. In case "a" the first gambler has won the game and should receive 20 ducats. If the second gambler has won, then both of them are in equal situation and have to share the total stake equally, i.e. each of them gets 10 ducats. So, the first gambler has 50 chances of 100 to get 20 ducats after the first casting and even in the case of mishap has 50 chances of 100 to get 10 ducats after the second casting. Thus, the first gambler has to get

[34] **Fra Luca Bartolomeo de Pacioli** (1445-1517), Italian mathematician and collaborator with Leonardo da Vinci. ***For more details see Chapter "Pantheon" in Book 3.***

$20 \times \frac{1}{2} + 10 \times \frac{1}{2} = 15$ ducats, and the second gambler has 5 ducats.

One more outstanding scientist of that time also took part in solution of the fair sharing stake problem. It was Christiaan Huygens[35] who heard about the problem when he came to Paris. He was interested in the problem though neither Pascal, nor Fermat published their results, Huygens was forced to solve the problem from the beginning. When Blaise Pascal was read Huigens work, he advices him to write a book on the subject what apparently would be the first book on probability theory. That Pascal's advice inspired Huigens and he accepted proposal of his teacher van Schooten[36] to participate in a book titled "Mathematical etudes". In 1657 this book was published with a special appendix "*De ratiociniis in ludo aleae*" ("*Calculations in hazard games*") submitted by Huigens. Since Huigens did not know Latin enough to translate his manuscript, it was done by his teacher, though the author expressed some dissatisfaction about the Schooten's translation. Nevertheless, the first work on probability theory was published!

It is interesting to notice that in 1687 it was published Spinoza's[37] work "Notes about mathematical probability" – ten years after the great philosopher death. So, even great philosophers were not strangers to this intriguing problem.

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35 **Christiaan Huygens** (1629-1695), Dutch mathematician, astronomer and physicist. His name is commonly associated with the scientific revolution. ***For more details see Chapter "Pantheon".***

36 **Franciscus** (**Franz**) **van Schooten** (1615-1660), Dutch mathematician.

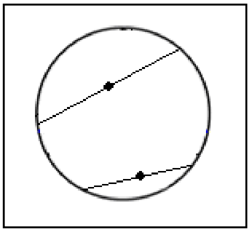
37 **Baruch de Spinoza**, or in Latin **Benedictus de Spinoza** (1632-1677), outstanding Dutch philosopher of Portuguese Jewish origin.

3. Paradoxes of Probability

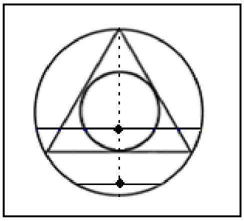
3.1. Bertrand's Paradox

For some the Bertrand's paradox
Is worse than the Pandora's Box[38]?
(Unknown author)

The problem called as "Bertrand's[39] Paradox" can be formulated in the following way: one randomly chooses a chord on a circle. The question is: what is the probability that this random chord is larder than a side of right triangle inscribed into this circle?



Let us choose in random a point within the circle. Stop! What does it mean "in random"? Okay, let it be a point from uniform distribution defined in the area of the circle. Then assume that this point is the middle of the chord. Then construct the second chord in the same manner.



Let us locate these chords in such a way that their centers lay on the diameter that originates in the top vertex of the right triangle.

It is easy to note that the chord is larger than a right triangle side if a random point locates within a circle inscribed in the triangle; otherwise the chord is smaller than a triangle side. Thus, the probability that a chord, constructed in such a way, will be larger than a triangle side is equal to the ratio of the area of the small circle to the area of the larger circle. A diameter of the larger circle is twice as much as a diameter of the small circle, as it can be easily obtained with the help of simple geometrical constructions.

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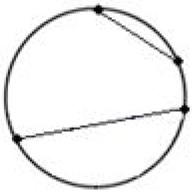
[38] In Greek mythology, **Pandora** ("all-gifted", or "she-who-sends-up-gifts") was the first woman, created by Zeus as part of the punishment of mankind for Prometheus' theft of the secret of fire. According to the myth, Pandora opened a sealed jar (pithos) releasing all the evils of mankind.

[39] **Joseph Louis François Bertrand** (1822-1900), French mathematician who worked in the fields of number theory, differential geometry and probability theory.



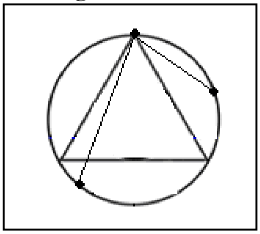
. So, since the ratio of circles areas equals to the ratio of squares of their diameters, one gets that the probability to get a random chord larger than a triangle side is equal to ¼.

So, where is a paradox? It was so simple and understandable geometrical construction…

Well… However, a "random chord" might be constructed in other way: take two random points on the circle and connect them by a line!

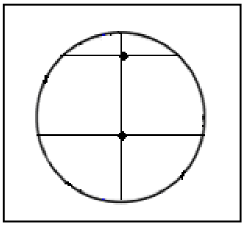
Of course, in this case we also have to define: what does it mean a "random point"? Let us assume that each point has uniform distribution on the circle. Again let us construct two chords.

It is clear that the first point can be chosen arbitrarily on the circle and only the second should be chosen in random. Assume that we choose the first point for a chord again at the vertex of a triangle. Then both chords constructed above could be presented in the next figure.



From the figure, one can see that if a random point locates on the circle part lying under the triangle base side, the chord will be larger than a triangle side. Thus, the probability that a random chord is larger than a triangle side is equal 1/3... 1/3? But we just a minute ago find that this probability is ¼ …

What has happened?!



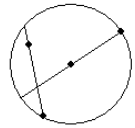
Okay, let's try another way. Take a diameter (for convenience again starting at the triangle vertex). Choose a random point (again uniformly distributed, of course) on this diameter and construct a random chord that is perpendicular to the diameter.

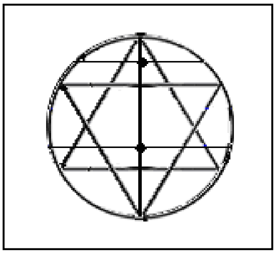
For explanation, let us construct a hexagram[40] constructed with the help of a right

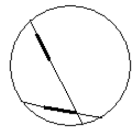
[40] This figure is widely known as The Shield of David (or Magen David in Hebrew), or The Star of David, who was, in accordance with the Bible, the King of Israel about 1000 B.C.

triangle and an upside down right triangle.

From the figure, one can see that the diameter is divided into three parts in proportion 1:2:1. It is clear that a random chord is larger than a triangle side if a point locates in the middle portion of the diameter. Thus, the probability that a random chord is larger than a triangle side equals ½ !







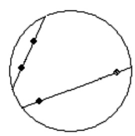
So… What is it? “Mathematical hallucinations”? Black magic?..

Of course, you have already guessed that there is no a paradox at all! In each of this case the uniform distribution was defined on *different* domains! In all three cases we met with three different problems, and each of them was solved – and solved correctly – in accordance with its formulation!

Of course, one can suggest other ways of construction “random chords”, for instance:

(1) One chooses in random two independent points and use them for construction of a “random chord”:

(2) One chooses the first point at the circle and the second point in the area of the circle.



(3) One throws within the circle randomly “Buffon’s needles” and continues lines to get a random chord.

In conclusion, notice that Josef Bertrand liked to invent paradoxes. For instance, there is another well known Bertrand’s Paradox in economics.

This paradox describes a situation in which two players reaching a state of equilibrium in economic competition find themselves with no profits. Suppose two firms, A and B, sell an identical product. Naturally, customers choose the product solely on the basis of price. If one of the companies will set a higher price than the other, it would yield the entire market to the rival. If both set the same price, the companies will share both the market and profits. On the other hand, if either firm were to lower its price, even a little, it would gain the whole market and substantially larger profits. Since both firms know this, they will each try to undercut their competitor until the product is selling at zero economic profit.

The economical Bertrand's paradox rarely appears in practice because real products are almost always differentiated in some way other than price.

3.2. Birthdays coincidence

Probability theory is unusually rich of paradoxes, i.e. true statements that contradict our common sense. It is difficult to believe in the result even when its correctness has been strongly proved.

One of such examples is the so-called Birthday Coincidence Paradox. Let us choose, say, 23 people in random. What do you think about chances that at least two of them were born on the same day? (Of course, we exclude possibility that in that group there are twins ☺.) Just for convenience, consider non-bissextile year and forget non-uniformity of births during a year (it is known that most conceptions happen in spring). Our intuition hints us that chance should by small: only 23 people and 365 days! However, there are even more than 50% that such coincidence has place! More precisely, the probability equals ≈ 0.507.

Indeed, the probability that the birthday of the second chosen person differs from the birthday of a first randomly chosen equals 364/365 (since only in one case of 365 the birthdays coincide). The probability that the third person's birthday does not coincide with the two ones is 363/365. Indeed, there are only two chances that the third birthday coincides with one of two previous. For the fourth birthday the probability that it differs from three previous equals 362/365, and so on.

Finally, the probability that among 23 randomly chosen persons there is no such ones that their birthdays coincide is

$$1\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\frac{362}{365}\cdot\ldots\cdot\frac{342}{365}\cdot\frac{343}{365}\approx 0.493,$$

or it is equivalent to the statement that the probability that at last two birthdays come to the same day equals ≈ 0.507, i.e. slightly larger ½.

For a group of 30 randomly chosen people the analogous probability is 70%, for a group of 40 it reaches 90%, and for a group of 50 the probability equals 97%, and for a group of 60 the probability equals practically 100%!

It is interesting to observe how the probability of mismatching of birthdays depends on the size of a group.

Size of a group	**Probability of mismatching**	**Probability of matching**
1	1	0
2	0.998	0.002
3	0.992	0.008
4	0.984	0.016
5	0.973	0.027
6	0.960	0.040
7	0.944	0.056
8	0.926	0.074
9	0.906	0.094
10	0.884	0.116
11	0.859	0.141
12	0.833	0.167
13	0.806	0.194
14	0.777	0.223
15	0.748	0.252
16	0.717	0.283
17	0.685	0.315
18	0.654	0.346
19	0.621	0.379
20	0.589	0.411
21	0.557	0.443
22	0.525	0.475
23	0.493	0.507

Notice that naturally the sum of matching and mismatching for any fixed group equals 1. From the table one can see that for 22 people the probability of matching is still less than 0.5, however for 23 people it is already slightly higher.

By the way, the considered probability is growing very fast.

Size of a group	**Probability of mismatching**	**Probability of matching**
10	0.884	0.116
20	0.589	0.411
30	0.294	0.706
40	0.109	0.891
50	0.030	0.970
60	0.006	0.994
70	0.001	0.999
...	...	...

Thus, everything is calculated accurately, however, it is hardly easy to believe in these results, is not it?

For numerical illustration, let us consider birthdays of Presidents of the United States.

Number	***Name***	***Birthday***	***Day of death***
1	Washington	22 FEB 1732	14 DEC 1799
2	J. Adams	30 NOV 1735	4 JUL 1826***
3	Jefferson	13 APR 1743	4 JUL 1826***
4	Madison	16 MAR 1751	28 JUN 1836
5	Monroe	28 APR 1758	4 JUL 1831***
6	J. Q. Adams	11 JUN 1767	23 FEB 1848
7	Jackson	15 MAR 1767	8 JUN 1845
8	Van Buren	5 DEC 1782	24 JUL1862
9	W. H. Harrison	9 FEB 1773	4 APR 1841
10	Tyler	29 MAR 1790	18 JAN 1862
11	Polk	**2 NOV 1795**	15 JUN 1849
12	Taylor	24 NOV 1784	9 JUL 1850
13	Fillmore	7 JAN1800	8 MAR 1874*
14	Pierce	23 NOV1804	8 OCT 1869
15	Buchanan	23 APR 1791	1 JUN 1868
16	Lincoln	12 FEB 1809	15 APR1865
17	A. Johnson	29 DEC 1808	31 JUL 1875
18	Grant	27 APR 1822	23 JUN 1885
19	Hayes	4 OCT1822	17 JAN 1893

20	Garfield	19 NOV 1831	19 SEP 1881
21	Arthur	5 OCT 1829	18 NOV 1886
22	Cleveland	18 MAR 1837	24 JUN 1908
23	B. Harrison	20 AUG 1833	13 MAR 1901
24	Cleveland	18 MAR 1837	24 JUN 1908
25	McKinley	29 JAN 1843	14 SEP 1901
26	T. Roosevelt	27 OCT 1858	6 JAN 1919
27	Taft	15 SEP1857	8 MAR 1930*
28	Wilson	28 DEC 1856	3FEB 1924
29	Harding	**2 NOV 1865**	2 AUG 1923
30	Coolidge	4 JUL 1872	5 JAN 1933
31	Hoover	10 AUG 1874	20 OCT 1964
32	F. D. Roosevelt	30 JAN 1882	12 APR 1945
33	Truman	8 MAY 1884	26 DEC 1972**
34	Eisenhower	14 OCT 1890	28 MAR 1969
35	Kennedy	29 MAY 1917	22 NOV 1963
36	L. B. Johnson	27 AUG 1908	22 JAN 1973
37	Nixon	9 JAN 1913	22 APR 1994
38	Ford	14 JUN 1913	26 DEC 2006**
39	Carter	1 OCT 1924	-
40	Reagan	6 FEB 1911	5 JUN 2004
41	G.H.W. Bush	12 JUN 1924	-
42	Clinton	19 AUG 1946	-
43	G. W. Bush	6 JUL1946	-
44	Obama	4 AUG 1961	-

From the table, one can see that on the 29th line of the list the first such pair appearc: President Polk and President Harding (both of them were born on November 2nd).

Even more affective picture appears with days of death. Already among the first five Presidents we observe three coincided deaths (Jefferson, Adams and Monroe). All of them died on July Fourth, the Independence Day…

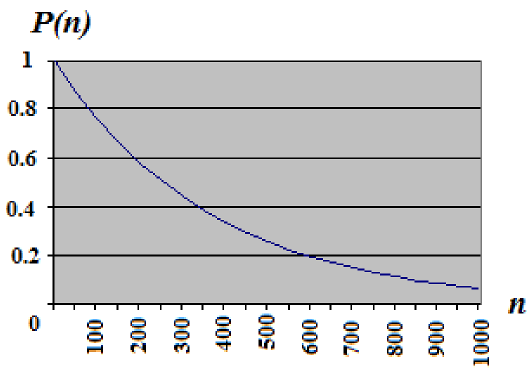
Now let us "slightly" change the question: what size has to be a group that somebody has the birthday exactly coinciding with yours with probability "fifty-fifty"? It is intuitively clear that it will be lower that the probability of coinciding of any two birthdays within a group. However, how much a new group will be larger? Fourty? Fifty? Maybe, 65?

Almost sure, your intuition in this case will also deceive you… We don't make a boring enough (though trivial) calculations. The result is that the group has to include … 253 people! Moreover, don't think that if there will be group of 365 people the probability will be close to 1. The probability of matching is only just about 37%...

In general, for an arbitrary large group of people there a tiny probability that there is no single person with the same birthday as yours. Why? Because this probability for any n is calculated by formula:

$$P(n) = \left(1 - \frac{1}{n}\right)^n,$$

And this value, of course, goes to zero, however always strictly positive!



Graph of the probability of mismatching namely with your birthdaydepending on the size of a group.

3.3. Is it possible to win in lottery regularly?

To win in a lottery is practically impossible. However, your chances slightly increase if you buy a ticket.

(Unknown author)

A fair game, or a "game with zero sum", roughly speaking, guarantees any gambler that after long-long series of games "on the average" will keep his original money. More exactly speaking, the mathematical expectation of loss (or profit) equals zero, i.e.

(loss $\times$ (probability to lose) = (profit) $\times$ (probability to win).

It is said that only stupid plays hazard games with the state... Mathematically speaking, it is correct (forgive me those who, nevertheless, play such games!). Indeed. Even if money does not "stick" to some dirty hands, organizers should cover their expenses and a good piece of profit is return to the state in form of taxes.

So, there is no a "zero-sum game" in any case! And at the same time the question arises: "Is it possible to win lottery in a regular way?" Why we ask this question if it was just recently explained that such a game is impossible?

So let us consider a situation when such "regular" win is possible. (Of course, "regular" in sense of "on the average".)

Let us consider a popular in 1970-s in the Soviet Union sport-lottery. You buy a ticket with a 7 by 7 table where in each cell there is a small picture of some kind of sportive competition: football, basketball, skiing, skating, and boxing, and so on. You look at these small pictures and imagine your beloved games and athletic competitions… Your hand unconsciously marks the "best" of them… Such is a human psychology!

Assume that we have to choose two cells of six where there are the following sports: football, marathon, hockey, shooting, basketball and biathlon. Most people would prefer to mark couple cells of the type: "football-hockey", "football-basketball" and "hockey-basketball"… Indeed, probably not everybody knows what biathlon means! So, among 15 possible combinations of two kinds of sport, only three might be most frequently met.

Imagine that 1500 people play such a lottery of type "Guess-two-of-six". Each buys a ticket for $1. If nobody has won, the lottery repeats again until somebody's success. Since there are 15 different outcomes a player can win with probability 1/15. Let 1499 players chose some "reasonable" sports (say, choosing at least one "beloved" sportive game). At the same time somebody has chosen one of three possible pairs of "unpopular" sports: "marathon-shooting", "marathon-biathlon" or "shooting-biathlon". For simplicity, consider that all 1499 players (except that one who chose "unreasonable ticket") are divided into 13 almost equal size groups, i.e. by about 115 people each. It means that about 100 people have tickets with the same choice of sports.

The win probability is equal for any outcome, though the expected profit is different! A player, who has chosen"reasonable" sports, should share his profit with 115 other people who have chosen the same sports, so he gets on the average

$$(-\$1)\cdot\frac{14}{15}+\left(\frac{\$1500}{115}\right)\cdot\frac{1}{15}\approx-\$0.13\,,$$

i.e. loose!

Now take a player, who chose one of "unreasonable" pair of sports. If he has won, he shares his profit with nobody! So, for him the analogous calculation gives:

$$(-\$1)\cdot\frac{14}{15}+(\$1500)\cdot\frac{1}{15}\approx\$99\ !$$

It is difficult to believe, is not it? Check yourself.

I did such an experiment personally. Lecturing probability theory at the UCSD (the University of California San Diego), I made an analogous experiment. I made a lottery ticket with 6 portraits: Washington, Lincoln, Jefferson, Kennedy, Chekhov (a Russian playwright) and Balzac (a French writer). I supplied students with small pieces of colored paper that imitated "money". After students had marked two portraits each, I gave numbers to each portrait and began to cast two dice. Those who had a "lucky ticket" after each casting got a stake, sharing it between four – six others. Of course, I chose Chekhov and Balzac. But … I won only half as much as I expected! Why? Because a smart Chinese guy chose the same portraits as I! He explained afterwards that if a professor asked an unexpected question, it means that there was some trap. Anyway, we both won a lot of pieces of colored paper!

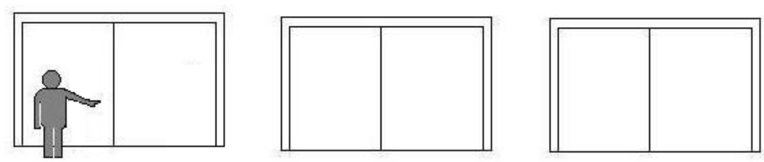
3.4. "Goatematics"

Even statisticians are trying to find a scapegoat!
(Unknown author)

In 1990 Marilyn vos Savant[41] published in Sunday column "Ask Marilyn" of American journal "Parade Magazine" solution of a problem, which later became known as the "Monty Hall paradox". It was a veridical paradox in the sense that the suggested solution was counterintuitive. When the problem with the correct solution (without proofs) appeared in the magazine, about 10,000 readers (including hundreds university professors!) wrote to the magazine claiming the published solution was wrong.

What was the problem? Let's give it in the form close to original.

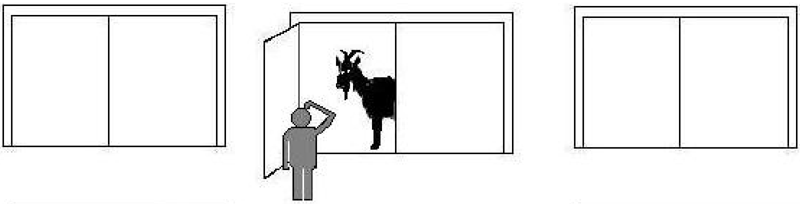
Suppose you are on a game show. There are three closed doors: behind one of them there is a car, and behind two other, goats. The host suggests you to choose a door. If you guess, you win a car. You pick a door, say, the left one.



Definitely, you wish to get a car (though… who knows?). Your choice is random, so there is one chance of three of your success, i.e. probability of choose a gate, behind whish there is a car, is 1/3.

After you made your choice, the host opens one of the closed doors and shows you a goat (there is always such a possibility, because there are two goats).

[41] **Marilyn vos Savant** (born 1946), an American magazine columnist, who in 1986 began the Sunday column "Ask Marilyn" in American "Parade magazine", in which she answered readers' questions on a variety of subjects.



You see a goat in, say, the middle gate. At this moment the host asks you: "Do you like to change your first choice and choose another close gate?"

It seems intuitively that such a suggestion has no sense at all!

Though Marilyn suggested solution: if you switch, the probability to get car will be 2/3!

Impossible! It was 1/3 and only because the host shows a goat the probability jumps from 1/3 to 2/3?! It absolutely contradicts to a common sense…

So, you understand that the magazine was overfilled with wrathful letters (we quote them from Marilyn's website though delete real names).

An intelligent professor wrote very gentle:

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?

Professor, University of Florida

Sometimes it was even politically incorrect:

Maybe women look at math problems differently than men.

Gentlemen, Oregon

Sometimes it was unacceptably rude:

You are the goat!

Professor, Western State College

Sometimes… Though it is not about Marilyn's solution, rather about American educational system:

You made a mistake, but look at the positive side. If all those Ph.D.'s were wrong, the country would be in some very serious trouble.

Ph.D., U.S. Army Research Institute

Do not think that everybody was so "educated" and "self-confident" – there were some other responses:

You could hear the kids gasp one at a time, "Oh my gosh. She was right!"
Teacher, Magnolia School, California

I must admit I doubted you until my fifth grade math class proved you right. All I can say is WOW!
Teacher, Westside Elementary River Falls, Wisconsin

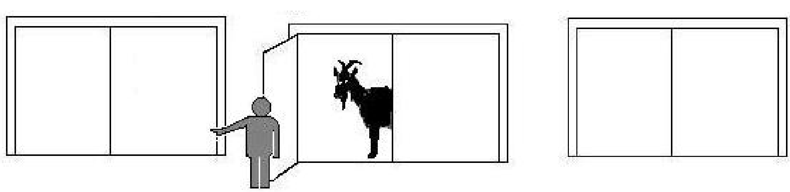
There were even good jokes:

One of my students wanted to know whether they were milk goats or stinky old bucks. Presumably that would redefine what a favorable outcome was!
Teacher, Bayview Christian School, Norfolk, Virginia

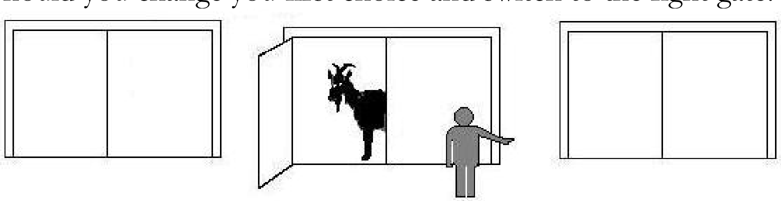
Interesting... Only children – who are not overloaded with knowledge and prejudice – solved the problem correctly! Not in vain, there is a proverb: The truth is said by a child's lips…

However, let's return to the Marilyn's problem.

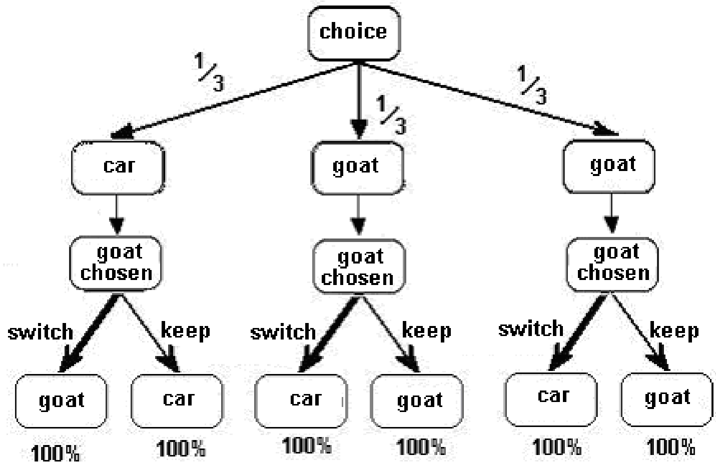
So, you have chosen the left gate. You are shown the middle gate with a goat.



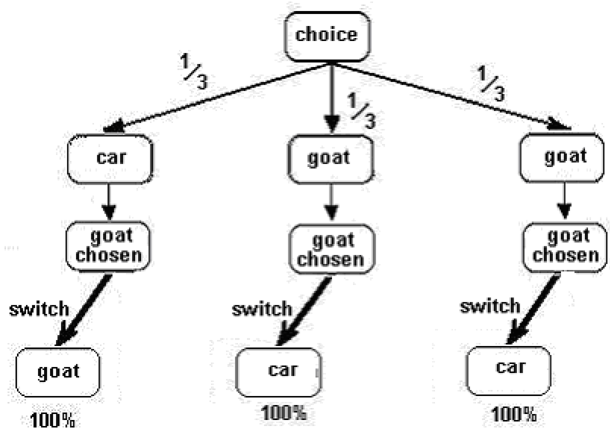
Should you change you first choice and switch to the right gate?



Now, we hope that you are intrigued by the correct answer of the problem. For this we choose a simple and demonstrative way: we will use the so-called "tree of outcomes" that sometimes is also called as "tree of solutions".



On the last level of the "tree" bold lines show outcomes when you switch to another closed gate and thin lines show outcomes if you keep your first decision. To make this figure even more "transparent, let us redraw it.



It is clear that if you switch the gate you have two chances of three to win a car.

As a matter of fact, smart professors should not rely on their intuition and, drinking a cup o coffee, criticize Marilyn: they should write a simple formula, which almost on verbal level sound like the following:

$$\Pr\{\textit{to win a car}\} = \Pr\{\textit{a car is chosen}\}\Pr\{\textit{a car is won / switch the gate}\} + \Pr\{\textit{a goat is chosen}\}\Pr\{\textit{a car is won / switch the gate}\}.$$

In the probability theory by vertical line one denotes expression "under condition that". So, what we have in our case?

If the chosen gate is rejected (switching) after a goat has been shown:

$\Pr\{\textit{a car is chosen}\} = 1/3$
$\Pr\{\textit{a car is won / switch}\} = 0$
$\Pr\{\textit{a goat is chosen}\} = 2/3$
$\Pr\{\textit{a car is won / switch}\} =1$,

From where we have $(1/3)\cdot 0 + (2/3)\cdot 1 = 2/3$.

3.5. Formula of complete probability

Complete probability is better
than incomplete chance.
(Unknown author)

Probability calculus began with simple things: addition and multiplication. In the beginning, people did not calculate probabilities, they rather calculated numbers of chances of one or another outcome. Naturally, there is only a small step to come to a frequency of appearance of a random event: one should divide the number of some event chances on the total number of possible outcomes. Nevertheless, though Girolamo Cardano, Blaise Pascal, Pierre Fermat and Jacob Bernoulli were so close to the formulation of the theorem of probability summation, only Thomas Bayes had formulated the theorem. This theorem states that occurrence of any event of a set several independent events equals to the sum of its probabilities.

Returning to a dice game, we can say that probability to occur "one" or "five" is equal to $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$ (i.e. it is two chances of six possible outcomes).

The work by Bayes, where the theorem had been proved, was published only after his death by his friend Richard Price[42].

Thomas Bayes
(1702-1761)

British mathematician, member of the Royal Society. He got home education and at 17 entered Edinburgh University. Afterwards, he assisted to his father who was a Presbyterian minister and soon became a minister himself. He discovered the theorem of probabilities summation. His name bears a well-known so-called Bayesian theorem (though actually he did not formulate it).

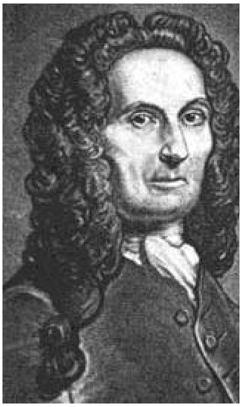
The theorem of probability multiplication was formulated in strong terms only by Abraham de Moivre in 1718 in his book "*The Doctrine of Chances*". There he introduced concept of event dependence: " ... two events are independent when occurrence of one of them did not influence on occurrence of another one". In the same book he gave the probability multiplication rule, accompanying it with a simple example. Assume that one has two sets of cards of the same suit from two to ace (i.e. 13 cards each set). Then probability to choose in random an ace from the first deck and an ace from another deck equal $\frac{1}{13}\times\frac{1}{13}=\frac{1}{169}$.

Further he defined a conditional probability, again supplying it with a simple example> two aces are chosen in random from the same

[42] **Richard Price** (1723-1791), Welsh moral and political philosopher.

deck. The first one is taken with probability $\frac{1}{13}$, and the second one with probability $\frac{1}{12}$, since the deck became smaller on one card. Thus, the probability those two first cards taken in random from the same deck are equal to $\frac{1}{13} \times \frac{1}{12} = \frac{1}{156}$.

He gave the following definition for dependent trials: "... probability of two independent events occurrence is equal to product of the probability of the first event occurrence and the probability of another one occurrence if the first one would have occurred. This rule could be extended on several events".



Abraham de Moivre
(1667- 1754)

French mathematician who had been working his entire life in Great Britain. Fellow of the Royal Society, foreign member of Paris and Berlin Academies of Sciences. He was on a shirt term with Sir Isaac Newton. He worked in the fields of the set theory, probability theory and complex numbers. In probability theory he had proved an important theorem now bearing his name – the Moivre-Laplace Theorem. In theory of complex numbers he gave rules of raising complex numbers to the higher power and extract roots from them (Moivre's formulae).

De Moivre gave a definition of probability of occurrence of two dependent events A and B in the form:

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B|A) = \Pr(B) \cdot \Pr(A|B)$$

where sign $\cap$ denotes in mathematical logic "and". At the same time, Bayes guessed how to find a conditional probability:

$$\Pr(B \mid A) = \frac{\Pr(AB)}{\Pr(A)} \text{ or } \Pr(A \mid B) = \frac{\Pr(AB)}{\Pr(B)}.$$

However, neither de Moivre, nor Bayes knew the formula of complete probability that is written in the form

$$\Pr\{A\} = \sum_{k=1}^{n} \Pr(A \mid B_k\} \cdot \Pr\{B_k\},$$

where $\Pr\{B_k\}$ is the probability of event B_k, $\Pr(A \mid B_k\}$ is the probability that event *A* occurs under condition that event B_k would have already realized. Notice that all events B_k form the so-called full set of events (i.e. all events that might influence on event *A* occurrence).

Actually, Pierre Laplace was the first who derived the Bayesian Formula in the form familiar to us in his work" *Théorie Analytique des Probabilités*" where he formulated his so-called "principle of causes probabilities". The content of this principle is like this.

Let some event *A* can occur with one of other of *n* events B_1, B_2,…, B_n, which Laplace called "causes". What is the probability that namely some cause B_i has realized if it is already known that event *A* has been occurred?

Laplace answered on this question approximately as follows: the probability of a cause of interest equals a ration whose numerator is the probability of the event *A* that sourced from this cause and denominator equals the sum of all analogous probabilities. This verbal definition leads to the formula that is called Bayesian:

$$\Pr(B_i \mid A) = \frac{\Pr(B_i) \cdot \Pr(A \mid B_i)}{\sum_{j=1}^{n} \Pr(B_j) \cdot \Pr(A \mid B_j)}.$$

However, after this short but probably a little bit boring excursion into a history of specific subject, let us as Frenchmen say, "retournons a nos moutons": continue the problem of switching gates for winning a car.

3.6. High "Goatematics"

> If you are deadly thirsty, probably, you will prefer to choose a she-goat than a car...
> ***(Unknown author)***

So, we understand what to do standing in front of three gates when a host shows you a goat…But what if there will be N gates with M goats behind them? Is it still reasonable to switch the chosen gate? We hope that armed with the Bayesian Formula discovered by Laplace, anybody will solve this problem correctly.

Nevertheless, let us consider the situation. With probability $p = \dfrac{N-M}{N}$ one chooses the gate with a car behind. What happened if you decided to switch the gate? You have a choice of $N – 2$ gates, excluding a gate of your choice and the open gate with a goat:

$$\Pr(\textbf{\textit{win a car}}) = \Pr(\textbf{\textit{a car is chosen}}) \times \Pr(\textbf{\textit{win a car / switch the gate}}) + \Pr(\textbf{\textit{a goat is chosen}}) \times \Pr(\textbf{\textit{win a car/ switch the gate}}).$$

Notice if you initially chose a gate with a goat behind, then all $N – M$ cars are among remaining $N – 2$ gates; if you initially chose a car, then behind remaining gates there are only $N – M – 1$ cars. Taking this into account we get:

$$\Pr(\textbf{\textit{win a car}}) = \frac{M}{N} \times \frac{N-M}{N-2} + \frac{N-M}{N} \times \frac{N-M-1}{N-2} =$$

$$= \frac{M(N-M) + (N-M)(N-M-1)}{N(N-2)} = \frac{(N-M)(N-M)(N-1)}{N(N-2)}.$$

Now it remains to check if the new probability is larger than original one:

$$\frac{(N-M)(N-M)(N-1)}{N(N-2)} > \frac{N-M}{N}.$$

After reduction to the common denominator, we have:

$$(N - M)(N - 1) > (N - M)(N - 2),$$

And this is correct, since N – 1is always larger than N – 2.

Speaking about "High Goatematics" it is interesting to consider couple of more complicated and rather interesting problems.

Assume that a host is your friend who wantc to help you to win. Behind the gates there are two goats – one black and another white. The host has no right to give you a direct hint though he informed you in advance that if you have to change a gate you chose initially, he will show you a black goat, and if you already have a right choice, he will show you a white goat, that is you have not to change the gate.

It seems that you got a fortune in your hands! However, before fell into euphoria, maybe it is reasonable to perform a simple analysis of opened opportunities:

1. With probability 1/3 you chose a gate behind which there is a car. A host, your secret friend, shows you a white goat and you don't switch – car is your!
2. With probability 1/3 you chose a gate behind which there is a white goat. Your friend shows you a black goat. You switch the gate and win!
3. With probability 1/3 you chose a gate behind which there is a black goat. By the rule, the host has to show you a goat but… There is only a white goat available! So, the host is forced to show you a white goat and you proudly reject suggestion to switch the gate! What is the result? You got a black goat…

Thus, the probability to win a car even with a friendly host is still 2/3. It seems that additional information gave you nothing…

Assume that you suspect that the host is your ill-wisher, not a friend!

Let us repeat analogous arguments in this case.

1. With probability 1/3 you chose a gate behind which there is a car. A host, your secret friend, shows you a white goat, hinting you that you should keep your choice. However, you don't trust him and switch the gate. In the result you get a goat (a black one)…

2. With probability 1/3 you chose a gate, behind which there is a white goat. By the rule, the host has to show you a goat but… There is only a black goat available! So, the host is forced to show you a black goat. Since you don't trust the host, you think: "He advises me to switch but he is a liar, so I am keeping the gate!" And you get a white goat…

3. With probability 1/3 you chose a gate behind which there is a black goat. Your friend is forced to show you a white goat that means a "don't-switch-signal". Though now you don't trust the host now, you switch and … win!

So, if the host is your friend and you trust him you still have probability to win a car equal to 2/3. If you don't trust the host who is actually is a friend of yours, you decrease the probability half as much!

Now consider a case when the host is your ill-wisher but says you that he is you friend, and you ingeniously trust him.

What happens in this case?

1. With probability 1/3 you chose a gate behind which there is a car. A host, your ill-wisher, shows you a black goat, deceiving you and forcing you to switch the gate. In the result you get a goat (a white one)…

2. With probability 1/3 you chose a gate, behind which there is a

white goat. By the rule, the host has to show you a black goat (the only one he has to show). So, the host is forced to show you a black goat. You change the gate, and you get a car!

3. With probability 1/3 you chose a gate behind which there is a

black goat. The host shows you a white goat that means a "don't-switch-signal". You keep the gate and… loose! You get a black goat.

Again your chance to get a car equals 1/3!

One can construct the following table:

Host indeed	**Your attitude to the host**	**Probability to win a car**
Your friend	You trust him	2/3
Your friend	You don't trust him	1/3
Your ill-wisher	You trust him	1/3
Your ill-wisher	You don't trust him	2/3

Of course, in cases with multiple gates all these arguments will be clumsier (not more difficult!), though the moral is still the same: trust your friends and don't trust your ill-wishers!

<u>For curious people</u>

Let us without detailed explanations consider a case with four gates, behind which there are three goats and a car. Let us to use the rule: always switch a chosen gate after a host shows you a goat. In the table below: G = goat, C = car.

	1^{st} gate	2^{nd} gate	3^{rd} gate	4^{th} gate
Prob. of choice	¼	¼	¼	¼
Behind	G	G	G	C
Shown	G	G	G	G
Decision	switch	switch	switch	switch
Prob. of win	½	½	½	0

Total probability to win a car is

$$\Pr\{win\ a\ car\} = \frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot 0=\frac{3}{8}$$

Notice that probability to choose a car in random is only ¼! So, if you switch the first gate you have chosen, your chances to win a car arise in 50%.

Now let us change conditions: let the host is your buddy and you trust him. In addition there are two white goats and one black. You friend promise you to open gate with a black goat behind if you have to switch the chosen gate, and to show a white goat if you have to keep the chosen gate. In this case the table of outcomes will be as follows.

	1st gate	2nd gate	3rd gate	4th gate
Prob. of choice	1/4	1/4	1/4	1/4
Behind	WG	WG	BG	C
Shown	BG	BG	WG	WG
Decision	switch	switch	not switch	not switch
Prob. of win	1/2	1/2	0	1

(Here WG is a white goat and BG is a black goat.)

When you chose the 3rd gate, you friend has no chance to show you that you should switch the gate!

The total probability to get a car with your friend hints is

$$\Pr\{win\ a\ car\} = \frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot 0+\frac{1}{4}\cdot 1=\frac{1}{2}.$$

This probability twice as much as in the first case when you just blindly switch!

Now let us change insignificantly the conditions: the host has two black goats and one white. Will anything change? “What the difference can be due to the goats color?” – you might ask, though after all this "strange" results above you, probably, try to think before giving the answer.

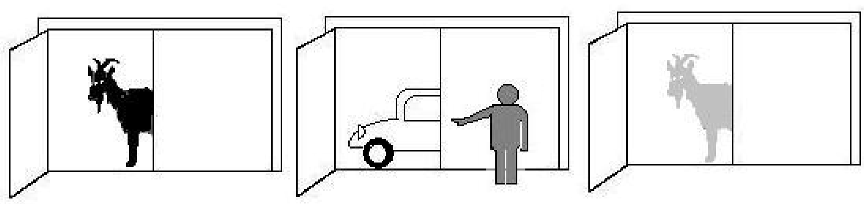
Let us make a table of outcomes for this case.

	1st gate	2nd gate	3rd gate	4th gate
Prob. of choice	¼	¼	¼	¼
Behind	BG	BG	WG	C
Shown	BG	BG	BG	WG
Decision	switch	switch	not switch	not switch
Prob. of win	½	½	½	1

In this case, the total probability to win a car is

$$\Pr\{win\ a\ car\} = \frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot 1=\frac{5}{8}\ !$$

So, let us stop on these examples with goatematics, though it is so interesting!



4. FROM CASINO TO COMPUTATIONAL MATHEMATICS

4.1. Is everything really random?

Most often almost impossible things really happen
(Unknown author)

It is interesting to mention that probabilistic ideas were not so transparent and clear even for brightest brains of that not so far time. For instance, a famous French mathematician D'Alembert in his well known work "*Croix ou Pile*" ("*Heads and Tails*") assumed mistakenly that a series of successes (say, heads) led to increasing the probability of failure at the next trial (i.e. increased a tail appearance), and vise versa. The D'Alembert authority was so high that the so-called "D'Alembert Rule" was very popular among gamblers

Jean le Rond D'Alembert
(1717-1783)

French mathematician, mechanic, physicist and philosopher. He was co-editor with Diderot of "*Encyclopédie, ou dictionnaire raisonné des sciences, des arts et des métiers*" ("*Encyclopedia, or a systematic dictionary of the sciences, arts, and crafts*") which was published in France between 1751 and 1766.

He was invited by Russian Empress Catherine the Great to tutor her son but rejected the proposal.

D'Alembert was an illegitimate child and his mother left him on the steps of the Saint-Jean-le-Rond de Paris. According to custom, he was named after the patron saint of the church. D'Alembert was placed in an orphanage and was soon adopted by the wife of a glazier.

The sense of this rule is in as follows: double your stake after a loss and put a half of a stock after win. It seems very natural: if probability changes from step to step, you should change your tactics! (By the way, this rule is still popular now though gamblers had never heard who D'Alembert was!)

Evidently, such argumentation appears from observation of statistical stability and arguments of the kind: if on average there should be 50-50 for head appearance, then if one observes a long series of tails, then definitely will appear "more than usual" tails – otherwise how to keep the balance?!

It is interesting that statistical stability of outcomes of independent random events irritated many first-class mathematicians.

Could you imagine a serious (and in addition well-educated) man tossing a coin and recording the results to convince themselves that the probability of getting a particular outcome (say, a head) is equal to ½? If you don't trust that it is possible look at the following table

Experimenter*	**Number of tests**	**Frequency of tails appearance**
Buffon [43]	4040	0,5085
De Morgan[44]	4092	0,5005
Feller[45]	10000	0,4979
Pearson[46]	12000	0,5046
Jevons[47]	20480	0,5068
Pearson[47]	24000	0,5005
Romanovsky[48]	80640	0,4923

[43] **Georges-Louis Leclerc, Comte de Buffon** (1707-1788), French naturalist, mathematician, biologist, and cosmologist.
[44] **Augustus De Morgan** (1806 - 1871), Indian-born Scottish mathematician and logician. He formulated De Morgan's law in mathematical logic.
[45] **William Feller** (1906 - 1970), outstanding American statistician.
[46]**Karl Pearson** (1857-1936), outstanding British statistician, Fellow of the Royal Society. He established the discipline of mathematical statistics. In 1911 he founded the world's first university statistics department at University College London.
[47] **William Stanley Jevons** (1835-1882), English economist, statistician and logician.
[48] **Vladimir Ivanovich Romanovsky** (1879 - 1954), Russian statistician, founder of the Tashkent Probability theory school.

Let us make our own experiment: calculating weighed average of all those trials that equals 0.4974. Frankly speaking, one could expect better result for almost 150 thousand tossing in total !

4.2. "Buffon's Needle"

George Buffon opened a way to a new calculation method, namely, to the method of statistical simulation, or Monte Carlo method as it is known now. However, the method he discovered was never considered seriously by mathematicians. Probably, for most people it was just a curious case than scientific method: why one should throw a needle if there is a possibility to find number "pi" with any needed accuracy using recurrent formula?

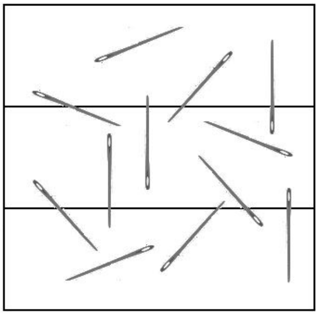
Georges-Louis Leclerc,
Comte de Buffon
(1707-1788)

French naturalist, mathematician, biologist, cosmologist and author. Buffon's views influenced the next two generations of naturalists, including Jean-Baptiste Lamarck (1744-1829) and Charles Darwin (1809-1882).

Buffon is best remembered for his great work "Histoire naturelle, générale et particulière" 36 volumes (8 additional volumes published after his death). It included everything known about the natural world up until that date.

He was first who introduced differential and integral calculus into probability theory. The problem of Buffon's needle in probability theory is named in his honor.

The experiment is in the following. One draws parallel lines on a plane. Distance between lines is equal to *G*.



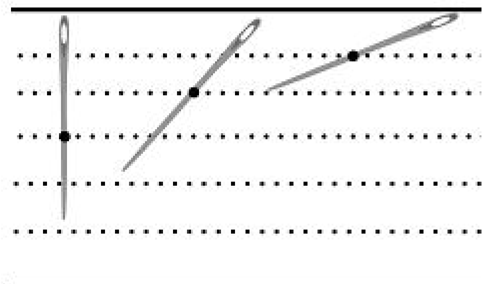
Случайное бросание иглы на разлинованную поверхность

.

Then from above one drop in random way a needle. The needle's length is $g < G$. "Randomness" the needle position is formally defined as follows: the center of a needle has a tw0-dimensional uniform distribution on the plane and angle of the needle is also uniformly distributed from 0° to 180°.

After each such experiment it is observed if the needle intersects with any of lines or not, and the number of such events is recorded. After experiment completion, one calculated the frequency of the observed event.

It is clear that if the center of the needle locates on a distance more that $g/2$, an intersection has never occurred.



Примеры возможных положений иглы.

Avoiding computations, let us give you the result: the probability of intersection is equal to:

$$P = \frac{2}{\pi} \cdot \frac{g}{G}$$

Since the frequency of intersection is $\frac{m}{n}$, then for n going to infinity this frequency approaches the probability of the event. So, one can write approximately:

$$\pi = 2 \cdot \frac{g}{G} \cdot \frac{n}{m}.$$

Buffon himself made the experiment with $g = G$, i.e. in his case he used approximation

$$\frac{m}{n} = \frac{2}{\pi}.$$

Buffon's experiment intrigued a number of naturalists. However. Needle is not a coin, the experiment was harder that coin tossing, so the number of enthusiasts was much smaller. In the book "Probability Theory" Boris Gnedenko[49] cited an examples in the following table

Experimenter	Year	Number of experiments	Result of experiment
R. Wolf	1850	5000	3.1596
W. Smith	1855	3204	3.1553
D. Fox	1894	1120	3.1419
M. Lazzarini	\`90\`	3408	3.1415929

On the last line you see the phenomenal result of a certain Mario Lazzarini who in 1901 got the accuracy of six (!!!) decimal digits. (Remind that $\pi \approx 3.1415927$. Though it is a well known truism that about the deceased say only good or nothing, we notice that Lazzarini is a pure charlatan: his experiment is a simple fraud. Let us chek, how many experiments should produце Lazzarini to get this number in 3408 experiments. For this purposes, use the formula given above. Denote unknown number of experiments by *x*. Then

$$\frac{x}{3408} = \frac{2}{3,1415929}$$

from where

$$x = \frac{2 \cdot 3408}{3,1415929} \approx 2169.6.$$

Excuse me, but the number of experiments *x* has to be integer! Let us try closest integers to 2169.6, they are 2169 and 2170. For the first number we have result 3.142462, and for the second one we have 3.141014. In other words, real results are far from those gotten by Lazzarini! How eager was a wish of "experimenter" to find himself in annals of history of mathematics!

For comparison let us show some results gotten by computer simulations:

[49] **Boris Vladimirovich Gnedenko** (1912-1995), outstanding Russian statistician, long time Chair of the Probability Theory Department at the Moscow State University in Russia.

Number of experiments	1.000	10.000	100.000	1.000.000
Empirical value π	3,1104	3,1363	3,1523	3,1439

From the table, one can see that convergence takes place but the process is very slow. Even 1 million experiments gives the value of π with accuracy that is worse than that found by great Ancient mathematician Archimedes who found it in the form of a very simple fraction $\frac{22}{7} \approx 3.142857$.

* * *

George Buffon factually gave a birth to a new numerical method of calculation that now is called Monte Carlo method. At the same time he began a new direction in the probability theory – the theory of geometrical probabilities. .

4.3. Monte Carlo simulation

In less than two hundred years, the method invented by Buffon began to burst forth into a powerful method of numerical calculations

Then once in the middle of last century a set of extremely complex calculation problems appeared during development the Manhattan Project. That was a code name for a project concerning an atomic bomb creation developed at one of the most secret laboratory of the time – Los Alamos Scientific Laboratory. One of key persons of the project was outstanding American mathematician John von Neumann who had been working at the time over atomic bomb detonation. Calculations Neumann had to perform were so complex that he was forced to find something more effective those classical analytical and computational methods.

John von Neuman
(1903-1957)

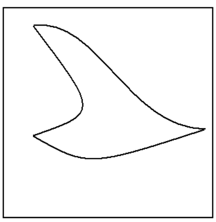
Hungarian mathematician who is reputed by a forefather of modern computer architecture and who is known for his achievements in quantum mechanics, functional analysis, set theory, economics and other important theoretical fields..

In 1930 when fascists came to power, he emigrated in USA and became a scientist of the Princeton Institute for Advanced Study.

Architecture of two first generations of computers is named in his name "Neumanian". ***For more details see Chapter Pantheon.***

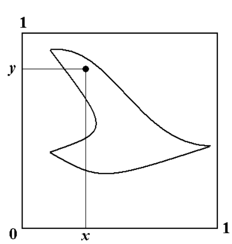
The method invented by Neumann was based on Buffon's needle experiment! Neumann replaced real model with a statistical one which after multiple trials would deliver a "statistical" solution with the confidence level depending on the number of experiments.

Consider a very simple example of finding some exotic solid embedded into a unit square.



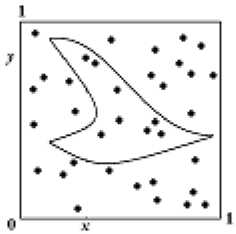
It is clear that if one drops in random points into the square, the frequency of targeting the solid will be "almost" proportional to the portion of the square area occupied by this solid. However, knowing Bertrand's Paradox, we should define what does it mean "in random".

Let us say that each random point (x, y) is generated in the following way: on segment [0, 1] on axe X one chooses point x that has uniform distribution, afterwards the same one does with segment [0, 1] on axe Y.



In such a way, the statistical trials continue.

The main problem in performing such an experiment was the absence of tables of random numbers and then there were no computers!



As John's colleagues told, he suggested rent several roulettes at one of Monte Carlo casino and using them as generators of random numbers. Though such an idea seemed more than extravagant, US Department of Defense permitted necessary fond probably being under the pressure of the outstanding scientist's name.

John von Neumann with his colleague Stanislaw Ulam[50] began to simulate on the roulettes the processes that are imitated an atomic bomb detonation.

Due to the place of birth of this new method it was called Monte Carlo Method.

Neumann definitely understood that such a Neanderthal way of random umber generation, the idea would have no development as it happened with Buffon's idea. He realized that there was a need n a new computational tool that could generate random numbers that could be then used for simulation. Just this situation pushed Neumann to the idea of creating a computer.

Let us notice that random number generation is not so simple problem. Most of modern computer programs are based on various recurrent procedures that usually leads to cycles, i.e. from some moment generated sequence of random numbers begins to repeat itself!

The first and probably simplest generator was designed by Neumann himself. He used the following computational procedure. Choose an arbitrary $2n$-digital number and find its square, in the result you get $4n$-digital number. In this new number, choose a "central segment": n-digital number starting with $(n+1)$-th digital and finishing with 3n-th digital. Then initial number is changed by some rule for instant by adding

[50] **Stanislaw Martin Ulam** (1909–1984) was a Polish mathematician, pupil of famous Polish mathematician Stefan Banach (1892-1945). Moved to Princeton in 1934 and later took part in Manhattan Project in Los Alamos. Made a significant input into development of Monte Carlo method.

some constant. After multiple repetitions of such calculations you have a table pseudo-random numbers that, unfortunately, has distribution distinguished from uniform one.

Examples of construction of random number tables in such a manner are given below.

Initial number	Square		Initial number	Square
4763	22**6861**69		5956	35**4739**36
6861	47**0733**21		4739	22**4581**21
0733	00**5372**89		4581	20**9855**61
5372	28**8583**84		9855	97**1210**25
8583	73**6678**89		1210	01**4641**00
. . .	. . .		. . .	. . .

In the table generated pseudo-random numbers are marked out by bold fonts. Of course one of the problems is how to choose numbers for squaring. In the first part of the table initial number in column A is 4567 that is chosen arbitrary.

Of course, the obtained numbers are rather “disordered” but who knows how much they are random in sense of uniform distribution). Moreover, as soon as the some pseudo-random number appears at the second time, it begins the cyclic repetition of previously calculations.

Now there are many perfect programs for random numbers generation. For instance, program RANDU has a length of cycle equal to 2^{29}, that is more than enough for practically any practical application.

4.4. What is a roulette?

Roulette (sometimes known as “rulet”) is a casino game named after a French diminutive for "little wheel". What is the procedure of the game? Players choose to place bets on number and color (red or black). Then a croupier spins a wheel in one direction, then spins a ball in the opposite direction around a tilted circular track running around the circumference of the wheel. The ball eventually loses its speed and falls into one of colored and numbered pockets on the wheel.

There is a version (though not confirmed documental) that a roulette was invented and built by Blaise Pascal who once was seized by idea to invent a perpetuum mobile. Somebody even mentioned that ir was invented in 1665. However, there is no confirmations that Pascal played roulette himself though he liked hazard games (remember at least his friendship with chevalier de Mere).

It seems that roots of a roulette are in Ancient times. It is known that soldiers of Ancient Greece turn a shield on the blade end of a sword for casting lots. The same did Roman Emperor August with a wheel: in one of the rooms of his castle there was a wheel on the vertical axis that he used for making decision.

I some sources, one can find mentions about similar use of a wheel in Old China and among American Indians.

First version of this game appeared in France by the name of "hoca". A casino for playing hoca was open in France in the end of XVIII century. It was a very profitable gambling business: 3 of 40 pockets were marked with "0" (zero) and if the ball stopped at those pockets, the winner was a casino.

Those first roulettes were primitive and imperfect: observing the game for a long period of time, one can find that some pockets are lucky more often than others. (Of course, casino had a possibility to use these pockets to mark them with zero!)

The Wheel of Fortune had been stopped during French Revolution of 1789-1799. In 1791 National Assembly abandoned all hazard games and severe punished not only the casino owners but as well those who knew about gamblers and did not inform Police about them. Only Napoleon opened casino back. At this time, two enterprising and even a bit roguish brothers Francois and Louis Blanc appeared on the gambling horizon. They both were gamblers and crooks: they constructed a roulette with controlled wheel that could stop a ball in a pocket with zero.

Not in vain, there were rumors that the brothers had a deal with Satan to exchange their souls for such a fortunate roulette. (Somebody noticed that the sum of all numbers from 1 to 36 equaled «666», that was – as the Bible tells – "the Beast Number".)

However, in 1839 all casinos in France had been closed again. Brother Blancs were forced to leave France for Germany, and soon they open casino in Hamburg. Here they change the rule: mark "zero" was only on a single pocket. It decrease the casino profit per a session from original 3/40 = 7.5% down to 1/37 = 2.7%. However, the total profit jumped up, because rumors about frequent wins stimulated more active visits to the casino! Roulette became a very popular and soon with triumph returned to France.

In 1861 the Duke of Monaco invited the brothers to open casino there. The Duke was not a gambler himself, however he understood that such enterprise could help his dwarfish state.

So, Francois Blanc got concession for casino opening in Monaco. As you understand such deal was not done for free: Blanc paid "dwarfish: monarch 2 million francs (at that time it was a huge money!), had built the casino on his own money and gave 15% of his profit. In other words, Blanc bought the entire state: he replace entire state government implanting everywhere his buddies.

And again the Wheel of Fortune turns to the Blanc: in 1873 all European states closed their casinos, except Monaco, evidently. It makes Francois Blanc a real King of Gambling.

Not without Blanc's influence, it was open steamer line Nice-Monaco and a railway had been built from France, In addition, Blanc family established stock-company *Societe des Bains de Mer* (Society of Sea Bathing)), that was one of the first health resort in the World.

The building of Monte Carlo was absolutely fabulous. It also worked to make a casino very popular.

Its victorious procession in the USA roulette begins in New Orleans at the beginning of XIX centuries, where this gave was introduced by French immigrants. At the beginning there was a real fight against roulette but Blanc's creature won.

Americans added one more pocket – the so called "double zero" that was initially shyly hidden behind an image of American Eagle.

* * *

Talking about roulette, it is almost impossible to leave aside the so called "Russian roulette". This is a name of extreme hazard game: in a revolver barrel is put a single bullet. A second multiply rolls the barrel and gives the revolver to the first gambler who brings it to his temple and pulls the trigger. If he is lucky, the other gambler does the same, and so on.

It is interesting that there is no single mentioning of this "game" (actually a form of a duel) in classical Russian literature! The first time such an expression appeared in American magazine «Collier» in 1937, and afterwards began to be a very popular on many different languages though it has no relation to Russians!

At the same time, what is called in America as "Russian hills" in Russia is called "American hills". O, there are many funny names with no reason.

4.5. Frequency and probability

The frequency of an event occurrence converges to the probability. It was shown above on examples with a coin tossing and a Buffon's needle. So, the probability is a limit to which frequency tends for "regular" events.

When such a serious mathematician like Carl Pearson tossed a coin 12 thousand times and afterwards repeated such an experiment again 24 thousand times, one becomes thoughtful… If each toss and registration of the result takes, say, 5 seconds, then during 24 hours Pearson could make about only 15 thousand experiments (without eating, drinking and doing his deeds ☺).

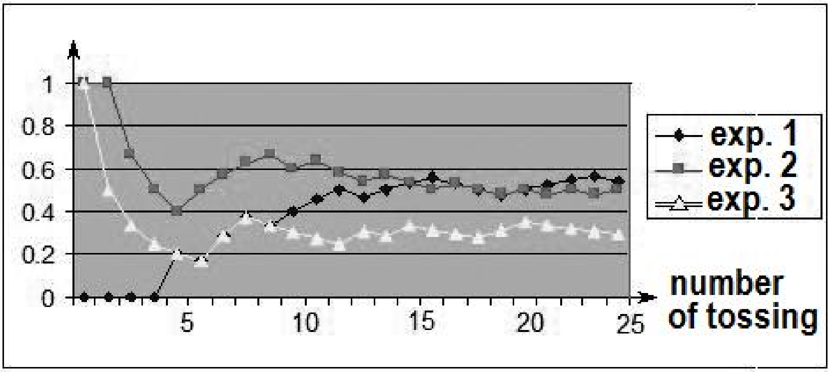
Actually, the phenomenon of frequency convergence to probability had been proven by Jacob Bernoulli[51] almost 200 years before Pearson's experiments!!

It seems more verisimilar that those who played "heads or tails" were more interested in the speed of convergence rather than the phenomenon of convergence. Though nobody lefts empirical functions of the rate of convergence. However, it is understandable: such work is too time-consuming.

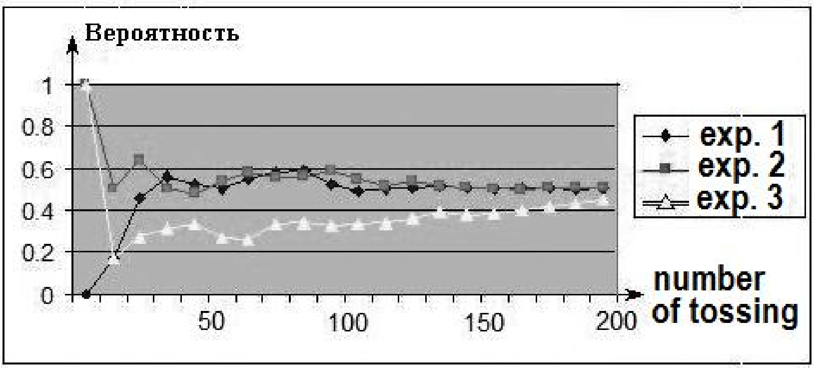
Such analysis began to be possible only after appearance modern computers. The author with the help of a simple mathematical model

[51] **Jacob Bernoulli** (1654-1705), Swiss mathematician, professor of the University of Basel. ***For more details see Chapter "Pantheon".***

based on Microsoft Excel imitated such processes. There are only illustrations, though even these simple graphs show a clear picture.



Of course, after 25 tossing, one can observe only a general tendency. If the number of tossing is increased to 100, the effect of convergence is really visible.

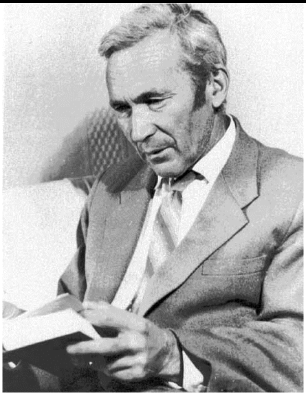


Nevertheless, it was almost evidently that pure statistical interpretation of randomness is insufficient. The probability theory had to be turned into a strong mathematical discipline with its own axiomatic, theorems and specific methods of analysis.

The problem o such transformation of the probability theory was formulated by David Hilbert as the Six Problem when he made his historical presentation in the very beginning of the XX century about problems those were waiting its solution in the forthcoming century. He wrote:

> "*The investigations on the foundations of geometry suggest the problem: To treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part; in the first rank are the theory of probabilities and mechanics*".

This Hilbert's problem has been solved by the great Russian mathematician Andrei Kolmogorov.



Andrei Nikolaevich Kolmogorov
(1902 - 1987)

Great Russian mathematician , the founder of modern Probability Theory. He also worked in areas of mathematical topology, mathematical logics, theory of turbulence, theory of algorithm complexity.

He was a founder and first Chair of the Probability Theory Department of the Moscow State University. He was an Academician of the Soviet Academy of Sciences as well as a Honorable member of a number other Academies in the World.

For more details see Chapter "Pantheon".

Laying in the basis Theory of sets and Theory of measure, he suggested in 1929 strong mathematical Theory of probability.

Before Kolmogorov, there were attempts to create axiomatic Theory of probability. One should mention works by Sergei Bernstein[52] who in 1917 made first steps in that direction.

[52] **Sergei Natanovich Bernstein** (1880-1968), Russian mathematician, Academician of the Soviet Academy of Sciences. Honorable member of several European Academies of Sciences.

Speaking about axiomatic theory of probability, one needs to mention a role of Richard von Mises who was the strongest of frequency theory.

Kolmogorov, whose axiomatic theory was recognized as the best one, wrote:

> "*Basis for implementation of result based on mathematical theory of probability to real stochastic phenomena should depend on some forms of statistical concepts of probability, which was so inspiringly established by von Mises.*"

Richard Edler von Mises
(1883-1953)

Austrian scientist and mathematician who worked on solid mechanics, fluid mechanics, aerodynamics, aeronautics, statistics and probability theory.
When Nazis came to the power in Germany in 1933, he emigrated in Turkey, where he founded and headed Department of Mathematics in Istanbul University. In 1939 he emigrated to the United States and became a Professor of Harvard Uiversity.

Mises himself being a witty man, once pronounce the following pun: "There are a forced lie, which is forgivable, intended lie, which icannot be excused, and statistics". (By the way, this aphorism is often ascribed to Benjamin Disraeli[53] or Mark Twain[54], or even O.Henry[55], though without correct referring.)

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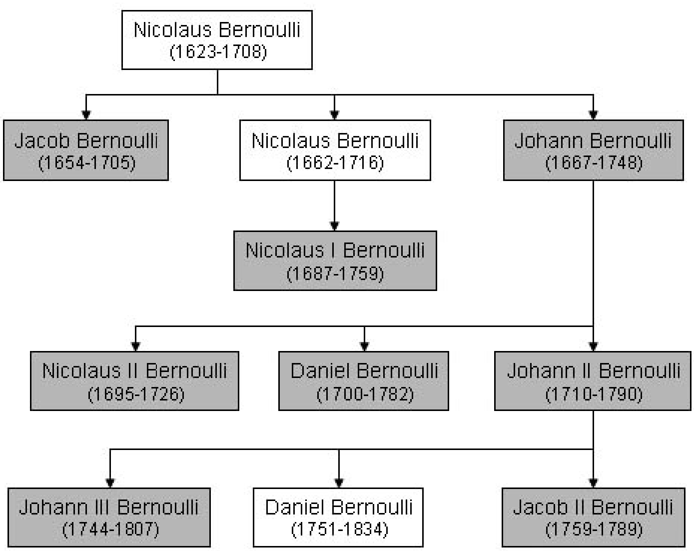
53 **Benjamin Disraeli** (1804-1881) was a British Prime Minister, parliamentarian, Conservative statesman and literary figure. He served in government for three decades, twice as Prime Minister of the United Kingdom.
54 **Mark Twain** was a penname of **Samuel Langhorne Clemens** (1835-1910) was an American author and humorist.
55 **O. Henry** was the pseudonym of the **William Sydney Porter** (1862-1910) was an American writer of short stories well known for their wit.

PANTHEON

Bernoulli's Genealogic Tree

Probably, there is no other family in the entire manhood history, for which mathematics was a "family business". Possibly there will be no such family in the future, since nowadays pure business or dirty politics are more fashionable… Less and less place for pure mathematics!



Genealogic Tree of "mathematical branch" of Bernoulli family. (Mathematicians are highlighted by grey color.

Protestant family of the Bernoulli were living in Antwerp, the largest town of Belgian province of Flanders. Because of multiple religious pogroms, the family was forced to leave the country, and most of them flew to Germany where there was a religious tolerance. In 1582 the Bernoulli came to Frankfurt-on-Main. On the next year the head of the family, Jacob had been died.

However, in Germany there was an animosity between Protestants and Catholics. The family was forced to find a more reliable place, so in 1622 Jacob Bernoulli, the grandson of the first Jacob, moved to Basel and became a citizen of the Basel Republic. Here the Bernoulli settles for a long time.

By family tradition, first names are repeated from generation to generation. So, you can find a number attached to the name, just as it is done for royal families.

Below is the genealogy of Bernoulli's family starting from the Basel period:

- **Nicolaus** (1623-1708), Jacob's son was born in Basel. He was a famous Basel pharmacist, member of the City Counsel and member of the Court. He had 11 children.
- **Jacob I** (1654-1705), Nicolaus's son,. Professor of mathematics of Basel University since 1687 His pupils were is younger brother Johann I, his nephew Nicolaus I, who became afterwards a member of Sankt-Petersburg.
- **Nicolaus** (1662-1716), Nicolaus's son. He was a painter. (No number because he was not nathematician.)
- **Johann I** (1667-1748), Nicolaus's son. A medical doctor by education, since 1695 became Professor of mathematics at the Gronigen University in Holland and since 1705 was Professor of the Basel Uiversity. Honorable member of the Sanct-Peterburg Academy of Sciences (Russia).
- **Nicolaus I** (1687-1759), son of the "second:L Nicolaus in the list. He was a lawyer by education but later became Professor of mathematics at the University of Padua.
- **Nicolaus II** (1695-1726), son of Johann I. A lawyer by education, he was Professor of mathematics at the Sanct-Peterburg Academy of Sciences (Russia).

- **Daniel** (1700-1782), son of Johann I. A medical doctor by education, in 1725 became Professor of mechanics at the Sanct-Peterburg Academy of Sciences (Russia). In 1733 he returned to Basel and headed the Department of Mechanics at the University. Honorable member of the Sanct-Peterburg Academy of Sciences.
- **Johann II** (1710-1790), son of Johann I. A lawyer by education, he was Professor of mathematics at the Basel University.
- **Johann III** (1744-1807), son of Johann II. A lawyer by education, he was an astronomer at Berlin Academy of Sciences and simultaneously lecturing mathematics,
- **Jacob II** (1759-1789), сын Иоганна II. A lawyer by education, he was Professor of mathematics at the Sanct-Peterburg Academy of Sciences.

Representatives of the Bernoulli family lives in Basel from those times up to now.

* * *

Below the most influential mathematicians of that great family are presented.

Jacob Bernoulli

(1654-1705)

He made a substantial input into differential and integral calculus, analytical geometry, probability theory and variation calculus. He introduced the polar coordinates and investigated logarithmic spiral.

He was the first who proved the Large Numbers Law.

Jacob Bernoulli was graduated from the Basel University where he studied philosophy, theology and languages (German, French, English, Italian, Latin and Greece) since his father wished him to be a Protestant priest. In 1671 he got his Magister degree in philosophy.

After work in France, Belgium and Great Britain, he returned to Basel and in 1682 founded seminary on experimental physics being at the same time a priest.

He was self-educated mathematician and got some recognition in thie area, so when in 1686 it was open a position of Professor of mathematics at Basel University, he was invited there. It was the beginning of the "reign" of Bernoulli's dynasty in mathematics.

Let us notice that members of Bernoulli's family possessed professorship (not necessarily in mathematics) in various universities almost quarter of thousand years till the second half of XX century.

Jacob made a substantial input into differential and integral calculus, analytical geometry, probability theory and variation calculus. He introduced the polar coordinates which afterwards were very effectively used in the theory of complex variables. In 1685 he had formulated the Law of Large Numbers and later proved it. (The name of the Law of Large Numbers was coined later by Poisson.)

Jacob Bernoulli died in Basel in 1705.

His main work "The Art of Conjecturing» (in Latin «*Ars conjectandi*»), had been published only 8 years after his death.

Johann Bernoulli

(1667-1748)

In 1697 Johann published his work on the so called exponential calculus, I which he first formulated the brachistochrone[56] curve ("least-time path").

He also authored several important discoveries in the area of differential and integral calculus.

One of his multiple pupils was the great mathematician Leonard Euler.

When in 1682 Johann graduated from school, and his father sent him in French-speaking part of Swiss to improve his spoken French and to get some practice in commerce. However, in a year Johann returned home and entered the university. In a short time he defended his bachelor dissertation written in Latin verses and afterwards in 1685 he got his Magister degree for dissertation again written in verses though this time in Greece.

The same year he began to study mathematics led by his elder brother Jacob. In two years he reach the level of his brother and they together began to develop calculus.

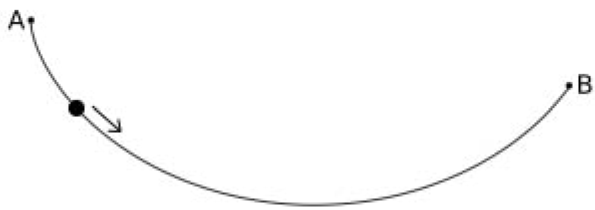
Simultaneously Johann studied medicine and in 1690 he got Doctoral degree in medicine. In several days after this he married and in

[56] **Brachistochrone** is the curve of the fastest descent: it is a curve connecting two points in a vertical plane with such a properties that a material dot, moving under gravity, reaches the final point during the minimum time.

1695 entire his family moved to Groningen where he got professorship I mathematics and experimental physics.

When his elder brother died, Johann was invited to head the Department of mathematics at Basel University. Here he got a brilliant student – Leonard Euler who became one of the greatest mathematicians of the all times.

In 1697 Johann published the work on the so-called exponential calculus, in which he introduced and solved the problem of brachistochrone.



He also made several important discoveries in integral and differential calculus.

Johann Bernoulli died in Basel in 1748.

Daniel Bernoulli

(1700-1782)

He first introduced a concept of work and coefficient of efficiency. He first wrote equation of stationary moving of ideal liquid and developed ideas of kinetics of gas. He made a great input into mathematics developing numerical methods of algebraic equations solution and set theory.

He is particularly remembered for his applications of mathematics to mechanics, especially fluid mechanics, and for his pioneering work in probability and statistics.

Daniel Bernoulli was born in Groningen where his father lectured mathematics at the University. In 1705 Johann's family moved back to Basel because he was invited to take a position of Professor of mathematics that was vacant after the death of the elder brother Jacob.

Daniel took classes at the Basel gymnasium. After graduation, his father sent him – just like Johann's father did with himself – in French-speaking part of Swiss to practice in commerce.

The story repeated: in a year Daniel returned home, entered the Basel university, and in 1716 earned his Magister degree in philosophy. However, his father insisted him to study medicine, more practical profession than mathematics. Daniel went to Heidelberg and then Strasburg, and in 1720 he got medical diploma.

Nevertheless, mathematics remained his inspiration. In 1724 he wrote his first scientific tractate "*Mathematical Exercises*" ("*Exercitationes*"). At the same year he became a member of Bologna Academy of Sciences, and later he got an invitation to head Genoa Academy of Sciences. Almost simultaneously, he got invitation from Russian Emperor Peter the Great to join recently established Sankt Petersburg Academy.

It was very seducing invitation but Daniel hesitated and decided to respond that he could not leave his younger brother Nicolaus. The decision of the Sankt Petersburg Academy was amazingly simple: he invited both brothers!

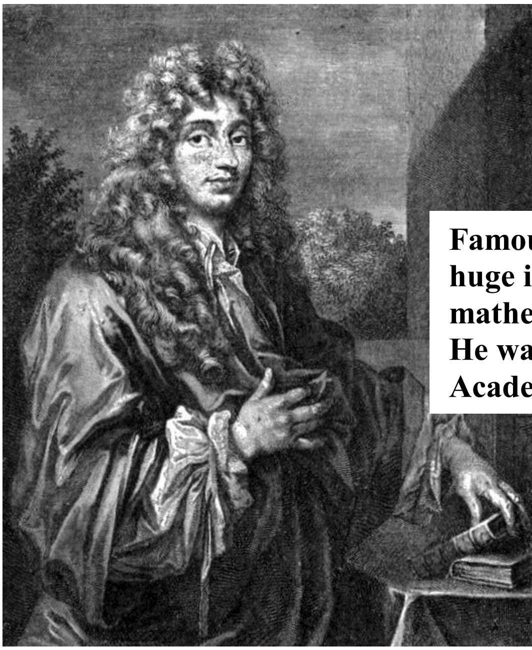
Sending his sons to Russia, Johann Bernoulli told them: "It is better to endure cold in the country where muses are reigning than to starve though with a good weather in the country where muses are treated badly and despised".

In October 1725 brothers Bernoulli arrived in Sankt Petersburg. Daniel got Department of mathematics and Nicolaus got Department of Physiology. The brothers began active scientific life. Daniel was a contemporary and close friend of Leonhard Euler.

However, severe Russian climate occurs too severe: less than in a year Nicolaus got a cold and died… Daniel was very much upset and tried to fight with his grief by overworking. He was elected academician. However, his grief was so strong that and he was unhappy there, and a temporary illness in 1733 gave him an excuse for leaving. He returned to the University of Basel, where he successively held the chairs of medicine, metaphysics and natural philosophy until his death

Daniel Bernoulli's scientific authority was extremely high, he was a member of Berlin and Paris Academies of Sciences and London Royal Society.

Christiaan Huygens
(1629 -1695)



Famous Dutch universal scientist who made huge input in various fields from mathematics and physics to engineering. He was the first President of the Paris Academy of Sciences.

Christiaan Huygens was born in Hague in the prosperous family: his mother was a daughter of prosperous Amsterdam merchant who occupied a high position of the Secretary to Prince Frederick Hendrik and later to his son William II [57].

[57] Prince of Orange **Frederic Hendrik** (1584–1647) was stadtholder of the United Provinces of the Netherlands. After his death his son **William II** (1626 -1650) led the country.

He was a talented man, a writer and playwright, even a bit scientist. Among his close friends were famous mathematicians René Descartes[58] and Marin Mersenne[59],

Christiaan got a perfect home education: in science and languages. At nine he knew Latin, French and Greek and at twelve played violin and harpsichord. He made himself a simple lathe and got a fun making fancy trinkets… In other words, he was an exceptionally gifted boy. However, mathematics was his real passion.

Parents decided to educate their son for diplomat career and he as sent to University of Leiden at the Department of Law. Though, young Christiaan spent most of his time learning mathematics. At the time he began correspondence with Descartes whom he knew since his childhood. During his studying at the University he began also a short correspondence with Mersenne who lived in Paris. That correspondence continued until Mersenne's death. Mersenne was astonished by Christiaan's talents so much that in one of letters to his parents compared a young man with Archimedes. Probably, since that time in the Huygens family it was coined Christiaan's nickname "our Archimedes".

After two years Christiaan changed University of Leiden for the College of Orange where, as his father hoped, he would continue his education. However, he and his brother, who learned at the same college, were urgently returned home because Christiaan's brother took part in a duel with one of students.

Nevertheless, in spite of incomplete education, 22-year old Christiaan in 1651 published his first work "Theorem on the square hyperbole, ellipse and terms". Three years later published his work "Opening the value of circle", in which he improved the value of "pi", using algebraic approach. These two works brought Huygens recognition in mathematicians' circle, and he decided to dedicate himself completely to mathematics without holding any official position. Only once he participated in a diplomatic mission and went to Holland with the help to

58 **René Descartes**, or in Latin **Renatus Cartesius** (1596-1650), famous French philosopher, mathematician, scientist, and writer.

59 **Marin Mersenne** (1588-1648), French theologian, philosopher, mathematician and music theorist, often referred to as the "father of acoustics".

meet with Descartes. However, that meeting never occurred: Descartes died just before Huygens arrival…

In 1750-s Huygens became interested in physics. He described the process of colliding of solids and reported it at the Royal Society in London in 1655. (This work was published only after its author death…)

Another area of his interest was optics. Here he was interested not only in theory but rather in a practical problem solution: improvement of telescope. Since he was dissatisfied by lenses he could find, he began to smooth lenses by himself and reached phenomenal successes – his lenses were much better any other. As a byproduct of those works was construction of an ocular consisting of two lenses (the so-called Huygens; ocular).

Using a telescope of his own construction, in 1955 Huygens discovered Saturn's satellite, which later was called Titan. He defined the period of its rotation and later on the basis of his observations explained the enigmatic nature of Saturn's rings.

In 1655 Huygens visited Paris where he met many outstanding scientists of that time, discussed with them various mathematical and physical problems. Here he first time heard about new mathematical branch – probability theory.

Blaise Pascal, who read Huygens manuscript on the problem, advised him to write the first book on probability theory. And in 1657 Huygens' tractate "*De ratiociniis in ludo aleae*" ("*Calculations in hazard games*") has been published in Latin. It was the first publication on the Probability Theory.

Coming back to Holland, Huygens was deeply involved in the probability theory and simultaneously began developing a precise watch that was an actual problem for sea navigation. By the way, a substantial prize for the problem solution was promised by the King of Spain, the country that had the largest and strongest fleet in the World.

In 1657 a pendulum watch was build by the Huygens draft. The idea to use pendulum for a watch was not new – Galileo already mention it. However, it was Huygens who built first operating watch of this type. In a year he published his book "*Horologium*" ("*Watch*") on the problem.

Developing the watch construction, in 1665 he published "Brief Manual for Using Watches for a Longitude Determination" and in 10 years he made a watch for using at a ship. In this type of watch he used a balance instead of a pendulum and a spiral spring instead of poise.

Do you recognize it? It is the same construction that is used in our time and still successfully competes with electronic watches. Anyway, Huygens supply the manhood with precise time for more than half a millennium!

When Huygens in 1660 went again to Paris, he was so famous as a great scientist that the King of France Louis XIV[60] gave him an audience. Moreover, Huygens was among a few of foreigner scientists who got from the King a stipend of 1200 livres that was a substantial amount of money at that time.

In the work, presented to the competition at the Royal Society in London in 1669, Huygens investigated colliding of elastic solids and found its laws. After the victory at the competition, 34year old Huygens was invited to Great Britain where he was elected a Fellow of the recently formed Royal Society.

Meanwhile, the Sun King decided to glorify France, trying to make it a center of European culture and science. He established the Academy of Sciences, actually copying the Royal Society. Huygens was invited to lead the Academy with a salary 6000 livres a year, he was gifted with a luxurious apartment at the Royal Library that was chosen as the Academy location.

Becoming the President of the Paris Academy, Huygens long time lived in Paris leaving it only twice to go to Holland for recreation after a severe illness.

In 1673 Huygens published in Paris his fundamental' work on pendulum ocscilation, where he considered moving of heavy solids and gave a new construction of watches with round balance.

In his work "Treatise on Light", which Huygens presented to the Paris Academy of Sciences in 1678, an absolutely new wave theory of light was presented. Within the frame of this theory he explained many

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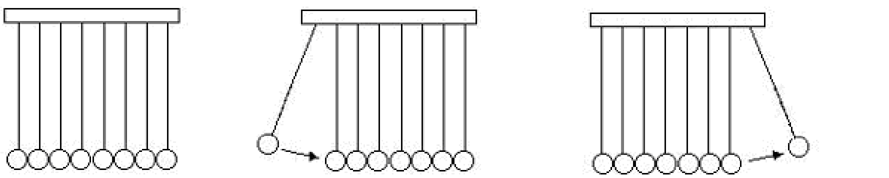
[60] **Louis XIV** known as **The Sun King** (in French **Le Roi Soleil**) or as **Louis the Great** (in French **Louis le Grand**) (1638 - 1715), King of France whose reign spanned seventy-two years – the longest reign of any major European monarch. During that period of time he increased the power and influence of France in Europe.

"enigmatic" at the time optical properties: light propagation, reflection and refraction of the light, and so on.

He suggested the existence in the Universe a specific type of "intermediate" substance – "ether" that was consisting of tensely packed small solid fractures. In that context he used his own theory of solids collision: the light propagation was considered as a chain collision of such solids. Probably, everybody knows a simple gadget that in a sense reflects Huygens' theory of light propagation.



Huygens suggested a similar though 3-dimensional model of spherical propagation of light.

In 1676 Rømer [61], basing his astronomical observations of Jupiter satellites, concluded that the light has finite speed. Huygens not only support that revolutionary scientific idea but using Rømer's records of observations estimated that the light speed is lightly over 200 thousand kilometers in second (a real value is 299 792, 458 km/sec).

Concluding a story about Huygens' genius discoveries, notice that he invented diascope projector, or the so-called "Magic Lantern".

In 1681 Huygens became so severely sick that he was forced to move to Holland. In two years he repaired his health but during this time Franc became really sick: religious intolerance of Catholics to Protestants increased "to the boiling point"… Huygens was a Protestant, so he preferred not to return to France.

[61] **Ole Christensen Roemer** (1644 - 1710), Dutch astronomer who in 1676 made the first quantitative measurements of the speed of light. Long time had been working in Paris and even participated in Versailles's fountains construction.

In 1689 Huygens undertook a long journey to England where he several times met with Isaac Newton. They have absolutely opposite viewpoints on positions in the light nature: Newton supported corpuscular theory while Huygens believed in wave nature of the light. In some discussions, the truth is on the side of those who are stronger. After Newton-Huygens discussions due to Newton's strong scientific authority, Huygens; theory had been rejected.

Only in the beginning of XIX century, basing on experiments, scientists again came to the wave theory of light. And now it is believed that actually both light properties are exist and don't contradict to each other.

His last years, Huygens was interested in life problem in the Universe. In 1968, already after the author's death, the book "*Cosmotheoros*", further entitled "The Celestial Worlds Discovered, or Conjectures Concerning the Inhabitants, Plants and Productions of the Worlds in the Planets" exposed Huygens' concepts. He believed that the Earth cannot be the only place in the Universe where the life exists. He even made n assumption that a form of live on other planets should not dramatically distinguished from our.

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NASA[62], European Space Agency and Italian Space Agency produced a joined Cassini–Huygens robotic spacecraft mission currently studying the planet Saturn and its moons. This spacecraft was named after Giovanni Cassini[63] and Christiaan Huygens. The spacecraft was launched on October 15, 1997 from Kennedy Launch Center at Cape Canaveral, Florida. It reached Saturn's system on July 1, 2004 and became the first artificial Saturn's satellite.

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[62] NASA is abbreviation for National Aeronautics and Space Administration of the USA.

[63] **Giovanni Domenico Cassini**, or in French **Jean-Dominique Cassini** (1625-1712), Italian-French astronomer, engineer and astrologer. He discovered the Great Red Spot on Jupiter and four Saturn's moons.

Launching "Cassini-Huygens"

Johann Karl Friedrich Gauss (1777-1855)

I have had my results for a long time: but I do not yet know how I am to arrive at them.

K. Gauss

Gauss joined deep pure mathematical investigation with pragmatic applied science, classical physics with its practical implementations.

Wideness of his spectrum of scientific approaches is unbelievable: high algebra, theory of numbers, differential geometry, probability theory, gravity theory, theory of electricity and magnetism, astronomy and geodesy…

Gauss was born in Brunswick to the uneducated mason. By a legend, his mathematical gift was shown when at age of 3-year old he corrected, in his head, an error his father had made on paper while calculating finances.

At seven, Karl went to a primary school. When he was 10-years old, he solved a problem suggested by a teacher to the class: to calculate sum of integers from 1 to 100. The young Karl produced the correct answer within seconds by a flash of mathematical insight, to the astonishment of his teacher. Gauss had realized that pairs of two integers from opposite ends yielded the same sums: 1 + 100 = 101, 2 + 99 = 101, 3 + 98 = 101, and so on, that gave a total sum of 50 × 101 = 5050. So, the boy discovered for himself a formula of arithmetic progression. Naturally, the teacher distinguished Karl and began to give him individual classes.

In four years Karl left the parent's house to continue his education. His father was not supportive of Gauss's schooling in mathematics, he saw him following in his footsteps and becoming a mason.

Being 14-year old, he was introduced to Karl Wilhelm Ferdinand, Duke of Brunswick[64] as one of the most gifted young citizens. Since then Gauss was invited to the court and got a stipend of 10 talers that was a substantial amount of money for the youth.

The Duke also awarded Gauss a fellowship to the Collegium Carolinum (now Technische Universität Braunschweig), which he attended from 1792 to 1795. During this time he studied works by Newton, Euler and Lagrange.

In 1795 Gauss with the protection of the same Duke entered Göttingen University. At the beginning, he attended lectures on philology, though continue to study mathematics by him. At that time he developed the method of inferences unequal observed data (that was actually a basis for his famous method least squares).

While in university, Gauss independently go some important results in geometry. In 1796, studying regular polygons with a number of sides which are Fermat primes[65], he tried to find method of constructing

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[64] **Brunswick** is a dukedom in Lower Saxon, Germany, that kept the status of independent state up to 1918. The city of Brunswick is one of university cities of Germany.

[65]A **Fermat number** is a positive integer of the form $F_n = 2^n + 1$where n is a nonnegative integer. The first three Fermat numbers are: 3, 5, 17... A **Fermat prime** is a Fermat number $F_n = 2^{2^n} + 1$ that is prime. Fermat primes are therefore near-square

them by divider and ruler. These construction problems had occupied mathematicians since the Ancient Greeks time. And once, as it said, awaking in the morning, Karl clear realized the rule construction of a regular heptadecagon (polygonal with 17-sides). That turned Gauss to mathematics irrevocably. He himself was so proud of this discovery that requested that a regular heptadecagon be inscribed on his tombstone. (Though a master, who should engrave the tomb, declined doing it, stating that a complicated figure would essentially look like a simple circle, drawn by unskilled hand.)

This was a real major discovery in an important field of mathematics: Gauss in addition found all numbers of sides *n* of polynomials, for which such construction is possible.

Since that moment Gauss began to diarize his mathematical discoveries that he considered important.

1796 was probably the most productive year for both Gauss and number theory. He found the general law allowing mathematicians to determine the solvability of any quadratic equation in modular arithmetic[66]. The Gauss' prime number theorem gave a good understanding of how the prime numbers are distributed among the integers. Gauss also discovered that every positive integer[67] is representable as a sum of at most three triangular numbers[68]. In his diary there were jotted famous words: "Eureka! Num = Δ + Δ + Δ."

However, in the fall of 1798 Gauss by unknown reasons left Göttingen without a diploma and came back to Brunswick. The Duke

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primes. The only known Fermat primes are first five, where first three coincide with Fermat numbers 3,5,17.

[66] **Modular arithmetic** (sometimes called ***modulo arithmetic***, or ***clock arithmetic***) is a system of arithmetic for integers, where numbers "wrap around" after they reach a certain value – the modulus. A familiar use of modular arithmetic is its use in the 24-hour clock: for instance, if the time is noted at 8 o'clock in the evening (or 20:00 in the 24-hour system), then again 10 hours later, then in usual addition the time being 30:00 but in the considered modular arithmetic the time will actually be denoted as 06:00, albeit in the next day.

[67] The **positive integers** are the numbers 1, 2, 3, ..., sometimes called the ***counting numbers*** or ***natural numbers***. They are the solution to the simple linear recurrence equation $a_n = a_{n-1} + 1$ with $a_1 = 1$.

[68] A **triangular number** is the sum of the n natural numbers from 1 to n. The sequence of triangular numbers for n = 1, 2, 3, 4, 5... is: 1, 3, 6, 10, 15... .

increased his stipend up to 158 talers a year. Coming back Gauss published a series of serious works that brought him notoriety in scientific Europe.

In his 1799 dissertation, "*A New Proof That Every Rational Integer Function of One Variable Can Be Resolved into Real Factors of the First or Second Degree*", Gauss gave a proof of the fundamental theorem of algebra. This important theorem states that every polynomial over the complex numbers must have at least one root.

Gauss also made important contributions to number theory with his 1801 book "*Investigations in Arithmetics*", which contained a clean presentation of modular arithmetic and the first proof of the law of quadratic reciprocity.

In the end of the book Gauss exposed a theory of equations of type $x^n - 1 = 0$, which forestalled ides of Galois[69] Theory.

And at the moment Gauss was only 24-year old!

This Gauss' work is estimated as mathematical chef d'oeuvre due to richness of the material, excellent mathematical discoveries and witness of given proofs. This book in many senses determined further development number theory and higher algebra.

On January 1, 1801, Piazzi[70] discovered a stellar object that moved against the background of stars. At first he thought it was a fixed star, but once he noticed that it moved, he became convinced it was a planet, or as he called it, "a new star". Piazzi named it "*Ceres*", after the Roman goddess of grain

Piazzi had only been able to track Ceres for about two months, following it only for three degrees across the night sky. Then it disappeared temporarily behind the glare of the Sun. Several months later, when Ceres should have reappeared, Piazzi could not locate it. The problem was that from such a scant amount of data (three degrees represent less than 1% of the total orbit) there were no mathematical tools able to extrapolate a position of a celestial body.

[69] **Evariste Galois** (1811 -1832), French mathematician. His work laid the foundations for Galois Theory, a major branch of abstract algebra. He died from wounds suffered in a duel at the age of twenty. ***For more details see Chapter "Pantheon" in Book 4.***.
[70] **Giuseppe Piazzi** (1746 - 1826), Italian monk, mathematician, and astronomer. He established an observatory at Palermo.

Many prominent astronomers tried to calculate the new planet orbit but no success... Karl Gauss, never being an astronomer, participated in mathematical investigations. Working on Ceres motion, based on Piazzi's extremely limited data, Gauss developed a theory of the motion of planetoids (dwarf planets) disturbed by large planets.

In September of 1801 Gauss predicted correctly the position at which it could be found again. And indeed, Ceres was rediscovered by Zach[71] on December 31, 1801 and one day later by Olbers[72]. Zach noted that "…without the intelligent work and calculations of Doctor Gauss we might not have found Ceres again."

After its orbit was better determined, it was clear that Piazzi's assumption was correct and this object was not a comet but more like a small planet. Under the terms of a 2006 International Astronomical Union resolution, Ceres can be called a dwarf planet.

In March of 1802, astronomers made a new set of partial observations of an analogous planetoid Pallas[73]. In this case, Gauss also was the first who calculated the planetoid's orbit.

Being in doubts that pure mathematics could be enough to get deserve financial support for a life, in 1807 Gauss took a position of Professor of Mathematics and Astronomy at the University of Göttingen. It gave him automatically also a position of Director of the astronomical observatory. He held these positions for the remainder of his life.

The results of this research were published later, in 1809 under the name "*Theory of Motion of the Celestial Bodies Moving in Conic Sections around the Sun*". For supporting of the strength of suggested method, Gauss repeated calculations made by Leonard Euler that he made for tracking a comet of 1769. In his time, Euler astounded his contemporaries by making huge calculation in three days of extremely extensive work. (He even lost his vision after this.) To repeat the same calculations Gauss needed only an hour!

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[71] **Franz Xavier Freiherr von Zach** (1754 - 1832), Hungarian astronomer

[72] **Heinrich Wilhelm Matthäus Olbers** (1758-1840), German astronomer, physician and physicist..

[73] **Pallas** is the second by size asteroid (about 500 km in diameter) that is also considered as a dwarf planet. It is named after Pallas Athena, the Greek Goddess of Truth.

For his works in astronomy Gauss was awarded a Prize o Paris Academy of Sciences and a Golden Medal from the London Royal Society. A number of European Academies honored him with their membership.

Notice that the famous 1812 year comet astronomers also observed using Gauss' calculations.

Gauss had been asked in 1818 to carry out a geodesic survey of the state of Hanover to link up with the existing Danish grid. Gauss was pleased to accept the job and took personal charge of the survey, making measurements during the day and making calculations at night.

To aid in the survey, Gauss invented the heliotrope[74], which worked by reflecting the Sun's rays using a design of mirrors and a small telescope.

How astonishing is that a number of mathematicians made various technical tools by their hands to confirm the correctness of their ideas! It is said that Gauss once told: "Theory attracts practice as the magnet attracts iron.".

And again this especially practical problem led to creation of geodesy theory. He developed the so-called "inner geometry of surfaces" that led to development of *n*-dimensional Riemann[75] geometry.

Since geodesic measurements are subject of significant random instrumental errors. Even multiple measurements left the question: what should be taken as a "real" value? For minimization of random errors influence, Gauss developed the so-called Least Squares Method that is widely used in modern statistics. Here on also grounded one of the most important probability distribution – Normal Law, that frequently is called now Gaussian distribution.

In 1828 Gauss published his fundamental tractate on geometry "*General Discussions about Curved Surfaces*", in which the author forestalled some ideas of Nikolai Lobachevsky[76], where he discovered the possibility of non-Euclidean geometries. This discovery was a major paradigm freed

[74]**Heliotrope** term is formed from Greek Helios (Sun) + Tropos (direction), a geodesic tool for triangulation.
[75] **Georg Friedrich Bernhard Riemann** (1826 –1866), German mathematician who made important contributions to analysis and differential geometry.
[76] **Nikolai Ivanovich Lobachevsky** (1792–1856), Russian mathematician, creator of non-Euclidian Lobachevsky's geometry.

mathematicians from the mistaken belief that Euclid's axioms were the only way to make geometry consistent and non-contradictory.

Here he also proved an important theorem, *theorema egregium* ("remarkable theorem" in Latin) establishing a property of the notion of curvature. Informally saying, the theorem states that the curvature of a surface can be determined entirely by measuring angles and distances on the surface.

In 1828, visiting Alexander Humboldt[77], Gauss met there the physics professor Wilhelm Weber[78]. At the moment both of them were interested in problems of electrodynamics and Earth magnetism. They developed a fruitful collaboration that it led to new knowledge in the field of magnetism. They created the system of electromagnetic units, and – as a byproduct! – in 1833 they constructed the first electromagnet telegraph in Germany , which connected the observatory with the institute for physics in Göttingen.

Five years later Gauss published another fundamental work in new scientific sphere – "*General Theory of the Earth's magnetism*".

It is interesting to notice that in 1832 Gauss created absolute system of measures on the basis of three units: second, millimeter and kilogram. As you van see Gauss' system insignificantly differs from the main modern system MKS (meter-kilogram-second)!

Gauss died in 1855 at age 78. Until his last days he had kept youthful thrust for knowledge and exceptional curiosity. For instance, being 62-year old, he learned Russian to read Lobachevsky's works in origin.

Private life of the scientist was not as intriguing as his scientific career, though it was not simple.

In 1805 Gauss married Johanna Ostoff who was a daughter of a tanner. Happy life was grievely interrupted: in 1808 his father died, and a year later his wife Johanna died after giving birth to their son, who was to

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77 **Friedrich Wilhelm Heinrich Alexander Freiherr von Humboldt** (1769-1859), Prussian naturalist and explorer.

78 **Wilhelm Eduard Weber** (1804 -1891), known German physicist.

die soon after her. Gauss was shattered and even wrote to Olbers asking him to give him a home for a few weeks.

Gauss was married for a second time the next year, to Minna the best friend of Johanna. . He had been offered a position at Berlin University and Minna and her family was keen to move there. Gauss, however, never liked change and decided to stay in Göttingen. In 1831 Gauss's second wife died after a long illness. After her death, Gauss' sons emigrated to America left their father alone…

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One of Gauss's last doctoral students was Dedekind[79]. He wrote a fine description of his supervisor:

> "... usually he sat in a comfortable attitude, looking down, slightly stooped, with hands folded above his lap. He spoke quite freely, very clearly, simply and plainly: but when he wanted to emphasize a new viewpoint ... then he lifted his head, turned to one of those sitting next to him, and gazed at him with his beautiful, penetrating blue eyes during the emphatic speech. ... If he proceeded from an explanation of principles to the development of mathematical formulas, then he got up, and in a stately very upright posture he wrote on a blackboard beside him in his peculiarly beautiful handwriting…"

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Many Gauss' scientific results were unfinished or dispersed in blusterous correspondence with friends and colleagues. His scientific heritage was a subject of careful study in Göttingen Scientific Society, which prepared 12-volume series.

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In Germany in Gauss honor there were issued stamps. His portrait one can find on German banknotes.

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[79] **Julius Wilhelm Richard Dedekind** (1831-1916), German mathematician who did important work in abstract algebra and algebraic number theory. .

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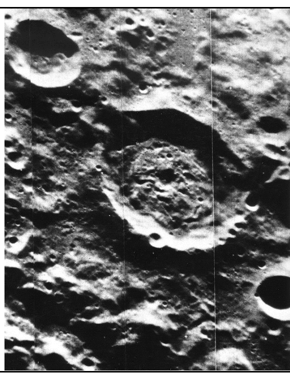
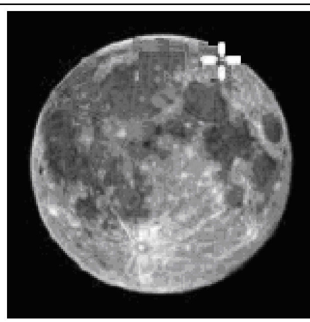
Honoring Gauss' great achievements in astronomy, his name bears one of the largest craters on the Moon.

	
Gauss' Crater on the Moon	**Location of the crater**

Siméon-Denis Poisson
(1781 – 1840)

Life is good for only two things, discovering mathematics and teaching mathematics.

S. Poisson

French mathematician, geometer, and physicist.

He published over 300 first class mathematical works.

Poisson got international recognition: he was a member of almost all scientific societies as well as Academies of Sciences of Europe and America.

Poisson was born in small town Pithiviers that belongs to the Loire Department, in central France, about 50 miles south of Paris. His father had been a soldier, who after retiring from active service was appointed to a position of notary.

Siméon-Denis was not the first child in the family but several of his older brothers and sisters had failed to survive. Indeed his health was also very fragile as a child and his mother, fearing that her young child would die, entrusted him to the care of a nurse.

When Simeon had grown up, his father taught him at home, believing that his son will be a notary. However, the son showed no success and parents found that notary to difficult profession for him and decided to teach him a "simpler profession". By their imagination, it was a profession of physician!

Siméon-Denis was eight years old when on July 14th 1789 the French Revolution began. His father enthusiastically supported it and became president of the district of Pithiviers. From this position he was able to influence the future career of his son.

Poisson's father decided that a profession of physician would provide his son's secure future. His brother was a surgeon in Fontainebleau and Siméon-Denis was sent there to study and practice. However, Poisson found that he was ill suited to be a surgeon. Firstly he lacked coordination to quite a large degree which meant that he completely failed to master the delicate movements required. Moreover, one patient died after the first blood-letting Siméon-Denis performed… It was a real shock.

Poisson literally ran home from Fontainebleau and his father had to think again to find a career for him. Simeon-Denis' father noted that his son easily solved mathematical problems published in "Magazine d'Ecole Polytechnique" and decided a new trial.

In 1796 Poisson was sent back to Fontainebleau by his father, this time to enroll in the École Centrale, where he showed that he had great talents for learning, especially mathematics. His mathematics teacher was extremely impressed by his abilities and encouraged him to take the entrance examinations for the École Polytechnique in Paris. He sat these examinations and proved his teachers right.

Ecole Politechniq had been established by Revolution Convent in 1704 for preparing engineering and military specialists for Republic of France. The teaching procedure was new: two years of intensive teaching by high level professors. And in the School teaching was performed by outstanding French mathematicians, physicists, chemists of the time:

Monge[80], Lagrange[81], Legendre[82], Furie[83], and Carno[84]. Pierre Laplace was Chair of the School Council and one of the lecturers.

Few people can have achieved academic success as quickly as Poisson did.

Once Laplace, who had been teaching Celestial Mechanics, gave students non-trivial problem for solving. He got an elegant and absolutely new way of solution from Poisson. Since then Laplace and after him Lagrange quickly saw his mathematical talents. They were to become friends for life with their extremely able young student and always gave him support in a variety of ways.

A memoir on finite differences, written when Poisson was 18, attracted the attention of Legendre. However, Poisson found that descriptive geometry, an important topic at the École Polytechnique because of Monge, was impossible for him to succeed with because of his inability to draw diagrams.

In his final year of study he wrote an excellent paper on the theory of equations. This work allowed him to graduate in 1800 without taking the final examination. He proceeded immediately to the position of répétiteur in the École Polytechnique with the strong recommendation of Laplace.

Hardly 20-year old, Poisson was known to the European mathematicians due to two bright publications in leading scientific journal.

In two years, Poisson had a position of deputy professor at the École Polytechnique, and in 1806 he was appointed to the professorship, when Fourier left the School.

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80 **Gaspard Monge** (1746-1818), French mathematician and inventor of descriptive geometry.

81 **Joseph-Louis Lagrange**, baptized in the name of **Giuseppe Lodovico Lagrangia** (1736-1813), Italian-French mathematician and astronomer who made important contributions to all fields of analysis and number theory and to classical and celestial mechanics.

82 **Adrien-Marie Legendre** (1752-1833), French mathematician. He made important contributions to statistics, number theory, abstract algebra and mathematical analysis.

83 **Jean Baptiste Joseph Fourier** (1768 - 1830), French mathematician and physicist who is best known for initiating the investigation of Fourier series. The Fourier transform is also named in his honor.

84 **Nicolas Léonard Sadi Carnot** (1796 - 1832), French physicist and military engineer who gave theoretical account of heat engines (known as the Carnot cycle).

In 1808 and 1809 Poisson published three important papers with the Academy of Sciences. In the first "*Sur les inégalités des moyens mouvements des planets*" he looked at the mathematical problems which Laplace and Lagrange had raised about perturbations of the planets.

In 1809 he published two papers: "*Sur le mouvement de rotation de la terre*" and "*Sur la variation des constantes arbitraires dans les questions de mécanique*". The second one developing of Lagrange's method of variation of arbitrary constants, which had been inspired by Poisson's 1808 paper.

In 1811 Poisson published his two volume treatise "*Traité de mécanique*" that was based on his lectures at the École Polytechnique.

After his election to the Paris Academy of Sciences, he was appointed a member of the Council of the University of Paris. He became an examiner for the École Military and in the following year he became an examiner for the final examinations at the École Polytechnique.

It is remarkable how much work Poisson put in to his research, to his teaching and to organization of mathematics in France.

In 1813 Poisson produced major work on electricity and magnetism, followed by work on elastic surfaces. Papers followed on the velocity of sound in gas, on the propagation of heat, and on elastic vibrations.

Most of Poisson's work was motivated by results of Laplace, in particular his work on the relative velocity of sound and his work on attractive forces.

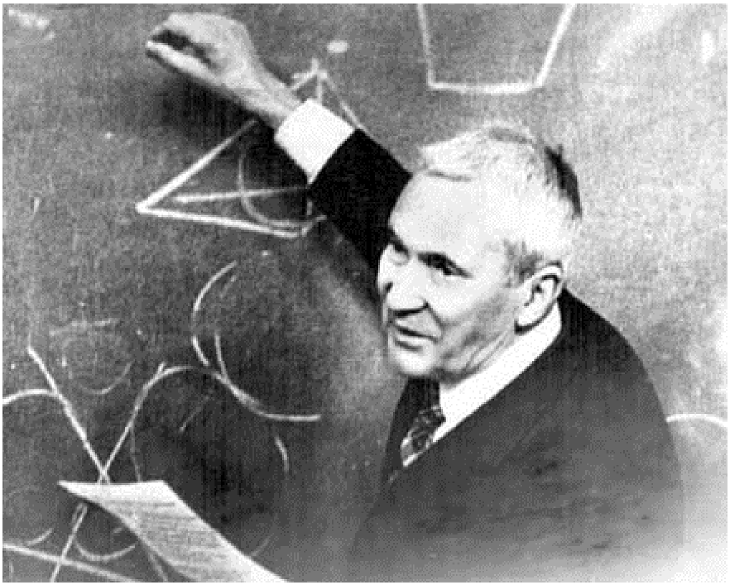
The famous Poisson distribution first appears in 1837 in an important work on probability "*Recherches sur la probabilité des jugements en matière criminelle et matière civile*". The Poisson distribution describes the probability that a random event will occur in an interval (time or space) under the conditions that the probability of the event occurring is very small, but the number of trials is very large so that the event actually occurs a few times. He also introduced the expression "Law of large numbers". Although now great importance of this work is understood, at the time it found little favor. The exception was the Russian probabilistic school where Chebyshev developed Poisson's ideas. Poisson's results lay in the foundation of the modern probability theory.

Poisson published over 300 first class mathematical works. Despite this exceptionally large output, he worked on one topic at a time.

Poisson got international recognition: he was a member of almost all scientific societies as well as Academies of Sciences of Europe and America.

Many mathematical objects have Poisson's name: Poisson's distribution, Poisson's stochastic process, Poisson's coefficient (Elasticity theory), Poisson's brackets (theory of differential equations), Poisson's integral and Poisson's summation formula calculus), Poisson's equation (electrostatic), and so on.

Andrei Nikolaevich Kolmogorov (1903-1987)



Russian mathematician who made major advances in different scientific fields (among them probability theory, topology, intuitionist logic, turbulence, classical mechanics and computational complexity).

Kolmogorov is widely considered one of the most prominent mathematicians of the 20th century.

Hardly it is possible to find another mathematician of XX century who had made fundamental input in such various areas of mathematical knowledge: probability theory, geometry, stochastic processes, mathematical statistics, set theory, mathematical logics, theory of functions, functional analysis, theory of approximation, topology, differential equations, theory of turbulence, theory of algorithms and automats, dynamic systems, classical mechanics, information theory, cybernetics, mathematical linguistic, and so on.

A.N. Kolmogorov was not the so-called “cabinet scientist”. Significant time he devoted to various applications in physics, biology, geology, crystallography, oceanography, meteorology, and so on.

Like Euler, Newton, Leibniz he was world-widely recognized he was a member of various scientific societies, he was a Fellow of the Royal Society, foreign member of Academies of Sciences of USA, France, Germany, Hungary, Netherlands, Poland, Finland…

In the world of science, mathematics is not covered by Nobel Prizes[85].

To correct this situation Bolzano Prize[86] has been established. The very first Bolzano Prize laureate became A.N. Kolmogorov. In addition, he was a laureate of Lobachevsky[87] Prize and Chebyshev[88] Prize established by the Russian Academy of Sciences for the outstanding achievements in mathematics.

* * *

Andrei Nikolayevich Kolmogorov was born on Aril 25, 1903 in Tomboy to the family Nikolai Katie who was agronomist. His mother Maria Yakovlevna died during delivery and a boy was adapted by his aunt Vera Yakovlevna Kolmogorov. Andrei’s father was killed during Russian Civil War.

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[85] The Nobel Prize was established by **Alfred Bernhard Nobel** (1833-1896), Swedish chemist who invented dynamite. He bequeathed entire his heritage for awarding scientists and writers for outstanding scientific approaches.

[86] Named after **Bernhard Placidus Johann Nepomuk Bolzano** (1781-1848), Bohemian mathematician, theologian, philosopher and logician

[87] **Nikolai Ivanovich Lobachevsky** (1792-1856), prominent Russian mathematician who developed the non-Euclidean geometry (hyperbolic geometry). Before him, mathematicians were trying to deduce Euclid's fifth postulate from other axioms. He replaced Euclid's parallel postulate with the postulate that there is more than one parallel line through any given point.

[88] **Pafnutiy Lvovich Chebyshev** (1821- 1894), famous Russian mathematician. Works in theory of approximation, numbers theory, probability theory, and mechanics. Member of St. Petersburg, Paris, Berlin, Swedish and Bologna Academies.

His childhood Andrei spent in a village near Yaroslavl, where his grand father had homestead. His aunts organized a school for local children where they practiced newest pedagogical methods. They issued a one-copy handwritten magazine “Spring Swallows” where published their pupils' creations – drawings, poems, stories… Andrei put there his first “scientific” opuses containing invented him arithmetical problems. Kolmogorov once wrote in his reminiscences about that time:

“ I cognized the happiness of a mathematical “discovery” at very early age, when being 5 or 6 years old found legitimacy:

$1 = 1^2$

$1 + 3 = 2^2$

$1 + 3 + 5 = 3^2$

and so on…”

Another problem invented by 6-year old boy sounds as follows: “By how many ways one can sew a button?” (It was assumed that none of four holes will be without a thread and none of the holes will be sewed twice.)

When Andrei became 7, Vera Yakovlevna moved to Moscow where Andrei was sent to a private lyceum that was led by young progressive teachers. Here boys and girls were taught together. In this school Andrei met Anna Egorova who became later his wife and life-long friend. Already in those early years Andrei realized his mathematical abilities though he showed interest to history and sociology and even had a dream to become a forester.

It was a tough time, young teenager was forced to earn his own salt: he even interrupted education and worked as a railroad builder, preparing to final exams by himself. He wrote later: “I was deeply disappointed that I was given a school certificate even without exam, just for nothing.”

He entered Moscow University where his teacher was Nikolai Luzin[89] . During first months he prepared for junior exams and jumped to

[89] **Nikolai Nikolaevich Luzin** (1883- 1950) was a Soviet/Russian mathematician known for his work in descriptive set theory and aspects of mathematical analysis with strong connections to point-set topology.

the second year. It was very important since sophomores got a "stipend" of special type: 30 pounds of bread and two pounds of butter a month! For that time, in the country suffering after Civil War consequences it was a real wealth.

One of the most important persons in Kolmogorov's life was Pavel Sergeevich Alexandrov. Kolmogorov wrote: "Probably, I would become a mathematician, though my human qualities were formed under Pavel Sergeevich's influence. He was an absolutely exclusive man due to his rich and wide nature. His knowledge of music, fine art and his attitude to people were amazing."

Beginning with 1933 in parallel with phenomenal scientific approaches Kolmogorov had been performing titanic organizational activity.

Here is only a brief list of his duties and occupied positions.

1933. Director of Institute of Mathematics and Mechanics at the Moscow State University.
1933. Founder and Editor-in-Chief of the journal "*Progress of Mathematical Sciences*".
1935. Founder and first Chair of the Probability Theory Department at the Moscow State University.
1938. Chair of Probability Theory Department at the Steklov[90] Mathematical Institute of the Soviet Academy of Sciences
1939. Academician-Secretary of Physics and Mathematics Branch of the Soviet Academy of Sciences and a member of Presidium of the Academy of Sciences.
1946. Director of Laboratory of Turbulence at the Institute of Theoretical Geography of the Academy of Sciences.
1954. Dean of the School of Mechanics and Mathematics at the Moscow State University.
1956. Founder and Editor-in-Chief if journal "Probability Theory and Its Applications"

[90] **Vladimir Andreevich Steklov** (1864-1926) was Russian mathematician. In 1921 he petitioned for the creation of the Institute of Physics and Mathematics. Upon his death the institute was named after him.

1960. Founder and Director of Laboratory of Statistical Methods at the Moscow State University.
1963. Founder of Physics & Mathematics boarding school at the Moscow State University (since 1989 it was named after Kolmogorov).
1964. President of the Moscow Mathematical Society.
1970. One of the founders and Deputy Editor of popular mathematical journal "Quant".
1976. Founder and Chair of Department of Statistics at the Moscow State University.
1980. Chair of Department of Mathematical Logic at the Moscow State University.
1983. Director of Department of Mathematical Statistics and Information Theory at Steklov Mathematical Institute of the Academy of Sciences.

Kolmogorov himself taught in special mathematical school and many his schoolchildren became perfect mathematicians. However, it is needed to mention that schoolchildren for that school were selected among hundreds if not thousands youngsters around the country! Every summer, a number of Kolmogorov's colleagues and Ph.D. students went to remote places of the country, spreading 1/6 of the World land, to find young mathematical talents.

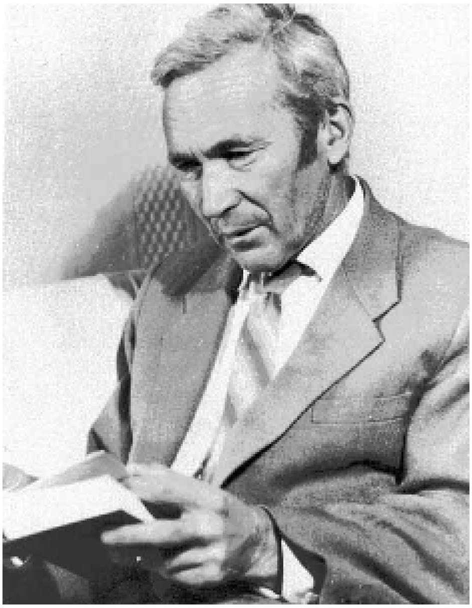
Kolmogorov's idea was to gather in special boarding-school talented children from all over the former Soviet Union.

On the blackboard at the class of this school there was a phrase written by Kolmogorov's hand: "***MEN ARE CRUEL BUT A MAN IS KIND*"**. His pupils kept and preserved this inscription from erosion covering with a special lacquer.

By the way, Kolmogorov and Alexandrov organized at Moscow University a "club" of music lovers: bi-weekly they presented lectures accompanied with records of classical music. Later they bought a country house that became a little scientific and cultural center where they gathered their senior students and colleagues.

In the beginning of 1970-s Kolmogorov took art in two oceanography expedition on the scientific-research sip "Dimitri Mendeleyev". With a group of his pupils, he performed investigation of ocean turbulence. He used to repeat: "In my belief, mathematicians should always take a part in experiments alongside with physicists."

Kolmogorov died in Moscow on October 20th of 1987.

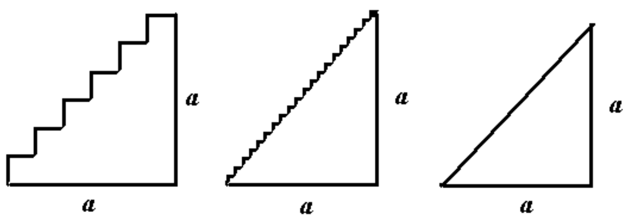


Some real stories about Kolmogorov

<u>What is the staircase perimeter?</u>

Boris Gnedenko – a beloved pupil and afterwards life-long friend of Kolmogorov – told once the following story about his teacher.

In Moscow State University a professor on calculus explained juniors about infinitesimally small values. He drew a rectangle steps staircase and began to decrease steps to zero.



Having made a ketch, he pronounced: "Obviously, the staircase perimeter goes to diagonal $a\sqrt{2}$, where a is the side length of the triangle…

At the same moment a strong young voice shouted from the audience: "It is not obvious! Moreover, you statement is erroneous!" Everybody was astonished: a young junior contradicted to a venerable professor. As you probably guessed it was Kolmogorov's voice. Professor asked a student to clarify his remark. And he explained: "With decreasing a size of a step a rectangular staircase perimeter remains constant. Indeed, projections of all vertical sides on axes y equals a, and projections of all horizontal sides on axes x also equals, so the staircase perimeter is always equal to $2a$."

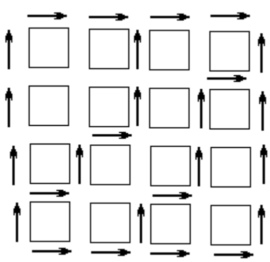
The fact that was found by Kolmogorov is now known many of us: it is the so-called Manhattan metric, in which distance $d(P_1,P_2)$ between two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ defines not as in Euclidian space by formula

$$d(P_1,P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} ,$$

but by formula

$$d(P_1,P_2) = |x_1| + |x_2|.$$

Why this metric is called "Manhattan"? Imagine that you are on Manhattan in New York. There are avenues from SW to NE and streets from SE to NW (excluding Broadway crossing Manhattan from N to S). Walking from, say, N to S you are forced to make your way in zigzags. By the way, there are several different ways of the same length. Examples are given on the figure below.



In American literature it has sometimes name "taxicab driver metrics".

<u>My meetings with the Giant</u>

Such books like this one are usually written basing on bookish sources. Because I had a privilege and luckiness to meet a real genius in my life, I cannot afford not to share with you some facts that you never find in other books: this is my own remembrance…

First time I met Andrei Kolmogorov in 1963. There was an International Bernoulli Symposium on Probability and Statistics in Tbilisi, Georgia, attending which was not a simple task: invitations were given only to distinguished mathematicians and I was at the time a young Ph.D. My friend, Yuri Belyaev who was a Kolmogorov's pupil asked him if he has a spare pass. He answered that unfortunately he had no but in a second told: "Take mine. I hope that I don't need it: the organizers know me in person".

So, I became a possessor of Kolmogorov's pass!

When a group of us took our places in a coupe and the train moved, in the door appeared an angry aged man and asked irritatingly: "Who bought me the ticket?!" One of assistants of the Probability Theory Department responded that it was he. "So, go yourself to this comfortable solitary confinement!" (The ticket was for a luxury place preferred by "Important people".)

Cooled a bit, Kolmogorov told: "Okay, okay… Excuse me… But may I stay here? We could stay here five of us here and you – he pointed to the unhappy assistant – all day long though you will go to my compartment only to sleep".

The way to Tbilisi from Moscow was not too short, so we, youngsters, spent with Kolmogorov almost two days. It was' probably first time in my life when I felt myself so excited: to sit next to a live legend!

Kolmogorov told a lot interesting stories. We were all attention. Sometimes he asked us about something. Since I was the only engineer, he asked me how it happened that I am interested in probability and statistics. Then I told him a fanny story. I worked at an aviation construction bureau designed ground-to-ground missiles. Once we got strange results from the field: splinters of the missile formed unusual groups – five-seven close to each other, then another group far away, and so on. Constructors were in panic: no Gaussian distribution, some enigmatic phenomenon… I had no idea also but I asked, why not to request splinters for additional examination. The idea was so simple that was accepted immediately. When splinters were delivered, we found that they were accurately sawing up pieces! An answer was so simple: each soldier who found a splinter shared it with his friends by cutting it in smaller pieces!

Kolmogorov laughed more than others and commented: "It was an excellent statistical approach!"

I brightly remember the answer that Kolmogorov gave to one of our questions: how he got always practically needed results solving seemingly pure mathematical problem. His answer was simple: "If I understand a problem and have a vision where and what for the solution could be useful, fist of all, I am guessing the needed result and then move from both side: this is like making a tunnel from two sides – if you dig correctly, two ends will meet! If you would solve a pure mathematical problem your solution could be perfectly elegant thing though sometimes having no common sense. Of course the main problem remains: you should guess the right solution!"

Afterwards I met Kolmogorov at least once a year at Boris Gnedenko birthday with whom I had almost weekly contacts: I was his deputy in two engineering journals, we led a Consulting Center on Reliability and Quality Control, etc. Gnedenko was the beloved Kolmogorov's pupil and hearted friend, so sometimes Kolmogorov came to Gnedenko (they were neighbors) and I had a chance to meet him.

So, once at Gnedenko's birthday (I was one of few invited guests) Andrei Kolmogorov came up to me and said: "Boris told me that you are a radio engineer. Could you help me with a minor problem? My player has broken…"

I was really scared: my engineering experience was negligible, nevertheless I was forced by circumstances to promise. The player was at his country house ("dacha"), so he gave me an address and we decided about forthcoming Sunday. Immediately I came up to Gnedenko's son who, as I knew was a good amateur in electronics. In addition, he was a good driver and perfectly knew a direction to the "Uncle Andrei's dacha". We were with him on good terms, so he gladly agreed.

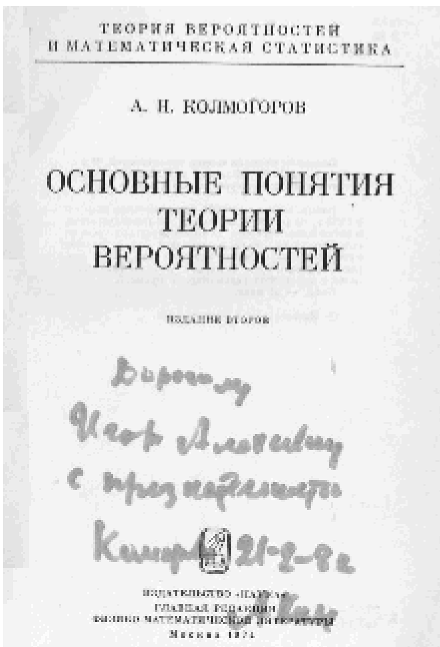
Coming home from the birthday party, I said my son: "Slava, on Sunday you'll go with me. There will be a rare opportunity for you to meet one of the most brilliant mathematicians of this century and probably the greatest Russian mathematician of all times". Slava, who was 14-year old at the time, heard from me a lot of stories about an extravagant genius and so was happy to accompany me.

Next Sunday, three of us (Gnedenko's son, my son and I) were at the Kolmogorov's country house. Andrei Nikolayevich heartily met us, introduced with Slava and invited us into the house. Then he showed me a player.

I decided to involve Gnedenko's son after my brief examination of the player. Bravely came up to it and with a professional surety touched a turntable of the player. It was deadly locked… I use some force and take it out. The axes were swarmed around with dead insects and grease… I had cleaned it, drop of oil into the axes compartment and… it began to work properly!

Kolmogorov immediately put on the player a disc with Bach or Handel (I could not distinguish those composers at the time). Kolmogorov was happy. Probably, I was even happier than he was.

Then he invited us to share a lunch with him. On the way to the dinner table, he asked me: "Igor Alexeevich, I don't know how I can repay for what you done for me!" (In Russia professors always use patronymic talking to their students, though I was astonished that Andrei Nikolayevich knew mine…)


ТЕОРИЯ ВЕРОЯТНОСТЕЙ
И МАТЕМАТИЧЕСКАЯ СТАТИСТИКА

А. Н. КОЛМОГОРОВ

ОСНОВНЫЕ ПОНЯТИЯ
ТЕОРИИ
ВЕРОЯТНОСТЕЙ

ИЗДАНИЕ ВТОРОЕ

21-2-82

ИЗДАТЕЛЬСТВО «НАУКА»
ГЛАВНАЯ РЕДАКЦИЯ
ФИЗИКО-МАТЕМАТИЧЕСКОЙ ЛИТЕРАТУРЫ
МОСКВА 1974

At the moment we walked near bookshelves with a long row of newly published classical Kolmogorov's book "Main Concepts of Probability Theory". I was brave enough to ask: "May I ask you to gift a copy of your book with autograph?" – "With my great pleasure! What you would like me to write? Something like 'To respectful Professor Ushakov?' Or something like that?" I was really confused and murmured: "Maybe something simpler…"

Kolmogorov took a book from the shelf and wrote: "To dear Igor Alexeevich with gratitude for a help. Komarovka. 21-2-82. A. Kolmogorov." (Komarovka was the name of a village where was his "dacha".)

I blushed… "Andrei Nikolaevich, I could not show such an autograph to anybody… Could anybody imagine how Ushakov could help Kolmogorov?" – "But first, it is right: I could never did it myself. And the second: tell the story if somebody will be interested why I made such a sign".

Then we had a lunch. Kolmogorov suggested Slava to sit next to him. They started a conversation and Gnedenko's son and I literally disappeared from his eyeshot. -

Naturally, their talk began with mathematics. Kolmogorov asked if Slava liked school textbooks on the subject. (Kolmogorov was the ideologist and main author of new math textbooks.) Abashed Slava mumbled, putting his eyes down: "My dad told that the textbooks are written by you…" –"So?.." – "We don't like it…" – "Why?" – "It is too complicated… Fortunately, I have textbooks left from my elder sister…" – "It is good that you told me a truth. My colleagues sometimes are afraid to do so. Thank you. Definitely, we should make some corrections in the textbooks."

It is to notice that actually the textbooks were written excellently but for too intelligent teachers and smart schoolchildren. But it was too formal and too difficult for average persons.

Kolmogorov was criticized by his colleagues for these textbooks but accepted all remarks very painfully. I remember how once, discussing the textbooks, Kolmogorov said Gnedenko: "Boris, you understand nothing in school teaching of math!"

Kolmogorov continue to ask Slava. "And what are you interests?" – "Drawing and painting!" – "It is perfect! Let me show you my collection of drawings!" They had a long and vivid excursion over the place with tens engravings and aquarelles. Kolmogorov explained who was the authors, when it was painted, etc.

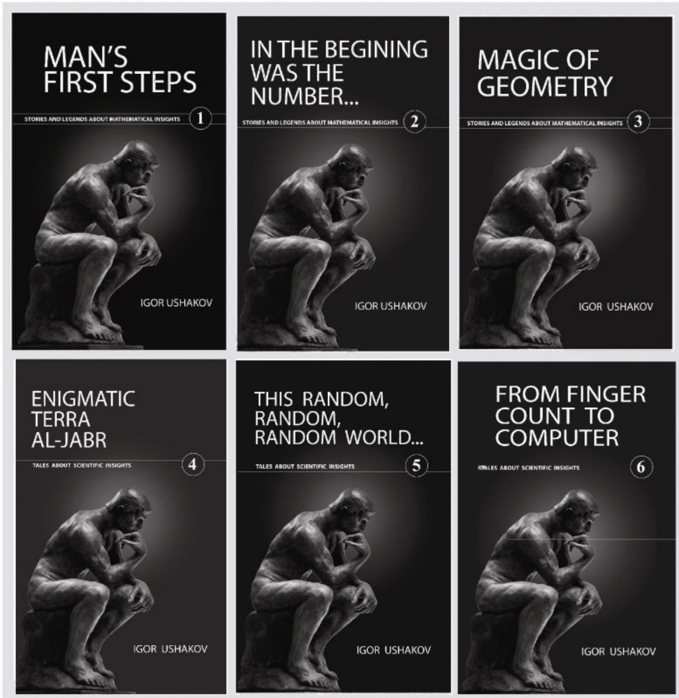
Gnedenko's son and I were sitting as spectators in a theater…

Seeing us of, Kolmogorov said me: "You have a good son. He is sincere. He feels a fine art. Probably, he will never need mathematics at all!"

Kolmogorov was right: my son became an animator, and he got several National and International prizes for his animation movies.

Self-advertising page

You can buy the following my books of series "Legends and Stories about mathematical Insights" in English:



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Professor Igor Ushakov, Doctor of Sciences. He led R&D departments at industrial companies and Academy of Sciences of the former USSR. Simultaneously, was a Chair of department at the famous Moscow Institute of Physics and Technology. Throughout his career he had the pleasure of acting as the Scientific Advisor for over 50 Ph.D. students, nine of which became Full Professors.

In 1989 Dr. Ushakov came to the United States as a distinguished visiting professor to George Washington University (Washington, D.C.), later worked at Qualcomm and was a consultant to Hughes Network Systems, ManTech and other US companies..

The author has published roughly 30 scientific monographs in English, Russian, Bulgarian, Czechoslovakian, and German.

In addition to scientific writings, the author has published several book of prose, poems and lyrics (in Russian).